

## Power system harmonic analysis based on adding Hamming window FFT and db24 wavelet packet transform

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### ABSTRACT

At present, the traditional harmonic analysis method is mainly Fourier transform (FFT), but it has the problems of spectrum leakage, slow identification and slow positioning. In this paper, a time-frequency analysis method based on windowed FFT and db24 wavelet packet transform is proposed, in combination with good time-frequency localization characteristics of wavelet packet transform and good frequency domain analysis characteristics of windowed Fourier transform. Firstly, analyze and obtain the accurate steady-state spectrum of signal Hamming windowed Fourier transform, using db24 wavelet packet analysis of singular points of signal recognition and extraction time, initial positioning of transient signal occurred and termination of transient harmonic occurred in windowed Fourier transform to calculate the amplitude of the transient signal. Simulation results show that the method can analyze the steady-state and transient harmonics in power system accurately.

**Keywords:** FFT, hamming window, db24 wavelet packet, singularity, steady-state harmonic, transient harmonic

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### I. INTRODUCTION

Fast Fourier transform(FFT) is the mainly method in power system harmonics analysis presently, which belongs to traditional harmonic detection algorithm. Although it can accurately determine every harmonic amplitude and other power parameters for stationary signal, FFT process local signal by making average, resulting a loss of local characteristics and inability to confirm the spatial distribution of singularity, i.e. unable to confirm the generation time of transient harmonic[1]. Therefore, FFT is only applied to analyze the steady-state harmonic, even the improved hamming windowed FFT can not solve this problem.

Wavelet packet transform has flexible time-frequency characteristics, and it can ascertain the moment of signal mutation accurately, filter out interfering signals and extract the information effectively from the signals. Wavelet packet transform can make multilayer division for the frequency band and further decomposition of high frequency part, which has been not analyzed in multi-resolution analysis. It can also choose the frequency band self-adaptively to match the signal frequency and improved the time-frequency resolution according to the features of signal. Therefore, wavelet packet transform has a wider range of application area[2]. In literature [3] proposed the spectrum of steady-state signal based on hamming window FFT by using the characteristic of

the suppressing spectrum leakage of hamming window FFT. While literature [4] and [5] proposed the harmonic detection of power system based on FFT and wavelet packet transform.

In this paper, the harmonic detection of power system has been analyzed by the combination of the advantages of window FFT, which can suppress the spectrum leakage, and wavelet packet transform, which can accurately determine the moment of signal mutation and make multilayer division for the frequency band and have higher resolution. This method can not only reduce the spectrum leakage in signal analysis and receive more accurate steady-state harmonic amplitude, but also analyze the amplitude and beginning moment and ending moment of transient signal. The results of simulation confirmed the feasibility of this method in the harmonic analysis of power system. And this method provides the basis for power system harmonic analysis and treatment.

### II. HARMONIC DETECTION OF POWER SYSTEM BASED ON FFT AND WAVELET PACKET TRANSFORM

#### 2.1 spectrum leakage[6]

Fourier Transform can accurately determine the different kinds of frequency involved in steady-state signals. For sine wave with a single line, which is infinitely long, the right single line spectrum can be calculated with FFT. But in the actual harmonic analysis, all the signals are after sampling and A/D conversion. The duration of the analyzed signal is

$T = N\Delta t$ , which is similar to using a rectangular window to make a signal truncation from infinite long signal to the limited time-frequency signal.

If the signal contains only the fundamental frequency only, then

$$x(t) = A_0 \sin(\omega_0 t) \quad (1)$$

The rectangular window function is

$$w(t) = \varepsilon(t) - \varepsilon(t - T) \quad (2)$$

The Fourier Transform in the signal is

$$X(\omega) = \frac{\pi}{j} A_0 [\delta(\omega - \omega_0) - \delta(\omega + \omega_0)] \quad (3)$$

The Fourier Transform for the rectangular window function is

$$W(\omega) = T \text{Sa}\left(\frac{\omega T}{2}\right) e^{-j\frac{\omega T}{2}} \quad (4)$$

The Fourier Transform for the discrete input signal is

$$\begin{aligned} \overline{X(\omega)} &= \frac{1}{2\pi} \frac{\pi}{j} A_0 [\delta(\omega - \omega_0) - \delta(\omega + \omega_0)] \\ &\quad * T \text{Sa}\left(\frac{\omega T}{2}\right) e^{-j\frac{\omega T}{2}} \\ &= \frac{TA_0}{j2} \left[ \text{Sa}\left(\frac{\omega - \omega_0}{2} T\right) e^{-j\frac{(\omega - \omega_0)T}{2}} \right. \\ &\quad \left. - \text{Sa}\left(\frac{\omega + \omega_0}{2} T\right) e^{-j\frac{(\omega + \omega_0)T}{2}} \right] \quad (5) \end{aligned}$$

Function is also called a sampling function, when  $t \neq 0$ , with the absolute value of  $t$  increases, the absolute function's value oscillation decrease continuously and approach to zero. As can be seen,  $\overline{X(\omega)}$  spread to the whole spectrum instead of a single line, i.e. Which indicates energy is no longer concentrated and, spectrum leakage happened.

### 2.2 Add window Fourier Transform

The general signals of power system contain mainly the integer harmonics, so the research focuses on the window function of cosine function. The feature of this kind of function is, if the observation time is selected as the integer times of signal cycle, the amplitude of all integer harmonic frequency is 0, thus no leakage is happened between harmonic.

The expression of window function is

$$w(n) = \frac{1}{N} \sum_{k=0}^n a_k \cos\left(\frac{2\pi}{N} kn\right), \quad n = 0, 1, 2, 3 \dots N-1 \quad (6)$$

$a_k$  determines the different windows, and to meet the interpolation theorem, then

$$\sum_{k=0}^n |a_k| = 1 \quad (7)$$

$$\sum_{k=0}^n a_k = 0 \quad (8)$$

The discrete Fourier Transform of window function is

$$\begin{aligned} W(\theta) &= \sum_{k=0}^n \frac{a_k}{2} \left[ e^{-j\pi(\theta-k)\frac{N-1}{N}} \frac{\sin(\pi(\theta-k))}{N \sin\left(\frac{\pi}{N}(\theta-k)\right)} \right. \\ &\quad \left. + e^{-j\pi(\theta+k)\frac{N-1}{N}} \frac{\sin(\pi(\theta+k))}{N \sin\left(\frac{\pi}{N}(\theta+k)\right)} \right] \end{aligned}$$

$$\theta = 0, 1, 2, 3 \dots N-1 \quad (9)$$

The principles of choosing window function are a. the main board is as narrow as possible and b. to minimizing the relative amplitude of window spectrum side lobe. Here hamming window was chosen, as follows:

$$w(n) = 0.54 - 0.46 \cos\left(\frac{2n\pi}{N-1}\right), \quad n = 0, 1, 2, 3 \dots N-1 \quad (10)$$

The harmonic content and the main distribution range of the harmonic can be derived from window FFT, then the details are gained through wavelet packet transform.

### III. DETECTION METHOD WITH WAVELET PACKET ANALYSIS

There are two steps in the process of signal wavelet packet analysis: the first one is signal decomposition and calculation of the coefficient in the corresponding scale space of fundamental wave and harmonic, the second step is reconstructing the fundamental signal and harmonic signals according to the coefficient in the corresponding scale space of fundamental wave and harmonic for completing the harmonic detection. Defining the subspace  $U_j^n$  as closure space of function  $w_n(t)$ , and  $U_j^{2n}$  as the closure space of function  $w_{2n}(t)$ , and making

$w_{2n}(t)$  to meet the double scale equations as follows:

$$w_{2n}(t) = \sqrt{2} \sum_k h(k) w_n(2t-k) \quad (11)$$

$$w_{2n+1}(t) = \sqrt{2} \sum_k g(k) w_n(2t-k) \quad (12)$$

in the expression  $k \in Z$ ,  $g(k) = (-1)^k h(1-k)$ .

Reconstruction algorithm of wavelet packet is

$$d_{j+1,j}^n = \sum_{k \in Z} h_{l-2k} d_{j,k}^{2n} + g_{l-2k} d_{j,k}^{2n+1} \quad (13)$$

In (13),  $h_{l-2k}$ ,  $g_{l-2k}$  are low-pass and high-pass filter group of the wavelet packet reconstruction, and the total decomposition layers, respectively.

Now the harmonics in the power system is mainly low harmonics i.e., 2-25. The design requires the analysis of the harmonics within 31. According to the sampling theorem, we know, that  $f_s$  must be higher or equal to  $2 * 50 * 31 = 3100$ , here

$f_s = 3200\text{Hz}$ . For the four layers signal analysis with wavelet packet based on db24, in the process of decomposition of wavelet packet analysis, all frequencies for  $(0 - f_s / 2)$  are analyzed in high resolution. The calculation of harmonic detection will be increase and the real-time capability of detection will be reduced.

In This design chose db24 wavelet packet to do four layer decomposition. The original signal is decomposed into a series of subband signals with wavelet packet, using  $f_s$  to describe the band of each node in the decomposition of wavelet packet. The Figure 1 shows the division of 16 bands, which come from 4 layers the wavelet packet decomposition for the actual signals with frequencies in range of 0-1600Hz[7].

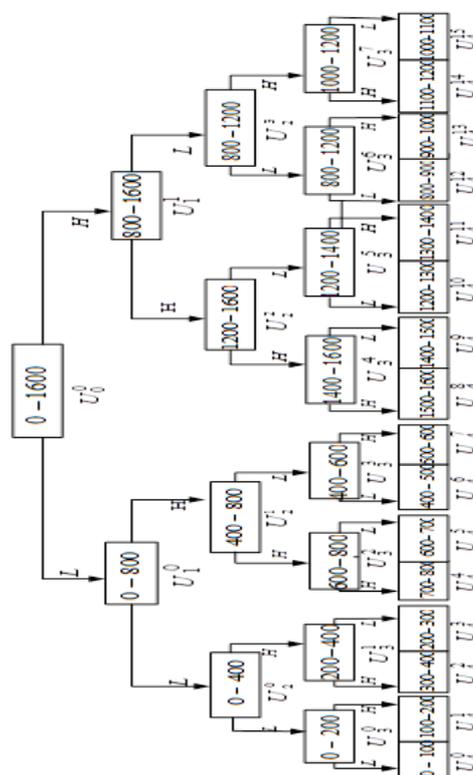


Fig. 1 wavelet packet decomposition in 4 layers

From figure 1 we can see, that the wavelet packet decomposition in 4 layers has a complete tree structure, the decomposition is not only for low frequency part but also for high frequency part. Furthermore, after decomposition has every band  $U^4$  has same width, i.e. in every band contains the same harmonic frequency. The space division of wavelet packet can decompose the original signal into different wavelet subspaces. The wavelet packet decomposition improves the resolution of signal in high frequency part.

#### IV. SIMULATION OF HARMONIC DETECTION METHOD

In this method, using firstly window FFT detects the amplitude of harmonic with steady-state and different frequency. For transient harmonic the beginning moment and ending moment can be detected by wavelet packet transform, while for transient harmonic the amplitude can be obtained by using window Fourier transform to analyze the duration of the transient harmonic. At the same time wavelet packet transform can separate the decay quantity signal from the detecting signal in the signal analysis.

If the signals from power system in 0 to 1 second contain steady-state harmonic with 1,3,5,7 times and 21 times harmonic with decay, then

$$x_1(t) = 1.414 \times [(10 \sin(100\pi t) + 5 \sin(300\pi t) + 35 \sin(500\pi t) + 2 \sin(700\pi t) + 10 \sin(2100\pi t)e^{-10t}] + 0.03 \text{randn}(1,3200)$$

(14)

During the time of (1500-2000) /3200 are there 11 times harmonics. Its function's expression is

$$x(t) = \begin{cases} x_1(t) + 2.828 \sin(1100\pi t); t \in ((1500 - 2000) / 3200) \\ x_1(t); t \in \text{others} \end{cases}$$

(15)

The time domain waveform of original voltage signal x(t) is shown in figure 2:

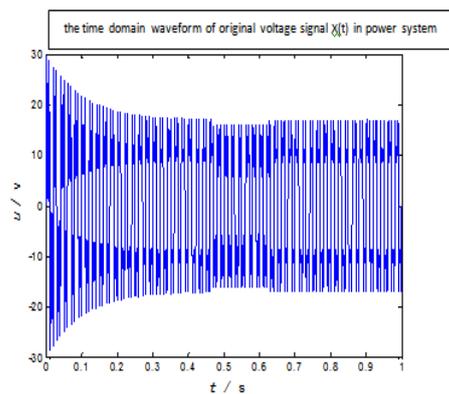


Fig. 3 the time domain waveform of original voltage signal

#### 4.1 The result of hamming window FFT

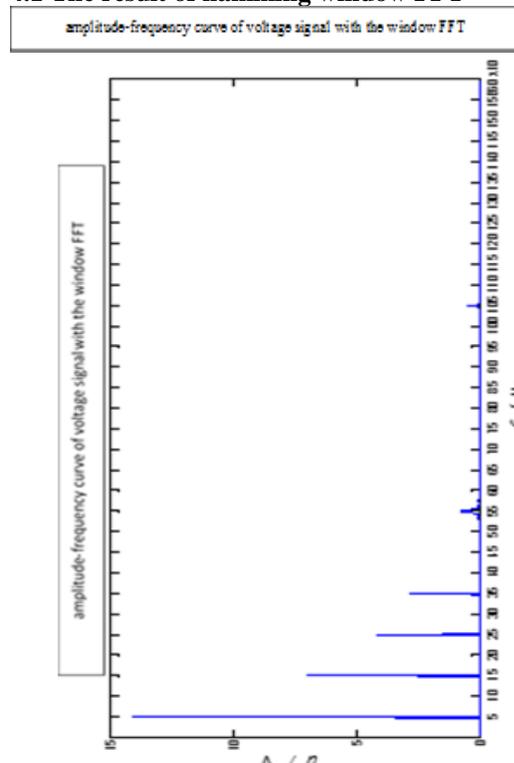


Fig. 3 amplitude-frequency curve of voltage signal in power system with the window Fourier Transform

Times of harmoni c	Setting value of harmonic amplitude	Amplitude with FFT algorithm	Amplitude differences/ %
1	14.14	14.1381	0.013
3	7.07	7.0710	0.014
5	4.242	4.2403	0.040
7	2.828	2.8324	0.160
11	2.828	0.7905	72.047
21	14.14	0.5458	96.140

Table 1 the relationship between setting harmonics and harmonic amplitude calculated with hamming window FFT

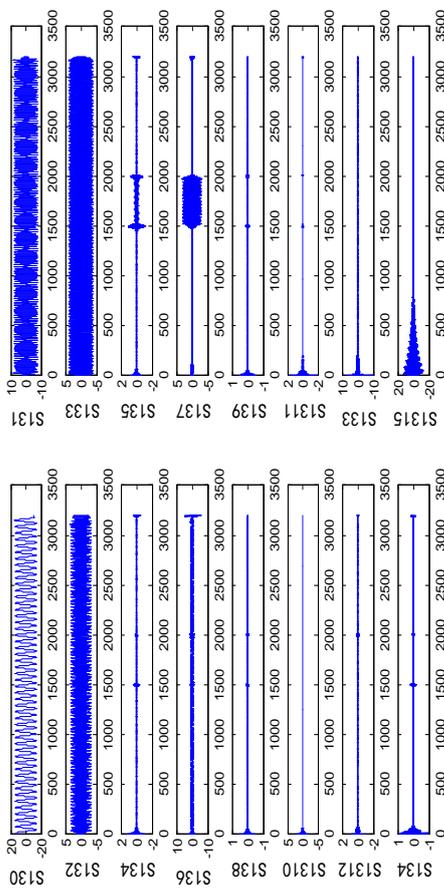
In Fig.3 we can see, that the signals of power system contain the signals with 50Hz , 150Hz, 250Hz, 350Hz, 550Hz, 1050Hz.

As known from the testing results, our experiment can calculate the harmonic amplitude of times 1,3,5,7 with hamming window Fourier Transform. But for abrupt signals, which contain 11 times harmonics and for decay signals, which contain 21 times harmonics, the error of amplitude testing is very large. This suggests that the window FFT is only suitable for harmonic analysis of steady-state signals and is not suitable for the one of transient harmonic.

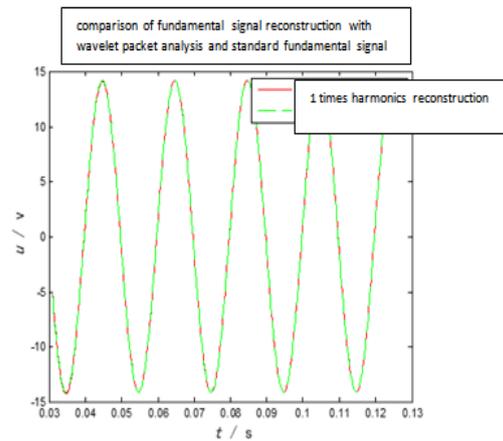
#### 4.2 wavelet packet analysis based on db24[8][9]

Both wavelet transform and wavelet packet transform can well detect and analyze decay signals and abrupt signals [10]. By using 4 layers wavelet packet transform the simulation results are shown in Fig. 4-10. In Fig. 4 the reconstruction signal S130-S1415 has a corresponds frequency band, which correspond to the frequency band in the fourth  $U_0^0 - U_0^{15}$  in figure 4. Then we can know, that S130 corresponds to the reconstruction signal with (0-100) Hz, and in our design, to fundamental frequency with 50 Hz. Likewise, S131, S132, S133, S137 are corresponding to the reconstruction harmonic signal with times 3,5,7,11. The decay signal with 21 times harmonic correspond to the reconstruction signal S1315. From S1315 we can see ,that it appears modulus maxima, when n=1500 and n=2000[8], it shows, that the transient signals with

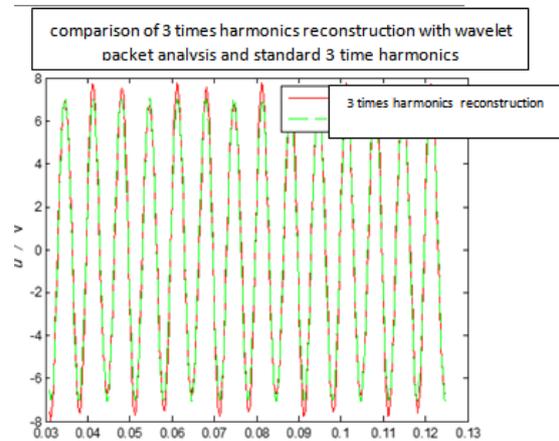
11 times harmonic start at  $1500/3200s \approx 0.469s$  and end at  $2000/3200s = 0.625s$ . In Fig. 5-9 we can see, wavelet packet decomposition can reconstruct accurately fundamental signal, but for 3,5,7,11 times harmonics it has reconstruction minor errors. Certainly, we know from S1313, that it has a little leakage of decay harmonics with 21 times, but it doesn't affect the analysis. The frequency of decay harmonics with 21 times focus on 1050Hz, we can directly reconstruct the high frequency part (800-1600) of wavelet packet decomposition in 1 layer and get more accurately waveform of decay signals. In figure 11 we can see the waveform of reconstruction for the high frequency part of wavelet packet decomposition in 1 layer.



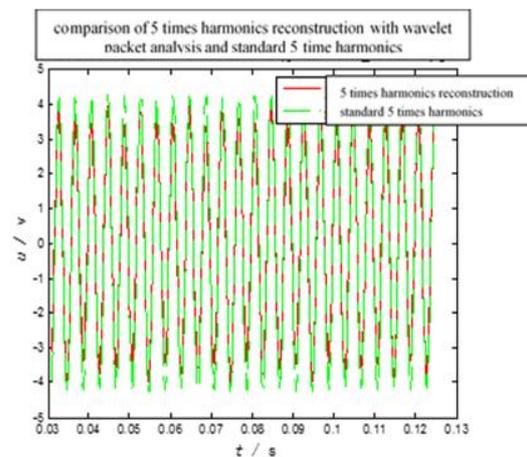
**Fig. 4** waveform of reconstruction for signal, which has been decomposed by using wavelet packet based on db24 with 4 layers



**Fig.5** comparison of fundamental signal reconstruction and standard fundamental signal



**Fig. 6** comparison of 3 times harmonics reconstruction and 3-time harmonics



**Fig.7** comparison of 5 times harmonics reconstruction and 5 time harmonics

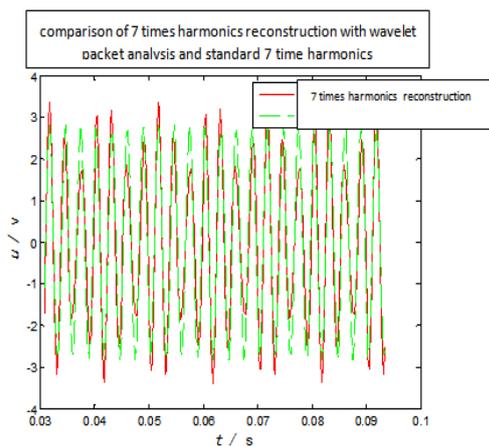


Fig. 8 comparison of 7 times harmonics reconstruction and 7 time harmonics

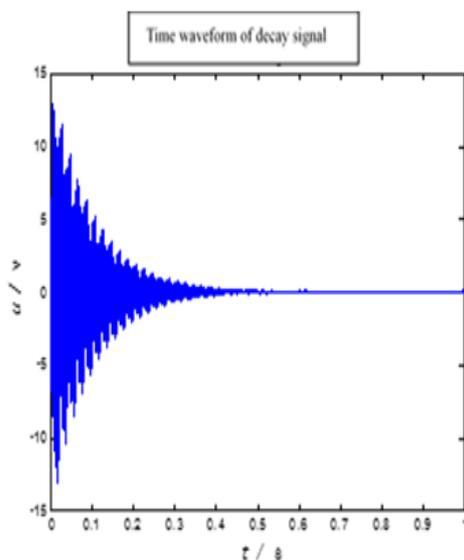


Fig. 10 decay signal of reconstruction

The starting moment and ending moment of transient signals with 11 times harmonics can be obtained by wavelet packet decomposition and reconstruction signal. Then we can make the hamming window Fourier transform for signals at the time  $(1500-2000) / 3200s$ . During this time the 11 times harmonics are steady-state harmonics. Based on simulation, the amplitude of 11 times harmonics during the time of  $(1500-2000) / 3200s$  is 2.8288, the difference between 2.8288 and our setting 2.828 is only 0.028%.

## V. CONCLUSION

Wavelet packet analysis has a lot of advantages and been applied widely. In this paper a easy and effective method of harmonics analysis in power system has been proposed. The simulation with MATLAB demonstrates that with this method the fundamental harmonics and all the steady-state harmonics have been easily and effectively extracted. At the same time, the transient harmonics can also be extracted, while the amplitude and starting moment and ending moment of transient harmonics can be accurately located as well.

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