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Model-free Adaptive Control with Compact Form Dynamic Linearization for Nonlinear SISO Objects

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ABSTRACT
This paper explained the dynamic linearization data models of a nonlinear SISO object class linearized in compact form CFDL. It proposed the model-free adaptive control algorithm CFDL-MFAC based on the pseudo partial derivative parameter estimation. From the solution of the objective function optimization problem by the extended differential tool, the modified CFDL-MFAC control law was synthesized and guaranteed mathematically.
***Keywords*-**Model-free adaptive control, pseudo partial derivative, compact form dynamic linearization.

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I. INTRODUCTION

Model-Free Adaptive Control (MFAC) is a data-driven control technique that uses system input/output data to design the controller. MFAC was first introduced for a class of nonlinear objects in 1994 [1]. Since then, MFAC has continuously developed and been continuously improved [2-6], forming a framework system of MFAC methods [7], [8].

For MFAC, only the measured input/output data of the closed-loop control system are used for controller design without knowing explicit information about the object model. Because there is no need for a dynamic model or training process, MFAC hasa simple structure and low computational cost. Therefore, recently, quite a few adaptive control problems for a nonlinear system with time-varying parameters and structures can be implemented using this method [9], [10].

The starting idea of the MFAC design is to build a purely mathematical model based on data, also known as a dynamic linearization model for the nonlinear system at each sampling time. The dynamic linearization model couldbe divided into three forms [11]: Compact Form Dynamic Linearization (CFDL), CPDL Partial Form Dynamic Linearization, and Full Form Dynamic Linearization (FFDL). CFDL compression is most commonly used to synthesize MFAC algorithms. Since the purely mathematical model deals with data, when data loss occurs due to sensor or network connection errors, the stability and sustainability of the system will be affected. In [12], MFAC was built using the swarm optimization method. Research [13] proposed using RBF neural network and evolutionary optimization algorithm to improve the robustness and convergence time for MFAC. Some studies have shown that applying MFAC ensures convergence bias [12], [13]. In addition, the stability and robustness of the system can be guaranteed by certain assumptions and statistical methods [14].

This paper studies and describes the dynamics of a class of dynamically linearized SISO nonlinear objects in compressed form and proposes a modified CFDL-MFAC control law, which is strictly mathematically guaranteed based on the solution of The optimization problem minimizes the objective function using an extended differential tool, helping to overcome the effects of data shortages due to network or sensor errors. The article also clarifies some essential assumptions and theorems as a premise for estimating parameters and synthesizing the proposed CFDL-MFAC control law. The parameters of the dynamic linearization model are estimated online using PPD (Pseudo Partial Derivative) theory.

II. COMPACT FORM DYNAMIC LINEARIZTON MODEL (CFDL)

Let us consider a class of SISO discrete-time nonlinear systems with its input/output information. The data model of the system is represented as equation (0):

$$y_{k+1} = f\left(y_k, y_{k-1}, \dots, y_{k-n_y}, u_k, u_{k-1}, \dots, u_{k-n_u}\right) \quad (0)$$

Where:

$u_k \in R, y_k \in R$ are the input and output of the system at time step k.

$n_u, n_y \in N$ arepositive integers representing the unknown orders of the input and output.

$f(*) \in R^{n_u+n_y+2}$is an unknown nonlinear function.

k represents time; the interval from step k to $k+1$ corresponds to one sampling period.

Assumption 123: Assumption 123 consists of 3 sub-assumptions as follows:

1) The inputs and outputs of the system (0) can be observed and controlled.

2) The partial derivatives of $f(*)$ with respect to the control input u_k are continuous or smooth.

3) The system (0) satisfies a general Lipschitz condition, meaning that for any k and $\Delta u_k \neq 0$, it holds that:

$$|\Delta y_{k+1}| \leq b|\Delta u_k|;\ b > 0 \quad (1)$$

Where: $\Delta y_{k+1} = y_{k+1} - y_k$; $\Delta u_k = u_k - u_{k-1}$

Remarks:

From a practical perspective, Assumptions 123 for system (0) is reasonable and acceptable. The second sub-assumption represents a specific condition for designing a control system for a general nonlinear system. The third sub-assumption limits the rate of output change allowed by the system before applying the control law [10]. It defines an upper bound to restrict the rate of output change based on the tracking error. From an energy perspective, the rate of energy change within a system cannot go to infinity if the rate of input energy change is finite.

Theorem 1:

If the SISO nonlinear system (0) satisfies Assumption 123, then there exists a PPD ϕ_k. Moreover, if $\Delta u_k \neq 0$, the system (0) can be described by the compressed linearized model (2):

$$\Delta y_{k+1} = \phi_k \Delta u_k;\ \phi_k \leq b, b > 0 \quad (2)$$

Proof:

According to the third sub-assumption, the difference equation (2) can be written as:

$$y_{k+1} = y_k + \phi_k \Delta u_k \quad (3)$$

In that case, the output differential of the system is given by (4):

$$\begin{aligned}\Delta y_{k+1} = & f\left(y_k, \dots, y_{k-n_y}, u_k, \dots, u_{k-n_u}\right) \\ & - f\left(y_k, \dots, y_{k-n_y}, u_{k-1}, u_{k-1}, \dots, u_{k-n_u}\right) \\ & + f\left(y_k, \dots, y_{k-n_y}, u_{k-1}, u_{k-1}, \dots, u_{k-n_u}\right) \\ & - f\left(y_{k-1}, \dots, y_{k-n_y-1}, u_{k-1}, \dots, u_{k-n_u-1}\right)\end{aligned} \quad (4)$$

According to the second assumption and the Cauchy mean value theorem [15], Eq. (4) can be written in the form of (5):

$$\Delta y_{k+1} = \frac{\delta f}{\delta u_k} \Delta u_k + \Upsilon_k \quad (5)$$

Where $\frac{\delta f}{\delta u_k}$ is the partial derivative of the function $f(*)$to the variable $n_y + 2$ at a certain point lying between$\left(y_k, \dots, y_{k-n_y}, u_{k-1}, u_{k-1}, \dots, u_{k-n_u}\right)^T$ and $\left(y_k, \dots, y_{k-n_y}, u_k, u_{k-1}, \dots, u_{k-n_u}\right)^T$.

And Υ_k is determined as follows:

$$\begin{aligned}\Upsilon_k = & f\left(y_k, \dots, y_{k-n_y}, u_{k-1}, u_{k-1}, \dots, u_{k-n_u}\right) \\ & - f\left(y_{k-1}, \dots, y_{k-n_y-1}, u_{k-1}, \dots, u_{k-n_u-1}\right)\end{aligned} \quad (6)$$

At each specific time step k, we have:

$$\Upsilon_k = \Gamma_k \Delta u_k \quad (7)$$

When $\Delta u_k \neq 0$, the equation (8) has a solution $\Gamma_k = \frac{\Upsilon_k}{\Delta u_k}$, thus (7) always exists. By combining (5) and (7), we obtain:

$$\Delta y_{k+1} = \frac{\delta f}{\delta u_k}\Delta u_k + \Gamma_k \Delta u_k = \left(\frac{\delta f}{\delta u_k} + \Gamma_k\right)\Delta u_k \quad (8)$$

Let: $\frac{\delta f}{\delta u_k} + \Gamma_k = \phi_k$, we obtain:

$$\Delta y_{k+1} = \phi_k \Delta u_k \quad (9)$$

According to Assumption 123, we have $|\Delta y_{k+1}| \leq b|\Delta u_k|, b > 0$, from which we can conclude $\phi_k \leq b$.

The theorem has been proven. ■

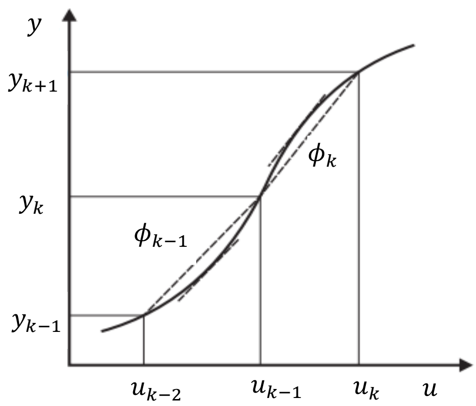


Fig. 1: Illustration of the concept of PPD

Fig. 1 showsthat the Pseudo Partial Derivative (PPD) is determined at three points: (u_{k-2}, y_{k-1}), (u_{k-2}, y_{k-1}) and (u_{k-2}, y_{k-1}).Accordingly, the SISO nonlinear system is dynamically linearized in the compact form with the gradients ϕ_{k-1} and ϕ_k.

III. MODEL-FREE ADAPTIVE CONTROL BASED ON COMPACT FORM DYNAMIC LINEARIZATION (MFAC-CFDL)

To synthesize the control law, commonly, a cost function $J(*)$ representing the squared difference between the system output and the model output is used. However, the parameters estimated using the derivatives of this type of cost function often exhibit high sensitivity to inaccurate data sources caused by noise or sensor errors. Studies [11] have employed the cost function $J(u_k)$, but for mathematical convenience, the proposed paper introduces and interprets the cost function $J(\Delta u_k)$ as the differential function of the control signal Δu_k and the tracking error e_{k+1}.

$$J(\Delta u_k) = e_{k+1}^2 + \gamma \Delta u_k^2 \quad (10)$$

Where: $e_{k+1} = y_{k+1}^* - y_{k+1}$
y_{k+1}^* is the desired output signal.
γ is a hyperparameter, $\gamma > 0$.

Moreover, according to Theorem 1 and Assumption123, we have:

$$y_{k+1} = y_k + \phi_k \Delta u_k \quad (11)$$

Combining (10) and (11), we obtain:

$$J(\Delta u_k) = (y_{k+1}^* - y_k - \phi_k \Delta u_k)^2 + \gamma \Delta u_k^2 \quad (12)$$

To minimize the objective function (11), we take the derivative of $J(\Delta u_k)$ to Δu_k, which leads to the equation (13):

$$\frac{\partial J(\Delta u_k)}{\partial \Delta u_k} = 2\phi_k^2 \Delta u_k - 2(y_{k+1}^* - y_k) + 2\gamma \Delta u_k = 0 \quad (13)$$

Solving equation (13) results in:

$$\Delta u_k = \frac{\phi_k}{\gamma + \phi_k^2}(y_{k+1}^* - y_k)$$
$$\Rightarrow u_k = u_{k-1} + \frac{\phi_k}{\gamma + \phi_k^2}(y_{k+1}^* - y_k) \quad (14)$$

Therefore, the control signal u_k at time k depends on the sum of the control signal u_{k-1} at time $k-1$and an additional control term that depends on the measured data and the unknown parameter ϕ_k. To adjust the amount of the additional control term (related to the speed of the control signal), we introduce a parameter $\rho > 0$ in Eq. (14), representing the step size of the control signal u_k. Then, Eq. (14) becomes:

$$u_k = u_{k-1} + \frac{\rho \phi_k}{\gamma + \phi_k^2}(y_{k+1}^* - y_k) \quad (15)$$

In order to make u_k explicit, the unknown parameter ϕ_k needs to be estimated. With a similar structure as (10), the paper uses the objective function $J(\hat{\phi}_k)$to estimate the PPD for ϕ_k, which takes the form of (16).

$$J(\hat{\phi}_k) = (y_k - y_{k-1} - \hat{\phi}_k \Delta u_{k-1})^2 + \mu(\hat{\phi}_k - \hat{\phi}_{k-1})^2 \quad (16)$$

Where $\hat{\phi}_k$ is the estimation of ϕ_k at step k, and μ is a hyperparameter, $\mu > 0$.

Taking the derivative of $J(\hat{\phi}_k)$to $\hat{\phi}_k$in (12), get equation (17):

$$\frac{\partial J(\hat{\phi}_k)}{\partial \hat{\phi}_k} = 2(\Delta u_{k-1}^2 + \mu)\hat{\phi}_k - 2\mu\hat{\phi}_{k-1} - 2\Delta u_{k-1}(y_k - y_{k-1}) \quad (17)$$

$$\frac{\partial J(\hat{\phi}_k)}{\partial \hat{\phi}_k} = 0 \quad (18)$$

$$(\Delta u_{k-1}^2 + \mu)\hat{\phi}_k - \mu\hat{\phi}_{k-1} - \Delta u_{k-1}(y_k - y_{k-1}) = 0 \quad (19)$$

From (19), $\hat{\phi}_k$ is computed:

$$\hat{\phi}_k = \hat{\phi}_{k-1} + \frac{\Delta u_{k-1}(\Delta y_k - \hat{\phi}_{k-1}\Delta u_{k-1})}{\mu + \Delta u_{k-1}^2} \quad (20)$$

To adjust the estimation step size for the parameter (the rate of parameter adaptation), we introduce a parameter $\eta > 0$, representing the magnitude of the estimation step $\hat{\phi}_k$. In this case, Eq. (20) becomes:

$$\hat{\phi}_k = \hat{\phi}_{k-1} + \frac{\eta\Delta u_{k-1}(\Delta y_k - \hat{\phi}_{k-1}\Delta u_{k-1})}{\mu + \Delta u_{k-1}^2} \quad (21)$$

The estimated PPD $\hat{\phi}_k$ is obtained from the recursive formula (21), where the hyperparameter μprevents singularity when the denominator of (21) becomes zero. The PPD parameter is updated at each time step k using the measured input/output data during the operation of the control system.

The sufficient condition for $\hat{\phi}_k$: If $|\phi_k| \le \epsilon$ or $|\Delta u_{k-1}| \le \epsilon$, with $\epsilon > 0$ then $\hat{\phi}_k = \hat{\phi}_1$. Here, $\hat{\phi}_1$is an initial value.

Remarks:

- The parameter estimation $\hat{\phi}_k$ at time k is wholly determined as it only depends on the data (Δy_k), the control signal (Δu_{k-1}), and its value at the previous time $k-1$.
- The reset mechanism $\hat{\phi}_k = \hat{\phi}_1$ to provide the parameter estimation algorithm (21) with a reference parameter closely following the parameter changes over time.
- According to the separation principle [15], by combining a stable observer with a stable controller, we obtain a stable dynamical system that includes both the observer and thecontroller. Therefore, it is possible to replace $\hat{\phi}_k$ with ϕ_k in equation (15), resulting in the synthesized control signal:

$$u_k = u_{k-1} + \frac{\rho\hat{\phi}_k}{\gamma + \hat{\phi}_k^2}(y_{k+1}^* - y_k) \quad (22)$$

Theorem 2:

Suppose the system (0) satisfies Assumption 123. If the CFDL-MFAC control law (15) is designed with $\gamma > \gamma_{min} > 0$, then the closed-loop control system is asymptotically stable.

Proof: According to the assumption, if one of the conditions $|\phi_k| \le \epsilon$ or $|\Delta u_k| \le \epsilon$ is satisfied, then $\hat{\phi}_k$ is bounded. Using the error in estimating the PPD, denoted by $\tilde{\phi}_k = \hat{\phi}_k - \phi_k$, and the parameter estimation algorithm (21), the error in estimating the PPD $\tilde{\phi}_k$ is obtained.

$$\tilde{\phi}_k = \left(1 - \frac{\eta\Delta u_{k-1}^2}{\mu + \Delta u_{k-1}^2}\right)\tilde{\phi}_{k-1} + \phi_{k-1} - \phi_k \quad (23)$$

Taking the absolute value of both sides of (23), we obtain the inequality (24).

$$|\tilde{\phi}_k| \le \left|\left(1 - \frac{\eta\Delta u_{k-1}^2}{\mu + \Delta u_{k-1}^2}\right)\right| |\tilde{\phi}_{k-1}| + |\phi_{k-1} - \phi_k| \quad (24)$$

Therefore, the term $\frac{\eta\Delta u_{k-1}^2}{\mu + \Delta u_{k-1}^2}$ is monotonically increasing to Δu_{k-1}^2, and its minimum value is given by (25):

$$\frac{\eta\epsilon^2}{\mu + \epsilon^2} \quad (25)$$

We can choose η suitably. For example, if we consider $0 < \eta \le 2$ and $\mu > 0$, then there exists a number α such that:

$$0 \le \left|\left(1 - \frac{\eta\Delta u_{k-1}^2}{\mu + \Delta u_{k-1}^2}\right)\right| \le 1 - \frac{\eta\epsilon^2}{\mu + \epsilon^2} = \alpha < 1 \quad (26)$$

Suppose $|\phi_k| \le b$; in the proof of **Theorem 1**, we have already established:

$$|\phi_{k-1} - \phi_k| \le 2b \tag{27}$$

Combining (24), (26), and (27), we obtain:

$$\begin{aligned}|\tilde{\phi}_k| &\le \alpha|\tilde{\phi}_{k-1}| + 2b = \alpha|\tilde{\phi}_{k-1}| + \frac{2b(1-\alpha)}{1-\alpha} \\ &\le \alpha^2|\tilde{\phi}_{k-2}| + 2b(\alpha+1) \\ &= \alpha^2|\tilde{\phi}_{k-2}| + \frac{2b(1-\alpha^2)}{1-\alpha} \\ &\le \alpha^3|\tilde{\phi}_{k-3}| + 2b(\alpha^2+\alpha+1) \\ &= \alpha^3|\tilde{\phi}_{k-3}| + \frac{2b(1-\alpha^3)}{1-\alpha} \\ &\le \cdots \\ &\le \alpha^{k-1}|\tilde{\phi}_1| + 2b(\alpha^{k-2}+\cdots+\alpha+1) \\ &= \alpha^{k-1}|\tilde{\phi}_1| + \frac{2b(1-\alpha^{k-1})}{1-\alpha}\end{aligned} \tag{28}$$

The formula (28) shows that:

$$|\tilde{\phi}_k| \le \alpha^{k-1}|\tilde{\phi}_1| + \frac{2b(1-\alpha^{k-1})}{1-\alpha}$$

So, $\tilde{\phi}_k$ is bounded.

Denoting the output error as:

$$e_{k+1} = y_{k+1}^* - y_{k+1} \tag{29}$$

Substituting the output value of the compact model (11) into (29), we have:

$$\begin{aligned}|e_{k+1}| &= |y_{k+1}^* - y_{k+1}| \\ &= |y_{k+1}^* - y_k - \phi_k \Delta u_k| \\ &\le \left|1 - \frac{\rho\phi_k\hat{\phi}_k}{\gamma + \hat{\phi}_k^2}\right| \times |e_k|\end{aligned} \tag{30}$$

Suppose $\gamma_{min} = \frac{b^2}{4}$. Since $\hat{\phi}_k$ is bounded (as inferred from the assumption of the sufficient condition), there exists a constant $0 < M < 1$ such that:

$$\begin{aligned}0 < M &\le \frac{\phi_k\hat{\phi}_k}{\gamma + \hat{\phi}_k^2} \\ \le \frac{b\hat{\phi}_k}{\gamma + \hat{\phi}_k^2} &\le \frac{b\hat{\phi}_k}{2\hat{\phi}_k\sqrt{\gamma}} < \frac{b}{2\sqrt{\gamma_{min}}} = 1\end{aligned} \tag{31}$$

Based on (31) and the condition $\gamma > \gamma_{min}$, then there exists a number β such that:

$$\begin{aligned}\left|1 - \frac{\rho\phi_k\hat{\phi}_k}{\gamma + \hat{\phi}_k^2}\right| &= 1 - \frac{\rho\phi_k\hat{\phi}_k}{\gamma + \hat{\phi}_k^2} \\ &\le 1 - \rho M = \beta < 1\end{aligned} \tag{32}$$

Combining (30) and (32), the tracking error converges to 0, and the bound on the error satisfies the condition:

$$|e_{k+1}| \le \beta|e_k| \le \beta^2|e_{k-1}| \le \cdots \le \beta^k|e_1| \tag{33}$$

The theorem has been proven. ■

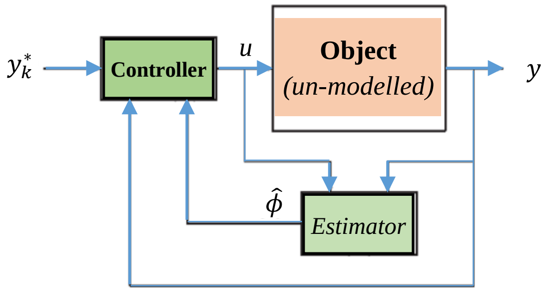
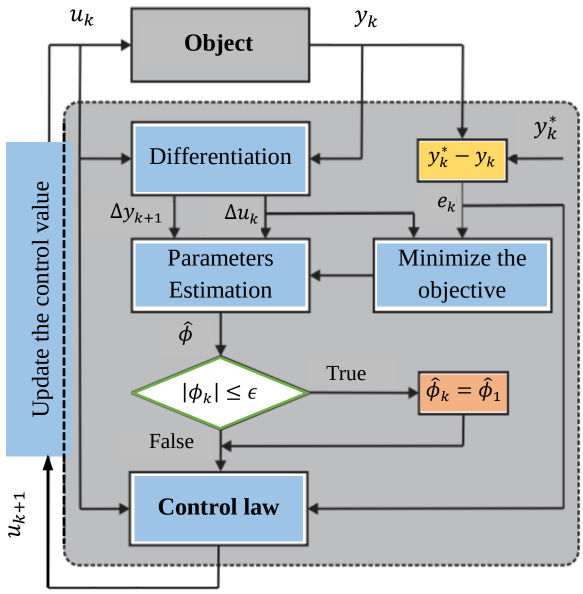


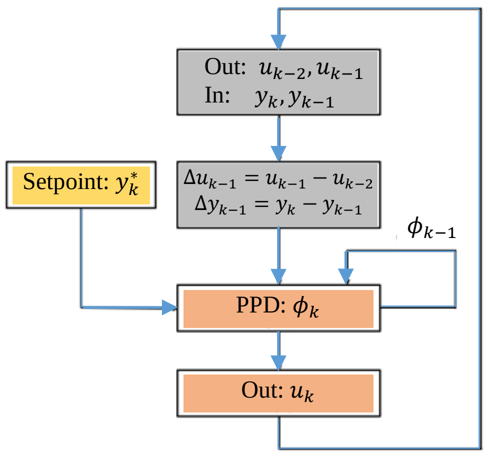
Fig.2: Block Diagram of a Closed-loop Nonlinear SISO System

Remarks:

Figure 2 illustrates the simple structure of the CFDL-MFAC controller. The unknown dynamic model parameters $\hat{\phi}$ are estimated using input/output data. The data is also used to synthesize the control input u. Figure 3a presents the detailed structure of the control structure diagram in Figure 2, highlighting the estimation algorithm based on the optimization results of the objective function. Figure 3b demonstrates the input/output data flow diagram of the system.



a) Structure diagram of CFDL-MFAC



b) Data stream Diagram of CFDL-MFAC

Fig. 3: Flowchart of the Model-free Adaptive Control Algorithm for the CFDL-MFAC

IV. MODIFIED CFDL-MFAC METHOD

In practice, the difference between consecutive sampling instances is usually minimal and does not significantly impact. Furthermore, in cases where there is a lack of data due to network errors or sensor malfunctions, the difference between consecutive sampling instances can be zero. Therefore, an extended derivative is proposed to synthesize the improved MFAC controller.

The extended derivative of the tracking error is given by:

$$\begin{aligned}\Delta e_{k+1} &= e_{k+1} - e_k \\ &= (y_{k+1}^* - y_{k+1}) - (y_k^* - y_k) \\ &= \Delta y_{k+1}^* - \Delta y_{k+1}\end{aligned} \tag{34}$$

The proposed objective function is as (35):

$$J(u_k) = \begin{bmatrix} e_{k+1} \\ \Delta e_{k+1} \end{bmatrix}^T S \begin{bmatrix} e_{k+1} \\ \Delta e_{k+1} \end{bmatrix} + \gamma (u_k - u_{k-1})^2 \tag{35}$$

With $S = \begin{bmatrix} 1 & 0 \\ 0 & s \end{bmatrix}$ is the weighting matrix, and $s$is a selected constant parameter during design. Substitute the output of the compression model (11) into (35):

$$\begin{aligned}J(u_k) &= A_0 \Delta u_k^2 - 2B_0 \Delta u_k \\ &+ (y_{k+1}^* - y_k)^2 + s(\Delta y_{k+1}^*)^2\end{aligned} \tag{36}$$

Where:

$$\begin{aligned}A_0 &= \gamma + (1+s)\phi_k^2 \\ B_0 &= \phi_k (y_{k+1}^* - y_k + s\Delta y_{k+1}^*)\end{aligned} \tag{37}$$

Consider the expanded differential quantity according to (38) with N as the sampling interval.

$$\begin{aligned}\Delta e_{k+1} &= (y_{k+1}^* - y_{k-N+1}^*) - (y_{k+1} - y_{k-N}) \\ &= \Delta y_{k+1}^* - \Delta y_{k-N}\end{aligned} \tag{38}$$

By applying the expanded differential (38) and solving the minimizing problem for the objective function (36), we obtain the control law (39) and the estimated value of ϕ_k as given by (40):

$$\begin{aligned}u_k &= u_{k-1} + \frac{\rho \hat{\phi}_k (y_{k+1}^* - y_k)}{\gamma + (1+s)\hat{\phi}_k^2} \\ &+ \frac{s[y_{k+1}^* - y_{k-N+1}^* - (y_k - y_{k-N})]}{\gamma + (1+s)\hat{\phi}_k^2}\end{aligned} \tag{39}$$

Where: $\gamma > 0,\ \rho \in (0,1]$.

$$\hat{\phi}_k = \hat{\phi}_{k-1} + \frac{\eta \Delta u_{k-1} (\Delta y_k - \hat{\phi}_{k-1} \Delta u_{k-1})}{\mu + \Delta u_{k-1}^2} \tag{40}$$

Where: $\mu > 0,\ \eta \in (0,1]$.

V. CONCLUSION

The paper has presented the compact form dynamic linearization model for a nonlinear system and synthesized an adaptive control law based on the estimated partial derivative parameter (PPD) for a class of SISO nonlinear objects. To synthesize the CFDL-MFAC control law, the article proposed a new representation of the objective function for the optimization problem of minimizing the objective function, applying extended differential theory to solve the problem of loss and lack of data due to network connection or sensor error.

Mastering the data has brought the conventional model based on physical and chemical laws to a purely mathematical model thanks to the dynamic linearization tool. The simple structure of the MFAC algorithm and dynamic linearized data model provide adaptive control solutions for many complex systems. As long as data is available, systems with strong nonlinearities, non-minimum phases, and time-varying delays become apparent with explicit mathematical representations. In the following studies, the authors apply CFDL-MFAC to an optical control system with measurement delay, adding visual simulation as a basis for further algorithm comparison and evaluation.

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