RESEARCH ARTICLE OPEN ACCESS

Effect of Baffle on Mixed Convection Flow in a Vertical Channel Filled With Electrically Conducting Fluid

M. Karuna Prasad*, J. Prathap Kumar and J.C. Umavathi
1 Department of Mathematics, GITAM School of Technology, Bangalore, India.
2&3 Department of Mathematics, Gulbarga University, Kalaburagi-585 106, Karnataka, India
Corresponding Author: M. Karuna Prasad

ABSTRACT
The magnetohydrodynamic flow in a vertical double passage channel using Robin boundary conditions in the presence of applied electric and magnetic field has been investigated. The fluid is electrically conducting while the channel plates are electrically insulated. The general equations that describe the discussed problem are coupled nonlinear ordinary differential equations and hence closed-form solutions cannot be found. However approximate analytical solutions are found using regular perturbation method valid for small values of Brinkman number and using differential transform method valid for all values of Brinkman number. The channel is divided into two passages by means of a solid, thin, perfectly conducting baffle and hence the velocity will be individual in each stream. The governing equations which are coupled nonlinear ordinary differential equations are solved analytically using regular perturbation method valid for small values of Brinkman number and using differential transform method valid form all values of brinkman number. The effects of electrical field load parameter, Hartman number, mixed convection parameter on velocity and temperature fields are shown graphically for equal and unequal Biot numbers at different positions of the baffle. It is also found that the perturbation method and differential transform method solutions agree very well for small values of Brinkman number and error increases as Brinkman number increases.
Keywords: Mixed Convection, Vertical channel, Baffle, Magnetohydrodynamic,

---

Date of Submission: 30-03-2019 Date of acceptance: 13-04-2019

Nomenclature

P : Dimensional pressure
K : Thermal conductivity
B_0: Applied magnetic field
Bi_1 Bi_2: Biot numbers $\left(qD/k \right)$
Br: Brinkman number $\left(\mu U_0^2 / k\,\Delta T \right)$
h_1 : Width of region-I
h_2 : Width of region-II
C_p : Specific heat at constant pressure
u : Velocity
θ: Temperature
θ_1: Temperature in region-I
θ_2 : Temperature in region-II
Gr : Grashof number $\left(g\,\beta h^3 \Delta T / \upsilon^2 \right)$
Re: Reynolds number $\left(h\,U_0 / \upsilon \right)$

Br : Brinkman number $\left(\mu U_0 / K \Delta T\right)$
Λ : Dimensionless parameter $\left(Gr/\mathrm{Re}\right)$
g: Acceleration due to gravity
U_0 : Reference velocity
T : Reference temperature
M: Hartman number
E_0: Dimensional applied electric field
E: Dimensionless electric field load parameter $\left(E_0 / B_0 U_0\right)$

Greek Symbols
α : Thermal diffusivity
β : Coefficient of thermal expansion
ΔT : Difference in temperature $\left(T_2 - T_1\right)$
ε : Dimensionless parameter
θ : Non dimensional temperature
μ : Viscosity
υ : Kinematics viscosity
ρ : Density of fluid
σ : Electrical conductivity

I. INTRODUCTION

The flow and heat transfer of electrically conducting fluids in channels and circular pipes under the effect of a transverse magnetic field occurs in magnetohydrodynamic (MHD) generators, pumps, accelerators and flow meters, and have applications in nuclear reactors, filtration, geothermal system and others. The interest in the outer magnetic field effect on heat-physical processes appeared seventy years ago. Research in magnetohydrodynamics grew rapidly during the late 1950s as a result of extensive studies of ionized gases for a number of applications. Blum et al. [1967] carried out one of the first works in the field of heat and mass transfer in the presence of magnetic field. The increasing interest in the study of MHD phenomena is also related to the development of fusion reactors where plasma is confined by a strong magnetic field (Hunt and Holroyd [1977]). Many exciting innovations were put forth in the areas of MHD propulsion (Davidson [1993]), remote energy deposition for drag reduction (Tsinober [1990]), plasma actuators, radiation driven hypersonic wind tunnel, MHD control of flow and heat transfer in the boundary layer (Boric et al. [2009], Obrovic et al. [2009], Xu et al. [2007] and Nikodijevic et al. [2009]), enhanced plasma ignition (Kessel et al. [2002]) and combustion stability.

The MHD research extended gradually to new applied problems. The MHD devices for liquid metals attracted the attention of metallurgist. It was shown that the effect of magnetic field could be very helpful in the modernization of technological processes. Barletta et al. [2008] considered the problem of buoyant magnetohydrodynamical flows with Joule and viscous heating effects in a vertical parallel plate channel which results have important industrial applications in metallurgy and in growing of pure crystals, as well as in energy engineering. Umavathi [1996] studied the combined effect of viscous and Ohmic dissipations on magneto convection in a vertical enclosure in the presence of applied electric field. Later on, Umavathi and her group analyzed magnetohydrodynamic flow and heat transfer for various geometries (Malashetty et al. [2000, 2001a and 2001b], Umavathi and Malashetty [2005], Umavathi et al. [2011] and Prathap Kumar et al. [2011a]).

Hasnaoui et al. [1990], Ben-Nakhi and Chamka [2006] and Dagtekin and Oztop [2001] investigated the natural convection in enclosures with a partition. The presence of a partition was the effective parameter on heat transfer. Tansmim and Collins [2004] did a numerical study on heat transfer in a square cavity with a baffle located on the hot wall. The study showed that the baffle has a significant effect on increasing the rate of heat transfer compared with a wall without baffle. Recently Prathap Kumar et al. [2011b] analyzed free convection in a double passage wavy channel using Walter`s fluid. Chang and Shiau [2005] studied the effects of horizontal baffle on the heat transfer characteristics of pulsating opposing mixed convection in a parallel vertical open channel.

Above mentioned works concern first and second type boundary conditions on temperature in studying mixed convection flow through vertical channels. There are few works available in literature concerning the third type temperature boundary conditions. Heat transfer in laminar region of a flat rectangular channel with the temperature boundary condition of third kind was first explored by Javeri [1977]. Javeri [1978] have studied the laminar heat transfer in the thermal entrance region of a rectangular channel for the temperature boundary conditions of third kind. Recently, Zanchini [1998] investigated the effect of viscous dissipation in a vertical channel with temperature boundary conditions of third kind. In this chapter we have analyzed the flow behavior of mixed convection electrically conductive fluid in a vertical double passage channel using Robin boundary conditions.

II. MATHEMATICAL FORMULATION

Consider a steady and laminar electrically conducting flow in the fully developed region of a parallel plate vertical channel as shown in Fig. 1. The channel is divided into two passages by inserting a perfectly conductive thin baffle. The X -axis lies on the axial plane of the channel, and its direction is opposite to the gravitational field. The Y -axis is orthogonal to the walls. The electric field is applied normal to the flow and the magnetic field is applied perpendicular to the flow as shown in the following figure. The fluid properties such as thermal conductivity, thermal diffusivity, dynamic viscosity and thermal expansion coefficient are assumed to be constant. The Boussinesq approximation and the equation of state are also adopted. For fully developed flow it is assumed that the transverse velocity and temperature gradient in the axial direction are zero.

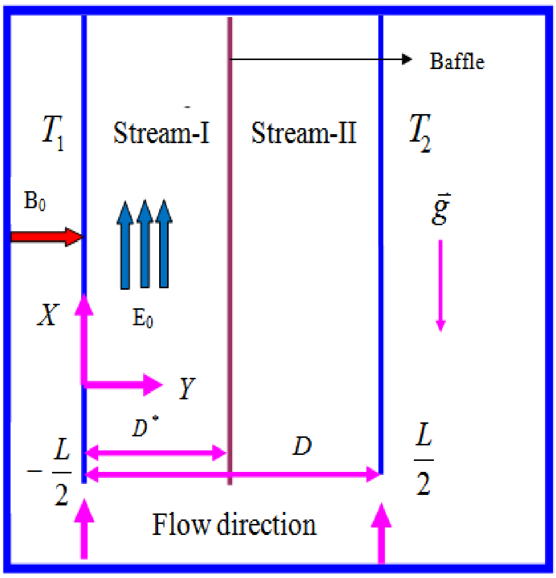

Figure 1
Physical Model

Under these assumptions the momentum balance equations along X and Y yield

Stream-I

$$\rho\beta g\left(T_1-T_0\right)-\frac{\partial P}{\partial x}+\mu\frac{d^2U_1}{dy^2}-\sigma_e(E_0+B_0U_1)B_0=0 \tag{1}$$

Stream-II

$$\rho\beta g\left(T_2-T_0\right)-\frac{\partial P}{\partial x}+\mu\frac{d^2U_2}{dy^2}-\sigma_e(E_0+B_0U_2)B_0=0 \tag{2}$$

and the Y - momentum balance equation in both the streams can be written as

$$\frac{\partial P}{\partial Y}=0 \tag{3}$$

where

$$P=p+\rho\,g\,X \tag{4}$$

(assuming equal pressure gradient in both the streams) is the difference between the pressure and the hydrostatic pressure. Equation (4) insists that P depends only on X , and hence Eqns. (1) and (2) can be written as

Stream-I

$$\rho\beta g\left(T_1-T_0\right)-\frac{dP}{dx}+\mu\frac{d^2U_1}{dy^2}-\sigma_e(E_0+B_0U_1)B_0=0 \tag{5}$$

Stream-II

$$\rho\beta g\left(T_2-T_0\right)-\frac{dP}{dx}+\mu\frac{d^2U_2}{dy^2}-\sigma_e(E_0+B_0U_2)B_0=0 \tag{6}$$

Differentiating Eqns. (5) and (6) with respect to X and Y , one obtains

Stream-I

$$\frac{\partial T_1}{\partial X}=\frac{1}{\rho\beta g}\frac{d^2P}{dX^2} \tag{7}$$

$$\frac{\partial T_1}{\partial Y}=-\frac{\mu}{\rho\beta g}\frac{d^3U_1}{dY^3}+\frac{\sigma_e B_0^{\,2}}{\rho\beta g}\frac{dU_1}{dY} \tag{8}$$

$$\frac{\partial^2 T_1}{\partial Y^2}=-\frac{\mu}{\rho\beta g}\frac{d^4U_1}{dY^4}+\frac{\sigma_e B_0^{\,2}}{\rho\beta g}\frac{d^2U_1}{dY^2} \tag{9}$$

Stream-II

$$\frac{\partial T_2}{\partial X}=\frac{1}{\rho\beta g}\frac{d^2P}{dX^2} \tag{10}$$

$$\frac{\partial T_2}{\partial Y}=-\frac{\mu}{\rho\beta g}\frac{d^3U_2}{dY^3}+\frac{\sigma_e B_0^{\,2}}{\rho\beta g}\frac{dU_2}{dY} \tag{11}$$

$$\frac{\partial^2 T_2}{\partial Y^2}=-\frac{\mu}{\rho\beta g}\frac{d^4U_2}{dY^4}+\frac{\sigma_e B_0^{\,2}}{\rho\beta g}\frac{d^2U_2}{dY^2} \tag{12}$$

Both the walls of the channel will be assumed to have a negligible thickness and to exchange heat by convection with an external fluid. In particular, at $Y=-L/2$ the external convection coefficient will be considered as uniform with the value h_1 and the fluid in the stream-I will be assumed to have a uniform reference temperature T_1. At $Y=L/2$ the external convection coefficient will be considered as uniform with the value h_2 and the fluid in the stream-II will be supposed to have a uniform reference temperature $T_2 \geq T_1$. Therefore, the boundary conditions on the temperature field can be expressed as

$$-k\frac{\partial T_1}{\partial Y}\bigg|_{Y=-\frac{L}{2}} = h_1\left[T_1 - T(X,-L/2)\right] \tag{13}$$

$$-k\frac{\partial T_2}{\partial Y}\bigg|_{Y=\frac{L}{2}} = h_2\left[T(X,L/2) - T_2\right] \tag{14}$$

On account of Eqns. (8) and (11), Eqns. (13) and (14) can be rewritten as

$$\left[\frac{d^3U_1}{dY^3} - \frac{\sigma_e B_0^2}{\mu}\frac{dU_1}{dY}\right]\bigg|_{Y=-\frac{L}{2}} = \frac{\rho\beta g\, h_1}{\mu k}\left[T_1 - T(X,-L/2)\right] \tag{15}$$

$$\left[\frac{d^3U_2}{dY^3} - \frac{\sigma_e B_0^2}{\mu}\frac{dU_2}{dY}\right]\bigg|_{Y=\frac{L}{2}} = \frac{\rho\beta g\, h_2}{\mu k}\left[T(X,L/2) - T_2\right] \tag{16}$$

Thus on account of Eqns. (7) and (10), there exists a constant A such that $\frac{dP}{dX} = A$ (17)

For the model defined as above in the formulation, the energy balance equation in the presence of viscous dissipation can be written as

Stream-I

$$\frac{d^2T_1}{dY^2} = -\frac{\mu}{k}\left(\frac{dU_1}{dY}\right)^2 - \frac{\sigma_e\left(E_0 + U_1B_0\right)^2}{k} \tag{18}$$

Stream-II

$$\frac{d^2T_2}{dY^2} = -\frac{\mu}{k}\left(\frac{dU_2}{dY}\right)^2 - \frac{\sigma_e\left(E_0 + U_2B_0\right)^2}{k} \tag{19}$$

$$\frac{d^4U_1}{dY^4} = \frac{\rho\beta g}{k}\left(\frac{dU_1}{dY}\right)^2 + \frac{\rho\beta g\sigma_e}{\mu k}\left(E_0 + B_0U_1\right)^2 + \frac{\sigma_e B_0^2}{\mu}\frac{d^2U_1}{dY^2} \tag{20}$$

Stream-II

$$\frac{d^4U_2}{dY^4} = \frac{\rho\beta g}{k}\left(\frac{dU_2}{dY}\right)^2 + \frac{\rho\beta g\sigma_e}{\mu k}\left(E_0 + B_0U_2\right)^2 + \frac{\sigma_e B_0^2}{\mu}\frac{d^2U_2}{dY^2} \tag{21}$$

The boundary conditions for the velocity in stream-I and stream-II are

$$U_1\left(-\frac{L}{2}\right) = U_2\left(\frac{L}{2}\right) = 0 \tag{22}$$

and together with Eqns. (15) and (16), which on account of (5) and (6) can be written as

$$\left[\frac{d^3U_1}{dY^3} - \frac{h_1}{k}\frac{d^2U_1}{dY^2} - \frac{\sigma_e B_0^2}{\mu}\frac{dU_1}{dY} + \frac{h_1\sigma_e B_0}{k\mu}\left(E_0 + U_1B_0\right)\right]_{Y=-L/2} = -\frac{Ah_1}{\mu k} - \frac{\rho\beta g\, h_1}{\mu k}\left(T_0 - T_1\right)$$

(23)

$$\left[\frac{d^3U_2}{dY^3} + \frac{h_2}{k}\frac{d^2U_2}{dY^2} - \frac{\sigma_e B_0^2}{\mu}\frac{dU_2}{dY} - \frac{h_2\sigma_e B_0}{k\mu}\left(E_0 + U_2B_0\right)\right]_{Y=L/2} = \frac{Ah_2}{\mu k} - \frac{\rho\beta g\, h_2}{\mu k}\left(T_2 - T_0\right)$$

(24)

In addition we consider the continuity of temperature $\left(T_1 = T_2\right)$ and continuity of heat flux $\left(\frac{dT_1}{dY} = \frac{dT_2}{dY}\right)$ at the baffle position. Applying these conditions in Eqns. (5), (6), (8) and (11) yield

$$\frac{d^2U_1}{dY^2} - \frac{\sigma_e B_0^{\ 2}}{\mu} U_1 = \frac{d^2U_2}{dY^2} - \frac{\sigma_e B_0^{\ 2}}{\mu} U_2 \tag{25}$$

$$\frac{d^3U_1}{dY^3} - \frac{\sigma_e B_0^{\ 2} D^2}{\mu} \frac{dU_1}{dY} = \frac{d^3U_2}{dY^3} - \frac{\sigma_e B_0^{\ 2} D^2}{\mu} \frac{dU_2}{dY} \tag{26}$$

Equations (20) and (21) which determine the velocity distribution along with the boundary conditions at the channel walls and at the baffle positions are non-dimensionalised using the following dimensionless parameters:

$$u_i = \frac{U_i}{U_0},\ y = \frac{Y}{D}\ ,\ y^* = \frac{Y^*}{D^*},\ Gr = \frac{g\beta\,\Delta T\, D^3}{\upsilon^2},\ \mathrm{Re} = \frac{U_0 D}{\upsilon},\ Br = \frac{U_0^2\mu}{k\Delta T}\ ,\ \nu = \frac{\mu}{\rho}\ ,\ \Lambda = \frac{Gr}{\mathrm{Re}}\ ,$$

$$\theta = \frac{T_i - T_0}{\Delta T},\ Bi_1 = \frac{h_1 D}{k},\ Bi_2 = \frac{h_2 D}{k},\ S = \frac{Bi_1 Bi_2}{Bi_1 Bi_2 + 2Bi_1 + 2Bi_2}\ ,\ M^2 = \frac{\sigma_e B_0^{\ 2} D^2}{\mu},\ E = \frac{E_0}{U_0 B_0}$$

(27)

In the above equation $D = 2L$ is the hydraulic diameter and the reference velocity and the reference temperature are given by

$$U_0 = -\frac{AD^2}{48\mu}\ ,\quad T_0 = \frac{T_1 + T_2}{2} + S\left(\frac{1}{Bi_1} - \frac{1}{Bi_2}\right)\left(T_2 - T_1\right) \tag{28}$$

As in Barletta [1998] the reference temperature ΔT is given either by

$$\Delta T = T_2 - T_1 \quad \text{if} \quad T_1 < T_2 \tag{29}$$

or by

$$\Delta T = \frac{\mu^2}{\rho^2 C_p D^2} \quad \text{if} \quad T_1 = T_2 \tag{30}$$

Therefore the value of the dimensionless parameter R_T can be either 0 or 1. For asymmetric heating $\left(T_1 < T_2\right)$ R_T is equal to one and will be zero for symmetric heating $\left(T_1 = T_2\right)$. Equation (17) implies that A can be either positive or negative. If $A < 0$, then U_0, Re and Λ are positive, and the flow is upward. On the other hand, if $A > 0$, the flow is downward, so that U_0, Re and Λ are negative.

By employing the dimensionless quantities as defined in Eqn. (27), Eqns. (20)-(26) can be written as
Stream-I

$$\frac{d^4u_1}{dy^4} = \Lambda\, Br\left(\frac{du_1}{dy}\right)^2 + M^2 Br\Lambda\left(E + u_1\right)^2 + M^2\frac{d^2u_1}{dy^2} \tag{31}$$

Stream-II

$$\frac{d^4u_2}{dy^4} = \Lambda\, Br\left(\frac{du_2}{dy}\right)^2 + M^2 Br\Lambda\left(E + u_2\right)^2 + M^2\frac{d^2u_2}{dy^2} \tag{32}$$

and nondimensionalized boundary equations for both stream-I and stream-II are

$$u_1\left(\frac{-1}{4}\right) = 0\,,\ \left[\frac{d^2u_1}{dy^2} - \frac{1}{Bi_1}\frac{d^3u_1}{dy^3} + \frac{M^2}{Bi_1}\frac{du_1}{dy} - M^2 u_1\right]_{at\,y=-\frac{1}{4}} = -48 + M^2E + \frac{R_T\Lambda S}{2}\left(1 + \frac{4}{Bi_1}\right)$$

$$u_2\left(\frac{1}{4}\right) = 0\,,\ \left[\frac{d^2u_2}{dy^2} + \frac{1}{Bi_2}\frac{d^3u_2}{dy^3} - \frac{M^2}{Bi_2}\frac{du_2}{dy} - M^2 u_2\right]_{at\,y=\frac{1}{4}} = -48 + M^2E - \frac{R_T\Lambda\, S}{2}\left(1 + \frac{4}{Bi_2}\right)$$

$$u_1 = 0\,;\quad u_2 = 0\,;\quad \frac{d^2u_1}{dy^2} - M^2u_1 = \frac{d^2u_2}{dy^2} - M^2u_2\,;\quad \frac{d^3u_1}{dy^3} - M^2\frac{du_1}{dy} = \frac{d^3u_2}{dy^3} - M^2\frac{du_2}{dy}\,;\ \text{at}\quad y = y^* \tag{33}$$

III. SOLUTIONS

PERTURBATION METHOD

Equations (31) and (32) are coupled nonlinear differential equations and approximate solutions can be found by using the regular perturbation method. The perturbation parameter ε ($\varepsilon = \Lambda Br$) is small and hence regular perturbation method can be strongly justified. Adopting this technique, solutions for velocity are assumed in the form

$$u_i(y) = u_{i0}(y) + \varepsilon u_{i1}(y) + \varepsilon^2 u_{i2}(y) + ... \tag{34}$$

Substituting Eqn. (34) into Eqns. (31), (32) and (33) and equating the coefficients of like power of ε to zero and one, we obtain the zeroth and first order equations.

Stream-I, Zeroth order equations

$$\frac{d^4u_{10}}{dy^4} = M^2\frac{d^2u_{10}}{dy^2} \tag{35}$$

Stream-I, First order equations

$$\frac{d^4u_{11}}{dy^4} = \left(\frac{du_{10}}{dy}\right)^2 + M^2E^2 + M^2u_{10}{}^2 + 2M^2Eu_{10} + M^2\frac{d^2u_{11}}{dy^2} \tag{36}$$

Stream-II, Zeroth order equations

$$\frac{d^4u_{20}}{dy^4} = M^2\frac{d^2u_{20}}{dy^2} \tag{37}$$

Stream-II, First order equations

$$\frac{d^4u_{21}}{dy^4} = \left(\frac{du_{20}}{dy}\right)^2 + M^2E^2 + M^2u_{20}{}^2 + 2M^2Eu_{20} + M^2\frac{d^2u_{21}}{dy^2} \tag{38}$$

The corresponding boundary conditions for both stream-I and stream-II are

Zeroth order

$$u_{10} = 0\,,\quad -\frac{1}{Bi_1}\frac{d^3u_{10}}{dy^3} + \frac{d^2u_{10}}{dy^2} + \frac{M^2}{Bi_1}\frac{du_{10}}{dy} - M^2u_{10} = Bc_1 \ \text{ at }\ y = -\frac{1}{4}$$

$$u_{20} = 0\,,\quad \frac{1}{Bi_2}\frac{d^3u_{20}}{dy^3} + \frac{d^2u_{20}}{dy^2} - \frac{M^2}{Bi_2}\frac{du_{20}}{dy} - M^2u_{20} = Bc_2 \quad \text{at }\ y = \frac{1}{4}$$

$$u_{10} = 0\,, u_{20} = 0 \quad \text{at } y = y^*$$

$$\frac{d^2u_{10}}{dy^2} - M^2u_{10} = \frac{d^2u_{20}}{dy^2} - M^2u_{20}\,,\ \frac{d^3u_{10}}{dy^3} - M^2\frac{du_{10}}{dy} = \frac{d^3u_{20}}{dy^3} - M^2\frac{du_{20}}{dy} \quad \text{at } y = y^* \tag{39}$$

where $Bc_1 = -48 + M^2E + \Lambda\frac{R_T}{2}S\left(1 + \frac{4}{Bi_1}\right)$ and $Bc_2 = -48 + M^2E - \Lambda\frac{R_T}{2}S\left(1 + \frac{4}{Bi_2}\right)$.

First order

$$u_{11} = 0\,,\qquad -\frac{1}{Bi_1}\frac{d^3u_{10}}{dy^3} + \frac{d^2u_{10}}{dy^2} + \frac{M^2}{Bi_1}\frac{du_{10}}{dy} - M^2u_{10} = 0 \quad \text{at}\quad y = -\frac{1}{4}$$

$$u_{21}=0,\quad \frac{1}{Bi_2}\frac{d^3u_{20}}{dy^3}+\frac{d^2u_{20}}{dy^2}-\frac{M^2}{Bi_2}\frac{du_{20}}{dy}-M^2u_{20}=0 \quad \text{at} \quad y=\frac{1}{4}$$

$$u_{11}=0, u_{21}=0 \quad \text{at } y=y^*$$

$$\frac{d^2u_{11}}{dy^2}-M^2u_{11}=\frac{d^2u_{21}}{dy^2}-M^2u_{21},\ \frac{d^3u_{11}}{dy^3}-M^2\frac{du_{11}}{dy}=\frac{d^3u_{21}}{dy^3}-M^2\frac{du_{21}}{dy} \quad \text{at } y=y^* \tag{40}$$

The solutions of zeroth and first order Eqns. (35)-(38) using the boundary conditions (39) and (40) can be obtained are

Stream-I, Zeroth order

$$u_{10}=d_1+d_2\,y+d_3\,Cosh[m\,y]+d_4\,Sinh[m\,y] \tag{41}$$

Stream-I, First order

$$u_{11}=c_1+c_2\,y+c_3\,Cosh[m\,y]+c_4\,Sinh[m\,y]+p_{10}\,Cosh[2\,m\,y]+$$
$$p_{11}\,Sinh[2\,m\,y]+p_{12}\,y^2\,Cosh[m\,y]+p_{13}\,y^2\,Sinh[m\,y]+p_{14}\,y\,Cosh[m\,y]+ \tag{42}$$
$$p_{15}\,y\,Sinh[m\,y]+p_{16}\,y^4+p_{17}\,y^3+p_{18}\,y^2$$

Stream-II, Zeroth order

$$u_{20}=d_5+d_6\,y+d_7\,Cosh[my]+d_8\,Sinh[m\,y] \tag{43}$$

Stream-II, First order

$$u_{21}=c_5+c_6\,y+c_7\,Cosh[m\,y]+c_8\,Sinh[m\,y]+R_{10}\,Cosh[2\,m\,y]$$
$$+R_{11}\,Sinh[2\,m\,y]+R_{12}\,y^2\,Cosh[m\,y]+R_{13}\,y^2\,Sinh[m\,y]+R_{14}\,y\,Cosh[m\,y] \tag{44}$$
$$+R_{15}\,y\,Sinh[m\,y]+R_{16}\,y^4+R_{17}\,y^3+R_{18}\,y^2$$

Using the solutions of velocity ($u_1=u_{10}+\varepsilon u_{11}$ *and* $u_2=u_{20}+\varepsilon u_{21}$) fields, the solutions of temperature field obtained by Eqns. (5) and (6) are

Stream-I

$$\theta_1=-\frac{1}{\Lambda}\left(48-M^2\left(E+u_1\right)+\frac{d^2u_1}{dy^2}\right) \tag{45}$$

Stream-II

$$\theta_2=-\frac{1}{\Lambda}\left(48-M^2\left(E+u_2\right)+\frac{d^2u_2}{dy^2}\right) \tag{46}$$

The constants appeared in all the above equations

$$d_2=\left(\frac{(Bc_1-Bc_2)2s}{M^2}\right),\quad d_1=\left(\frac{d_2}{Bi_1}+\frac{d_2}{4}-\frac{Bc_1}{M^2}\right),$$

$$d_3=\frac{-\left(d_1Sinh[\frac{M}{4}]+Sinh[MYS]\right)-d_2\left(YS\,Sinh[\frac{M}{4}]-\frac{Sinh[MYS]}{4}\right)}{Sinh[MYS]Cosh[\frac{M}{4}]+Sinh[\frac{M}{4}]Cosh[MYS]},$$

$$d_4=\frac{d_1\left(Cosh[MYS]-Cosh[\frac{M}{4}]\right)-d_2\left(YS\,Cosh[\frac{M}{4}]+\frac{Cosh[MYS]}{4}\right)}{Sinh[MYS]Cosh[\frac{M}{4}]+Sinh[\frac{M}{4}]Cosh[MYS]},$$

$d_5 = d_1,\ d_6 = d_2,\ and\ m = M, r = R.$

$$d_7 = \frac{d_5(Sinh[\frac{M}{4}] - Sinh[MYS]) + d_6(YS\,Sinh[\frac{M}{4}] - \frac{Sinh[MYS]}{4})}{Sinh[MYS]Cosh[\frac{M}{4}] - Sinh[\frac{M}{4}]Cosh[MYS]},$$

$$d_8 = \frac{d_5(Cosh[\frac{M}{4}] - Cosh[MYS]) + d_6(YS\,Cosh[\frac{M}{4}] - \frac{Cosh[MYS]}{4})}{Sinh[\frac{M}{4}]Cosh[MYS] - Sinh[MYS]Cosh[\frac{M}{4}]}$$

$$p_1 = M^2(d_3{}^2 + d_4{}^2),\ p_2 = 2M^2 d_3 d_4,\ p_3 = 2Md_2d_4 + 2M^2 d_3 d_1 + 2M^2 d_3 E_1,$$

$$p_4 = 2Md_2d_3 + 2M^2 d_4 d_1 + 2M^2 d_4 E_1,\ p_5 = 2M^2 d_3 d_2,\ p_6 = 2M^2 d_4 d_2,$$

$$p_7 = M^2 d_2{}^2,\ p_8 = 2M^2 d_1 d_2 + 2M^2 E_1 d_2,\ p_9 = d_2{}^2 + M^2 E_1{}^2 + M^2 d_1{}^2 + 2M^2 d_1 E_1,$$

$$r_1 = M^2(d_7{}^2 + d_8{}^2),\ r_2 = 2M^2 d_7 d_8,\ r_3 = 2Md_6d_8 + 2M^2 d_5 d_7 + 2M^2 d_7 E_1,$$

$$r_4 = 2Md_6d_7 + 2M^2 d_8 d_5 + 2M^2 d_8 E_1,\ r_5 = 2M^2 d_7 d_6,\ r_6 = 2M^2 d_8 d_6,$$

$$r_7 = M^2 d_6{}^2,\ r_8 = 2M^2 d_5 d_6 + 2M^2 E_1 d_6,\ r_9 = d_6{}^2 + M^2 E_1{}^2 + M^2 d_5{}^2 + 2M^2 d_5 E_1,$$

$$p_{10} = \frac{p_1}{12M^4},\ p_{11} = \frac{p_2}{12M^4},\ p_{12} = \frac{p_6}{4M^3},\ p_{13} = \frac{p_5}{4M^3},\ p_{14} = \frac{p_4}{2M^3} - \frac{5p_5}{4M^4},$$

$$p_{15} = \frac{p_3}{2M^3} - \frac{5p_6}{4M^4},\ p_{16} = \frac{-p_7}{12M^2},\ p_{17} = \frac{-p_8}{6M^2},\ p_{18} = -\left(\frac{p_7}{M^4} + \frac{p_9}{2M^2}\right),$$

$$r_{10} = \frac{r_1}{12M^4},\ r_{11} = \frac{r_2}{12M^4},\ r_{12} = \frac{r_6}{4M^3},\ r_{13} = \frac{r_5}{4M^3},\ r_{14} = \frac{r_4}{2M^3} - \frac{5r_5}{4M^4},$$

$$r_{15} = \frac{r_3}{2M^3} - \frac{5r_6}{4M^4},\ r_{16} = \frac{-r_7}{12M^2},\ r_{17} = \frac{-r_8}{6M^2},\ r_{18} = -\left(\frac{r_7}{M^4} + \frac{r_9}{2M^2}\right),$$

$$k_1 = p_{10}Cosh[\frac{M}{2}] - p_{11}Sinh[\frac{M}{2}] + \frac{p_{12}}{16}Cosh[\frac{M}{4}] - \frac{p_{13}}{16}Sinh[\frac{M}{4}] - \frac{p_{14}}{4}Cosh[\frac{M}{4}]$$
$$+ \frac{p_{15}}{4}Sinh[\frac{M}{4}] + \frac{p_{16}}{256} - \frac{p_{17}}{64} + \frac{p_{18}}{16},$$

$$k_2 = r_{10}Cosh[\frac{M}{2}] + r_{11}Sinh[\frac{M}{2}] + \frac{r_{12}}{16}Cosh[\frac{M}{4}] + \frac{r_{13}}{16}Sinh[\frac{M}{4}] + \frac{r_{14}}{4}Cosh[\frac{M}{4}]$$
$$+ \frac{r_{15}}{4}Sinh[\frac{M}{4}] + \frac{r_{16}}{256} + \frac{r_{17}}{64} + \frac{r_{18}}{16},$$

$$k_3 = -2Mp_{10}Sinh[\frac{M}{2}] + 2p_{11}MCosh[\frac{M}{2}] + \left(\frac{(-M)p_{12}}{16} + \frac{p_{13}}{2} + \frac{Mp_{14}}{4} - p_{15}\right)$$
$$Sinh[\frac{M}{4}] + \left(\frac{Mp_{13}}{16} + \frac{-p_{12}}{2} - \frac{Mp_{15}}{4} + p_{14}\right)Cosh[\frac{M}{4}] - \frac{p_{16}}{16} + \frac{3p_{17}}{16} - \frac{p_{18}}{2},$$

$$k_4 = 2Mr_{10}Sinh[\frac{M}{2}] + 2r_{11}MCosh[\frac{M}{2}] + \left(\frac{Mr_{12}}{16} + \frac{r_{13}}{2} + \frac{Mr_{14}}{4} + r_{15}\right)$$
$$Sinh[\frac{M}{4}] + \left(\frac{Mr_{13}}{16} + \frac{r_{12}}{2} + \frac{Mr_{15}}{4} + r_{14}\right)Cosh[\frac{M}{4}] + \frac{r_{16}}{16} + \frac{3r_{17}}{16} + \frac{r_{18}}{2},$$

$$k_5 = 4M^2 p_{10} Cosh[\frac{M}{2}] - 4M^2 p_{11} Sinh[\frac{M}{2}] + \left(M^2 \frac{p_{12}}{16} + 2p_{12} - Mp_{13} - \frac{M^2 p_{14}}{4} + 2Mp_{15} \right) Cosh[\frac{M}{4}]$$
$$\left(-p_{13}\frac{M^2}{16} + p_{12}M - 2p_{13} - 2Mp_{14} + p_{15}\frac{M^2}{4} \right) Sinh[\frac{M}{4}] + p_{16}\frac{3}{4} - \frac{3}{2}p_{17} + 2p_{18},$$
$$k_6 = 4M^2 r_{10} Cosh[\frac{M}{2}] + 4M^2 r_{11} Sinh[\frac{M}{2}] + \left(M^2 \frac{r_{12}}{16} + 2r_{12} + Mr_{13} + \frac{M^2 r_{14}}{4} + 2Mr_{15} \right) Cosh[\frac{M}{4}]$$
$$\left(r_{13}\frac{M^2}{16} + r_{12}M + 2r_{13} + 2Mr_{14} + r_{15}\frac{M^2}{4} \right) Sinh[\frac{M}{4}] + r_{16}\frac{3}{4} + \frac{3}{2}r_{17} + 2r_{18},$$
$$k_8 = 8M^3 r_{11} Cosh[\frac{M}{2}] + 8M^3 r_{10} Sinh[\frac{M}{2}] + \left(6Mr_{13} + 3M^2 r_{14} + r_{12}\frac{6M^2}{4} + M^3\frac{r_{15}}{4} + r_{13}\frac{M^3}{16} \right)$$
$$Cosh[\frac{M}{4}] + \left(6Mr_{12} + 3M^2 r_{15} + r_{13}\frac{6M^2}{4} + M^2\frac{r_{14}}{4} + r_{12}\frac{M^3}{16} \right) Sinh[\frac{M}{4}] + 6r_{16} + 6r_{17},$$
$$k_7 = 8M^3 p_{11} Cosh[\frac{M}{2}] - 8M^3 p_{10} Sinh[\frac{M}{2}] + \left(6Mp_{13} + 3M^2 p_{14} - p_{12}\frac{6M^2}{4} - M^3\frac{p_{15}}{4} + p_{13}\frac{M^3}{16} \right)$$
$$Cosh[\frac{M}{4}] + \left(-6Mp_{12} - 3M^2 p_{15} + p_{13}\frac{6M^2}{4} + M^3\frac{p_{14}}{4} - p_{12}\frac{M^3}{16} \right) Sinh[\frac{M}{4}] - 6p_{16} + 6p_{17},$$
$$x_1 = \frac{k_5 - \frac{k_7}{Bi_1} + \frac{M^2 k_3}{Bi_1} - M^2 k_1}{M^2}, \ \ x_2 = \frac{1}{M^2}\left(k_6 + \frac{k_8}{Bi_2} - \frac{M^2 k_4}{Bi_2} - M^2 k_2 \right),$$

$$k_9 = p_{10} Cosh[2MYS] + p_{11} Sinh[2MYS] + p_{12} YS^2 Cos[MYS] + p_{13} YS^2 Sinh[MYS] + p_{14} YS\, Cos[MYS]$$
$$+ p_{15} YS\, Sinh[MYS] + p_{16} YS^4 + p_{17} YS^3 + p_{18} YS^2,$$
$$k_{10} = r_{10} Cosh[2MYS] + r_{11} Sinh[2MYS] + r_{12} YS^2 Cos[MYS] + r_{13} YS^2 Sinh[MYS] + r_{14} YS\, Cos[MYS]$$
$$+ r_{15} YS\, Sinh[MYS] + r_{16} YS^4 + r_{17} YS^3 + r_{18} YS^2,$$
$$k_{11} = 2Mp_{10} Sinh[2MYS] + 2p_{11} MCosh[2MYS] + p_{12} MYS^2 Sinh[MYS] + p_{13} YS^2 MCosh[MYS]$$
$$+ (2p_{12} + Mp_{15}) YS\, Cosh[MYS] + (2p_{13} + Mp_{14}) YS\ \ Sinh[MYS] + p_{14} Cosh[MYS] + p_{15} Sinh[MYS]$$
$$+ 4p_{16} YS^3 + 3p_{17} YS^2 + 2p_{18} YS,$$
$$k_{12} = 4M^2 p_{10} Cosh[2MYS] + 4M^2 p_{11} Sinh[2MYS] + M^2 p_{12} YS^2 Cosh[MYS] + M^2 p_{13} YS^2 Sinh[MYS]$$
$$+ 4Mp_{12} YS\, Sinh[MYS] + 2p_{12} Cosh[MYS] + 4Mp_{13} YS\, Cosh[MYS] + 2p_{13} Sinh[MYS]$$
$$+ M^2 p_{14} YS\, Cosh[MYS] + 2p_{14} M\ Sinh[MYS] + M^2 p_{15} YS\, Sinh[MYS] + 2p_{15} M\ Cosh[MYS]$$
$$+ 12p_{16} YS^2 + 6p_{17} YS + 2p_{18},$$

$$k_{13} = \left(6Mp_{12} + 3M^2 p_{15}\right) Sinh[MYS] + \left(6Mp_{13} + 3M^2 p_{14}\right) Cosh[MYS]$$
$$+\left(6M^2 p_{13} + M^3 p_{14}\right) YS\, Sinh[MYS] + \left(6M^2 p_{12} + M^3 p_{15}\right) YS\, Cosh[MYS]$$
$$+8p_{10} M^3 Sinh[2MYS] + 8p_{11} M^3 Cosh[2MYS] + p_{12} M^3 YS^2 Sinh[MYS]$$
$$+p_{13} M^3 YS^2 Cosh[MYS] + 24p_{16} YS + 6p_{17},$$
$$k_{14} = 2Mr_{10} Sinh[2MYS] + 2r_{11} MCosh[2MYS] + r_{12} M\, YS^2 Sinh[MYS] + r_{13} YS^2 MCosh[MYS]$$
$$+(2r_{12} + Mr_{15}) YS\, Cosh[MYS] + (2r_{13} + Mr_{14}) YS\ \ Sinh[MYS] + r_{14} Cosh[MYS] + r_{15} Sinh[MYS]$$
$$+4r_{16} YS^3 + 3r_{17} YS^2 + 2r_{18} YS,$$
$$k_{15} = 4M^2 r_{10} Cosh[2MYS] + 4M^2 r_{11} Sinh[2MYS] + M^2 r_{12} YS^2 Cosh[MYS] + M^2 r_{13} YS^2 Sinh[MYS]$$
$$+4Mr_{12} YS\, Sinh[MYS] + 2r_{12} Cosh[MYS] + 4Mr_{13} YS\, Cosh[MYS] + 2r_{13} Sinh[MYS]$$
$$+M^2 r_{14} YS\, Cosh[MYS] + 2r_{14} M\, Sinh[MYS] + M^2 r_{15} YS\, Sinh[MYS] + 2r_{15} M\, Cosh[MYS]$$
$$+12r_{16} YS^2 + 6r_{17} YS + 2r_{18},$$
$$k_{16} = \left(6Mr_{12} + 3M^2 r_{15}\right) Sinh[MYS] + \left(6Mr_{13} + 3M^2 r_{14}\right) Cosh[MYS] + \left(6M^2 r_{13} + M^3 r_{14}\right) YS\, Sinh[MYS]$$
$$+\left(6M^2 r_{12} + M^3 r_{15}\right) YS\, Cosh[MYS] + 8r_{10} M^3 Sinh[2MYS] + 8r_{11} M^3 Cosh[2MYS] + r_{12} M^3 YS^2 Sinh[MYS]$$
$$+r_{13} M^3 YS^2 Cosh[MYS] + 24r_{16} YS + 6r_{17},$$
$$g_1 = \frac{k_{16} - k_{13}}{M^2} + k_{11} - k_{14},\ g_2 = \frac{k_{12} - k_{15} - M^2 k_{10} + M^2 g_1 YS}{M^2},$$
$$c_2 = -2s\left(x_1 - g_2 - x_2 + g_1\left(\frac{1}{4} + \frac{1}{Bi_2}\right)\right),\ c_1 = x_1 + c_2\left(\frac{1}{4} + \frac{1}{Bi_1}\right),$$
$$c_6 = g_1 + c_2,\ c_5 = x_2 - c_6\left(\frac{1}{4} + \frac{1}{Bi_2}\right),$$
$$c_3 = \frac{c_1\left(Sinh[MYS] + Sinh[\frac{M}{4}]\right) + c_2\left(YS\, Sinh[\frac{M}{4}] - \frac{Sinh[MYS]}{4}\right) + k_1 Sinh[MYS] + k_9 Sinh[\frac{M}{4}]}{-\left(Cosh[\frac{M}{4}]Sinh[MYS] + Cosh[MYS]Sinh[\frac{M}{4}]\right)},$$
$$c_4 = \frac{c_1\left(Cosh[MYS] - Cosh[\frac{M}{4}]\right) - c_2\left(YS\, Cosh[\frac{M}{4}] + \frac{Cosh[MYS]}{4}\right) + k_1 Cosh[MYS] - k_9 Cosh[\frac{M}{4}]}{\left(Sinh[\frac{M}{4}]Cosh[MYS] + Sinh[MYS]Cosh[\frac{M}{4}]\right)}$$
$$c_7 = \frac{c_5\left(Sinh[MYS] - Sinh[\frac{M}{4}]\right) + c_6\left(\left(\frac{Sinh[MYS]}{4}\right) - YS\, Sinh[\frac{M}{4}]\right) + k_2 Sinh[MYS] + k_{10} Sinh[\frac{M}{4}]}{-\left(Cosh[\frac{M}{4}]Sinh[MYS] - Cosh[MYS]Sinh[\frac{M}{4}]\right)},$$

$$c_8 = \frac{c_5\left(Cosh[MYS]-Cosh[\frac{M}{4}]\right)+c_6\left(\frac{Cosh[MYS]}{4}-YS\,Cosh[\frac{M}{4}]\right)+k_2Cosh[MYS]-k_{10}Cosh[\frac{M}{4}]}{-\left(Sinh[\frac{M}{4}]Cosh[MYS]-Sinh[MYS]Cosh[\frac{M}{4}]\right)}.$$

DIFFERENTIAL TRANSFORM METHOD

The differential transform method was first introduced by Zhou[1986] who solved linear and nonlinear differential equations in electric circuit analysis. This method constructs a semi-analytical numerical technique uses Taylor series expansion for the solution of linear or nonlinear partial or ordinary differential equations in the form of a polynomial.

The differential transform of the k[th] derivative of the function $f(y)$ is defined as follows:

$$F[k]=\frac{1}{k!}\left[\frac{d^k f(y)}{dy^k}\right]_{y=y_0} \tag{47}$$

where $f(y)$ is the original function and $F[k]$ is the transformed function. Differential inverse transform of $F[k]$ is defined as follows:

$$f(y)=\sum_{k=0}^{\infty} F[k](y-y_0)^k \tag{48}$$

which implies that the concept of differential transform method is derived from Taylor series expansion, although this method is not able to evaluate the derivatives symbolically.

The Differential Transform Method (DTM) has been applied to obtain the solutions of equations (31) and (32).

$$\begin{aligned}&(k+1)(k+2)(k+3)(k+4)u_1[k+4]=M^2(k+1)(k+2)u_1[k+2]+\\&\varepsilon\sum_{r=0}^{k}(k-r+1)u_1[k-r+1](r+1)u_1[r+1]+\varepsilon M^2E^2\,\delta[k]+\\&\varepsilon M^2\sum_{s=0}^{r}u_1[k-r]\,u_1[r]+2\varepsilon M^2\,E\,u_1[k]\end{aligned} \tag{49}$$

$$\begin{aligned}&(k+1)(k+2)(k+3)(k+4)u_2[k+4]=M^2(k+1)(k+2)u_2[k+2]+\\&\varepsilon\sum_{r=0}^{k}(k-r+1)u_2[k-r+1](r+1)u_2[r+1]+\varepsilon M^2E^2\,\delta[k]+\\&\varepsilon M^2\sum_{s=0}^{r}u_2[k-r]\,u_2[r]+2\varepsilon M^2\,E\,u_2[k]\end{aligned} \tag{50}$$

where $\delta[k]=\begin{cases}1\ if\ k=0\\ 0\ \ if\ k>0\end{cases}$

The initial conditions are as follows

$$u_1[0]=c1,u_1[1]=c2\,,u_1[2]=c3\,,\ u_1[3]=c4\,,\ u_2[0]=d1,u_2[1]=d2\,,\ u_2[2]=d3,u_2[3]=d4 \tag{51}$$

The numerical values of the constants a1, a2, a3, a4, b1, b2, b3, b4 can be find by using the transformed equations with boundary and interface conditions. The system of equations leads to the solutions by substituting obtained numerical values of constants.

NUSSELT NUMBER

The non-dimensional heat transfer coefficient known as Nusselt number is given by

$$Nu=\frac{d\theta}{dy} \tag{52}$$

The Nusselt number at both left wall and right wall are calculated numerically for different values of several governing parameters and are given in table 1.

IV. RESULTS AND DISCUSSION

In this chapter, the flow behavior of electrically conducting fluid in a vertical double passage channel is analyzed by using Robin boundary conditions for equal and unequal Biot numbers with symmetric and asymmetric wall temperatures. The flow governing equations are coupled and nonlinear, and are solved by using perturbation method and differential transform method. The obtained values are agreed very well with small values of perturbation parameter ε $(\varepsilon < 1)$. The solutions of velocity field and temperature field are depicted graphically for non-dimensional parameters such as mixed convection parameter Λ , perturbation parameter ε , electric field load parameter E and Hartman number M for equal and unequal Biot numbers. A uniform magnetic field B_0 is applied normal to the walls and the uniform electric field E_0 is applied perpendicular to the walls. For electric load parameter, the case $E = 0$ implies short circuit and the case $E \neq 0$ implies open circuit. The values of non-dimensional parameters such as mixed convection parameter Λ and perturbation parameter ε are fixed as $\Lambda = 500$ and $\varepsilon = 0.1$ for upward flow $(\Lambda > 0)$, and $\Lambda = -500$ and $\varepsilon = -0.1$ for downward flow $(\Lambda < 0)$ (shown by dotted lines) in all figures.

The effect of applied electric field on the velocity field and the temperature field for equal Biot numbers at different positions of baffle is shown in Fig. 2 and Fig. 3. The effect of a negative E is to add the flow while the effect of positive E is to oppose the flow when compared to the case for $E = 0$. The flow velocity decreases in both the streams for both upward $(\Lambda > 0)$ and downward $(\Lambda < 0)$ flow as E increases. The optimal value of the velocity is in Stream-II for upward flow (Λ =500) when the baffle is placed near the cold wall, in Stream-I for downward flow (Λ = - 500) when the baffle is placed near the hot wall and the velocity profiles are symmetric when the baffle is placed in the center of the channel. It is seen from the Fig. 3 that the temperature profiles slightly vary (almost equal) for the effect of electric field load parameter E .

The effect of Hartman number on the velocity and the temperature fields for equal Biot numbers at different positions of the baffle is shown in Fig. 4 and Fig. 5. The velocity decreases in both the streams for both upward $(\Lambda > 0)$ and downward $(\Lambda < 0)$ flow with open circuit as M increases due to retarding effect of the Lorentz force. The optimal value of the velocity is in Stream-II for upward flow (Λ =500) when the baffle is placed near the left wall, in Stream-I for downward flow (Λ = - 500) when the baffle is placed near the right wall and the velocity profiles are symmetric when the baffle is placed in the center of the channel. It is observed from the Fig. 5 that the temperature profiles slightly vary (almost equal) for the effect of Hartman number M .

Figures 6 and 7 shows the effect of perturbation parameter ε on the velocity field and the temperature field for open circuit at different positions of the baffle. It is observed from the close view of figures 6 and 7 that the both velocity and temperature are increasing functions of ε and this is due to the fact that the increase in ε for Λ =500 implies increase of the Brinkman number and it is a known fact that as the Brinkman number increases the viscous dissipation also increases which helps to enhance the buoyancy force and hence the flow is promoted, and also seen that both velocity and temperature profiles are close when ε =0.1 and are distinguished when ε =5 and this difference is visible in stream-II when the baffle is placed near the right plate.

The velocity and the temperature fields of the flow are analyzed for the effect of mixed convection parameter Λ is shown in Fig. 8 and Fig. 9. It is observed clearly from the Fig. 8 that the velocity decreases in Stream-I and increases in Stream-II for both upward $(\Lambda > 0)$ and downward $(\Lambda < 0)$ flows at all baffle positions. The velocity reaches its optimum value in Stream-II for upward $(\Lambda > 0)$ flow when baffle is positioned near the left plate, in Stream-I for downward $(\Lambda < 0)$ flow when baffle is positioned near the right wall and the velocity profiles are symmetric when the baffle is placed in middle of the channel. It is seen form the Fig. 9 that the temperature profiles are not much affected by the effect of mixed convection parameter Λ . There is a small decrease in temperature profiles for both upward $(\Lambda > 0)$ flow and downward $(\Lambda < 0)$ flow is observed in a close view of the Fig. 9.

The velocity and temperature fields are depicted pictorially for different values of electric field load parameter E for unequal Biot numbers at different positions of the baffle in Fig. 10 and Fig. 11. From the Fig. 10 it can say that the velocity decreases in both the streams for both upward $(\Lambda > 0)$ flow and downward $(\Lambda < 0)$ flow at all positions of the baffle. The velocity profile reaches its optimum value in a wide passage when the baffle is positioned either near the left wall or near the right wall. The temperature field varied very slightly for unequal Biot numbers by the effect of E is observed from the Fig. 11. The velocity decreases in both the streams for both upward $(\Lambda > 0)$ flow and downward $(\Lambda < 0)$ flow for both equal and unequal Biot numbers with symmetric wall temperatures at all the baffle positions is observed form the figures 12 and 14. The temperature profiles for upward $(\Lambda > 0)$ flow are exactly opposite to the profiles for downward $(\Lambda < 0)$ flow for both equal and unequal Biot numbers with symmetric wall temperatures at different baffle positions is seen from the figures 13 and 15.

The rate of heat transfer for different values of non-dimensional parameters such as mixed convection parameter, Biot numbers, Hartman number and electric field load parameter is tabulated in Table-1. The rate of heat transfer decreases at left wall and increases at right wall as Λ increases when the baffle is placed near the left or right wall, and there is no significant variation at left wall and increases at right wall when the baffle is placed in center of the channel. The Nusselt number increases at both the walls as increases in Biot numbers for all positions of the baffle. The rate of heat transfer increases at both left wall and right wall when the baffle is placed near the left wall and it decreases at both the walls when the baffle is positioned near the right wall as M increases or E increases is observed from the table. And it varies not significantly when the baffle is placed in middle of the channel.

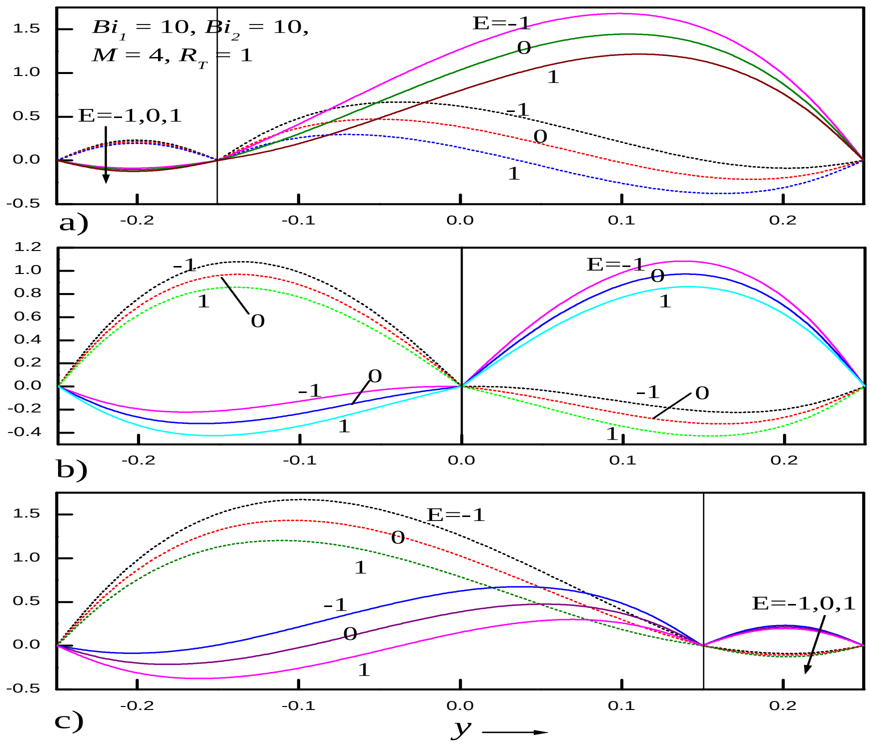

Figure 2 Velocity profiles for different values of E

a) y^*=-0.15 b) y^*=0 c) y^*=0.15

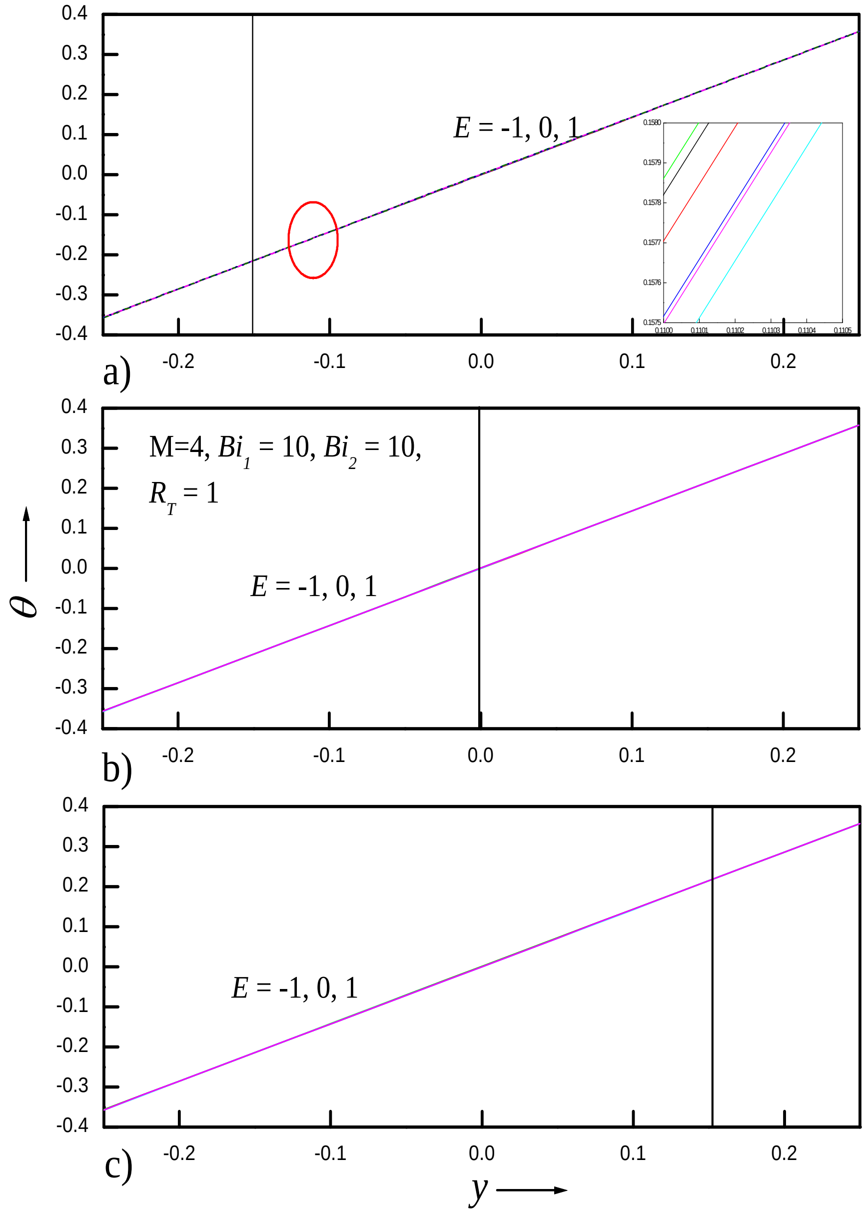

Figure 3 Temperature profiles for different values of E

a) $y^* = -0.15$ b) $y^* = 0$ c) $y^* = 0.15$

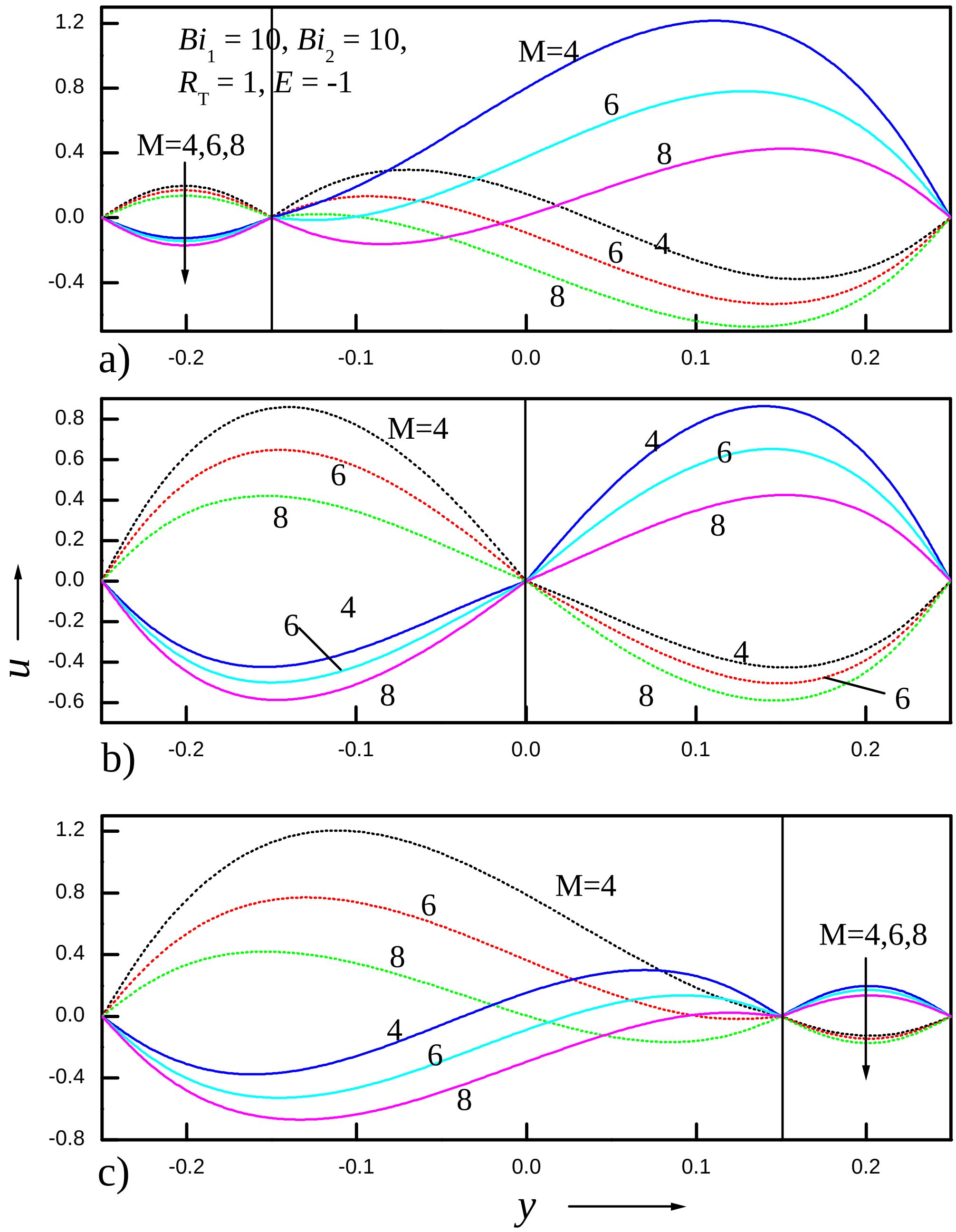

Figure 4 Velocity profiles for different values of M

a)y^*=-0.15 b)y^*=0 c)y^*=0.15

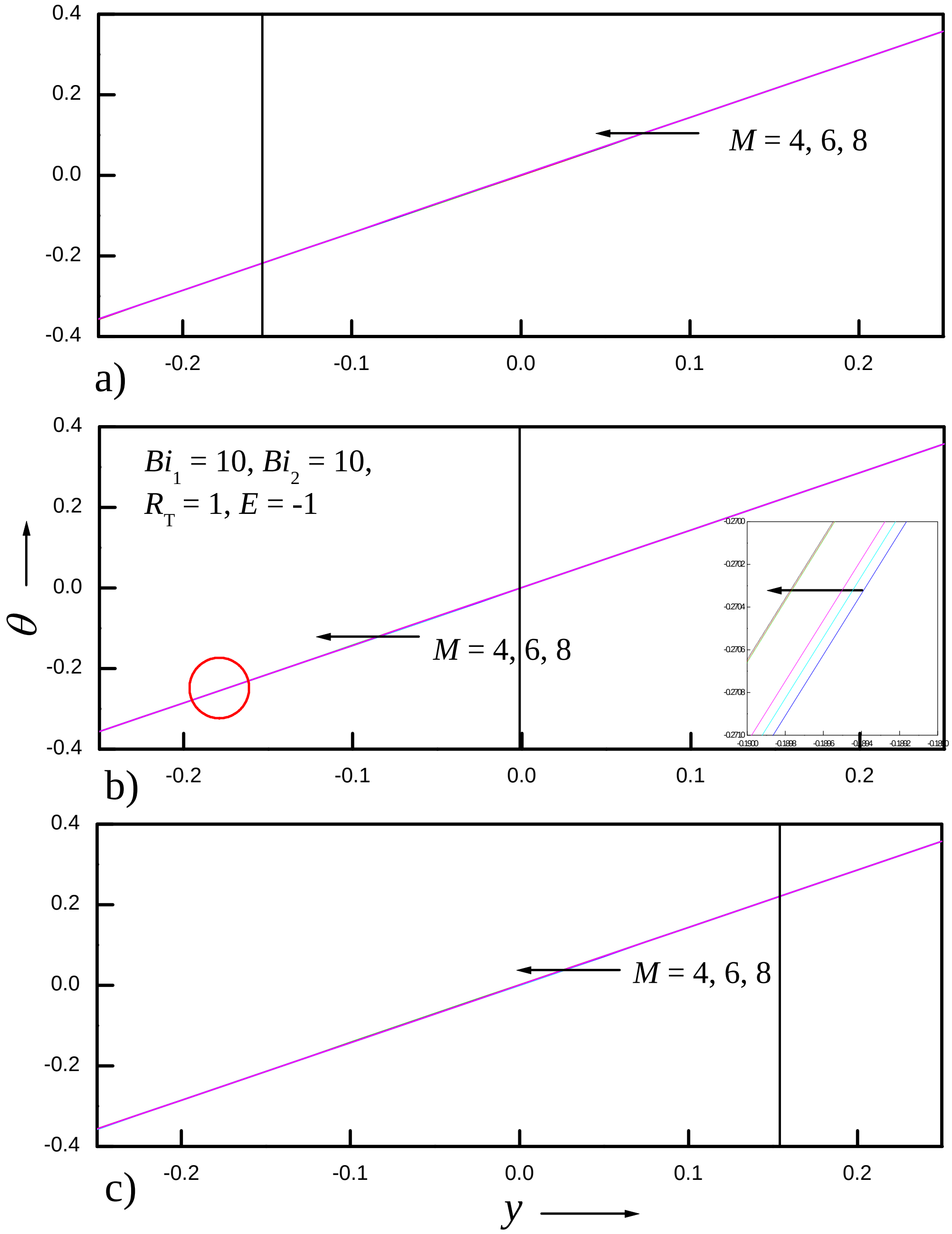

Figure 5 Temperature profiles for different values of *M*

a)y^*=-0.15 b)y^*=0 c)y^*=0.15

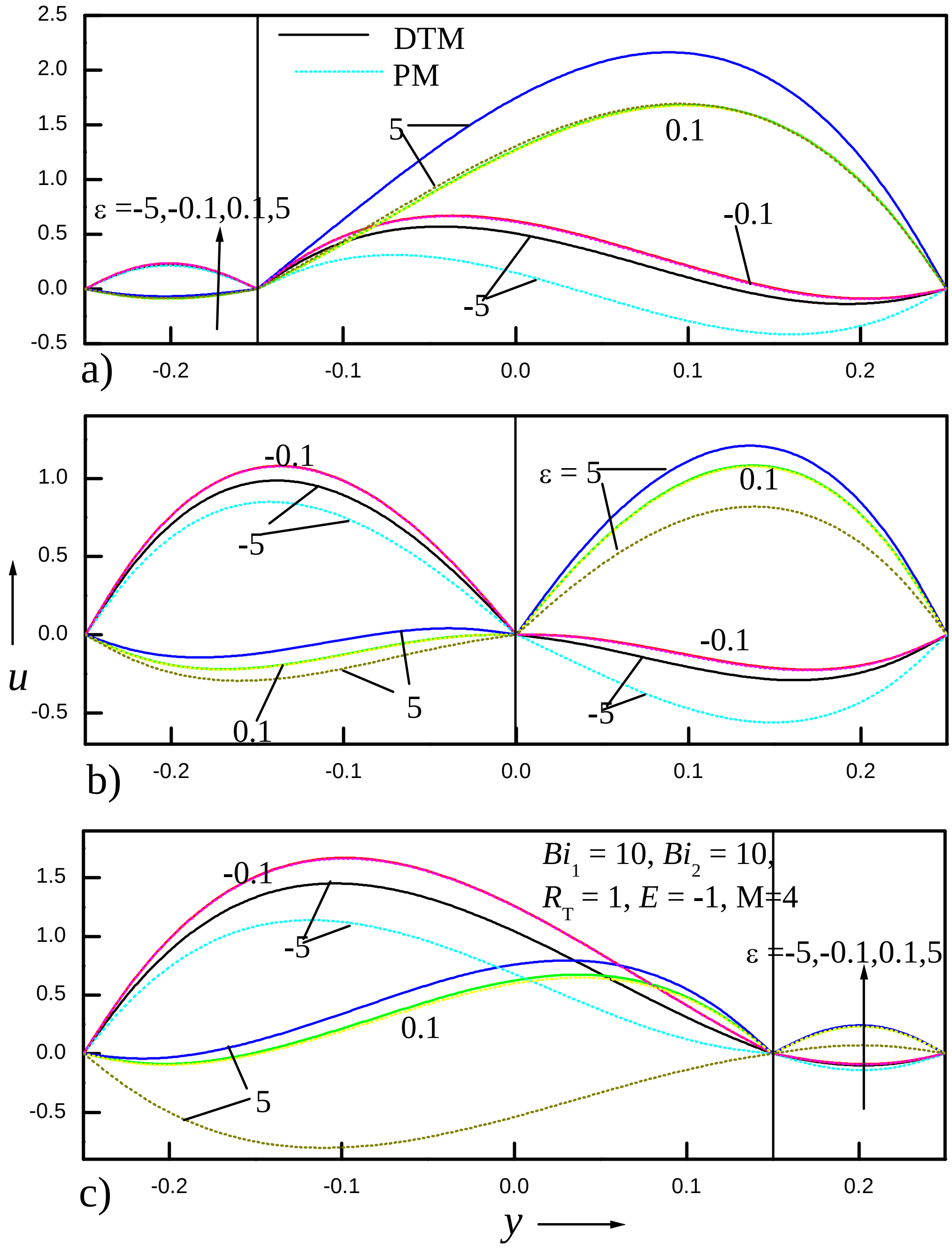

Figure 6 Velocity profiles for different values of ε

a)y^*=-0.15 b)y^*=0 c)y^*=0.15

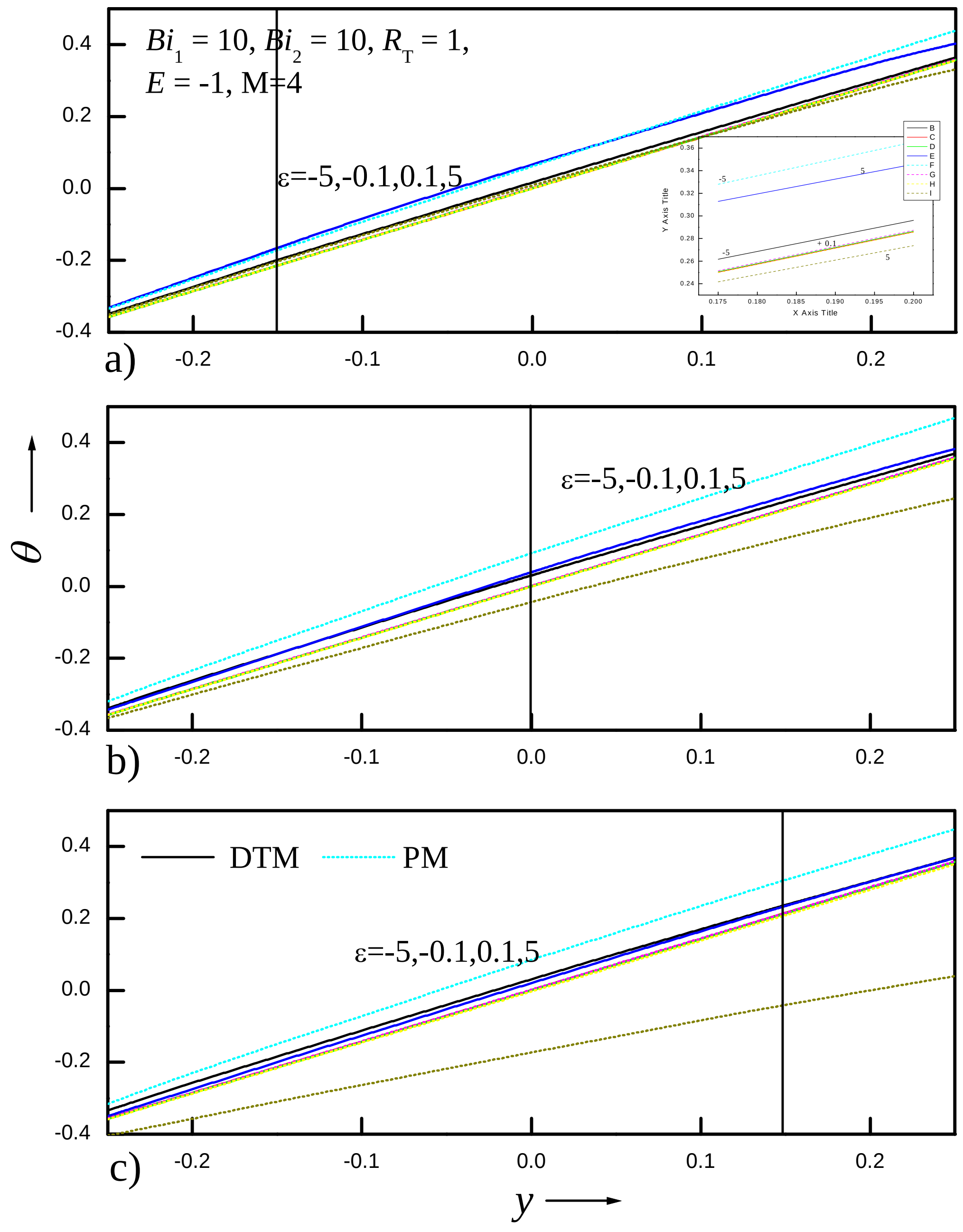

Figure 7 Temperature profiles for different values of ε

a)y^*=-0.15 b)y^*=0 c)y^*=0.15

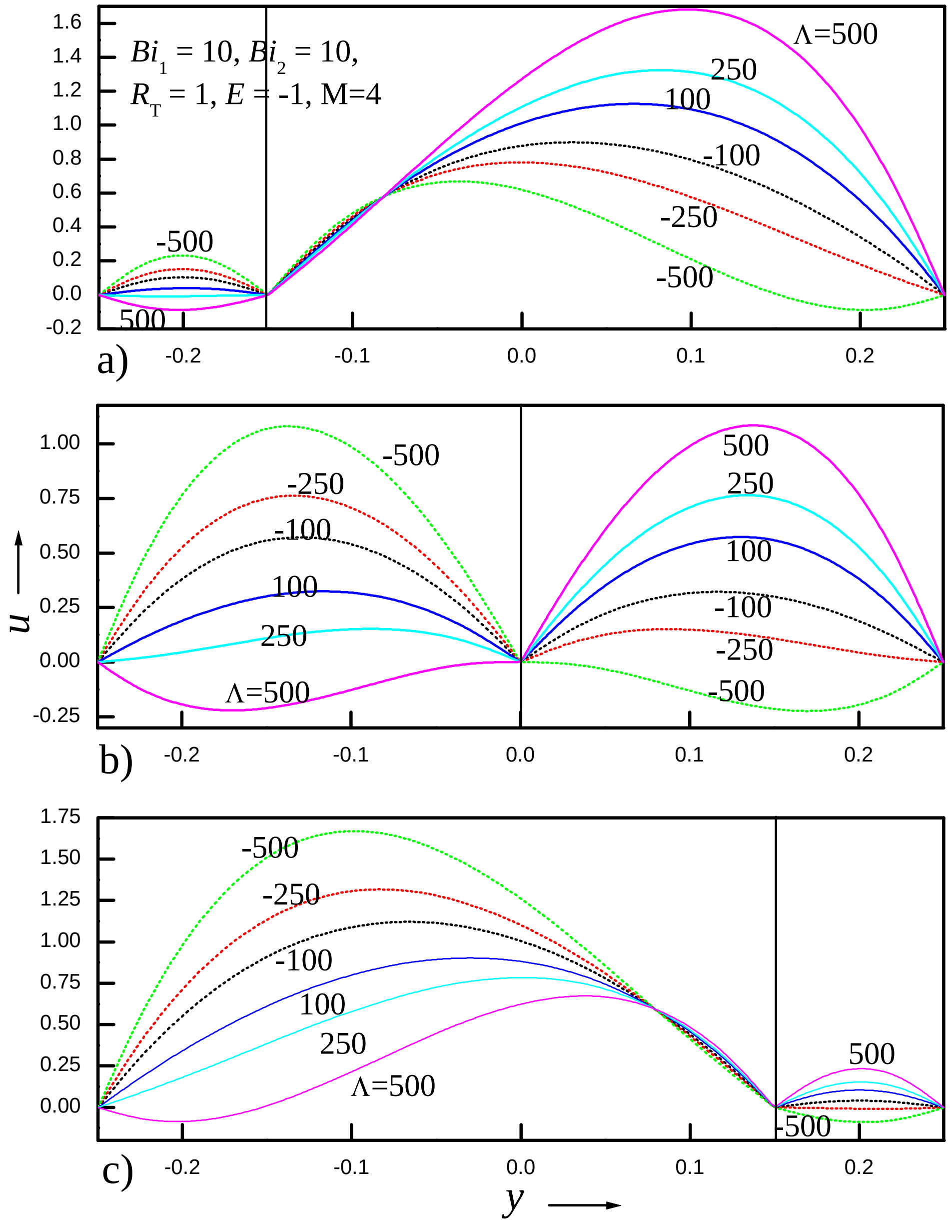

Figure 8 Velocity profiles for different values of *Λ*

a)y* =-0.15 b)y* =0 c)y* =0.15

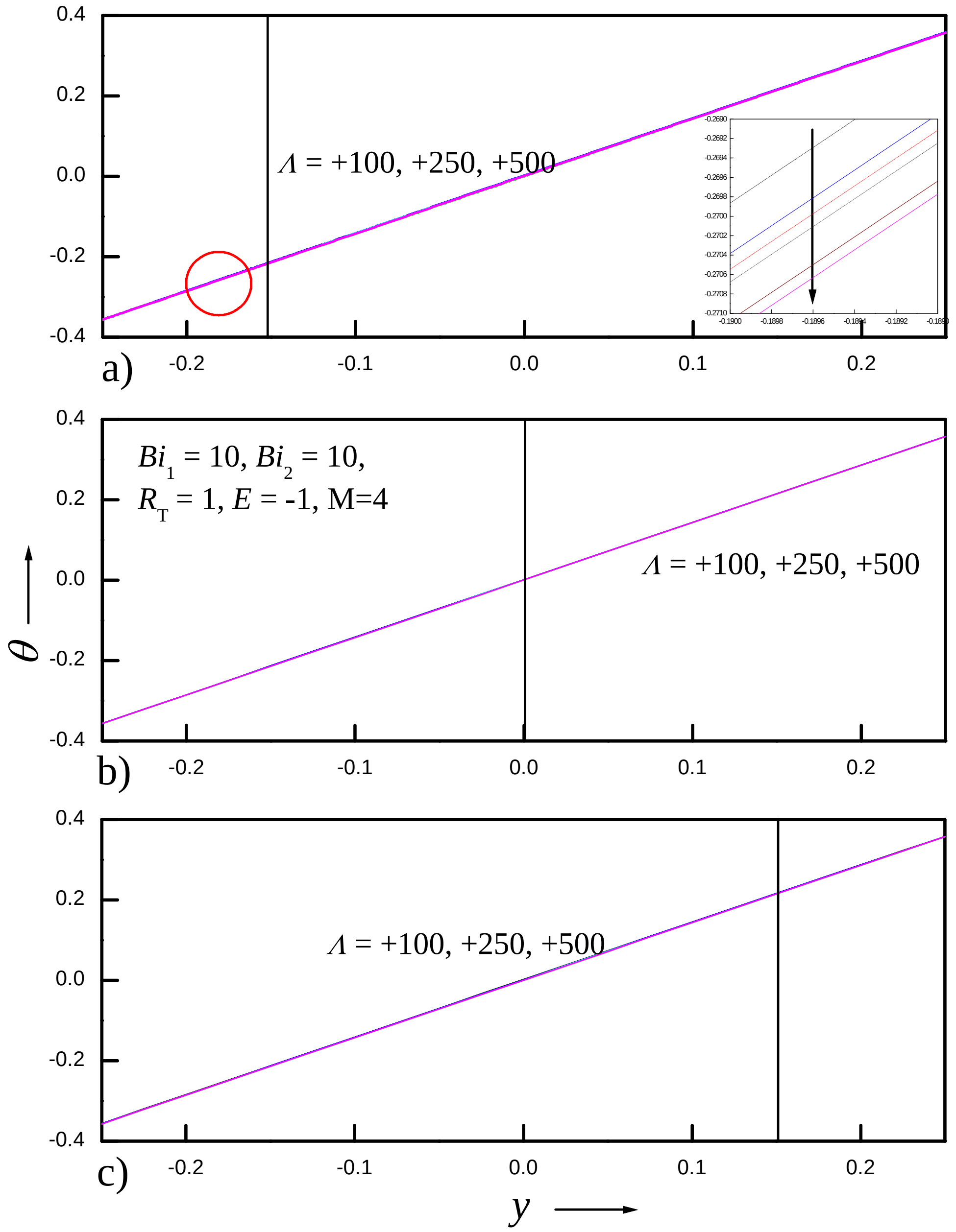

Figure 9 Temperature profiles for different values of Λ

a)y^*=-0.15 b)y^*=0 c)y^*=0.15

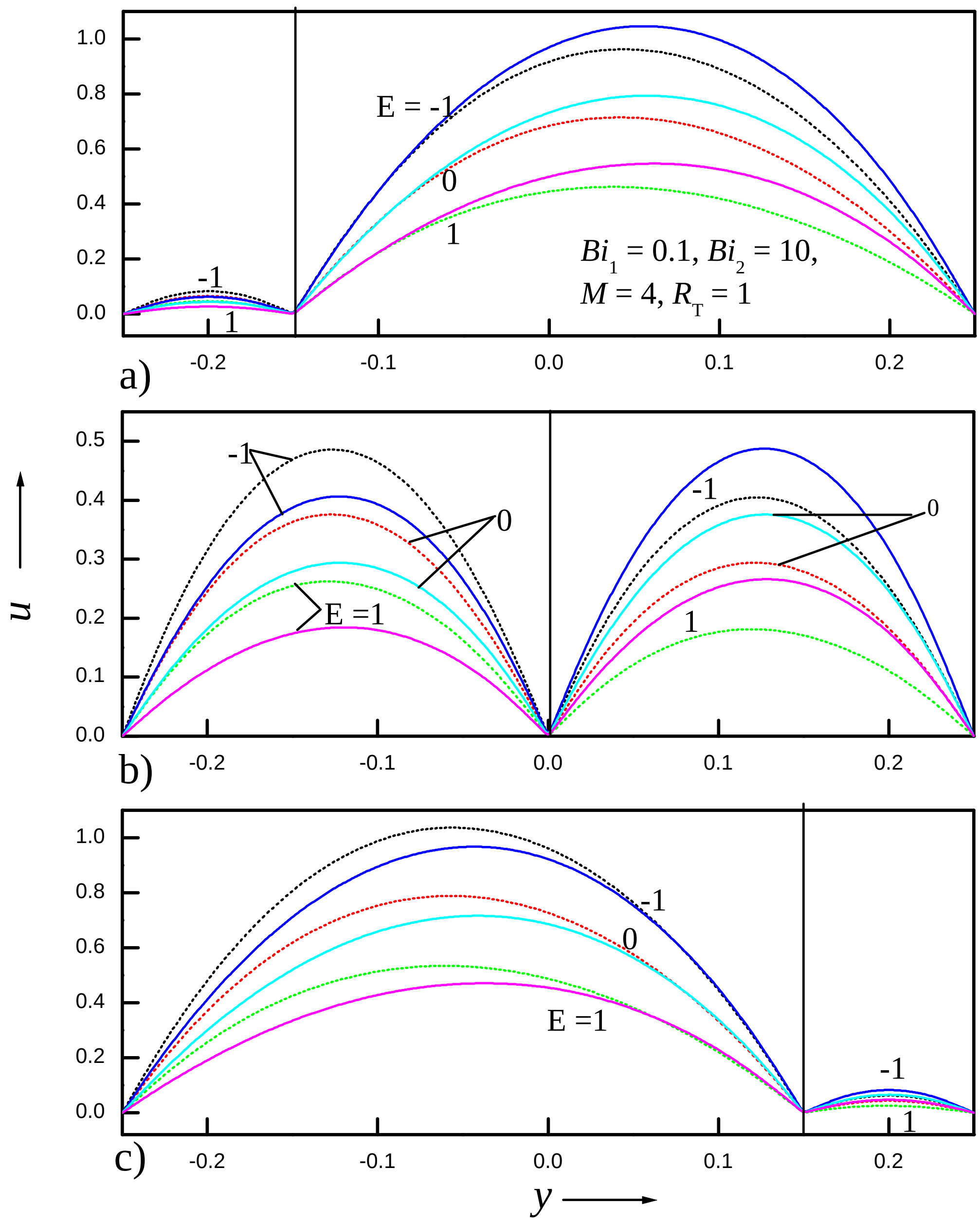

Figure 10 Velocity profiles for different values of E

a) y^*=-0.15 b) y^*=0 c) y^*=0.15

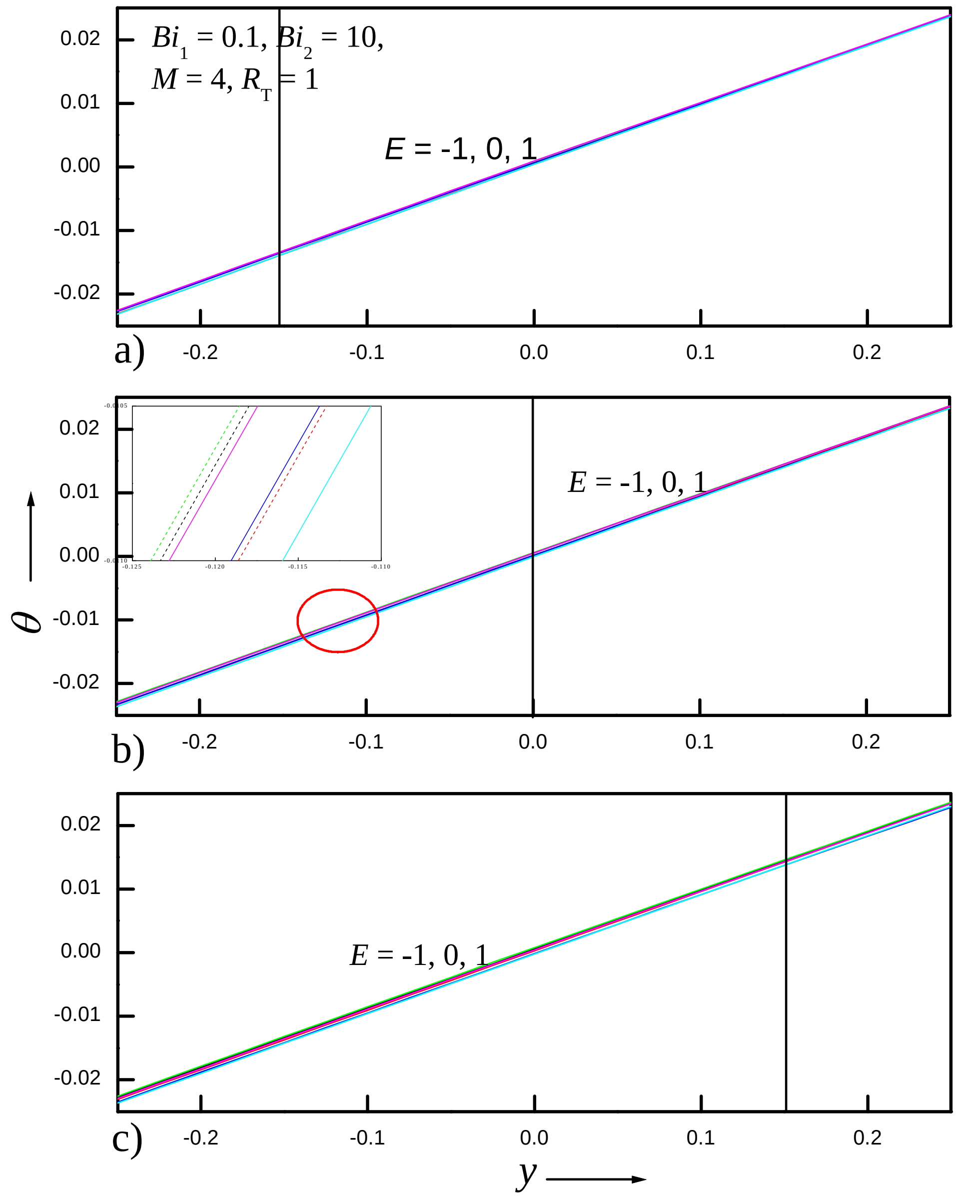

Figure 11 Temperature profiles for different values of *E*

a) y^*=-0.15 b) y^*=0 c) y^*=0.15

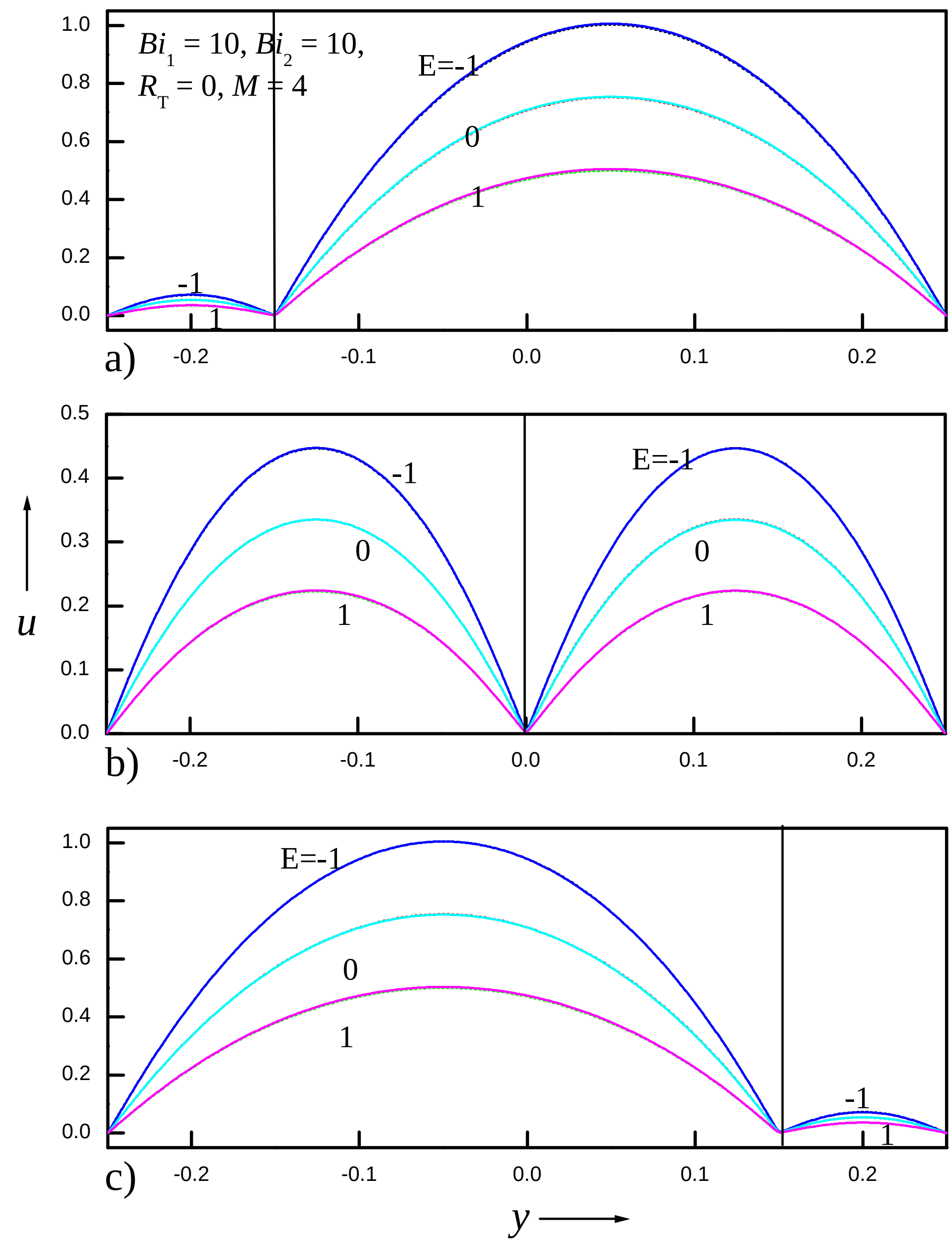

Figure 12 Velocity profiles for different values of E

a)y*=-0.15 b)y*=0 c)y*=0.15

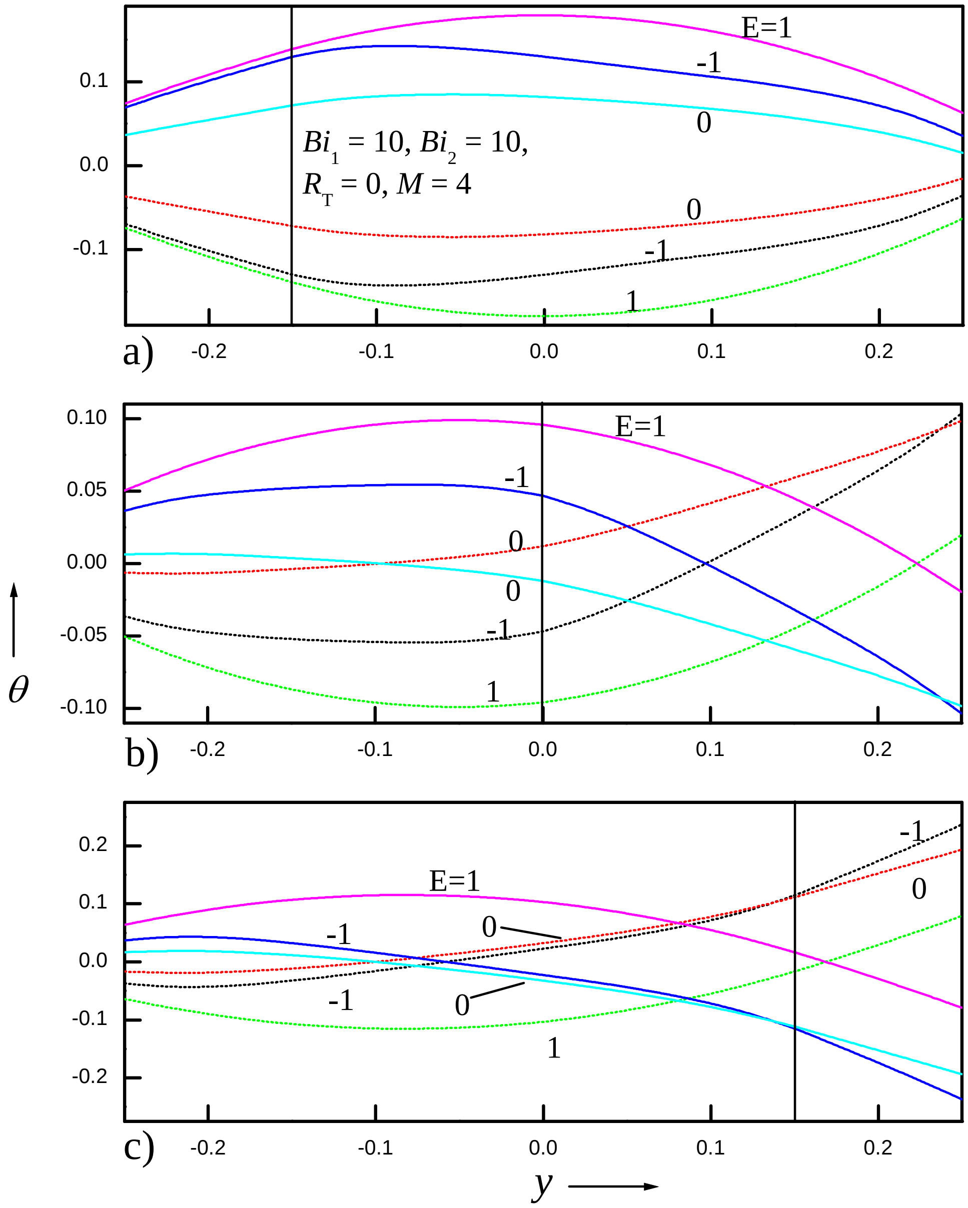

Figure 13 Temperature profiles for different values of E

a)y∗=-0.15 b)y∗=0 c)y∗=0.15

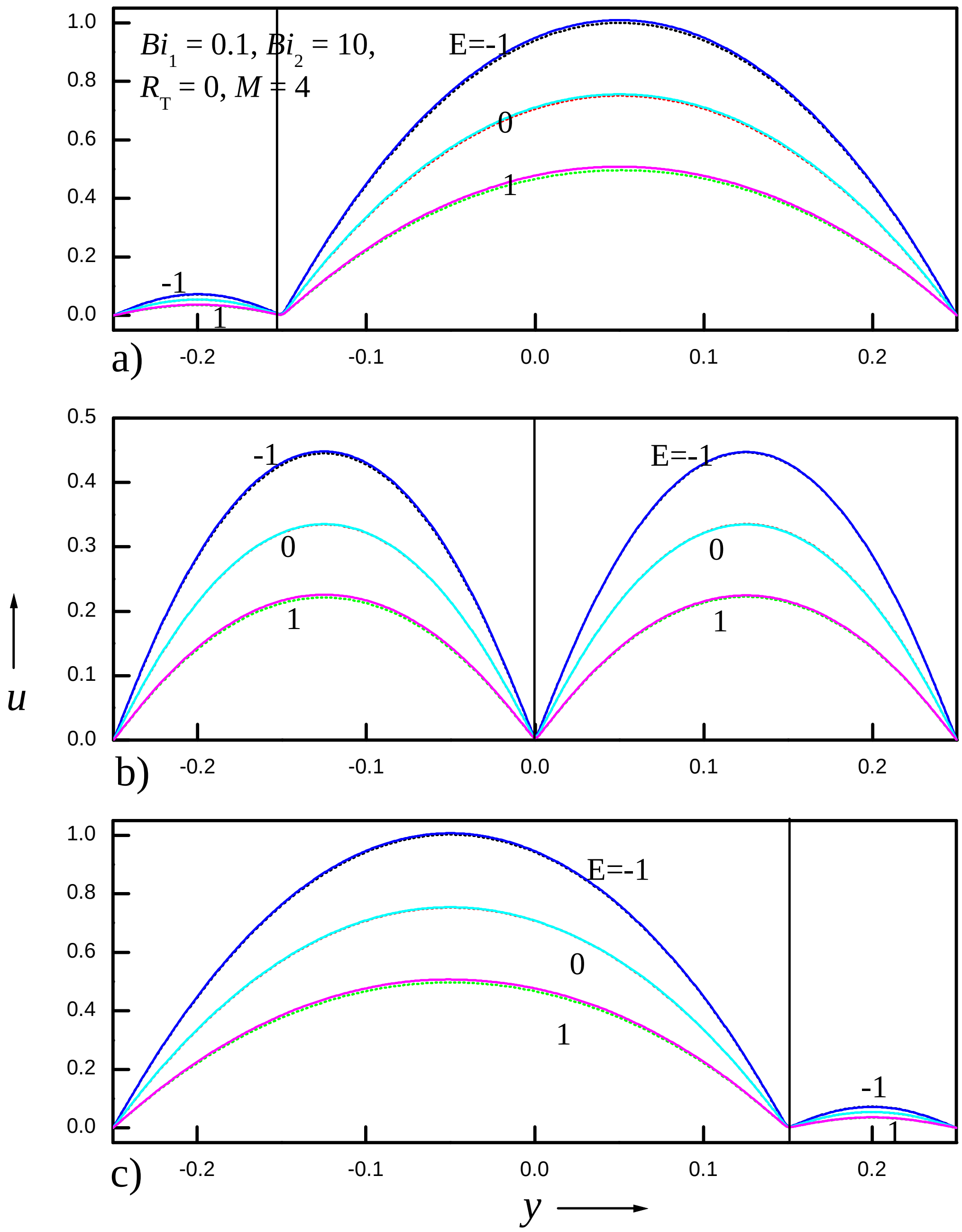

Figure 14 Velocity profiles for different values of E

a)y∗=-0.15 b)y∗=0 c)y∗=0.15

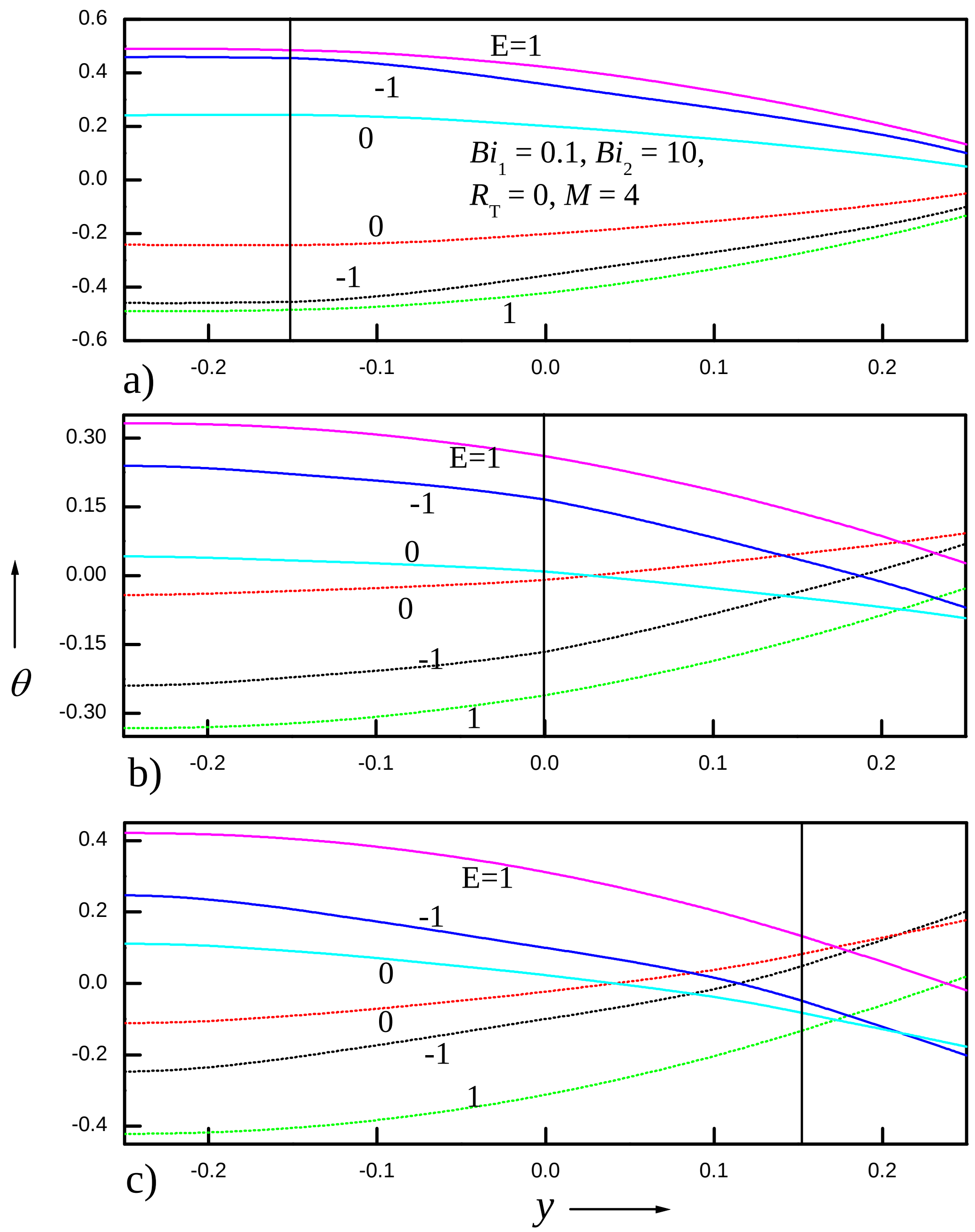

Figure 15 Temperature profiles for different values of *E*

a)y*=-0.15 b)y*=0 c)y*=0.15

Table 1

	$\frac{d\theta}{dy}$ **at y= -0.25**			$\frac{d\theta}{dy}$ **at y=0.25**		
	y^*= - 0.15	y^*=0	y^*=0.15	y^*= - 0.15	y^*=0	y^*=0.15
Λ						
100	1.43421	1.42789	1.42319	1.41410	1.41423	1.40845
250	1.43049	1.42632	1.42054	1.41966	1.41916	1.41509
500	1.42986	1.42661	1.41914	1.42101	1.41999	1.41566
Bi1						
1	0.62542	0.62458	0.62352	0.62034	0.62124	0.62078
5	1.25099	1.24661	1.24298	1.24313	1.24271	1.23983
10	1.42986	1.42661	1.41914	1.42101	1.41999	1.41566
Bi2						
1	0.624279	0.61759	0.60698	0.61919	0.61424	0.60425
5	1.25054	1.24563	1.23522	1.24268	1.23997	1.23206
10	1.42986	1.42661	1.41914	1.42101	1.41999	1.41566
M						
4	1.42986	1.42661	1.41914	1.42101	1.41999	1.41566
6	1.43068	1.42687	1.41504	1.42180	1.41919	1.41037
8	1.43122	1.42643	1.40639	1.42203	1.41747	1.40041
E						
-1	1.42986	1.42661	1.41914	1.42101	1.41999	1.41566
0	1.43056	1.42771	1.42101	1.42308	1.43029	1.41891
1	1.43266	1.42257	1.42422	1.42381	1.42367	1.42075

REFERENCES

[1]. Blum, E.L., Zaks, M.V., Ivanov, U.I. and Mikhailov, Yu.A., Heat and Mass transfer in the Presence of an Electromagnetic Field, (in Russian), Zinatne, Riga, 1967.
[2]. Hunt, J. C. R. and Holroyd, R.J., ‘‘Applications of laboratory and theoretical MHD duct flow studies in fusion reactor technology,’’ Technical Report CLM-R169, Culham Laboratory, 1977.
[3]. Davidson, P.A., ‘‘Pressure forces in the MHD propulsion of submersibles,’’ (in Russian), Magnetohydrodynamics, Vol. 29, No. 3, pp. 49–58, 1993.
[4]. Tsinober, A., ‘‘MHD-drag reduction, in: viscous drag reduction in boundary layers,’’ AIAA Progress in Aeronautics and Astronautics, Vol. 123, eds. Bushnell, D.M. and Hefner, J.N., pp. 327–349, 1990.
[5]. Boricˇic´, Z., Nikodijevic´, D., Obrovic´, B. and Stamenkovic´, Zˇ ., ‘‘Universal equations of unsteadytwo-dimensional MHD boundary layer whose temperature varies with time,’’ Theoretical and Applied Mechanics, Vol. 36, No. 2, pp. 119–135, 2009.
[6]. Obrovic´, B., Nikodijevic´, D. and Savic´, S., ‘‘Boundary layer of dissociated gas on bodies of revolution of a porous contour,’’ Strojnisˇki vestnik - Journal of Mechanical Engineering, Vol. 55, No. 4, pp. 244–253, 2009.
[7]. Xu, H., Liao, S.J. and Pop, I., ‘‘Series solutions of unsteady three-dimensional MHD flow and heat transfer in the boundary layer over an impulsively stretching plate,’ European Journal of Mechanics B/Fluids, Vol. 26, pp. 15–27, 2007.
[8]. Kessel, C.E., Meade, D. and Jardin, S.C., ‘‘Physics basis and simulation of burning plasma physics for the fusion ignition research experiment (FIRE),’’ Fusion Engineering and Design, Vol. 63–64, pp. 559–567, 2002.
[9]. Barletta A, Celli M, Magyari E, Zanchini E (2008) Buoyant MHD flows in a vertical channel: the levitation regime. Heat Mass Transf 44:1005–1013.
[10]. Umavathi, J.C., A note on magnetoconvection in a vertical enclosure, Int. J. Nonlinear Mechanics, vol. 31, pp. 371-376, 1996.
[11]. Malashetty, M.S., Umavathi, J.C., Two-phase magnetohydrodynamic flow and heat transfer in an inclined channel, Int. J. Multiphase Flow, vol. 23, pp. 545-560, 1997.
[12]. Malashetty, M.S., Umavathi, J.C., Prathap Kumar, J., Two-fluid magnetoconvection flow in an inclined channel, Int. J. Trans. Phenomena, vol. 3, pp. 73-84, 2000.
[13]. Malashetty, M.S., Umavathi, J.C., Prathap Kumar, J., Convective magnetohydro-dynamic two fluid flow and heat transfer in an inclined channel, Heat and Mass Transfer, vol. 37, pp. 259-264, 2001a.
[14]. Malashetty, M.S., Umavathi, J.C., Leela, V., Magnetoconvective flow and heat transfer between vertical wavy wall and a parallel flat wall, Int. J. Applied Mechanics and Engng., vol. 6, pp. 437-456, 2001b.
[15]. Umavathi, J.C., Malashetty, M.S., Magnetohydrodynamic mixed convection in a vertical channel, Int. J. Non-linear Mechanics, vol. 40, pp. 91-101, 2005.

[16]. Prathap Kumar, J., Umavathi, J.C., Basavaraj M Biradar, Mixed convection of magneto hydrodynamic and viscous fluid in a vertical channel, Int. J. Non-Linear Mechanics, vol. 46, pp. 278-285, 2011a.
[17]. Umavathi, J.C., Liu, I.C., Prathap Kumar, J., Pop, I., Fully developed magneto convection flow in a vertical rectangular duct, Heat Mass Transfer, vol. 47, pp. 1–11, 2011b.
[18]. Hasnaoui, M., Bilgen, E. and Vasseur, P., Natural convection above an array of open cavities heated from below, Numerical Heat Transfer, vol.18, pp. 436-442, 1990.
[19]. Ben-Nakhi, A. and Chamkha, A.J., Natural convection in an inclined partitioned enclosure, Int. J. Heat Mass Transfer, vol. 42, pp. 311-321, 2006.
[20]. Dagtekin, I. and Oztop, H.F., Natural convection heat transfer by heated partitions within enclosures, Int. J. Heat and Mass Transfer, vol. 28, pp. 823-834, 2001.
[21]. Tansmim, S.T. and Collins, M.R., Numerical analysis of heat transfer in a square cavity with a baffle on the hot wall, Int. Com. Heat Mass Transfer, vol. 31, pp. 639-650, 2004.
[22]. Prathap Kumar, J., Umavathi, J.C., Ali J. Chamkha and Prema, H., Free convection in a vertical double passage wavy channel filled with a Walters fluid (model B’), Int. J. Energy & Technology, vol. 3, No. 2, pp. 1–13, 2011.
[23]. Prathap Kumar, J., Umavathi, J.C. and Prema, H., Free convection of Walter’s fluid flow in a vertical double-passage wavy channel with heat source, Int. J. Eng. Sci. Techno., vol. 3, No. 1, pp. 136-165, 2011.
[24]. Chang, T.S. and Shiau, Y.H., Flow pulsation and baffle’s effects on the opposing mixed convection in a vertical channel, Int. J. Heat and Mass Transfer, vol. 48, pp. 4190-4204, 2005.
[25]. Nasiruddin and Siddiqui, M.H.K., Heat Transfer augmentation in a heat exchanger tube using a baffle, Int. J. Heat and Fluid Flow, vol. 28(2), pp. 318-328, 2007.
[26]. Yang, M.H., Yeh, R.H, and Hwang, J.J., Mixed convective cooling of a fin in a channel, Int. J Heat and Mass Transfer, vol. 53(4), pp. 760-771, 2010.
[27]. Barletta, A. Laminar mixed convection with viscous dissipation in a vertical channel, Int. J. Heat Mass Transfer, vol. 41 pp. 3501-3513, (1998)
[28]. Javeri, V. Heat transfer in a laminar entrance region of a flat channel for the temperature boundary condition of the third kind, Warme- and Stoffbertragung. Thermo- and- Fluid dynamics, vol. 10, pp. 137-144, (1977).
[29]. Javeri, V. Laminar heat transfer in a rectangular channel for the temperature boundary condition of the third kind. Int. J. Heat Mass Transfer, vol. 10, pp. 1029-1034, (1978).
[30]. Zanchini, E. Effect of viscous dissipation on mixed convection in a vertical channel with boundary conditions of third kind, Int. J. Heat and Mass Transfer, vol. 41, pp. 3949-3959, (1998).
[31]. Rashidi M. M., Anwar Bég O. and Rahimzadeh N., A generalized differential transform method for combined free and forced convection flow about inclined surfaces in porous media, Chem. Eng. Communication, Vol. 199(2), pp. 257–282, 2012.
[32]. Yaghoobi H. and Torabi M., The application of differential transformation method to non-linear equations arising in heat transfer, Int. Comm. Heat and Mass Transfer, Vol. 38, pp. 815–820, 2011.
[33]. Zhou J. K., Differential transformation and its applications for electrical circuits (in Chinese), Huazhong University Press, Wuhan, China, 1986.
[34]. Rashidi, M. M., The modified differential transforms method for solving MHD boundary-layer equations, Comp. Phy. Communication, Vol. 180(11), pp. 2210–2217, 2009.

M. Karuna Prasad" Effect of Baffle on Mixed Convection Flow in a Vertical Channel Filled With Electrically Conducting Fluid" International Journal of Engineering Research and Applications (IJERA), Vol. 09, No.04, 2019, pp. 12-40