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PID Controller Design Based On The Ziegler-Nichols Method For The Ball And Plate System

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ABSTRACT
This paper presents the design and simulation process of a PID controller for the ball and plate system. The ball's position is controlled to follow a predefined trajectory by adjusting the tilt angles of the plate along two coordinate axes. Given specific system parameters, the process of tuning and selecting the controller’s coefficients is discussed. Additionally, the linearization conditions for equation (2) are also presented. Simulation results show that the PID controller, with parameters determined using the Ziegler-Nichols method, stabilizes the system and ensures good steady-state performance.
***Keywords* -** Ball and plate, PID control, Ziegler-Nichols method

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I. INTRODUCTION

The ball and plate system is an extended version of the classic ball and beam system, which has only two degrees of freedom, where the ball moves in two directions based on the up-and-down motion of the beam. In contrast, the ball and plate system increases complexity as the ball rolls freely on a flat surface. This system is highly nonlinear and serves as a benchmark for testing various nonlinear control methods. Research on developing control algorithms for the ball and plate system, either to keep the ball at a specific position or to follow a predefined trajectory, can apply both classical and modern control techniques after system linearization. Numerous studies have explored different control algorithms for this system. For instance, Y. Wang, X. Li, Y. Li, and B. Zhao [1] employed neural networks to control the ball and plate system. While artificial intelligence-based methods achieve high accuracy, they are complex to design. Similarly, X. Fan, N. Zhang, and S. Teng [2], as well as J. Li and Z. Sun [3], designed fuzzy logic controllers (FLC) to stabilize the ball and plate system. Their results showed that the system remained stable using FLC. However, fuzzy controllers mainly rely on experience and human reasoning for inference, which is then implemented in computers using fuzzy logic principles. Other researchers, such as H. Bang and Y. S. Lee [4] and Hongwei Liu and Yanyang Liang [5], used sliding mode control (SMC) for the ball and plate system. They identified two sliding surfaces based on Lyapunov function definitions and combined them to stabilize the system. However, determining the sliding surface is not straightforward. Additionally, F. Zheng, X. Li, S. Wang, and D. Ding [6] applied a switching control mechanism to regulate the ball and plate system. However, this approach resulted in relatively large deviations in the ball’s angular position. In this paper, a PID controller is used to control the ball and plate system, offering a simpler design while providing better system response.

II. CONTROL METHOD

2.1 Theoretical Basis

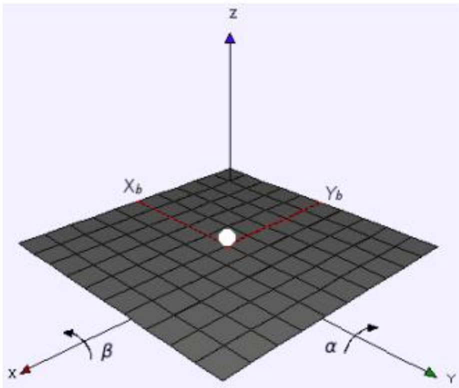

Figure 1. Ball and Plate System Model

The ball and plate system is illustrated in Figure 1, where a ball moves freely on a flat plate along the X and Y axes. The objective of this balancing system is to either keep the ball stable at a predefined position on the plate or allow it to follow a specified trajectory. To achieve this, the plate is controlled to tilt at two angles, α and β, using two servo motors, enabling the ball to move to the desired position.

The ball and plate system has four degrees of freedom: two representing the ball's movement relative to the plate and two representing the plate's tilting motion. The ball's movement along the X and Y axes is denoted as xbx_bxb and yby_byb, respectively, while the plate's tilt angles about the X and Y axes are denoted as α and β

Several initial assumptions are made for the ball and plate system: The ball always remains in contact with the plate, The ball can roll on the plate without slipping, The plate is rigid and uniform, The ball is a solid, homogeneous sphere, All frictional forces are. Based on these assumptions, the nonlinear differential equations governing the system are derived as follows [7]:

$$\begin{cases} \left(m_b+\dfrac{I_b}{r_b^2}\right)\ddot{x}_b-m_b\left(x_b\dot{\alpha}^2+y_b\dot{\beta}\dot{\alpha}\right)+m_b g\sin\alpha=0 \\ \left(m_b+\dfrac{I_b}{r_b^2}\right)\ddot{y}_b-m_b\left(x_b\dot{\alpha}\dot{\beta}+y_b\dot{\beta}^2\right)+m_b g\sin\beta=0 \end{cases} \tag{1}$$

With the initial assumption that the ball is a solid, homogeneous sphere, its moment of inertia is given by: $I_b=\dfrac{2}{5}m_b r_b^2$. Substituting this into equation (1), we obtain:

$$\begin{cases} m_b\left(\dfrac{5}{7}\ddot{x}_b-\left(x_b\dot{\alpha}^2+y_b\dot{\beta}\dot{\alpha}\right)+g\sin\alpha\right)=0 \\ m_b\left(\dfrac{5}{7}\ddot{y}_b-\left(y_b\dot{\beta}^2+x_b\dot{\alpha}\dot{\beta}\right)+g\sin\beta\right)=0 \end{cases} \tag{2}$$

Linearize the system of two equations above with respect to the tilt angle of the small plate (±5%): $\alpha \ll 1, \beta \ll 1$ deduce: $\sin\alpha\approx\alpha, \sin\beta\approx\beta$ and the slow movement speed of the plate: $\dot{\alpha} \ll 0$, $\dot{\beta} \ll 0$, suy ra: $\dot{\alpha}\approx 0, \dot{\beta}\approx 0, \dot{\alpha}\dot{\beta}\approx 0$:

$$\begin{cases} m_b\left(\dfrac{5}{7}\ddot{x}_b+g\alpha\right)=0 \\ m_b\left(\dfrac{5}{7}\ddot{y}_b+g\beta\right)=0 \end{cases} \tag{3}$$

2.2 PID Control for the Ball and Plate System

The PID controller has a simple structure and is easy to implement, making it widely used in feedback control systems (Figure 2). The primary function of the PID controller is to minimize the system error e(t) to zero, ensuring that the transient response meets fundamental quality requirements.

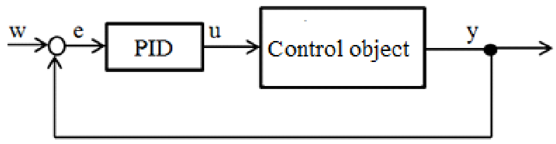

Figure 2. Feedback Control with PID Controller

The PID controller is described using an input-output model, as illustrated in the following diagram: Control object:

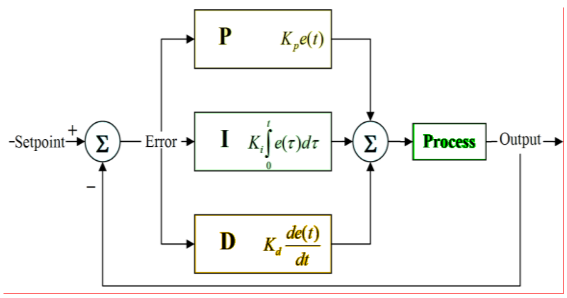

Figure 3. Block Diagram of the PID Controller

The diagram is mathematically expressed by the following equation:

$$u(t)=K_p e(t)+\frac{1}{T_I}\int_0^t e(t)dt+T_D\frac{de(t)}{dt} \tag{4}$$

Where:
e(t) - Input signal;
u(t) - Output signal;
K_p - Gain coefficiet;
T_I - Integral constant;
T_D - Derivative constant.

From the input-output model above, the transfer function of the PID controller is given by:

$$R(s) = K_P \left(1 + \frac{1}{T_I s} + T_D s \right) \qquad (5)$$

There are several methods for determining the parameters of a PID controller [1] [4]. In this paper, the authors use the second Ziegler-Nichols method.

The steps for determining the PID parameters using the second Ziegler-Nichols method are as follows:

Step 1: Set the gain K_I and K_D to zero, while gradually increasing the proportional gain K_P until the system reaches the ultimate gain k_{th} (in this case, 20, to achieve a fast response to the setpoint) until the response starts oscillating, from which the gain T_{th} and k_{th} are determined, as shown in Figure 4.

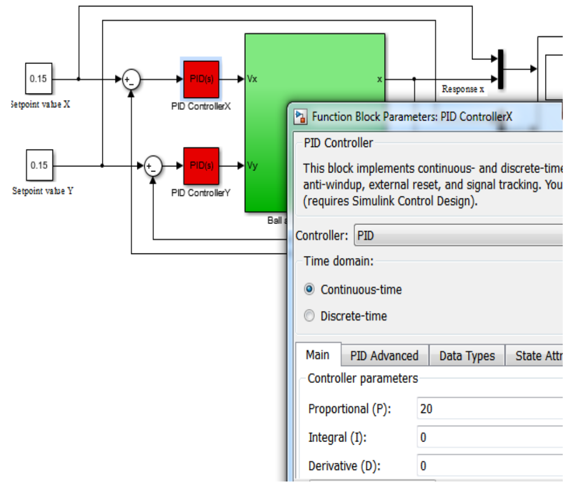

Figure 4. Setting Values for the PID Controller to Determine T_{th} and k_{th}

We obtain the system's oscillatory response to calculate T_{th} and k_{th}, as shown in Figure 5.

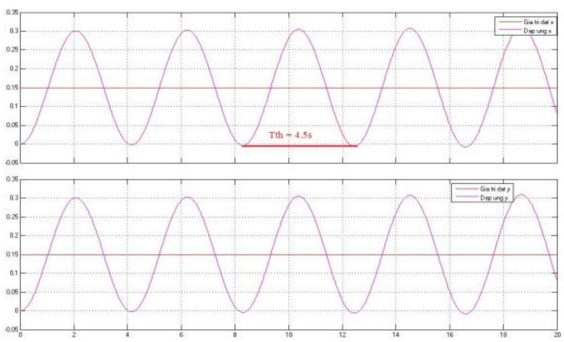

Figure 5. Oscillatory response when $K_p = 20$ and determine T_{th} of the system

k_{th} and the oscillation time T_{th} is used for calculations as shown in Table 1.

Table 1. Calculation Table of PID Controller Parameters

Parameter / Controller	k_P	T_I	T_D
P	$0{,}5k_{th}$	-	-
PI	$0{,}45k_{th}$	$0{,}85T_{th}$	-
PID	$0{,}6k_{th}$	$0{,}5T_{th}$	$0{,}125T_{th}$

Step 2: Calculate T_I and T_D of the system based on T_{th}, thereby obtaining a new PID controller suitable for the ball and plate system.

The controller only has a P block, we have:

$$K_P = 0{,}5 \times k_{th} \rightarrow \frac{K_P}{0{,}5} = \frac{20}{0{,}5} = k_{th} = 40 \quad (6)$$

With $T_{th} = 4{,}5$ (based on the response), we obtain the new parameters for the complete PID controller as follows.

$$K_P = 0{,}6 \times k_{th} = 24$$

$$T_I = 0{,}5 \times T_{th} = 0{,}5 \times 4{,}5 = 2{,}25 \rightarrow K_I = \frac{K_P}{T_I} = 10{,}6667 \quad (7)$$

$$T_D = 0{,}125 \times T_{th} = 0{,}5625 \rightarrow K_D = K_P \times T_D = 13{,}5$$

Step 3: Set the new coefficients for the controller and re-evaluate the system response.

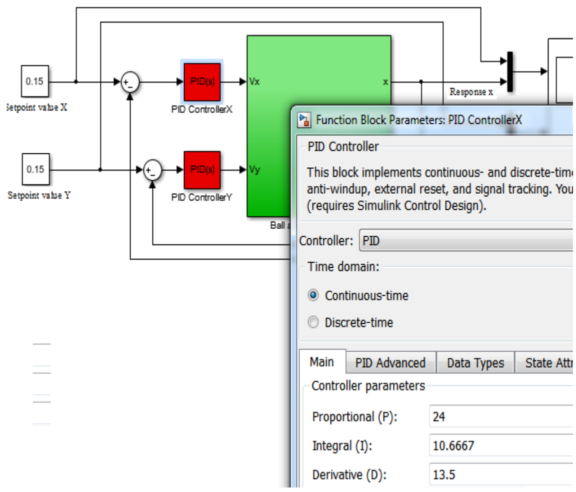

Figure 6. PID Controller with New Coefficients

Table 2. Physical parameters for Simulation

Parameter	Symbol	Value	Unit
Mass of the ball	m_b	0,1	kg
Radius of the ball	r_b	0,015	m
Moment of inertia of the ball	I_b	9.10^{-6}	kgm^2
Moment of inertia of the plate	I_p	0,02	kgm^2
Tilt angle along X-axis	K_t	15	Arc
Tilt angle along Y-axis	R	15	Arc
Ball position along X-axis	x_b	0	m
Ball position along Y-axis	y_b	0	m
Gravitational acceleration	g	9,81	m/s^2
Constant coefficient 1	K_t	0,022	
Constant coefficient 2	K_b	0,025	
Gear ratio	K_g	7	

III. SIMULATION RESULTS AND DISCUSSION

3.1 Investigate the PID controller for the ball and plate system with a constant input signal

3.1.1 Simulink diagram of the PID controller for the ball and plate system with a constant input signal

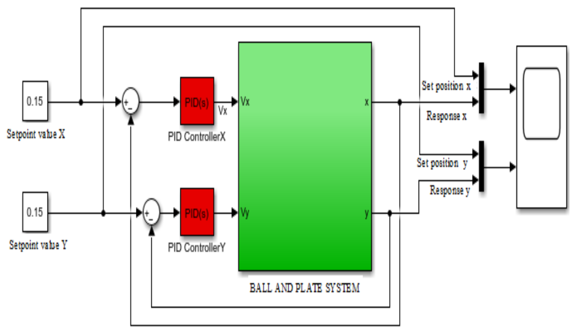

Figure 7. Simulink diagram of the PID controller for the Ball and Plate system with a constant input signal

PID parameters for the Ball and Plate system using the Second Ziegler–Nichols method: .

$$K_P = 24; K_I = 10,6667; K_D = 13,5$$

3.1.2. Simulation results

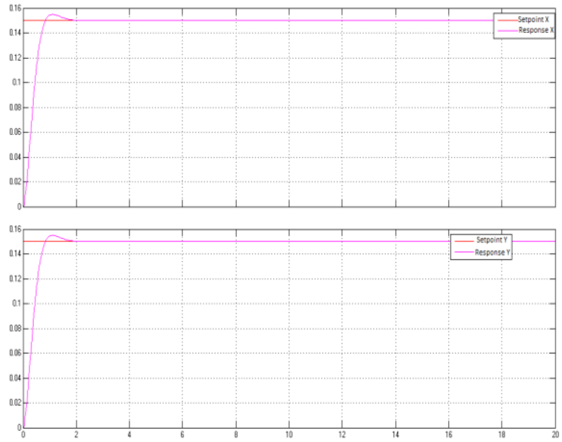

Figure 8. System response with a constant input signal

The obtained results show that the system responds well with an overshoot of 0.01, a response time of 0.8 seconds, and stabilizes at the 2nd second.

3.2. Investigate the PID controller for the Ball and Plate system with the input signal as a Repeating Sequence Stair function

3.2.1. Simulink diagram of the PID controller for the Ball and Plate system with Repeating Sequence Stair input signal

Considering the Ball and Plate system with the input signal as a Repeating Sequence Stair function, this function allows the generation of an arbitrary periodic signal with adjustable step changes, making it suitable for controlling the ball to follow a predefined trajectory.

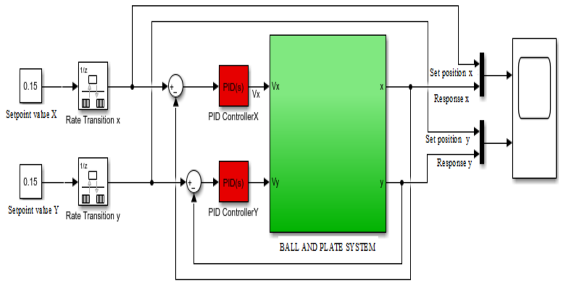

Figure 9. Simulink diagram of the PID controller for the Ball and Plate system with Repeating Sequence Stair input signal

3.2.2. Simulation results

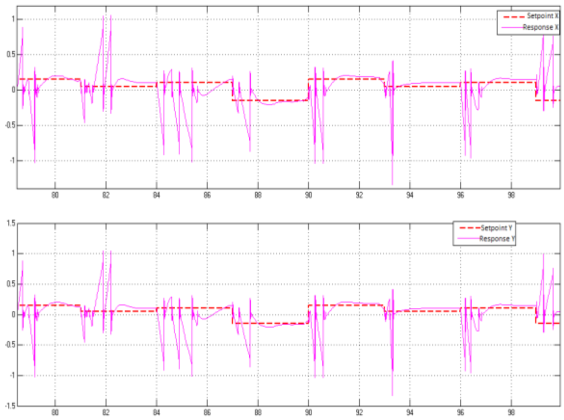

***Figure 10**. System response with Repeating Sequence Stair input signal using the same PID parameters*

With the same PID controller parameters as in the case of a constant input, the system fails to respond effectively to changes in the input signal, exhibiting large oscillations, especially at abrupt state transitions of the input signal.

To ensure the system response follows the reference signal, the PID parameters need to be re-tuned using the same method described earlier for the controller with a constant input signal.

Apply the new parameters to the controller: $K_P = 1,2; K_I = 0,2; K_D = 13,7.$

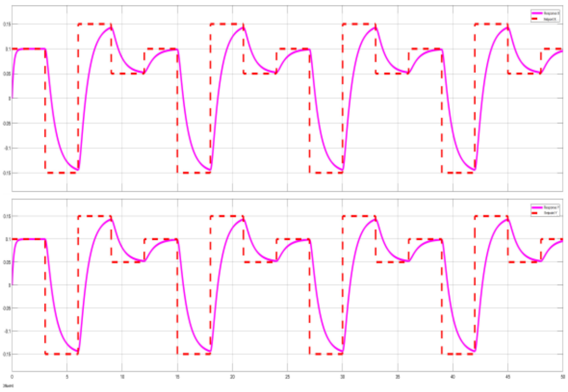

***Figure 11**. System response with Repeating Sequence Stair input signal*

The system output follows the reference signal with a small steady-state error (0.007), no overshoot, and a fast response even for large position changes.

IV. CONCLUSION

With the selected parameters for the PID controller using the Ziegler-Nichols method, the ball and plate system was simulated in MATLAB. The results show that, after parameter tuning, the system output closely follows the reference signal with overshoot reduced to zero and an acceptable response, as shown in Figure 11. Additionally, the linearized model of the ball and plate system was implemented in MATLAB Simulink, and the PID control parameters using the Ziegler-Nichols method were selected. However, in this paper, the effects of friction forces and the ball's inertia on the smooth surface were neglected to simplify the system's linearization. Therefore, to accurately assess the system's response while considering these effects, different control algorithms should be developed for further evaluation.

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