RESEARCH ARTICLE OPEN ACCESS

Mathematical modeling of moving boundaries of phase transition in the process of drying anisotropic plate

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ABSTRACT
A two-dimensional mathematical model of the process of convection drying of anisotropic porous materials has been onstructed, taking into account the movement of the boundary of phase transitions. Identified is the influence of the main components and the orientation of the main axes of the heat transfer tensor on the non-stationary temperature fields in the prismatic body. The analytical-numerical method as well as algorithms have been developed for implementation of a nonlinear mathematical model under variable temperature conditions of the environment. Integrals on the boundary of the phase transition must be calculated numerically. All other values included are calculated from the data of physical and thermal characteristics of a particular material. The research results can be used to optimize the process of convection drying of moist anisotropic materials, as well as to implement similar mathematical models in biology, medicine, geophysics and ecology.
Keywords **:** Mathematical model, heat-mass-exchange, anisotropy, phase transitions.

Date of Submission: 20-12-2018 Date of Acceptance: 04-01-2019

I. INTRODUCTION

Construction of adequate mathematical models of heat-and-mass transfer is essential for energy-, metallurgical-, chemical-, building-and-constructing and other industries. In particular, effective methods for optimizing the technological processes of convection drying should be based on reliable and accurate prediction of the kinetics and dynamics of the thermal moisture state of capillary-porous materials, taking into account the anisotropy of thermophysical characteristics. Intensification of drying technologies for anisotropic capillary-porous materials leads to further development of mathematical modeling of heat-and-mass transfer processes considering the boundary of phase transitions resulting from the presence of a moving moisture evaporation boundary, which could adequately describe the patterns of removing moisture in the materials being dried.

The presence of a moving boundary of phase transformations at the interface between phases with different thermophysical characteristics significantly complicates the mathematical models of heat-and-mass transfer processes during the drying of anisotropic capillary-porous materials. The simulation of heat-and-mass transfer with phase transitions in the drying process is reduced to solving the Stefan problems which are the most complicated even for minor changes in the density of the material in the evaporation zone. However, the evaporation of water causes a change in its volume of nearly a thousand times, and the removal of the vapor-gas mixture from the region of the evaporation zone requires significant energy consumption. With the deepening of the evaporation zone within the material being dried, there occurs a significant increase in pressure near the front of the evaporation. Therefore, the energy consumption of the kinetics of vapor transport and the convective transfer of heat to the evaporation zone are taken into consideration by different approaches to represent the model of the evaporation zone.

Therefore, there is an objective necessity for constructing two-dimensional mathematical models of heat-and-mass transfer during the drying of anisotropic capillary-porous materials, taking into account the movement of the evaporation zone for non-stationary drying regimes, as well as the development of effective analytical-numerical methods for their implementation. Such two-dimensional mathematical models and methods of analysis will enable to develop new and improve the existing technological processes of hydrothermal treatment of organic materials, in particular wood, since numerous applications of such materials require an understanding of the laws of the formation of the final product with given physical-and-mechanical and structural characteristics.

The purpose of this work is to mathematically simulate heat transfer in anisotropic capillary-porous bodies, taking into account the boundary of phase transitions, and to develop analytical-numerical methods for the implementation of such two-dimensional models for areas with moving boundaries of moisture evaporation zone. introduction of the paper should explain the nature of the problem, previous work, purpose, and the contribution of the paper. The contents of each section may be provided to understand easily about the paper.

II. ANALYSIS OF THE EXISTING STUDIES

The mechanism of the evaporation zone deepening in the material being dried was first investigated by T. Sherwood. Further theoretical and experimental studies confirmed the occurence of evaporation surface deepening. There are several approaches to simulating heat-and-mass transfer processes in materials during drying in view of the variable depth of evaporation zone [2-5]. In particular, the criterion of the phase transition, which varies with the coordinate of the body, is taken into account in the boundary conditions.

With another approach to modeling, the process of removing moisture is considered within the framework of the Stefan problems which are the most complicated even for minor changes in the density of the material in the evaporation zone [8]. However, the evaporation of water causes a change in its volume of almost a thousand times, and the removal of the vapor-gas mixture from the region of the evaporation zone requires significant energy consumption. With the deepening of the evaporation zone within the material being dried, a significant increase in pressure is observed near the front of the evaporation. Therefore, the energy consumption of the kinetics of vapor transfer and the convective transfer of heat to the evaporation zone is taken into account by different approaches to represent the model of the evaporation zone. In general, all the problems associated with determining moving boundaries of phase transitions belong to the class of essentially nonlinear problems with the available nodal gradient of temperatures at the phase boundary.

In well-known publications on heat-and-mass transfer in anisotropic materials [1,2,3], the methods of the analytical theory of thermal conductivity are mainly used. However, the existence of mixed derivatives essentially restricts the use of known methods which are well developed and suitable for isotropic case. Only a small amount of work is devoted to the study of heat-and-mass transfer in anisotropic bodies with regard to moving boundaries of phase transitions.

The use of numerical methods for multidimensional heat-and-mass transfer problems with phase transition is associated with algorithmic difficulties and significant computational costs. In order to find an approximate solution of wide application, methods of "pass-through" calculation have gained widespread application using the generalized formation of the classical Stefan problem in which the unknown is not the temperature, but enthalpy. Difference schemes are used for the numerical implementation of some mathematical models.

For numerical realizations of mathematical models of heat-and-mass transfer with phase transitions, two basic approaches are used. For the first approach, the methods of identifying the phase separation boundary in each time layer is used through the use of dynamic independent variables or the use of the dynamic network of constant structure with the fixation of nodes at the boundaries of the phase separation. For the second approach, methods are used without detecting the boundary of the phase transition or the methods of pass-through calculation [1]. Adaptation to the boundary of the phase separation is carried out by using the variable step in time (catching the front into the node of the spatial grid). In this aspect, the variational formation of mathematical models of heat transfer with the use of methods of penalty functions is important.

The most characteristic feature of processes for which mathematical models are nonlinear is the unknown in advance topology of the boundaries between different phases. Typically, the classical representations of one-phase and two-phase Stefan’s problems are considered. In these mathematical models, the energy conservation law is used at the interface of the phase separation in addition to the isothermal conditions, taking into account the latent heat. The main idea of the approach is to introduce an effective heat capacity which includes the heat of the phase transition. Using the Dirac delta function allows you to use a single energy equation for the entire region. This enthalpy form of representation of the energy equation is used to analyze Stefan's multidimensional models.

The literature presents few alternative approaches to modeling heat transfer in media with regard to phase transitions [5]. In particular, a cell-automaton algorithm for solving a one-dimensional Stefan’s problem for growing crystals and Boltzmann's lattice equations are described [4].

From the mathematical point of view, the boundary-value problems of the heat-and-mass transfer in anisotropic bodies are fundamentally different from classical problems. The dependence of the characteristic size of the evaporation zone on time, the presence of mixed derivatives in differential equations significantly complicates the application of classical methods for the separation of variables or integral transformations. For this purpose, methods of thermal potentials, contour integration, power series, "instant" Greenberg's eigenfunctions were used [1].

Obtaining analytical solutions to a boundary-value problem of a generalized type in the area with a moving boundary of the phase transition, according to an arbitrary law, was reduced to integro-differential equations, in particular the Volterra integral equations of the second kind with complex kernels. Therefore, only qualitative results of the behavior of such systems were obtained. Quite effective method for solving problems of heating and kinetics of drying of moist materials is the method of differential series. It allows obtaining numerical-analytic solutions to the boundary-value problem of heat-and-mass transfer for the boundary conditions of the third kind.

Let us consider the process of convection drying of anisotropic prismatic bar of rectangular cross-section with geometric dimensions $\{2L_1, 2L_2; -L_1 \le x_1 \le L_1, -L_2 \le x_2 \le L_2\}$. It is assumed that the drying conditions along the length of the bar are the same. Therefore, we consider the process of heat transfer taking into account the boundary of phase transitions for cross-section of the bar, the outer contour of which in the variables of the Cartesian reference system X_1 and X_2 is described by equations:

$$F_0 = \left(x_1^2 - L_1^2\right)\left(x_2^2 - L_2^2\right) = 0. \qquad (1)$$

In the process of heat exchange of the prismatic bar with a drying agent, a dried zone is formed that extends from the outer surface to the depth of the body. Let the dried and moist zones of the cross-section of the bar be separated by a cylindrical surface, the generatrices of which are parallel to the axis of the bar. Its contour is described by a continuous closed line in the cross-section whose equation takes the form:

$$F_m = F_0 - \varepsilon(\tau), \qquad (2)$$

where $\varepsilon(\tau)$ is unknown time function.

During the process of drying, the surface of the material contacts the gas environment which is a mixture of air and vapor, and the heat supplied by the drying agent is spent on evaporation of the moisture, heating the material and overcoming the bond of the moisture with the material. Therefore, the equation of heat transfer of a porous prismatic orthotropic body in a dried zone can be represented as:

$$\left[\Pi(C_v\rho_v + C_a\rho_a) + (1-\Pi)C_s\rho_s\right]\frac{\partial T}{\partial \tau} = \frac{\partial}{\partial x_1}\left(\lambda_{11}\frac{\partial T}{\partial x_1}\right) + \\ + \frac{\partial}{\partial x_2}\left(\lambda_{22}\frac{\partial T}{\partial x_2}\right) + (\lambda_{12} + \lambda_{21})\frac{\partial^2 T}{\partial x_1 \partial x_2} + F(x_1, x_2, \tau), \qquad (3)$$

Here, we denote by means of indices v, a, s the components of vapor, air and skeleton, and Π, C_v, C_a, C_s, ρ_v, ρ_a, ρ_s are porosity, heat capacity, density of vapor, air, skeleton, respectively; λ_{ij} are components of the tensor of thermal conductivity; T is temperature.

Boundary conditions on the heat transfer surface $x_i = \pm L_i \ (i = 1,2)$ take the form:

$$\frac{\partial T}{\partial n_i} = \pm H_i\left(T - T_a(\tau)\right), \qquad (4)$$

where n_i is external normal to the surfaces $x_i = \pm L_i$ respectively, $T_a(\delta)$ is temperature change of the drying agent with time; $H_i = \tilde{\alpha}_i / \lambda_{ii}$, $\tilde{\alpha}_i$ are heat transfer coefficients.

At the phase boundary $x_i = L_{mi}$, the temperature:

$$T = T_{mi}. \qquad (5)$$

In equation (3), we pass on to variables (x_1', x_2') that coincide with the main directions of the thermal conductivity anisotropy. As a result, we obtain:

$$c\rho\frac{\partial T}{\partial \tau} = \left(\lambda_1\frac{\partial^2 T}{\partial x_1'^2} + \lambda_2\frac{\partial^2 T}{\partial x_2'^2}\right), \qquad (6)$$

where λ_1, λ_2 are the main coefficients of thermal conductivity anisotropy;

$$x_1 = l_1 x_1' + l_2 x_2', \ x_2 = m_1 x_1' + m_2 x_2',$$

where $l_i = \cos\alpha_i$, $m_i = \cos\beta_i \ (i = 1,2)$, $l_i^2 + m_i^2 = 1$; $\cos\alpha_i, \cos\beta_i \quad (i = 1,2)$ are direction cosines of new variables.

If we go over to the variables $\xi_1 = (\lambda/\lambda_1)^{1/2} x_1', \ \xi_2 = (\lambda/\lambda_2)^{1/2} x_2', \quad (\lambda > 0)$ the equation (6) will take the form:

$$\frac{\partial T}{\partial \tau} = \bar{a}\left(\frac{\partial^2 T}{\partial \xi_1^2} + \frac{\partial^2 T}{\partial \xi_2^2}\right), \qquad (7)$$

$$\bar{a} = \frac{\lambda}{\left[\Pi(C_v\rho_v + C_a\rho_a) + (1-\Pi)C_s\rho_s\right]}.$$

The main coefficients of thermal conductivity are determined through the coefficients of thermal conductivity of the orthotropic material, and the mutually unambiguous transformation of coordinates is established:

$$\lambda_{1,2} = \lambda_{11} + \lambda_{22} \pm$$
$$\pm\sqrt{(\lambda_{11} + \lambda_{22})^2 + 4(\lambda_{12}\lambda_{21} - \lambda_{11} - \lambda_{22})}; \quad (8)$$
$$l_1 = \lambda_{12} / \Delta'; \ m_1 = (\lambda_{11} - \lambda_1) / \Delta';$$
$$l_2 = (\lambda_{22} - \lambda_2) / \Delta''; \ m_2 = \lambda_{21} / \Delta'';$$
$$x_1 = \lambda_{12} x_1' / \Delta' + \sqrt{(\lambda_{22} - \lambda_2)} \cdot x_2' / \Delta'';$$
$$x_2 = (\lambda_{11} - \lambda_1) x_1' / \Delta' + \lambda_{21} x_2' / \Delta''.$$
$$\Delta' = \sqrt{(\lambda_{11} - \lambda_1)^2 + \lambda_{12}^2}; \Delta'' = \sqrt{(\lambda_{22} - \lambda_2)^2 + \lambda_{21}^2}$$

It is important to obtain the boundary conditions (4) on the surfaces in the variables ξ_1, ξ_2:

$$\frac{\partial T}{\partial \xi_i} \pm H_i * [T - u(t)] = 0, \quad (9)$$

where $H_i^* = H_i \sqrt{\lambda_i / \lambda} \cdot l_1 / (m_1 l_2 + m_2 l_1)$. To determine the volume of the dried zone as a function of time, it is necessary to determine the coordinates of the contour of the cross-section of the bar in the coordinate system ξ_1 i ξ_2 (Fig.1). The cross-sectional equation in variables ξ_1 i ξ_2 takes the form

$$F_0(\xi_1, \xi_2) = (\xi_1^2 - \Delta_1^2)(\xi_1^2 - \Delta_2^2) \cdot$$
$$\cdot (\xi_2^2 - \Delta_3^2)(\xi_2^2 - \Delta_4^2) = 0. \quad (10),$$

where $\Delta_i \ (i = \overline{1,4})$ are coordinates of new vertices of the cross-section of the bar in the system (ξ_1, ξ_2) are determined from the relations (8).

From the surface (10), the drying process moves inside the body. On the side surfaces of the bar, the boundary conditions of heat exchange are given. Subsequently, a near-surface dried zone is formed. We assume that the surface separating the dry and moist zones will have an oval cylindrical shape, and when fully dried, it stretches to the line that is the axis of the bar.

Let us present the equation for the boundary of the dried and moist areas as:

$$F_m(\xi_1, \xi_2, \tau) = (\xi_1^2 - \Delta_1^2)(\xi_1^2 - \Delta_2^2) \cdot$$
$$\cdot (\xi_2^2 - \Delta_3^2) \cdot (\xi_2^2 - \Delta_4^2) - \delta\varepsilon(\tau) = 0, \quad (11)$$

where $\varepsilon(\tau)$ is yet unknown function of time τ.

Let us insert the following values: $(T(\xi_1, \xi_2, \tau) - T_m) / (T_\Pi - T_m) = \eta$, $\tau^* = \tau \overline{a}$, $\beta = \rho_m c_m \overline{a} / \lambda_m = \overline{a} / \overline{a}_m$, $\rho_m c_m / \lambda_m = 1 / \overline{a}_m$, T_Π, T_m are the temperatures on the contour of the cross-section of the bar and at the boundary of the phase transition.

Providing the continuity of the heat flow between the surfaces F_0 and F_m, we find the values η:

$$\eta = \left((\xi_1^2 - \Delta_1^2)(\xi_1^2 - \Delta_2^2)(\xi_2^2 - \Delta_3^2) \cdot \right.$$
$$\left. \cdot (\xi_2^2 - \Delta_4^2) - \delta\varepsilon(\tau)\right) / (-\delta\varepsilon(\tau)), \quad (12)$$

where $\delta = \Delta_1^2 \Delta_2^2 \Delta_3^2 \Delta_4^2$.

The equation for the contour of the bar (10) and the line dividing the dry and moist zones (11) is written as:

$$F_0 = \left[(\xi_1^2 - \delta_1)^2 - \delta_2^2\right] \cdot$$
$$\cdot \left[(\xi_2^2 - \delta_3)^2 - \delta_4^2\right] = 0, \quad (13)$$
$$F_m = \left[(\xi_1^2 - \delta_1)^2 - \delta_2^2\right] \cdot$$
$$\cdot \left[(\xi_2^2 - \delta_3)^2 - \delta_4^2\right] - \delta\varepsilon(\tau) = 0,$$

where $2\delta_1 = \Delta_1^2 + \Delta_2^2$, $2\delta_2 = \Delta_1^2 - \Delta_2^2$, $2\delta_3 = \Delta_3^2 + \Delta_4^2$, $2\delta_4 = \Delta_3^2 - \Delta_4^2$.

From (13) we define explicitly the equation of the phase transition curve in the cross-section of the bar:

$$\xi_2 = \pm\sqrt{\delta_3 \pm \sqrt{\delta_4^2 + \frac{\delta\varepsilon(\tau)}{(\xi_1^2 - \Delta_1^2)(\xi_1^2 - \Delta_2^2)}}}. \quad (14),$$

where the "+" and "-" signs in front of the radical correspond to random variables $\xi_2 > 0$ i $\xi_2 < 0$, respectively, and under the radical meet the conditions $\xi_2^2 - \delta_3$ (> or <) 0, respectively.

For further research, it is necessary to obtain the equation of thermal balance in the region bounded by the outer contour of the cross-section of the bar F0 and the contour of the boundary of the phase transition Fm. Using (2), defined is

$$\int \int_{F_m}^{F_0 = 0} \frac{d\eta}{d\tau^*} ds = \oint_{F_0} \frac{\partial \eta}{\partial n} dl + \beta\eta \frac{\partial V(F_m, F_0)}{\partial \tau^*}, \quad (15)$$

where $V(F_m, F_0) = \int \int_{F_m = 0}^{F_0 = 0} ds = \iint_{S_\Pi} ds - \iint_{S_\Phi} ds$

In order to calculate the integrals contained in formula (15), an explicit form is obtained for the equation of the contour line of phase transition, as well as the limits of the corresponding integrals are identified.

The double integrals in (15) on the surface between the closed contour and the outer contour will be found as the difference between the integral over the surface of the entire cross-section and the integral over the surface S_Φ, limited by the contour F_m.

The volume of dried zone per unit length of the bar which is located between the planes $F_0 = 0, F_m = 0$ is determined by the formulas:

$$V(F_0, F_m) = \iint_{F_m=0}^{F_0=0} dx_1\, dx_2 = 4L_1L_2 - -4\sqrt{\frac{\lambda_1}{\lambda}}\sqrt{\frac{\lambda_2}{\lambda}}\int_0^\gamma \Big(\Big[\delta_3\sqrt{(\xi_1^2-\Delta_1^2)(\xi_1^2-\Delta_2^2)} - -\sqrt{\delta_4^2(\xi_1^2-\Delta_1^2)(\xi_1^2-\Delta_2^2)+\delta\varepsilon}\Big]^{1/2} / /\big[(\xi_1^2-\Delta_1^2)(\xi_1^2-\Delta_2^2)\big]^{1/4}\Big)d\xi_1, \quad (16)$$

where

$$\gamma = \sqrt{\delta_1 - \sqrt{\delta_2^2 + (\delta\varepsilon)/(\delta_3^2-\delta_4^2)}} = = \sqrt{\delta_1 - \sqrt{\delta_2^2 + \Delta_1^2\Delta_2^2\varepsilon}}, \quad (17)$$

$$\Gamma(\xi_1) = \sqrt{\delta_3 - \sqrt{\delta_4^2 + \frac{\delta\varepsilon}{(\xi_1^2-\Delta_1^2)(\xi_1^2-\Delta_2^2)}}}.$$

Determine the derivative of the volume in time, taking into account the dependence on the time value $\varepsilon(\tau^*)$ and dependence on the time of the upper limit of the integral. After the cumbersome transformations we get:

$$\frac{\partial V}{\partial \tau^*} = \sqrt{\frac{\lambda_1}{\lambda}}\sqrt{\frac{\lambda_2}{\lambda}}\frac{d\varepsilon}{d\tau}\Big\{\int_0^\gamma \delta\Big/\Big(\delta_3\sqrt{(\xi_1^2-\Delta_1^2)(\xi_1^2-\Delta_2^2)} - -\sqrt{\delta_4^2(\xi_1^2-\Delta_1^2)(\xi_1^2-\Delta_2^2)+\delta\varepsilon}\Big)^{1/2}\cdot \cdot 1\Big/\Big(\Big(\sqrt{\delta_4^2(\xi_1^2-\Delta_1^2)(\xi_1^2-\Delta_2^2)+\delta\varepsilon}\Big)\cdot \cdot\sqrt[4]{(\xi_1^2-\Delta_1^2)(\xi_1^2-\Delta_2^2)}\Big)d\xi_1 - -\Big(\delta_3\sqrt{(\gamma^2-\Delta_1^2)(\gamma^2-\Delta_2^2)} - \sqrt{\delta_4^2(\gamma^2-\Delta_1^2)(\gamma^2-\Delta_2^2)+\delta\varepsilon}\Big)^{1/2} / /\Big(\sqrt[4]{(\gamma^2-\Delta_1^2)(\gamma^2-\Delta_2^2)}\cdot\sqrt{\delta_1^2-\sqrt{\delta_2^2+\varepsilon\Delta_1^2\Delta_2^2}}\cdot \sqrt{\delta_2^2+\varepsilon\Delta_1^2\Delta_2^2}\Big)\Delta_1^2\Delta_2^2\Big\}. \quad (18)$$

Here, the limit of integration is a function of time

$$\gamma(\varepsilon) = \sqrt{\delta_1^2 + \sqrt{\delta_2^2 + \varepsilon\Delta_1^2\Delta_2^2}} \quad (19)$$

The derivative of $\gamma(\tau^*)$ in time is equal to

$$\gamma'_{\tau^*} = \frac{1}{2\sqrt{\delta_1^2 + \sqrt{\delta_2^2+\varepsilon\Delta_1^2\Delta_2^2}}}\cdot \cdot\frac{\Delta_1^2\Delta_2^2}{2\sqrt{\delta_2^2+\varepsilon\Delta_1^2\Delta_2^2}}\frac{d\varepsilon}{d\tau^*} \quad (20)$$

In explicit form, defined are the integrals in the region S_Φ which is limited by the phase transition line and on the outer surface $S_п$. In particular

$$\iint_{S_\Pi}\frac{\partial\eta}{\partial\tau^*}ds = \sqrt{\frac{\lambda_1}{\lambda}}\sqrt{\frac{\lambda_2}{\lambda}}\Delta\frac{\varepsilon'}{\delta\varepsilon^2}\cdot \cdot\iint_{S_\Pi}(\xi_1^2-\Delta_1^2)(\xi_1^2-\Delta_2^2)\cdot \cdot(\xi_2^2-\Delta_3^2)(\xi_2^2-\Delta_4^2)d\xi_2 d\xi_1 = = 2\frac{\sqrt{\lambda_1\lambda_2}}{\lambda}\Delta\frac{\varepsilon'}{\delta\varepsilon^2}(J_1+J_2+J_3), \quad (21)$$

where $J_i(i=1,2,3)$ are integrals with variable limits. For their definition, analytical expressions are obtained.

$$\iint_{S_\delta}\frac{\partial\eta}{\partial\tau^*}ds = 4\frac{\sqrt{\lambda_1\lambda_2\Delta}}{\lambda}\frac{\varepsilon'}{\delta\varepsilon^2}\cdot \quad (22)$$

$$\cdot\int_{-\gamma}^{\gamma}\Bigg(\frac{\Big(\delta_3\sqrt{\Delta^*}-\sqrt{\delta_4^2\Delta^*+\delta\varepsilon}\Big)^{5/2}}{5\sqrt[4]{\Delta^*}} - -\frac{2\delta}{3}\Big(\delta_3\sqrt{\Delta^*}-\sqrt{\delta_4^2\Delta^*+\delta\varepsilon}\Big)^{3/2}\cdot \cdot\sqrt[4]{\Delta^*} + \Delta_3^2\Delta_4^2\Big(\delta_3\sqrt{\Delta^*}-\sqrt{\delta_4^2\Delta^*+\delta\varepsilon}\Big)^{1/2} \cdot\sqrt[4]{\Delta^{*3}}\Bigg)d\xi_1.$$

where $\Delta^* = \sqrt{(\varepsilon_1^2-\Delta_1^2)(\varepsilon_1^2-\Delta_2^2)}$

Since, at the boundary of the phase transition $\eta(\tau^*)=1$, the function $\varepsilon(\tau)$ at the initial moment of time is equal to zero ($\varepsilon(0)=0$, but in practice $(\xi_1=0,\ \xi_2=0)$ we have $\varepsilon(\tau)=1$), then $\Delta T = T(\xi_1,\xi_2,\tau)-T$ is reached on the intermediate surface F_m according to formula (11), and the difference T_0-T_m corresponds to the outer surface $\xi_1^2=\Delta_1^2$; $\xi_2^2=\Delta_2^2$; $\xi_2^2=\Delta_3^2$; $\xi_2^2=\Delta_4^2$.

Taking into account the above considerations, from the heat balance relation (15), we obtain an equation for determining $\varepsilon(\tau)$

$$\frac{\partial \varepsilon}{\partial \tau *} I \left| J_{s_{\Pi}} - J_{\Phi} \right| = \varepsilon \left[c_1 J_{BB_1} + c_2 J_{BD} \right] +$$
(23)
$$+ \varepsilon^2 I \beta \eta \left(J_V - A \right).$$

The values $J_{s_{\Pi}}, J_{\Phi}, J_v$ are calculated according to the known physical and thermal characteristics of a particular medium [6,7]. The integrals at the boundary of the phase transition are determined by numerical methods depending on the values ε .

The results of mathematical modeling are used to study the motion of the boundary of the phase transition in the process of wood drying.

The following parameters are taken as input data: wood species - pine; length of the plate L = 1 (m); initial moisture content U_0 = 0,4 (kg / kg); the initial temperature t_0 = 20 (^{0}C) = 293 (^{0}K); environment temperature t_c = 40 (^{0}C) = 313 (^{0}K); drying agent speed v = 2 (m / s) = 7,200 (m / h); relative humidity φ = 50%; density ρ = 581 (kg / m3).

Fig. 1 shows the dependence of the phase transition coordinate in time for different temperature regimes of the drying process.

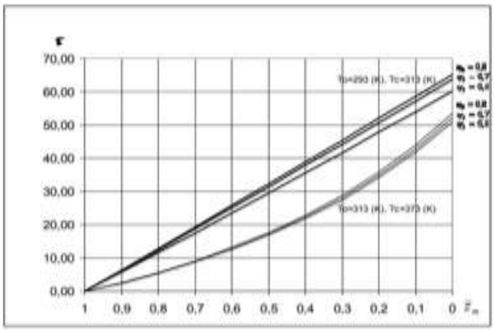

Fig. 1. Dependence of the coordinate of the phase transition in time for different temperature regimes of the drying process.

Fig. 2 shows the graphical dependences of moving boundaries of phase transitions depending on the change of the Fourier criterion for different species of wood (pine, beech, oak).

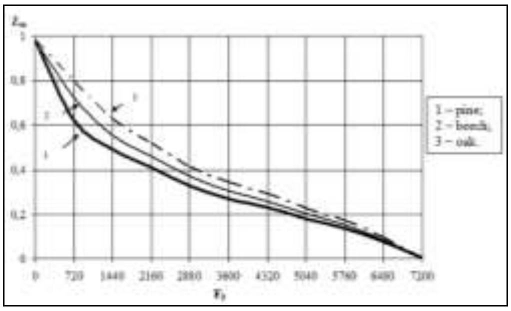

Fig. 2. Dependence of the function of the vaporation surface coordinate for different species of wood.

III. CONCLUSION

Synthesized is a nonlinear mathematical model of heat-and-mass transfer in capillary-porous anisotropic materials with regard to the boundary of the phase transition.

The random orientation of the main axes of the thermal conductivity tensor is taken into account and the influence of the main components and the orientation of the main axes of the thermal conductivity tensor and the non-stationary temperature fields in the anisotropic plate are determined.

The analytical-numerical method has been developed for establishing a moving boundary of phase transition in a rectangular anisotropic region with allowance for arbitrary axes of anisotropy.

A numerical modeling of the heat transfer dynamics in an orthotropic plate with a moving boundary of phase transitions was carried out and the dependences of motion of the front of evaporation in the middle of the plate for different species of wood were determined.

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Yaroslav Sokolovskyy" Mathematical modeling of moving boundaries of phase transition in the process of drying anisotropic plate" International Journal of Engineering Research and Applications (IJERA) , vol. 8, no.12, 2018, pp 57-63