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Analytical Approach for Soil Stability Downstream of Hydraulic structures Resting on Anisotropic Deep Soil Layer

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ABSTRACT
An Analytical model solution is found for a hydraulic structure with depressed foundation and two sheet piles, resting on anisotropic soil media. The solution is developed using Shwartz-Christoffel transformation. The elliptic integrals is used as part of the solution. The anisotropy of the soil media is considered by transforming the anisotropic flow field into a fictitious isotropic using rotation of axis according to the main principle hydraulic conductivity ellipse. In addition to flow field magnification according to the ratio of maximum and minimum hydraulic gradient.

The developed analytical solution is found highly compatible with finite difference solution. However in order to achieve the required accuracy for the finite difference solution a small discretization grid is required which make the required computer running time relatively high.For the case of depressed structure without sheet pile the exit gradient variation is found steep for the region near than the toe of the structure up to x1/b=2.5, when there after become very mild which may introduce a limiting value for the protection length design. This is true for different values of depressing ratio of the foundation of the structure. The degree of anisotropy has considerable effect on the exit gradient, as this ratio increases considerable reduction of exit gradient is observed, especially near the toe of the structure. This reduction is increase as the depressing ratio of the foundation increases. Similar variation is observed for the case of depressed structure with the existence of the sheet piles, as that case without sheet piles. The comparison of the two cases indicates that the existence of sheet pile will have considerable effect in reducing the exit gradient. Hence, these results recommends the use of small depressing depth with sheet piles which is more cost effective design of the foundation rather than increasing the depth of depressing. However this reduction is high for small successive s/b ratios than those for high s/b ratios. The analytical solution can be used for a reliable design of a structure resting on anisotropic soil media.

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I. INTRODUCTION

One of the most frequent failures of hydraulic structures is that of the foundation due to soil erosion at the downstream side developed by piping.

When the uplift pressure is greater enough to carry out the soil particles (i.e. exit gradient at the downstream side approaches the critical hydraulic gradient), soil piping occurs. The phenomena of piping will cause successive erosion of soil particles, and hence, extend below the hydraulic structures foundation on the downstream side causing settlement and, in the worst scenario, collapse failure of the hydraulic structure due to loss of support at the downstream side. In order to prevent pipingand to avoid failure, a protection is usually required. This protection consists of an impervious layer, such as, slab pavement or any kind of riprap protection provided with sand gravel filter.

The length of the required protection is usually decided upon the desired factor of safety against piping. The factor of safety is function of the exit gradient variation along the downstream side. Khosla (1954), Kochina (1952), Pavlovsky (1956), Harr (1962), and Karoufa (1964)had presented formulas for calculating the exit gradient for structures resting on an isotropic soil just at the toe of the structure, regardless of its variation along the downstream side and the properties of the soil, which is required for deciding the length of the required protection. For this purpose engineers usually use an approximate graphical methods, as mentioned by Karoufa (1964) that may present serious deficiencies in many cases due to disregarding variability of soil properties, Ahmed (2011).

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Alsenousi and Mohamed (2018) had used FE modeling to model seepage under hydraulic structure. The case is a dam with depressed foundation and single inclined sheet pile located anywhere in between the upstream and downstream edges of the structure base. The structure is resting on anisotropic homogeneous soil. The results show that as the K_x/K_y increases, the exit gradient increases. This observation was may be due to the lack of the required very small discretization grid required to achieve the convenient accuracy, because physically as this ratio increase the exit gradient should decresase. In other research, the comparison of the exit gradient obtained using finite differences by Abbas(1994) and an analytical solution presented by Mohamed and Agiralioglu (2005) showssignificant differences. Numerical solution for exit gradient needs a very small grid size to obtain reasonable values as will be shown later herein.

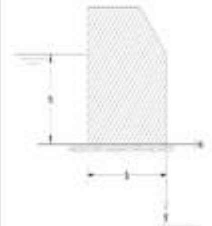
Jamal (2017) has used SEEP/W to obtain a data base for uplift pressure and the exit gradient of a two sheet piles structures laying on two layered soils. This data base is then used to obtain regression equations to find uplift force and the exit gradient as function of sheet pile depths, thicknesses of soil layers, hydraulic conductivities of the two layers and foundation width.

Khassaf et al (2009) had used GEO-SPLOE, SEEP/W finite element package to analyze seepage under a weir foundation with isotropic homogeneous soil. The structure has three sheet-piles: one at the upstream side, one at the downstream side and an intermediate with equal lengths. The effect of removing each sheet pile was investigated. It was also found that the downstream sheet pile has the most effect on toe exit gradient.

Alneamy and Alghazali (2015) had used ‘’SLIDE’’ program to analyze seepage under hydraulic structures though single and multi-layer soils with inclined cutoffs located at the downstream side and/or upstream side. The software used FE modeling. It was found that for the case of downstream cutoff only the exit gradient is minimum for an inclination angle120° from horizontal. For the case of two vertical sheet pile, the maximum exit gradient was found 0.053.

Al-Suhili et al (1988) presented a direct mathematical approach for exit gradient evaluation along the downstream side of hydraulic structures. The method was developed using Shwartz-Christoffel conformal mapping technique. Equations were developed for a dam with a single sheet pile. The location and length of the sheet pile were set as variables; so as many different configurations can be adopted. Different cases can be deduced by setting the values of depressed depth, length and location of sheet pile. A different approach of a simple ideal case of a dam without sheet pile and without depressed depth was develop by Kochina (1952),.Al-Suhili et al (1988) general solution is identical to Kochina(1952) solution, when the depth of depression and the length of sheetpile are both set to zero. The expressions for exit gradient variation along the downstream side of the structure for different cases of Al-Suhili et al (1988) solution are shown in Tables (1) and (2), for the infinite depth of porous media and for finite depth of porous media, respectively, these solutions are for homogeneous isotropic soil media. Goel and Pillai (2010) developed an exit gradient variation on the extended floor length for one intermediate pile of weirs of permeable foundations.

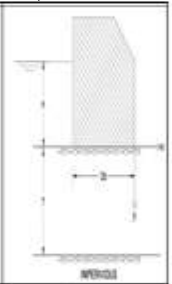
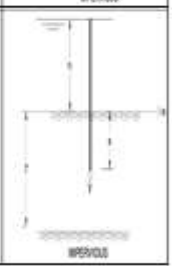
Table 1- Exit Gradient for the case of infinite porous media (Al Suhili et al 1988)

$I_e = \dfrac{h}{\pi\sqrt{x(x+b)}}$	
$I_e = \dfrac{h}{\pi s t}\sqrt{\dfrac{(t+1)(t-1)}{[t+\sqrt{1+(b_1/s)^2}][t-\sqrt{1+(b_2/s)^2}]}}$ $t=\sqrt{\dfrac{x^2+s^2}{s^2}}$	
$I_e = \dfrac{h}{\pi s t}\sqrt{\dfrac{t+1}{t+\sqrt{1+(b/s)^2}}}$	

Ie: exit gradient

t: intermediate t-plane variable of shwartz-Christoffel transformation all other symbols are the dimensions shown on table.

Table 2- Exit Gradient for the case of finite porous media (Al Suhili at al 1988).

$I_e=\frac{\pi h}{4kT}\sqrt{\frac{(t^2-1)(1-\beta_1)}{t(t+\beta_1)}}$ $\beta_1=\tanh\frac{\pi b}{T}$ $t=\tanh\frac{\pi x}{2T}$ $m=\sqrt{\frac{2\beta_1}{1+\beta_1}}$	
$I_e=\frac{\pi\mu h}{2kT\sigma 1}(1-t^2)\sqrt{\frac{\sigma^2-t^2}{(t^2-1)(t-\beta_2)(t+\beta)}}$ $t=\sqrt{\sigma^2+\sigma^{-2}\tanh^2\frac{\pi x}{2T}}$ $\mu=\frac{1}{2}\sqrt{(1+\beta)(1+\beta_1)}$ $m=\sqrt{\frac{2(\beta_1+\beta)}{(1+b_1)(1+b_2)}}$ $\beta_1=\cos\frac{\pi s}{2T}\sqrt{\tanh^2\frac{\pi b_1}{2T}+\tan^2\frac{\pi s}{2T}}$ $\beta_2=\cos\frac{\pi s}{2T}\sqrt{\tanh^2\frac{\pi b_2}{2T}+\tan^2\frac{\pi s}{2T}}$	
$I_e=\frac{\pi h(1+\sigma)}{4kT\sigma^2 t}\sqrt{(1-t^2)}$ $m=\sqrt{\frac{2\sigma}{(1+\sigma)^2}}$	

T: is the thickness of the soil strata.

In order to accountfor spatial variability of soils in hydraulic conductivity,Griffits and Fenton(1993), El –Fitiany et al (2003),Al-Suhili and Al-Joubori(2006), Alneamy and Alghazali (2015),Ahmnd and Oyedele(2016),and Jamel(2017) had developed numerical models to evaluate the exit gradient for anisotropic and heterogeneous soils. Mishra and Reddy (1983) proposed analytical solution for the exit gradient at the downstream side of a vertical sheet pilein anisotropic porous medium. Al-Suhili(2009), had developed an analytical solution for exit gradient variation downstream an inclined sheet pile. Results indicates that the angle of inclination of the sheet pile has considerable effect on the exit gradient variation. Al Suhili and Al Kaddi(1989) had developed a solution of the variation of exit gradient downstream of depressed dams resting on isotropic homogeneous soil. Al Suhili and karim(2014), had developed a seep/w based optimization model using a coupled Genetic Algorithm model GA with an Artificial Neural network model ANN to find the required optimum depths of two vertical sheet piles one at the upstream side and other at the downstream side. The model was verified by a physical tank model. Al Suhili et al. (2017), had developed a seep/w based optimization model using GA method coupled with an Artificial Neural network model to find the optimum dimensions and inclination angles of a two sheet piles one located at the upstream side and other at the downstream side. The seep/w model was verified using a physical tank model. The seep/w model uses finite element numerical modeling of the phenomena rather than analytical solution.

So far no analytical solution was proposed for variation of exit gradient at the downstream side of a depressed hydraulic structure with two sheet piles resting on anisotropic soil. This is a more realistic case that usually exist since all soils exhibits anisotropy.

The aim of this research is to develop an analytical solution for exit gradient variation with distance from the toe of the dam for a case that is not presented in previous studies: a depressed hydraulic structure with two sheet piles (equal length), one at the upstream side and the other at the downstream side, resting on infinite depthof anisotropic porous media.

It is a fact that increasing the depressed depth of the hydraulic structure foundation, and/or the length of the sheet pile, will eventually decrease the exit gradient, but will increase uplift pressure. This case is more realistic for the downstream sheet pile.

Recently, optimum design of the length of the sheet pile, depressing depth, and the length of protection is found in designing hydraulic structures foundation. Most of recent optimization models, such as GA (Genetic Algorithm), need an explicit mathematical expressions for calculating the uplift pressures variation along the base foundation of the structure and the exit gradient variation along the downstream side soil. For uplift pressure calculations, these explicit mathematical forms are exist, while for exit gradient variation along the downstream side are only available for limited geometrical cases, and isotropic soils only. The inclusion of different geometrical features such as dam foundation depressed foundation and the existence of both upstream and downstream sheet piles, and the consideration of anisotropy of soil layer, will impose much more difficulties to obtain such as explicit expressions.

II. MODEL DEVELOPMENT

As mentioned before the analytical solution will be developed for a depressed dam with two symmetrical piles as shown in Figure (1). The solution presented here adopts the Schwartz-Christoffel transformation, to map the physical z-plan, into the t-plan and w-plane, as shown in Figure (2).

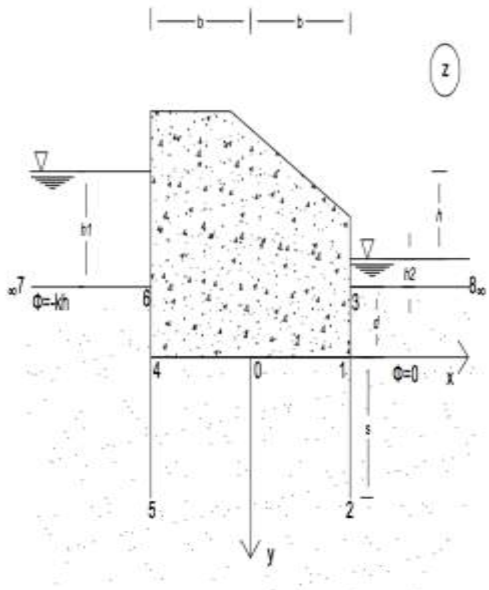


Figure 1- Depressed dam without sheet pile.

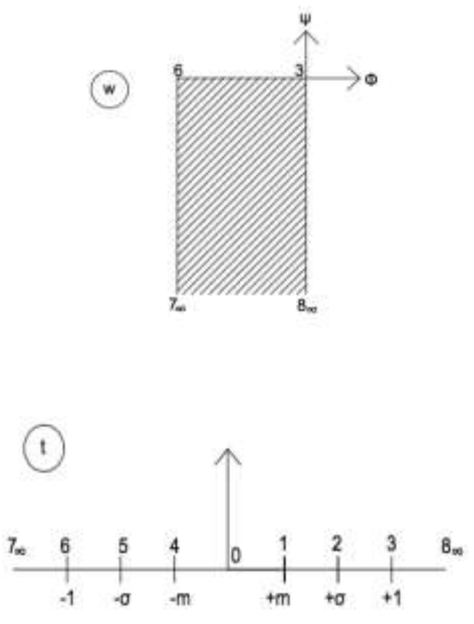


Figure 2- Plane t and w used in Schwartz-Christoffel conformal mapping.

For the physical plane, the following notations are adopted:

h1,h2: depth of water in the upstream and downstream sides respectively;

d: depressed depth;

h: difference in head between upstream and downstream sides;

ϕ, ψ: Potential and stream function;

Kx,Ky: the hydraulic conductivity of the porous media in the horizontal and vertical directions, respectively;

s: length of the sheet pile

2b: total width of hydraulic structure

d/b: ratio of depressed depth to the half-width of the hydraulic structure;

s/b: ratio of the length of sheet pile to the half-width of the hydraulic structure.

Following Harr (1962), the mapping of the z-plane on the t-plane is given by the following equation.

$$Z = M\int_0^t \sqrt{\frac{m^2-t^2}{1-t^2}}\,dt + N \tag{1}$$

Where Z=x+iy, is the complex coordinate of the physical plane, and t is the real axis coordinates of the t-plane.

Using equation (1), the following can be deduced for the derivative of Z with respect to t.

$$\frac{dZ}{dt} = M\frac{t^2-\sigma^2}{\sqrt{(1-t^2)(m^2-t^2)}} \tag{2}$$

Writing the numerator of the integrand as $(t^2 -$ m2+t2−σ2, the integral (1) can bewritten as:

$$Z = -M\int_0^t \sqrt{\frac{m^2-t^2}{1-t^2}}\,dt + \frac{M(m^2-\sigma^2)}{m}\int_0^t \frac{dt}{\sqrt{(1-t^2)(m^2-t^2}} \tag{3}$$

Integrating Equation (3), and using the mathematical definition of elliptic integrals, an expression for Z can be found as:

$$Z = -M[(\sigma^2-1)F(m,\theta) + E(m,\theta)] \tag{4}$$

Where $\theta = \sin^{-1}(t/m)$

F(m,θ):is the elliptic integral of the first kind,of Modulus m and amplitude θ ,and

E(m,θ):is the elliptic integral of the second kind,of modulus m and amplitude θ

According to Harr (1962),

$$\sigma^2 = \frac{E' + (\frac{d}{b})(K-E)}{K' + (\frac{d}{b})K} \tag{5}$$

And;

$$M = -\frac{2b}{\pi}[K' + \frac{d}{b}K] \tag{6}$$

Substituting Equation (5) and (6) into Equation (4) yields to:

$$Z = -\frac{2b}{\pi}[\left(E' - K' - \frac{d}{b}E\right)u + \left(K + \frac{d}{b}E\right)E(u)\Big]$$
(7)

where K is the Complete elliptic integral of the first kind,

E is the complete elliptic integrals of the second kind ,and

K^',E^',are the complete ellitic integrals of complemantary modulus m^',

of the first and the second kind resoectively,m^('^2)

$$= 1 - m^2.$$

$$\textit{and } u \textit{ is obtained from the Jacobian function } Sn\,u = t/m$$

The t-w transformation is given by:

$$t = cos\left(\frac{\pi w}{kh}\right) \qquad (8)$$

where, $w = \phi + i\psi$

(9)

w: is the complex representation of the flow stream and potential functions.

For a given s/b and d/b, m could be found from Fig 5-33 a) and b), Harr(1962).

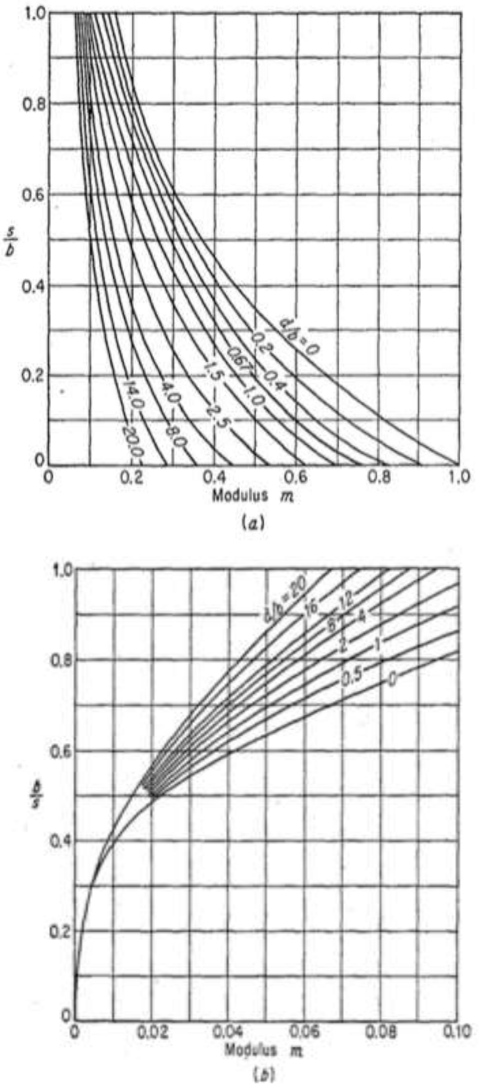


Figure 3- m values as a function of s/b or b/s from Harr (1962).

The derivation ofthe equation for the variation of the exit gradient along the downstream side, can be done as follows:

Along the downstream side the potential function,$\phi = 0$ which yield to $w = i\psi$; hence

$$\frac{dw}{dz} = i\frac{d\psi}{dz} = i\frac{d\psi}{dx} = iV = iI_x k \qquad (10)$$

Where V is the velocity of seepage; $i = \sqrt{-1}$, and I_x is the exit gradient.

Recall that

$$\frac{dw}{dZ} = \frac{dw}{dt} \cdot \frac{dt}{dZ} \qquad (11)$$

$$\frac{dw}{dt} = -\frac{kh}{\pi}\frac{1}{\sqrt{1-t^2}} \qquad (12)$$

Substituting Equations (2), (10) and (12) into Equation (11), it yields to:

$$-iI_x k = -\frac{kh}{\pi}\frac{1}{\sqrt{1-t^2}} \cdot \frac{1}{M}\frac{\sqrt{(1-t^2)(m-t^2)}}{t^2-\sigma^2} \qquad (13)$$

$$-iI_x = \frac{h}{\pi} \cdot \frac{1}{M}\frac{\sqrt{(t^2-m)}}{t^2-\sigma^2} \qquad (14)$$

Multiplying Equation (14) by i

$$-I_x = \frac{h}{\pi} \cdot \frac{1}{M}\frac{\sqrt{(m^2-t^2)}}{t^2-\sigma^2} \qquad (15)$$

Rearranging Equation (15):

$$\frac{I_x b}{h} = \frac{\sqrt{t^2-m^2}}{2[K'+\frac{d}{b}K](t^2-\sigma^2)} \qquad (16)$$

Along the downstream side $Z = x + b - id$. Substituting this relation into Equation (7), it yields to:

$$x + b - id = \frac{2b}{\pi}\left(K' + \frac{d}{b}K\right)[(\sigma^2 - 1)(F(m,\theta) + E(m,\theta) \qquad (17)$$

Setting,$x' = x + b$, Equation(17) can be written as following:

$$\frac{x'}{b} = \frac{2b}{\pi} Real\left[\left(K' + \frac{d}{b}K\right)[(\sigma^2 - 1)(F(m,\theta) + E(m,\theta) \qquad (18)$$

This is the equation that describes the relationship of the distance x'along the downstream side as a function of t.

To obtain the modulus m for any given values of s/b, and d/b, the following equation developed by Harr(1962), can be used:

$$\frac{s}{b} = \frac{\sigma^2 v + \left(m'^2\ Sn\, v'\frac{Cn\, v'}{dn\, v'}\right) - E(m', v)}{(\sigma^2 - 1)K + E}$$

Where:

$$v = dn'\left(\frac{m}{\sigma}, m'\right)$$

$Sn\, v', Cn\, v', and\ dn\, v', are\ the\ Jacobian\ functions\ of\ modulus\ v',$

To obtain the analytical solution for the variation of the exit gradient along the downstream side of such a structure resting on anisotropic soil media than on an isotropic one, the following derivation was modified from Mishara and Reddy(1983). A fictitious isotropic soil media is

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obtained by using a rotation of axis according to the ellipse of the hydraulic conductivity, and rescaling of the domain as shown in figure(4a). Figure (4a) shows an illustration of the method for a depressed dam with two equal length sheet piles. From this figure the direction of the maximum hydraulic conductivity μ makes an angle α with the x-axis, λ is the orthogonal direction to μ , i.e, the direction of the minimum coefficient of hydraulic conductivity, then the following equation for rotation of axis can be used;

$$\mu = x\cos\alpha - y\sin\alpha \ , and\ \lambda = x\sin\alpha + y\cos\alpha \quad (21)$$

Hence the following equations can be used to find the formula for the variation of the exit gradient downstream of the structure for anisotropic soil;

$$Ie = Ief\left[\frac{1+\tan^2\delta_1}{N+\tan^2\delta_1}\right]^{(0.5)} cosec\left[\frac{N^{0.5}\cot\delta_1+\tan\alpha}{1-N^{0.5}\cot\delta_1\tan\alpha}\right] \quad (22)$$

Where;

Ie: is the exit gradient for the anisotropic media with $N = \frac{K_\mu}{K_\lambda}$

Ief: is the exit gradient of the fictitious isotropic soil media with rescaled dimensions as will be shown later, and

$$\tan\delta_1 = \frac{\tan\alpha}{\sqrt{N}} \quad (23)$$

For the case under consideration , setting μ direction as x direction and λ direction as y direction, i.e, $\alpha = 0$, as shown in figure(4b);the following alternatives can be used to transform the real anisotropic soil domain into equivalent fictitious isotropic domain;

- Expansion of domain by multiplying each structure dimension in in λ direction using the following equation:

$$\lambda' = \lambda\,(\frac{K_\mu}{K_\lambda})^{0.5} \quad (24)$$

- Contraction of domain by multiplying each structure dimension in the μ direction using the following equation:

$$\mu' = \mu\left(\frac{K_\lambda}{K_\mu}\right)^{0.5} \quad (24)$$

Selecting the former and setting, $K_{max} = K_x = K_\mu$, and $K_{min} = K_y = K_\lambda$

And;

$$N = \frac{K_\mu}{K_\lambda} = \frac{K_x}{K_y} \quad (25)$$

The following equation can be deduced:

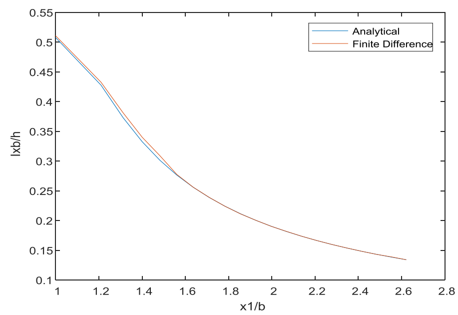
$$I_e = I_{ef}\left[\frac{K_y}{K_x}\right]^{1/2} \quad (26)$$

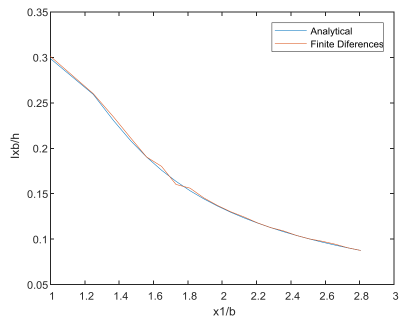
Where K_x, K_y , are the hydraulic conductivites in the x and y direction, respectively; I_e is the exit gradient for anisotropic porous media at any point at the downstream side of hydraulic structure, and I_{ef} is the exit gradient at any point of the downstream side of hydraulic structure for a fictitious isotropic soil media obtained from equation (16)with the following modification: $d' = d\left[\frac{K_y}{K_x}\right]^{1/2}$ and $s' = s\left[\frac{K_y}{K_x}\right]^{1/2}$.

Figures 4a and 4 b should be added here

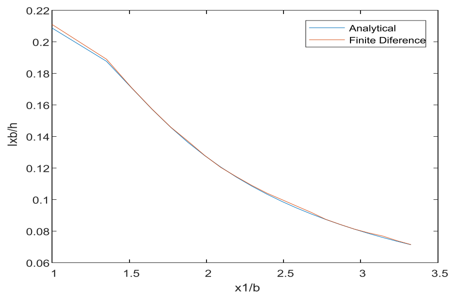
III. MODEL VERIFICATION

In order to verify the analytical solution obtained herein the results of the exit gradient variation obtained using the present analytical solution were compared to the results of a finite differences based model developed herein,. Three cases are selected as shown in Figure(5), each with different values of inputs. The results in the figure indicate high compatibility between the results of the developed solution and the finite difference model. It is worth here to mention that the accuracy of the results of the finite difference model for exit gradient are highly sensitive to the grid size. A sensitivity analysis is done by reducing the grid size until obtaining a stable solution (i.e, no further change in the accuracy), is obtained. The computer time required to run the finite difference model with the required accuracy for each case is found to be range from 8-12 hrs.





(a) (b)



(c)

Fig. (5) Comparison of exit gradient variation downstream of the structure,(a) s/b=0.2, d/b=0.1, kx/ky=1, (b) s/b=0.2, d/b=0.1, kx/ky=2,(c)s/b=0.4, d/b=0.1, kx/ky=2.

IV. RESULTS AND DISCUSSION

The developed analytical model is used to investigate the effect of s/b and d/b on the non-dimensional form of the exit gradient variation at the downstream side of structure$\frac{I_x b}{h}$. In addition to that, the anisotropic solution is used to investigate the effect of the degree of anisotropy on the variation of the exit gradient. In order to investigate these effects, the general case investigated herein is divided into two cases, the first one a depressed structure case without sheet piles obtained by setting s/b=0, to investigate the effect of depressing depth without the existence of sheet pile. The second case is a more generalized form to investigate the effects of both depressing structuredepth and sheet pile on this variation.

1.1. Depressed StructureResting on Permeable Soil of Infinite Extent.

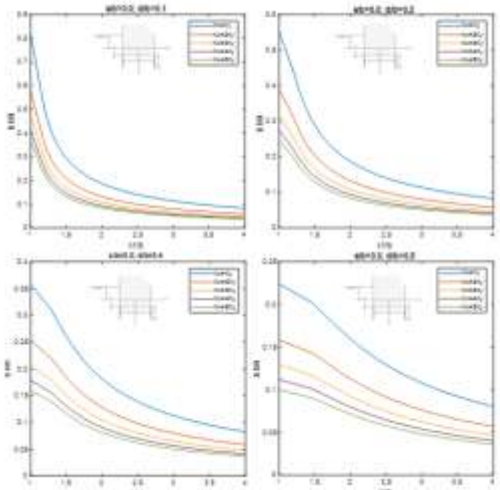


Figure 6–Variation of Exit Gradient of a depressed Structure with no sheet pile (s/b=0) and different depressed depth (d/b=0.1,0.2,0.4,0.8), for isotropic and anisotropic soil.

Figure (6) shows the exit gradient variation with downstream side distance for $s/b = 0$ (no sheet pile) and different values of depressed dam $d/b = 0.1, 0.2, 0.4, 0.8$. For each d/b value the effect of different degree of anisotropy are investigated $K_x/K_y = 1, 2, 3, 4, 5$. This figure shows that the exit gradient in general has a very steep descending variation for x_1/b almost up to 2, for $d/b = 0.1$ and $d/b = 0.2$, after which the variation become mild. Generally, it can be concluded that exit gradient reach a constant value at x_1/b almost equal to 2.5. For,$d/b \geq 0.4$, the descending gradient for low values of x_1/b is milder than that for the case of$d/b \leq 0.2$, but still it can be concluded that exit gradient has a low variation for $x_1/b > 2.5$. This figure also shows that, as expected, the exit gradient decreases with the increase of d/b. However, this reduction is more significantfor $1 < x_1/b < 2$. Table 3 shows the percentage reduction in exit gradient for different values of d/b.

Table 3- Percentage reduction in exit gradient for different values of d/b, s/b=0.

X1/b	d/b			
	0.1	0.2	0.4	0.8
1	0.82*	33%	56%	73%
1.5	0.28*	4%	13%	35%
2	0.18*	5%	8%	16%
*Values of IeB/h				

This figure also shows that for all d/b values, the exit gradient reduces when the degree of anisotropy increases. This is an expected behavior because when the degree of anisotropy increases, the vertical hydraulic gradient will decrease in comparison to horizontal gradient. However, an important conclusion can be drawn that is, at low successive degree of anisotropy this reduction is high, while the reduction will be insignificant between the high successive values of degree of anisotropy.

Table 4- Percentage Reduction in exit gradient for different values of x1/b for isotropic and anisotropic case Kx/Ky=1,and 5, s/b=0 (no sheet pile).

Kx/Ky	x1/b	d/b			
		0.1	0.2	0.4	0.8
1	1	0.82*	33%	56%	73%
	1.5	0.27*	4%	13%	35%
	2.5	0.14*	5%	8%	16%
5	1	0.37*	33%	56%	73%
	1.5	0.13*	9%	13%	35%
	2.5	0.14*	1%	4%	12%

*Values of IeB/h

1.2. Depressed Structure Resting on Permeable Soilof Infinite Extent with Two Symmetrical Sheet Piles

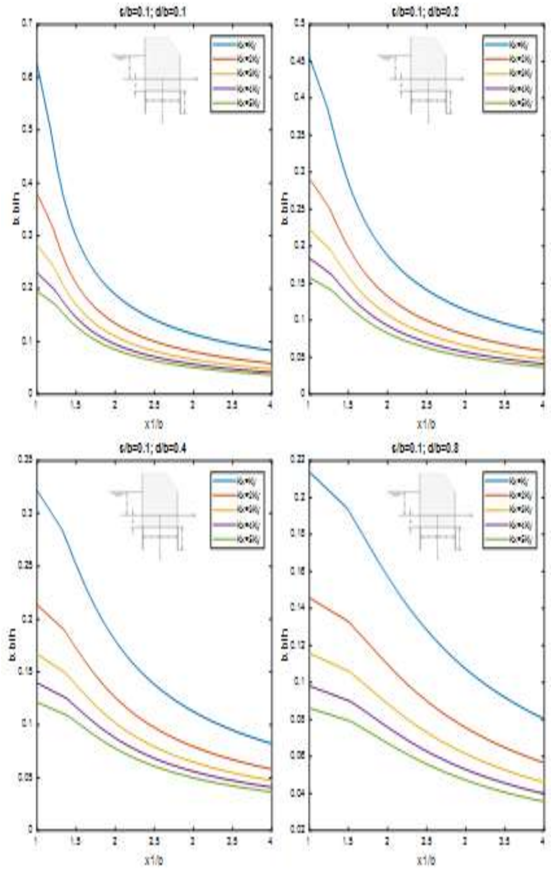


Figure 7- Variation of Exit Gradient of a Depressed Structure with sheet pile (s/b=0.1) and different depressed depth (d/b=0.1,0.2,0.4,0.8), for isotropic and anisotropic soil.

Figure (7) shows the exit gradient variation with downstream side distance for $s/b = 0.1$ (two sheet piles with equal length) and different values of depressed ratio $d/b = 0.1, 0.2, 0.4, 0.8$. For each $\mathrm{d/b}$ value the effect of different degree of anisotropy are investigated$\mathrm{K_x/K_y} = 1, 2, 3, 4, 5$. Similar behavior is presented as for the case $\mathrm{s/b} = 0$, mentioned above (Figure (6)). In order to compare this case and the previous one, and investigate the effect of the existence of sheet piles, the reductions in exit gradientdue to the presence of sheet piles $\mathrm{s/b} = 0.1$ for different $\mathrm{x_1/b}$ values and degrees of anisotropy are presented in Table (5).

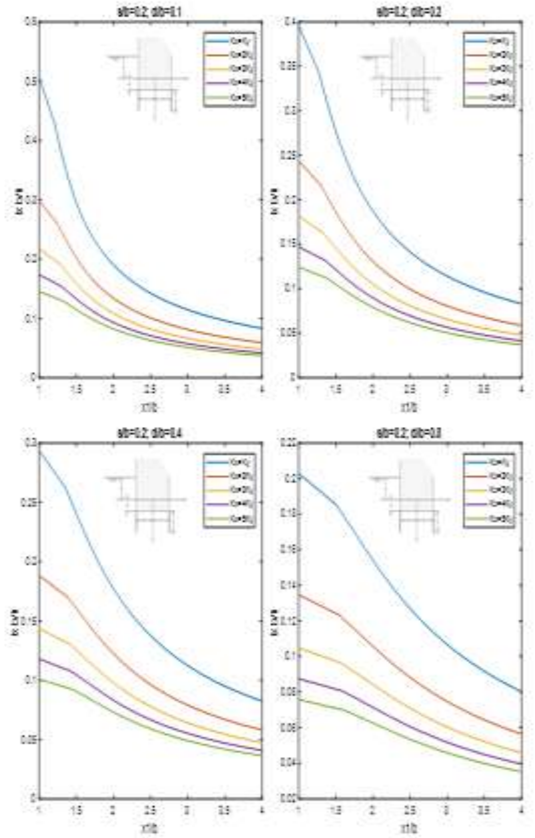


Figure 8 Variation of Exit Gradient of a dam with sheet pile (s/b=0.2) and different depressed depth (d/b=0.1,0.2,0.4,0.8), for isotropic and anisotropic soil.

Table 5-Percentage reduction in exit gradient, due to presence of sheet pile s/b=0.1 for different x1/b values and degree of anisotropy (Kx=Ky=5)

s/b=0.1			
x1/b	**d/b=0.1**		
	Kx/Ky=1	**Kx/Ky=5**	
	Value	**Value**	**% Reduction**
1	0.625384314	0.195519345	69%
1.5	0.291776831	0.129003893	56%
2	0.186760523	0.083423325	55%

Similar analyses can be done for the cases of $\mathrm{s/b} = 0.2, 0.4$ and 0.8 (Figures 8,9,and10). Table 6 shows the reduction of Exit gradient due to the presence of sheet piles for different $\mathrm{x_1/b}$ values and different degrees of anisotropy.

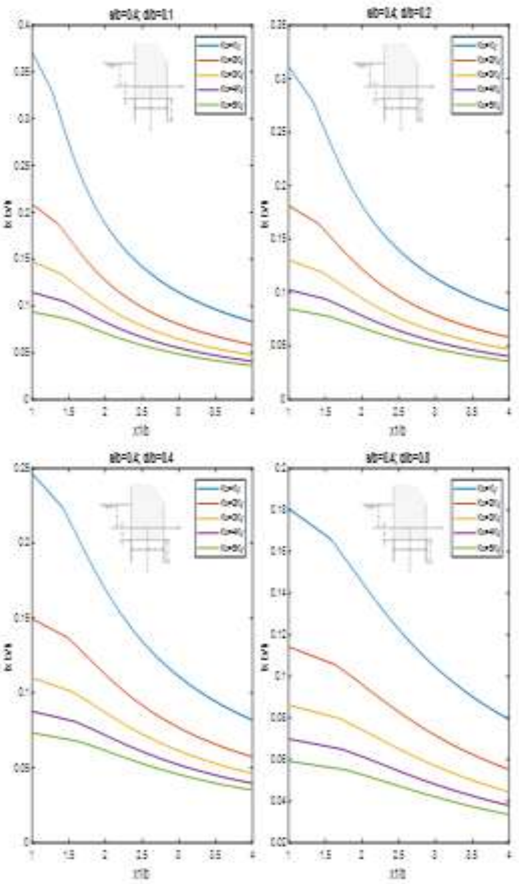


Figure 9- Variation of Exit Gradient of a Structure with sheet pile (s/b=0.4) and different depressed depth (d/b=0.1,0.2,0.4,0.8), for isotropic and anisotropic soil.

Table 6-Percentage Reduction in exit gradient due to presence of sheet pile s/b=0.1, 0.2, 0.4, 0.8 for different values of x1/b values and difference d/b values for isotropic and anisotropic soil (Kx/Ky=5).

s/b	x1/b	d/b=0.1		d/b=0.2		d/b=0.4		d/b=0.8	
		Kx/Ky=1	Kx/Ky=5	Kx/Ky=1	Kx/Ky=5	Kx/Ky=1	Kx/Ky=5	Kx/Ky=1	Kx/Ky=5
0.1	1	0.63	69%	0.46	65%	0.32	62%	0.21	60%
	1.5	0.29	56%	0.28	59%	0.26	61%	0.19	59%
	2	0.19	55%	0.19	56%	0.18	57%	0.09	20%
0.2	1	0.51	71%	0.40	69%	0.29	59%	0.21	60%
	1.5	0.30	61%	0.28	63%	0.08	-23%	0.19	59%
	2	0.19	56%	0.18	59%	0.18	58%	0.15	56%
0.4	1	0.37	75%	0.31	73%	0.25	70%	0.18	67%
	1.5	0.27	68%	0.25	69%	0.21	68%	0.17	67%
	2	0.18	61%	0.18	62%	0.17	62%	0.14	64%
0.8	1	0.24	79%	0.21	78%	0.18	76%	0.14	73%
	1.5	0.21	77%	0.20	76%	0.17	75%	0.13	72%
	2	0.16	72%	0.16	72%	0.14	72%	0.12	71%

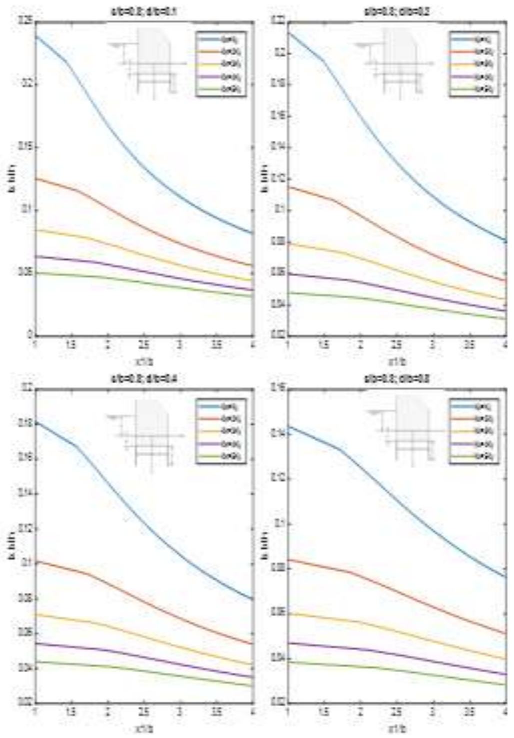


Figure 10- Variation of Exit Gradient of a Structure with sheet pile (s/b=0.8) and different depressed depth (d/b=0.1,0.2,0.4,0.8), for isotropic and anisotropic soil.

This variation are shown in Figures10 and 11 for different values of s/b and $d/b = 0.1, 0.8$.

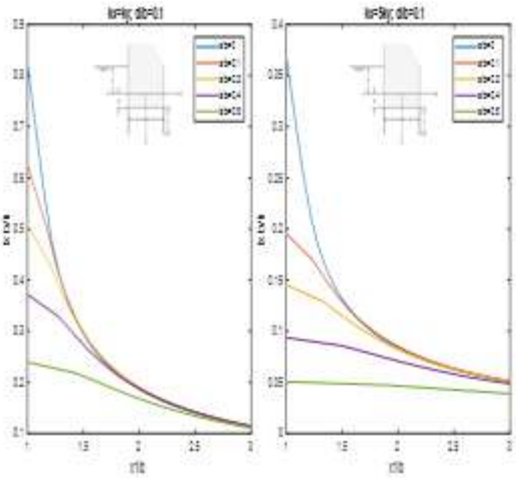


Figure 11 Effect of anisotropy ratio for small d/b ratio for a Depressed Structure without sheet piles.

Figure 11 indicates that the existence of sheet pile with short length ($s/b = 0.1$) will decrease considerably the maximum exit gradient ($s/b = 0$). Furthermore, it is clear that increasing s/b to 0.2, 0.4 and 0.8 will decrease the maximum exit gradient values by a constant quantity. That can be explained by the change in flow pattern due to existence of sheet pile of any length. However, for low exit gradient values located at $x_1/b > 2$, the effect of s/b can be neglected for isotropic case (as shown on the left plot of the figure), while for anisotropic case $K_x/K_y = 5$ the increase in s/b will still have effect as shown on the right plot of Figure 11.

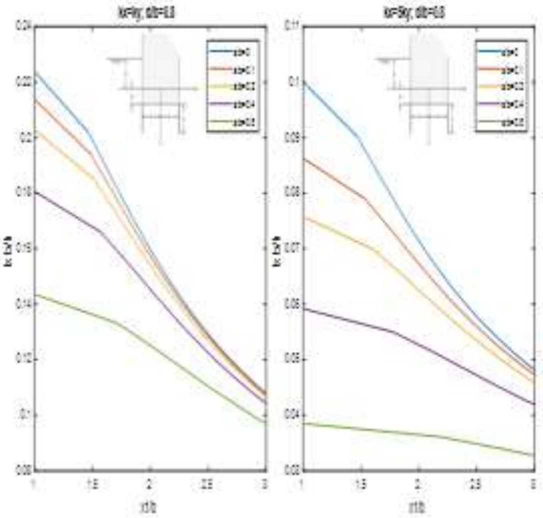


Figure 12 Effect of anisotropy ratio for High d/b ratio for a Depressed Structure without sheet piles.

Figure 12shows the same effect as Figure 11, for a depressed depth $d/b = 0.8$ rather than 0.1. This figure presents a different variation in the maximum exit gradient. The increase in s/b values causes a higher reduction in the maximum exit gradient for isotropic and anisotropic porous media.

V. CONCLUSIONS

The following conclusions can be deduced from the present research;

- The developed analytical solution is found highly compatible with finite difference solution. However in order to achieve the required accuracy for the finite difference solution a small discretization grid is required which make the required computer running time relatively high.
- For the case of depressed structure without sheet pile the exit gradient variation is found steep for the region near than the toe of the structure up to x1/b=2.5, when there after become very mild which may introduce a limiting value for the protection length design. This is true for different values of depressing ratio of the foundation of the structure.
- The degree of anisotropy has considerable effect on the exit gradient, as this ratio increases considerable reduction of exit gradient is observed, especially near the toe of the structure. This reduction is increase as the depressing ratio of the foundation increases.
- Similar variation is observed for the case of depressed structure with the existence of the sheet piles, as that case without sheet piles. The comparison of the two cases indicates that the existence of sheet pile will have considerable effect in reducing the exit gradient. Hence, these results recommends the use of small depressing depth with sheet piles which is more cost effective design of the foundation rather than increasing the depth of depressing. However this reduction is high for small successive s/b ratios than those for high s/b ratios.
- The analytical solution can be used for a reliable design of a structure resting on anisotropic soil media.

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