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Unsteady two Dimensional Accelerating Boundary Layer Flow Over a Wedge with Temperature Dependent Viscosity

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ABSTRACT
The present work is focused on the numerical solution of unsteady two dimensional accelerating laminar boundary layer flow and heat transfer of an incompressible fluid over a moving wedge in the presence of variable viscosity. The system of partial differential equations governing the flow and reduced to a system of non-linear ordinary differential equations by using similarity transformations, then solved numerically by an implicit finite difference scheme along with quasilinearization technique. The obtained numerical results are presented graphically in terms of velocity, temperature, as well as, for the skin friction and local nusselt number for several values of variable viscosity parameter (ε) and Falkner-Skan parameter (m) along with Prandtl number(Pr).

Keywords – Accelerating flow, Heat transfer, Skin friction, Variable Viscosity.

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I. INTRODUCTION

The two dimensional incompressible wedge flow investigated for the first time by Falkner and Skan[1]. Later, many investigators [2-10], have studied the classical Falkner-Skan problem employing various analytical and numerical methods for different flow as well as heat transfer situations. In all these studies, the fluid properties were assumed to be constant. However, in many engineering applications this assumption is not obeyed. As such, we have to consider such problems by assuming variable viscosity. It is known that the physical properties of the fluid may change significantly, when temperature changes (for example the viscosity of water decreases by about 24% when temperature increases from 10^0 to 50^0c).
The first attempt to solve the Falkner-Skan problem including the variation of viscosity with temperature was made by Herwing & Wickern[11]. Hossain et.al [12] studied the flow of a fluid with variable viscosity past a permeable wedge with uniform surface heat flux. Rudrakonta Deka et.al [13] studies the effect of variable viscosity on flow past a porous wedge with Suction or injection. A Pantokratorns et.al [14] presented Falkner-Skan flow with constant wall temperature and variable viscosity.

In all the above studies the effect of unsteadiness were not studied. The present study is aimed to analyzing the unsteady accelerating flow over a wedge with variable viscosity, where unsteadiness in the flow is due to time dependent free stream velocity.

II. MATHEMATICAL ANALYSIS

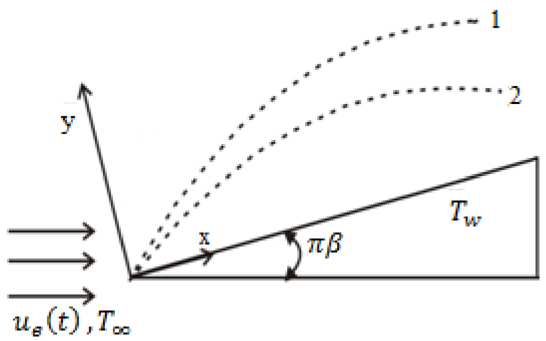


Figure 1. Physical model and co-ordinate system for MHD Falkner- Skan wedge flow, where 1 and 2 represent edge of thermal and momentum boundary layers, respectively.

Consider a two dimensional unsteady laminar incompressible boundary layer flow over a moving wedge, as shown in Fig.1. Where x is measured along the surface of the wedge and y is normal to it. The unsteadiness in the flow field is introduced by the free stream velocity u_e , varying inversely with time. The temperature of the wall T_w is uniform and constant and is greater than the free stream temperature (T_∞).The fluid is assumed to have constant physical properties except for the fluid viscosity (μ) which is assumed to be an inverse linear function of the temperature (T).

Under the aforesaid assumptions, the boundary layer equations governing the unsteady, forced convection flow over a moving wedge is given by,

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=0 \quad (1)$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=$$

$$\frac{\partial u_e}{\partial t}+u_e\frac{\partial u_e}{\partial x}+v\frac{\partial^2 u}{\partial y^2}+\frac{1}{\rho}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) \quad (2)$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2} \quad (3)$$

The boundary conditions are given by

$$at\;\; y=0 \quad u=v=0 \;\; and \;\; T=T_w$$

$$as\; y\to\infty: \;\; u\to U(x)=u_\infty(x/L)^m \;\; and \;\; T=T_\infty$$

$$at\;\; x=0: \quad u=u_\infty \;\; and \;\; T=T_\infty \quad (4)$$

In the present investigation, a semi-empirical formula for the viscosity of the form is

$$\frac{\mu}{\mu_\infty}=\frac{1}{1+\gamma(T-T_\infty)}$$

as developed by Ling and Dybbs [15] has been adopted, where μ_∞ is the viscosity of the ambient fluid and γ is a constant.

Introducing the following transformations:

$$u=\frac{\partial\psi}{\partial y};\; v=-\frac{\partial\psi}{\partial x};$$

$$f(\eta)=\sqrt{\frac{1+m}{2}\frac{L^m}{\upsilon u_\infty}}\left(\frac{\psi}{x^{(1+m)/2}}\right)$$

$$\eta=\sqrt{\frac{1+m}{2}\frac{u_\infty}{\nu L^m}}\left(\frac{y}{x^{(1-m)/2}}\right);\; G(\eta)=\frac{T-T_\infty}{T_w-T_\infty} \quad (5)$$

From equations (1) - (3), we find that continuity (1) is identically satisfied and (2) and (3) are, respectively transformed to:

$$(1+\varepsilon G)F''-\varepsilon G'F'+(1+\varepsilon G)^2\left(\frac{2m}{1+m}\right)(1-F^2)+$$

$$(1+\varepsilon G)^2\left[\lambda\left(\frac{2}{1+m}\right)\left(1-F-\frac{\eta}{2}F'\right)+fF'\right]=0 \quad (6)$$

$$\Pr^{-1}G''+fG'-\lambda\eta\left(\frac{1}{m+1}\right)G'=0 \quad (7)$$

Where

$$\frac{u}{u_e}=f'=F;\; f=\int_0^\eta Fd\eta$$

$$v=-\sqrt{\frac{2}{1+m}}\sqrt{\frac{\upsilon u_\infty}{L^m}}\left(1-\lambda t^*\right)^{-1/2}x^{(m-1)/2}\left[\frac{m+1}{2}f+\eta f''\frac{m-1}{2}\right]$$

$$\Pr=\frac{\upsilon}{\alpha}; \quad (8)$$

The transformed boundary conditions are:

$$F=0;G=1 \text{ at } \eta=0$$

$$F=1;G=0 \text{ as } \eta\to\infty \quad (9)$$

Here, $\varepsilon=(T_w-T_\infty)\gamma$ is termed as the viscosity variation parameter. ψ and f are dimensional and dimensionless stream functions, respectively; m is the Falkner-skan power law parameter ; F and G are respectively, dimensionless velocity and temperature of the fluid; λ is the unsteady parameter ; Re_L is the local Reynolds number; Pr is the Prandtl number; η is the transformed coordinate. Here prime (′) denotes derivative with respect to η.

The skin friction and heat transfer coefficient in the form of Nusselt number, can be expressed, respectively,as

$$C_f(\mathrm{Re}_L)^{1/2}==\frac{\tau_w}{\frac{1}{2}\rho u_e^2}=\frac{2\sqrt{\frac{1+m}{2}}(F')_{\eta=0}}{1+\varepsilon G}$$

$$Nu(\mathrm{Re}_L)^{-1/2}=-k\frac{\left(\partial T/\partial y\right)_{y=0}}{(T_W-T_\infty)}=-\sqrt{\frac{1+m}{2}}(G')_{\eta=0} \quad (10)$$

where the wall shear stress τ_w is given by $\tau_w=\mu\left(\frac{\partial u}{\partial y}\right)_{y=0}$ and with μ and k are, respectively, dynamic viscosity and thermal conductivity and $\mathrm{Re}_L=\frac{u_\infty L}{\nu}$ called the local Reynolds number.

III. METHOD OF SOLUTION

The system of nonlinear coupled ordinary differential equations (6) and (7) subject to the boundary conditions (9) have been solved numerically by using a very efficient and accurate implicit finite-difference scheme in conjunction with quasilinearization technique. Quasilinearisation technique can be viewed as a generalization of the Newton-Raphson approximation technique in functional space. An iterative sequence of linear equations is carefully constructed to approximate the non-linear (6) and (7) under boundary conditions (9) for achieving quadratic-convergence and monotonicity. This method, developed originally for ordinary differential equations by Inouye and Tate [16], has been successfully applied in a wide variety of thermo-fluid dynamic problems by many researchers. Following [16], we replace the non-linear ordinary differential equations (6) and (7) to the following sequence of linear ordinary differential equations:

$$F''^{(k+1)} + X_1^{(k)}F'^{(k+1)} + X_2^{(k)}F^{(k+1)} + X_2^{(k)}G'^{(k+1)} + X_4G^{(K+1)} = U_1^{(k)} \quad (11)$$

$$G''^{(k+1)} + Y_1^{(k)}G'^{(k+1)} = U_2^{(k)} \quad (12)$$

Where, the coefficient functions with iterative index k are known and functions with iterative index k+1 are to be determined. The boundary conditions become

$$F^{(k+1)} = 0, \quad G^{(k+1)} = 1 \quad \text{at} \quad \eta = 0$$

$$F^{(k+1)} = 1, \quad G^{(k+1)} = 0 \quad \text{at} \quad \eta = \eta_\infty \quad (13)$$

The coefficients in (11) and (12) are given by

$$X_1^{(k)} = -\frac{\varepsilon}{(1+\varepsilon G)}G' + (1+\varepsilon G)\left[\frac{-\lambda\eta}{2}\left(\frac{2}{1+m}\right) + f\right]$$

$$X_2^{(k)} = -(1+\varepsilon G)2F\left(\frac{2m}{1+m}\right) - (1+\varepsilon G)\lambda\left(\frac{2}{1+m}\right)$$

$$X_3^{(k)} = -\left(\frac{\varepsilon}{1+\varepsilon G}\right)F'$$

$$X_4^{(k)} = \left(\frac{\varepsilon}{1+\varepsilon G}\right)^2 G'F' + fF' + \varepsilon\left\{(1-F^2)\left(\frac{2m}{1+m}\right) + \lambda\left(\frac{2}{1+m}\right)\left[1 - F - \frac{\eta}{2}F'\right]\right\}$$

$$U_1^{(k)} = -\frac{\varepsilon}{1+\varepsilon G}F'G' - (1+\varepsilon G)\left(\frac{2m}{1+m}\right)(1+F^2) - (1+\varepsilon G)\lambda\left(\frac{2}{1+m}\right) + X_4 G$$

$$Y_1^{(k)} = \Pr f - \frac{\lambda\eta}{2}\left(\frac{2}{1+m}\right)\Pr$$

$$U_2^{(k)} = 0$$

Since the method is presented for ordinary differential equations by Inouye and Tate [16] and for partial differential equations in a recent study by Srinivasa and Eswara [17], its description is omitted here for the sake of brevity. The equations (11) and (12) along with boundary conditions (13) were expressed in difference form, considering central difference scheme in η-direction. In each iteration step, equations were then reduced to a system of linear algebraic equations with a block tri-diagonal structure which is later solved using [18]. To ensure the convergence of the numerical solution to the exact solution, step size $\Delta\eta$ is optimized and taken as 0.01. The results presented here are independent of the step size in η-direction at least up to the four decimal place. The value of η_∞ (i.e., the edge of the boundary layer) has been taken as 4.0 throughout the computation. Iteration is employed to deal with the nonlinear nature of the governing equations to become linear, locally. A convergence criterion based on the relative difference between the current and the previous iteration values of the velocity and temperature gradients at wall are employed. The solution is assumed to have converged and the iterative process is terminated when

$$Max\left[\left|(F'_w)^{(k+1)} - (F'_w)^{(k)}\right|, \quad \left|(G'_w)^{(k+1)} - (G'_w)^{(k)}\right|\right] < 10^{-4}$$

IV. RESULTS AND DISCUSSIONS

The numerical computations have been carried out for various values of temperature dependent viscosity $(0 \le \varepsilon \le 1.0)$, unsteady parameter ($\lambda$),and Falkner-Skan parameter (m). To validate the accuracy of our numerical method, we have compared skin friction (F'_w) and heat transfer (G'_w) parameters with those of Watanabe [7] for the range of m $(0 \le m \le 1.0)$ [as shown in Table 1] by taking Pr = 0.72 for accelerating flow. In fact, analysis has been carried out for entire range of realistic flow from $(0 \le m \le 0.2)$, corresponding wedge angle ranging from 0^0 to 60^0.

Table 1 Comparison of steady state (λ = 0.0) results for the range of m $(0 \le m \le 1.0)$ when ε = 0.0 with those of Watanabe [7].

m	F''_w		G'_w	
	Present	Watanabe [7]	Present	Watanabe[7]
0.0	0.4696	0.46960	0.4151	0.41512
0.014	0.5046	0.50461	0.4205	0.42051
0.042	0.5690	0.56898	0.4299	0.42984
0.090	0.6550	0.65498	0.4413	0.44125
0.142	0.7320	0.73200	0.4504	0.45042
0.2	0.8021	0.80213	0.4583	0.45826
0.333	0.9277	0.92765	0.4708	0.47083
1.0	1.2326	1.23258	0.4957	0.49571

Fig.3 shows the effect of variable viscosity parameter on the skin friction [$C_f (Re_L)^{1/2}$] and heat transfer coefficient [$N_u(Re_L)^{-1/2}$] for various values of m, $(0.0 \le m \le 0.2)$ corresponding to wedge angles ranging from 0^o to 60^0 when $\lambda = 1.0$ (accelerating flow) and Pr = 0.72. It is clear that both [$C_f (Re_L)^{1/2}$] and [$N_u(Re_L)^{-1/2}$] increases with the increase of variable viscosity parameter ($\varepsilon = 0, 0.5, 1.0$). Indeed, the percentage of increase of skin friction is about 93.2% at m = 0.1(32^0) and heat transfer is around 2.06% for an increase of ε in the range $0.0 \le m \le 0.2$.

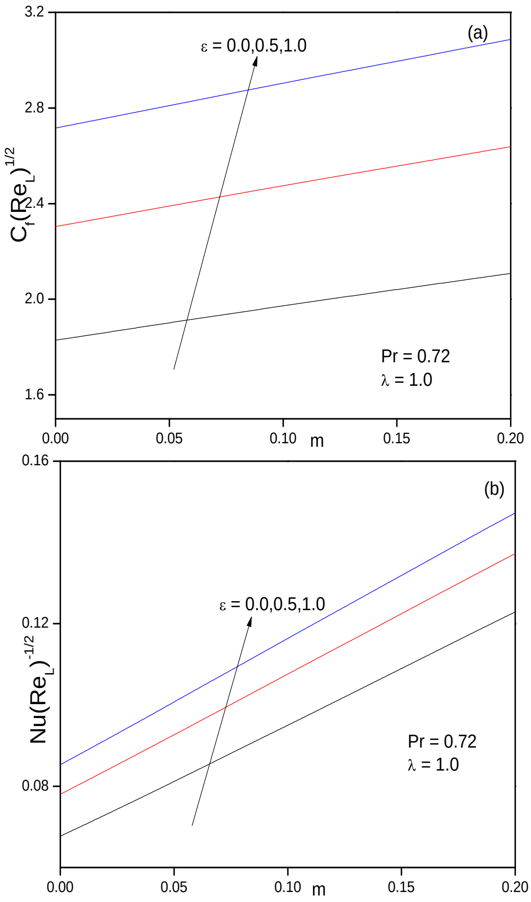


Figure 3. Effect of variable viscosity on (a) skin friction and (b) heat transfer coefficients for different values of m.

Fig.4 shows the effect of variable viscosity parameter (ε) on the skin friction $[C_f (Re_L)^{1/2}]$ and heat transfer coefficient $[N_u(Re_L)^{-1/2}]$ for different values of $\lambda > 0$ (accelerating flow) corresponding to wedge angle m = 0.2 (60^0) and Pr = 0.72. It is clear that both $[C_f (Re_L)^{1/2}]$ and $[N_u(Re_L)^{-1/2}]$ increases with the increase of variable viscosity parameter (ε =0.0,0.5,1.0). Indeed, the percentage of increase of skin friction is about 80.06% and heat transfer is around 3.01% for an increase of ε in the range $0.0 \le \lambda \le 1.0$.

The effect of variable viscosity (ε) on velocity [F] and temperature [G] profiles is shown in Fig. 5. It is seen that the velocity gradients increases and temperature gradient decreases with the increase of variable viscosity parameter. As the value of ε increases, the temperature difference of wedge within the boundary layer decreases. Thus the viscosity of air decreases, which results in the reduction of the thermal boundary layer thickness. Hence it is clear that the reduction of momentum and thermal boundary layer thickness, leads to the decrease in the velocity and temperature inside the boundary layer.

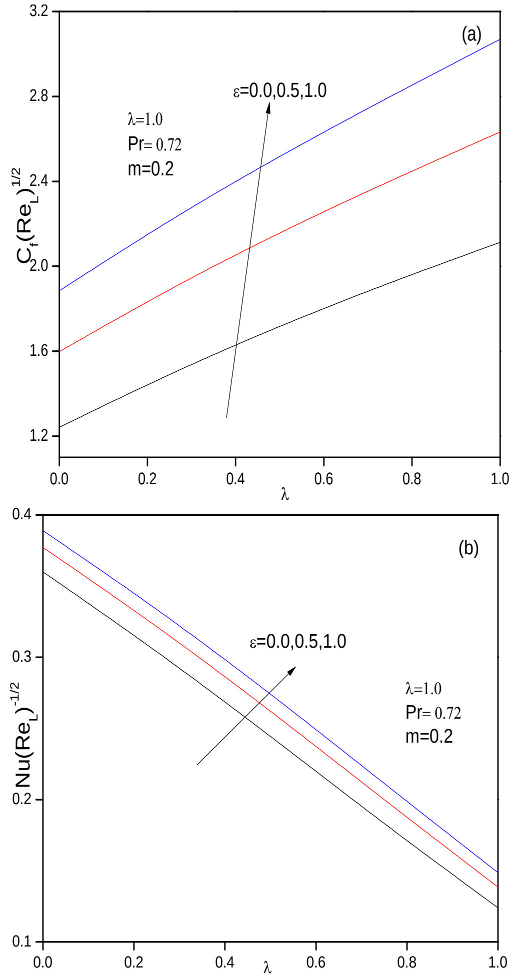


Figure 4. Effect of variable viscosity on (a) skin friction and (b) heat transfer coefficients

Fig. 6 illustrates the effect of Prandtl number (Pr =0.1, 0.72, 7.0) on skin friction $[C_f (Re_L)^{1/2}]$ and heat transfer coefficient $[N_u(Re_L)^{-1/2}]$ for various unsteady parameter $\lambda > 0$ (accelerating flow) corresponding to wedge angle m = 0.2 (60^0) and variable viscosity ε = 0.5. It is clear that both $[C_f (Re_L)^{1/2}]$ and $[N_u(Re_L)^{-1/2}]$ increases with the increase of Prandtl number . The percentage of increase of skin friction is about 8.276% and heat transfer is around 47.5% for an increase of ε in the range $0.0 \le \lambda \le 1.0$.

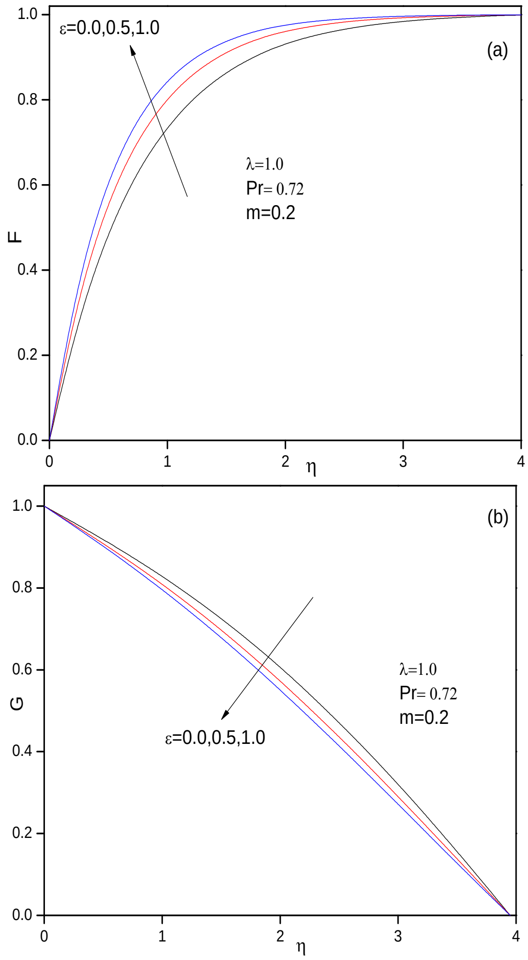
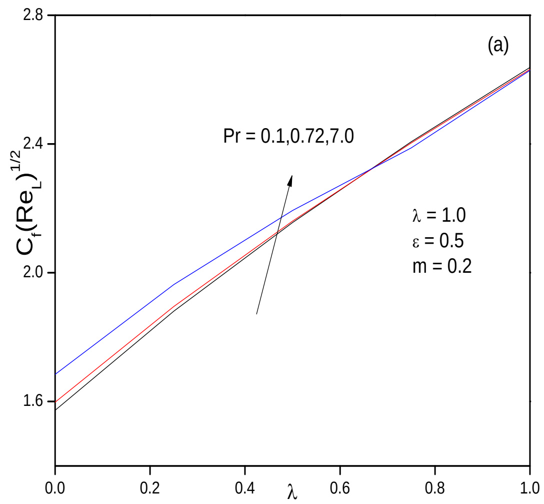


Figure 5. Effect of variable viscosity on (a) Velocity (F) and (b) Temperature (G) profiles



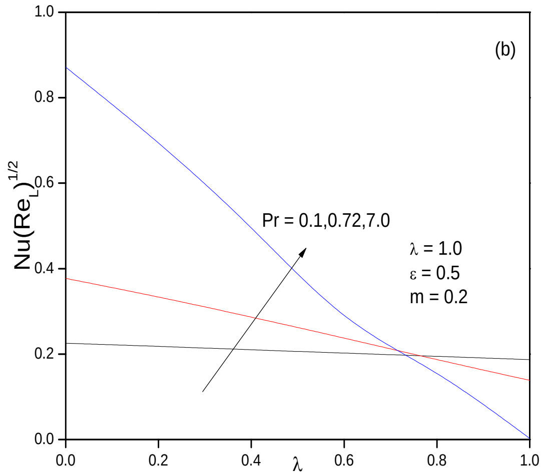


Figure 6. (a) skin friction and (b) heat transfer coefficients for different values of Prandtl numbers

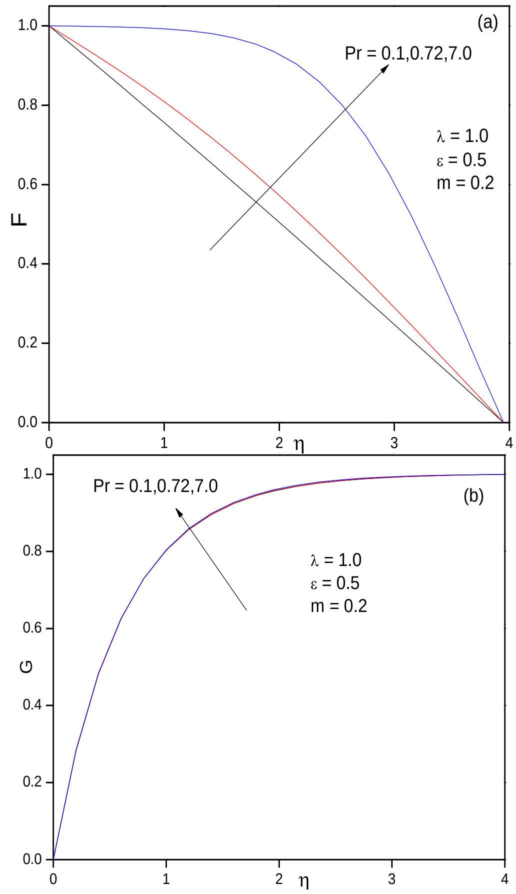


Figure 7. (a) Velocity (F) and (b) Temperature (G) profiles for different values of Prandtl numbers

Fig. 7 depicts the effects of different Prandtl number (Pr) on velocity [F] and temperature [G] profiles. It is seen that both the velocity and temperature profile increases with the increase of Pr. Numerical results from these figures shows that both momentum & thermal boundary thickness increases in terms of η at increasing distances from the leading edge. This is due to an increase trend in the dimensionless temperature in the thermal boundary layer.

V. CONCLUSION

In this paper the effect of variable viscosity on the unsteady two dimensional accelerating laminar boundary layer flow and heat transfer of an incompressible fluid over a moving wedge has been investigated.

1. The skin friction and heat transfer co efficient increases with the increase of variable viscosity parameter $(\varepsilon = 0, 0.5, 1.0)$ and the velocity profile increases but opposite trend in temperature profile for the fixed Prandtl number (Pr = 0.72) and wedge angle (m = 0.2) .
2. The skin friction, heat transfer, velocity & temperature profiles are found to be increased for the different Prandtl number (Pr = 0.1, 0.72, 7.0) for the fixed $\varepsilon = 0.5$ & $m = 0.2$.

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REFERENCES

[1]. V.M.Falkner and S.W.Skan, Some approximate solutions of the boundary layer equations, Philos.Mag.12,1931,865-896.
[2]. D.R.Hartree, On an equation occurring in Falkner and Skan's approximate treatment of the equations of the boundary layer, Proc.Cambridge Philos.Soc.,33, 1937,223-239.
[3]. K.Stewartson, "Further solutions the Falkner-Skan equation", Proc. Cambridge Phil. Soc., 50, 1954,454-465.
[4]. J.C.Y.Koh and J.P.Harnett, Skin friction and heat transfer for incompressible laminar flow over a porous wedge with suction and variable wall temperature, International journal of Heat Mass Transfer,2,1961,185-198.
[5]. K. K. Chen and P. A. Libby, Boundary layers with small departure from the Falkner-Skan profile, Journal of Fluid Mechanics ,33, 1968,273-282.
[6]. H. T.Lin and L. K. Lin, Similarity solutions for laminar forced convection heat transfer from wedges to fluids of any Prandtl number, International journal of Heat Transfer, 30,1987, 1111-1118.
[7]. T.Watanabe, Thermal boundary layer over a wedge with uniform suction and injection in forced flow, Acta Mechanica, 83, 1990,119-126.
[8]. A. Asaithambi, A finite –difference method for the Falkner- Skan equation, Appl.Math.Comp. 92,1998,135-141.
[9]. Eswara,A.T., A parametric differentiated finite-difference study of Falkner-Skan problem, Bull. Cal. Math. Soc., 90, 1998, 191-196.
[10]. G. Ashwini,C. Poornima, A.T.Eswara., Unsteady MHD accelerating flow past a wedge with thermal radiation and internal heat generation/absorption , International conference om Mechanival, Aeronautical and Manufacturing Engineering proceedings of world Academy of Science Engineering and Technology(WASET) Tokyo. Japan,65,2012, 687-690.
[11]. Herwig, H. and Wickern G, The effect of variable properties on laminar boundary layer flow, Warme und Stoffubertragung, 20, 1986, 47-57.
[12]. Houssain.M.A., Munir.M.Z., Hafiz.M.S., Takhar.H.S., Flow of viscous incompressible fluid with temperature – dependent viscosity past a permeable wedge with uniforms surface heat flux, Heat Mass Transfer, 36,2000,333 – 341.
[13]. Rudra Kanta Deka., Mwblib Basumatary., Effect of variable viscosity on flow past a porous wedge with suction or injection: new results, (African Mathematical Union and Springer-Verlag 2014).
[14]. Pantokratoras.A., The Falkner – Skan flow with constant wall temperature and variable viscosity, International journal of Thermal Sciences, 45,2006,378 – 389.
[15]. X. Ling, A. Dybbs, A forced convection over a flate plate submersed in a porous medium, variable viscosity case paper, 87-WA/HT-23, ASME, New York, (1987).
[16]. K. Inouye and A. Tate, Finite difference version of quasilinearization applied to boundary layer euations, A.I.A.A.J., 12, 1974, 558-560.
[17]. A. H. Srinivasa and A. T. Eswara, Unsteady free convection flow and heat transfer from an isothermal truncated cone with variable viscosity, International journal of heat and mass transfer,57,2013,411-420.
[18]. R. S. Varga, Matrix Iterative Analysis Prentice-Hall, (New Jersy, 2000).

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