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Study of the Coefficient of Rigidity by Physical Modeling of a Stiff Body for BicepsandTricepsanalysis

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ABSTRACT
This article presents studies on the analysis of the stiffness of the muscle of the upper limb of the human body. A physical model is formulated, which predicts the quantitative features around a balance position from the principle of the amount of angular momentum and the governing equations. Coordination around an axis of rotation is mathematically modeled by the definition of a non-linear governing motion function. The measurements of resistive forces of the biceps and triceps articulation were determined, and we obtained its linear stiffness coefficient in the model by means of linearization of the governing function using Taylor's series. To formulate physical problems, mathematical description is of extreme importance. There are exact solutions to many linear problems, but few for non-linear problems. Thus, deepening through mathematical artifices, such as expansions, are of fundamental importance in engineering, treated in specific components.
The mathematical modeling of mechanical vibration problems leads to differential equations. However, it is assumed that the modeling is only finished until appropriate calculations are applied so that a real solution is obtained. This work addresses the importance of interdisciplinarity to reach goals that are not always so evident in publications. This way we expose the need to link the knowledge acquired in multidisciplinarities in order to migrate results and models, giving space to a niche not always explored by exact sciences.
Keywords-Governing Equation, Linear Stiffness, Vibrational Movement, Biceps, Triceps.

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I. INTRODUCTION

Scientific and technological evolution lead to the need for applying concepts developed through usage of mathematical models and physical representations, as for the interpretation of aspects related to situations and phenomena involved in everyday life, making possible the use of the knowledge acquired in fields of engineering specific formations, limiting itself to the theory only.

However, such concepts are not contextualized in the daily life of the academy, but that literature is constantly changing for application to practical problems [1]. It is one of the reasons for students' lack of motivation in learning due to lack of contextualization related to physics and to mathematical models [2] and [3].

Problem contextualization can improve the understanding of physical and mathematical models, leading to the expansion of knowledge in specific subjects in courses, as a tool to externalize and represent behaviors and physical phenomena, whether they are micro or macroscopic ones. However, it is possible to observe in engineering textbooks applications in biophysics, medicine, and sensing, as well as their derivations.

Within this scope, we identified a case of biomechanics that enabled deepening with the pretension of developing new technological tests with active methodologies for new academic and professional opportunities. Still, in this conjuncture, we treat the human being as matter and structure, subject to the consequent limitations within existing physical quantities, as the force, degrees of freedom and kinetics [4]. Thus, imposed under the laws of physics [5].Being thus, tax under the laws of physics [5].

In works in the literature, we can observe the importance of modeling for virtual weights and their impact on the mass of prosthesis [6], [7], [8], [9], [10], [11].

Other articles treated in the literature seek to discuss, evaluate and measure by means of mathematical models, which are applied to the physiological treatment of the body and adaptations of the use of prostheses and thus to analyze the possible damages that the "structure" can suffer.

The circumferential deformation of the arm is used to represent the contraction intensity of the dominant flexor, biceps brachii, whose behavior is described by the theory described by [12].

Another biomechanical model was proposed for elbow flexions in order to quantify the torque generated as a function of the arm trajectory and influence factors of the elbow angle and the angular velocity [13].

In other articles, the relationship between the generated torque and the circumferential deformation of the upper arm is treated, thus providing a scientific basis for the anthropometric measurement technology, a research method related to the measurement of the body dimensions, as an alternative complement to the studies related to surface electromyography (SEMG) for monitoring the myoelectric signal associated with force generation and/or torque during elbow flexions [14], [15], [16] and [17].

Thus, the focus of this article is the study of biceps and triceps stretches from the point of view of vibrational mechanics presented in [18], with a structural and mathematical model to understand, interpret and simulate the physical model to analyze the characteristics related to the muscle's linear stiffness.

There are two types of models that are commonly used to simulate muscle biomechanics. The first of these is the purely mechanical muscle model built from mechanical engineering systems [12].

In addition, the first model that introduced the studies in the field of muscle mechanics was provided with little information about the internal structure and functioning of the muscle, being thus treated as a "black box" in the possibility of mapping the input-output characteristics of the muscle, aiming to formulate a mathematical model capable of predicting their general behavior [19].

In the second model, the sliding filament theory was proposed in 1957, after the first type of muscular model. It was constructed using an approach that considers the molecular structure of the muscle in an attempt to predict the developed tension, simulating the forces produced by the cross bridge accessories between actin and myosin molecules [19].

Therefore, multidisciplinary studies in engineering are important to stimulate the research related to the quantification of the physiology of the biceps and triceps muscles related to the coefficient of stiffness and weight of prostheses [20], [21].

Experimental observations of humans performing unrestricted voluntary movements in a horizontal plane are presented [22] and [23]. We start from an approach of the moment equilibrium method for systems in a degree of freedom [24]. This model is useful for obtaining the governing function. We thus aim to develop a relationship between the weight of the structure and the stiffness coefficient of the muscle.

The development is done by means of equations of movement for oscillations around the position of static equilibrium [4]. See Equation 1.

$$m\frac{d^2}{dt^2}x + c\frac{d}{dt}x + Kx = f(t) \qquad (1)$$

The theoretical analysis, as indicated in Equation 1, is based only on the kinematics of the movement, independent of the dynamics of the musculoskeletal system, and it is only successful when formulated in terms of hand movement in extracorporeal space.

The results of modeling should not extrapolate the parameters that are within the biological bands, i.e., a physical model modeled in the (virtual) computer, should not exceed the biological range of the real arm. For example, we cannot stress the muscle or add a force above its capacity.

The biomechanical model does not have to produce the exact behavior, but must be close, within a very acceptable range of errors.

In addition, simple models are desired with respect to their structure and development, seeking the capacity to reproduce the results, maintaining anthropomorphic realism.

The implications for the organization of movement are discussed and described below.

II. MATERIALS AND METHODS

Using as a reference the representation of Figure 1 to construct the mathematical model, we intend to conceive the structural model of the biceps and triceps biomechanics. We can observe that the model reproduces the behavior of the positions defined by θ.

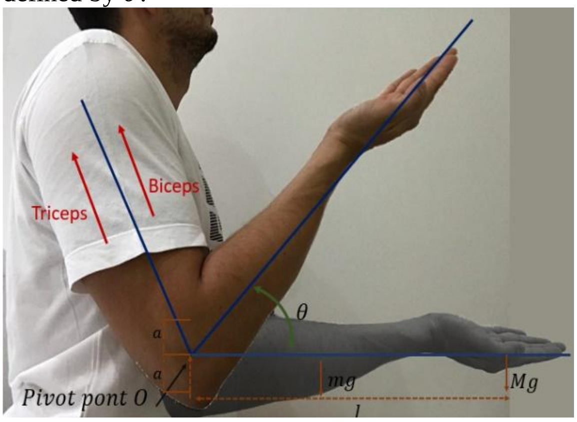

Figure 1. Forearm positions concerning the pivot point O. The authors.

Where m is the forearm mass, M is the hand mass, g is the gravity acceleration and θ is the angle around the pivot point.
It is assumed that Equation 1 is the non-linear governing equation of motion, obtained from the principle of the amount of angular movement around the center of the forearm at the pivot point O (elbow). We also assume that O is a fixed point.

As shown in Figure 2, clockwise, the total torque N of the external forces on the system, with respect to point P, is represented in Equation 2.

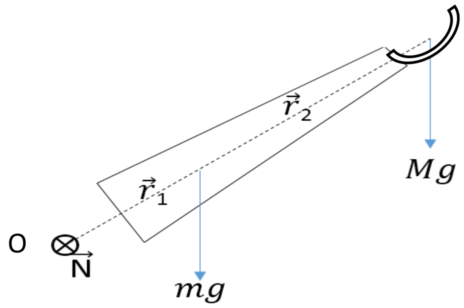

Figure 2. Total torque at point O. The authors.

The total torque N, with respect to the point O located at the elbow, is the sum of these quantities. The resulting muscle torque is the torques caused by the action of gravity on the forearm mass m and the mass M.

$$\vec{N} = \vec{r} \times \vec{F} = (\vec{N}_1 + \vec{N}_2) \qquad (2)$$

Where $\vec{r}$ is the position vector from the origin to the point of application of the force F, that is, the position vector of the point mass particle measured from the origin (pivot point), and M = M_1+ M_2, where M_1 is the mass ofhand (constant) and $M_2$is an external (variable) mass.

Therefore, based on Figure 1, the governing equations for vibration systems are based on Equation 1. This important principle will be used in the development of the mathematical model of the system, presented in Equation 3. Thus we encompass all forces acting on the system in question.

$$N = -Mgl\cos\theta\hat{k} - mg\frac{1}{2}\cos\theta\hat{k} + F_b a\hat{k} - F_t a\hat{k} \qquad (3)$$

The N torque, due to a force, produces a variation of the angular momentum (L) in a time interval. In other words, the torque represents the rate of change of angular momentum or even the variation of angular momentum in time. Although the unit N.m, the torque is not work or energy [25], these are scalar.

$$\vec{N} = \frac{d\vec{L}}{dt} \qquad (4)$$

All rotational quantities have a rotational analog. The analog of the linear momentum is the angular momentum L.
Being L the total Angular Motion, given by,

$$\vec{L} = \vec{r}x\vec{p} \qquad (5)$$

where $\vec{r}$ is the position vector of the fixed point O to the point where the mass M is located and $\vec{p}$ is the linear momentum of this mass M based on the absolute velocity of M.

Assuming the angular momentum related to the inertia momentum,

$$\vec{L} = \vec{J} \times \vec{\omega} \qquad (6)$$

where $\vec{\omega}$ is the angular speed and $\vec{J}$ is the inertia momentum.

The amount of angular movement of the rigid body relative to the fixed point O is,

$$L_O = J_O\dot{\theta}\hat{k} \qquad (7)$$

Using equation 4,

$$N - J_O\ddot{\theta}\hat{k} = 0 \qquad (8)$$

Being J_O the rotational inertia and N the resulting moment acting around pointO due to gravity and forces related to the biceps and triceps are described in equation 4 and represented in Figure 1.
Thus, by replacing Equation 3 in Equation 8,

$$-Mgl\cos\theta\hat{k} - mg\frac{1}{2}\cos\theta\hat{k} + F_b a - F_t a\hat{k} - J_O\ddot{\theta}\hat{k} = 0 \qquad (9)$$

Considering the rotation movement of the forearm in the X-Y plane with movement described by angle θ, Figure 3.

Using the simplified power distribution model described for the system in Figure 3, we have in the representation of the forces generated by the muscles the biceps and triceps, which provide forces of magnitude $F_b$and F_t, respectively.

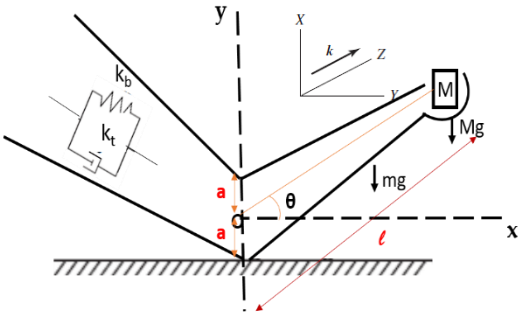

Figure 3. Structural representation of an arm with biceps and triceps magnitudes. The authors.

in which,

$$F_b = -K_b\theta \tag{10}$$

Where k_b is a constant given in N/m.
Related to the same representation, the triceps provides a force of magnitude

$$F_t = K_t v = K_t a\dot{\theta} \tag{11}$$

where K_t is constant and v is the speed magnitude with which the triceps is stretched, given by N.s / m.

$$J_O = \frac{1}{3}ml^2 + Ml^2 \tag{12}$$

Considering the first term of Equation 12, the rotational inertia of a thin bar to represent the forearm. The second term is the rotational inertia of a thin ring (hand)

Gathering all scalar coefficients of the vector terms of Equation 9,

$$\underbrace{\left(M+\frac{m}{3}\right)l^2\ddot{\theta}}_{1} + \underbrace{K_t a^2\dot{\theta}}_{2} + \underbrace{K_b a\theta}_{3} + \underbrace{\left(M+\frac{m}{2}\right)gl\cos\theta}_{4} = 0 \tag{13}$$

Where,

① The term of inertia is due to the rotational inertia of the forearm and the rotational inertia of the mass on the hand;
② Damping term is due to the triceps;
③ Stiffness term is due to the biceps;
④The fourth term is related to the gravity that makes the equation nonlinear due to the $\cos\theta$. It influences the static balance position and the stiffness of the system. This influence depends on the magnitude of θ.
Using Equation 11 for a position of balance in which the time-dependent terms are absent, the terms of velocity and acceleration are equal to zero and $\theta = \theta_0$ is the solution of the transcendental equation.

$$K_b a\theta_O + \left(M+\frac{m}{2}\right)gl\cos\theta_O = 0 \tag{14}$$

Using the concepts of series, such as those of Taylor, expressing functions as the sum of infinite terms is a very useful strategy, allowing for access to several contributions.
Thus, using this strategy, now considering the oscillations around the position of static equilibrium and expanding in terms of the angular variable of $\theta(t)$, we have,

$$\theta(t) = \theta_O + \hat{\theta}(t) \tag{15}$$

and linearizing the nonlinear term $\cos\theta$ in Equation 13, for this we carry out the expansion of the Taylor series, keeping only the linear terms in $\hat{\theta}$.

$$\cos\theta = \cos(\theta_O + \hat{\theta}) \approx \cos\theta_O - \hat{\theta}\sin\theta_O + \cdots \tag{16}$$

By making the first and second derivate in the time of Equations 15,

$$\ddot{\theta}(t) = \frac{d^2}{dt^2}(\theta_O + \hat{\theta}) = \ddot{\hat{\theta}}(t) \tag{17}$$

$$\dot{\theta}(t) = \frac{d}{dt}(\theta_O + \hat{\theta}) = \dot{\hat{\theta}}(t) \tag{18}$$

Using Equation 11, replacing Equations 13 and 15, we arrive at the linear equation of motion that governs the small oscillations of the forearm around the position of static balance,

$$\left(M+\frac{m}{3}\right)l^2\ddot{\hat{\theta}} + K_t a^2\dot{\hat{\theta}} + k_e\hat{\theta} = 0 \tag{19}$$

Being k_e part of the previous equation,

$$k_e = k_b a\left(M+\frac{m}{2}\right)gl\sin\theta_0 \tag{20}$$

Figure 3 shows that the linear stiffness k_e of the linearized system is influenced by the gravity load observed in the second term of Equation 20.

Therefore, the stiffness in muscle systems, using the simplified model for forces generated by muscles, here the biceps and triplicates, provides a force assuming that the forearm can be treated as a uniform rigid beam (see Figure 2), obtaining the governing nonlinear equation motion, linearized around a balance point.

III. RESULTS AND DISCUSSIONS

In our results for the experimental model, we considered the movement for small oscillations in θ_0 of the arm in relation to the horizontal line, limited by the position of the elbow. The initial position of the forearm is related to the angle of inclination relative to the horizontal where θ_0 = 30°, θ_0 = 45°, θ_0 = 60°, that is, t these reference angles are related to each coefficient of linear stiffness (k_e) and kept static in these points for each analysis.

Algorithm 1 is a generic model for building the graphs. This model can be used in programming platforms such as MatLaB, Scilab, MatCad and others. Thus, it is worth mentioning that computational modeling is of fundamental importance and, in our work, it is relevant for interdisciplinarity with the inclusion of simple computational programming algorithms. However, in complex models, this practice offers essential tools for solving more complex problems.

```
kb ⇐ 100000
a ⇐ 0.035
m ⇐ 1.72
l ⇐ 0.52
M1 ⇐ 0.575
M2 ⇐ 0 : 0.005 : 0.02;
pi ⇐ 3.1416
t1 ⇐ pi/6
t2 ⇐ pi/4
t3 ⇐ pi/3
g ⇐ 9.8
M ⇐ M1 + M2
ke2 ⇐ kb * a * ((M1 + M2) + m/2) * g * l * sin(t1)
subplot(3, 1, 1)
plot(M, ke2,' k*')
xlabel('externalmass(kg)')
ylabel('CoefficientofRigidity(uni)')
ke3 = kb * a * ((M1 + M2) + m/2) * g * l * sin(t2)
subplot(3, 1, 2)
plot(M, ke3,' r*')
xlabel('externalmass(kg)')
ylabel('CoefficientofRigidity(uni)')
ke4 = kb * a * ((M1 + M2) + m/2) * g * l * sin(t3)
subplot(3, 1, 3)
plot(M, ke4,' p*')
xlabel('externalmass(kg)')
ylabel('CoefficientofRigidity(uni)')
```

Algoritmo 1. Script para criar os gráficos. The authors

Using Equation 20, with k_b = 100 x10^3 N/m [26] and [27], M = M_1 0.575Kg (weight of the hand)+ M_2 (with external mass), l = 0.52m (Figure 1), a= 0.035m, m=1.72Kg (the mass of the forearm) [26] and [28].

Figure 4 represents the behavior of the coefficient of linear stiffness (k_e). It is observed that there is a request of the triceps muscle, because this dependence exists in Equation 19. Due to the interdisciplinary methodological focus, the results express mechanical quantities, but interconnected to topics related to anthropometry and physiology.

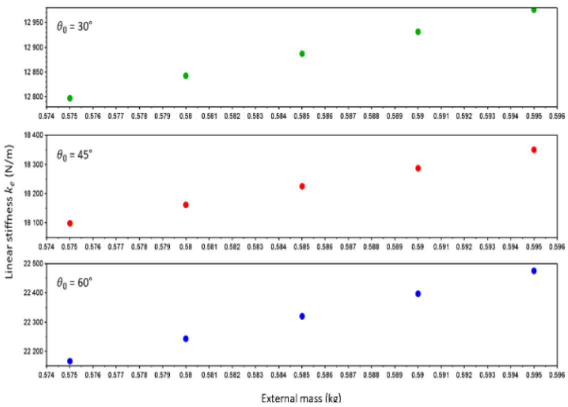

Figure 4. Coefficient of linear stiffness k_e as a function of the external mass. Source: The authors

Equation k_e will give us the possibility to measure the coefficient of linear stiffness as a function of external masses for studies and analyzes of the behavior of the myoelectric signal in relation to the variation of the weight of the external mass, in order to contribute with studies related to the physical structure and structural evaluation of myoelectric hand prosthesis. From the point of view of engineering, interdisciplinarity demands works that contribute to studies related to the rehabilitation of amputees of the arm stump, thus expanding the area of performance of these professionals.

IV. CONSIDERATIONS

Such study, with a structural and mathematical model to understand, interpret and simulate both physically and virtually the characteristics of the model, will soon enable research on muscular pathologies, as well as myoelectric prosthesis of the upper limbs and thus be able to compare and correlate temporal series of myoelectric signals , providing customization and adaptation of prosthesis of individuals.

To develop a prosthesis that allows a work of an external force, parallel to the muscles triggered due to some pathologies, that allows comfort to the individual.

Thus, a gap of possibilities of scientific research for the students of the engineering in relation to technologies directed to health and education is opened, mainly with the use of emergent technologies applied in transversal subjects, extending interdisciplinary studies.

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