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# Mathematical Tools for Quanta

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**Abstract**
The first section describes dihedrals as tool for Heegard decompositions of the SU(2) geometry, a 3-dimensional unit sphere. For gravity waves a new geometry and energy related interactions is invented as earthworm. It sets the stretching squeezing property of gravity on the earthworm. The tetrahedron configuration for nucleons and the *rgb*-gravitons as neutral color charge of nucleons is discussed. The use of $S_4$ permutation subgroups is described. In the last sections many examples are presented concerning quadruples such as coefficients of a Moebius transformation and triples such as rgb for the neutral color charge of nucleons and many other (often numerical) triples. General relativity is taken up using the tools of catastrophes, cross ratios and triple mappings by a cross ratio. Measuring tools are discussed.
***Keywords:*** *color charges, dihedrals, cross ratios, earthworm*

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## I. DIHEDRALS

Dihedrals use roots of unity for energy measures in equal distances on a circle $S^1$. If two frequencies hit in an xy-plane orthogonal in proportion 1:1, the energy fills out a circle or ellipse. The symmetry is the identity map id. In polar complex coordinates, the exponential function exp(iφ), φ polar angle, describes its points on a unit circle.

In setting n poles on $S^1$ as solutions of polynomials $z^n -1 = 0$, for many valued functions of energies, the first case is z - 1 = 0. The symmetry is a reflection σ and id = $\sigma^2$. There are in general Fibonacci like series as solutions of difference equations available. For this case, an initial value 0 sets 0 as series. It is used by the gravity potential where Higgs can set at a barycenter a mass scalar for a system P with mass. P has to be 3-dimensional, for lower spacial dimensions of P Higgs sets no mass. For other initial values b, the series is $c\cdot b^n$, n a natural number, c constant. For the initial value 1 the series is 1. This can set units for energy measures, using eigenvectors of symmetry matrices. If xy-coordinates of the $S^1$ plane are provided with eigenvectors (1 0) for x and (0 1) for y, it is possible to use them as 2x2-matrices for transformations of vectors. The identity has the first row (1 0), the second row (0 1). For σ the eigenvectors are (1 0) in the first row, (0 -1) in the second row as reflection on the x-coordinates line This introduces the second roots of unity $z^2 -1 = 0$ as quadric. The polynomial $z^2 - 1$ is for dipoles +1,-1. If dipoles arise for energies, they are joined by field lines or a flow around the poles. The 2 roll mill from catastrophe theory presents them driven by a common potential.

In another choice $\sigma_2$, complex numbers are introduced, the eigenvectors (0 1) as first row and (-1 0) as second row is used, the identity vectors are counterclockwise mpo rotated by $45^0$. The matrix is of order 4 for the powers of the imaginary unit i. Complex numbers are written as $x\cdot id + y\cdot\sigma_2$. Complex numbers z habe the Moebius transformations (az +b)/(cz +d), ad-bc ≠ 0, as symmetry.

There are two dimensional extensions mentioned for the case of b = -1. One is the 2 roll mill with a lemniscate as retract and two nonretractible circles as homology generaters. The second extension was for the number system in use. Real numbers have been doubled to complex numbers, the underlying vector space is blown up from one to two dimensionas by setting a new orthogonal base vector for i. Both extensions can be repeated, using for b roots of unitites which repreat as powers cyclic.

For coordinate extensions, the former two Pauli matrices $\sigma_j$ , j =3,2 are multiplied for getting $\sigma_1$. The quaternions are generated by adding the identiy id matrix. The complex plane is in quaternion notation extended to $z_1$ = z + ict, $z_2$ = x +iy. The space coordinates are x,y,z, t is time. For octonian numbers the quaternions q are doubled, also in form of 2x2-matrices with first row ($q_1$ $q_2$). Conjugation c(u) determines for both extensions the second row as (-c($u_2$) c($u_1$)). More details on constructing quaternionic and octonian (Cayley-Dickson) multiplications are available. The 8-dimensional case is not for the multilpication of GellMann SU(3) matrices for the strong interaction. Concerning number systems with multiplication, their is no extension for octonians.

The second geometrical extension from a circle to a lemniscate for homology generators can be arbitrarily repeated for n-th roots of unity. It uses Lissajous figures for two orthogonal hitting frequencies in proportion 1:n. This is heat bound, accoustics. A vibrating string is the model. The string is divides into (n-1) parts and generates for n = 1 one vibration of the whole string. A homology circle of a closed surface is obtained carrying the frequencies energies. The former sphere $S^2$ surface is replaced a torus of genus 1. This shows up int the Heegard decompositions of the unit sphere $S^3$ for the Hopf fiber bundle. A latitude circle $S^1$ of $S^2$ is geometrically blown up by the fiber $S^1$ to the torus. The sphere $S^3$ decomposes into two solid 3-dimensional tori having this surface. For the $S^2$ case the $S^3$ decomposes into two solid balls with surface $S^2$. The Lissajous figures for a vibrating string mean that the vibration generates different accoustic tones and the nodes k= (n-1) = 0,1,2,.. are repeated for (n-1) homology generators. For quarks as dipoles the lemniscate for a Heegard decomposition of $S^3$ is into two solid 3-dimensional quarks with surfaces as brezels of genus 2. For nucleons k = 3, for 4 roll mills k = 4, for 6 roll mills k = 6 and with k = 8 the computations get too complicate, already for the 8 roll mill. No higher genus Heegard decompositions of $S^3$ arise. Catastrophe theory has the computations available as seven cases (figure 7). The generation of new rolls jumps from 4 by two since there is a new motor generated which drives two rolls. The case of nucleons is an exception where three color charges of quarks

r,g,b as independent force vectors drive the rolls. For n = 2 the driving motor is POT, potential unified for the gravity and electromagnetic potentials. For n = 4 is added a driving motor WI of the weak interaction, it drives magnetic force and kinetic energy. For n = 6 a driving SI rotor is added by a motor which drives heat as force and rotational energy. This evolution of 6 energies occurs after a big bang or decay of a black hole and is described in other articles of the author. For the case n = 4 is added a motor which drives the first and last octonian coordinate as input and output energy for the evolution of six forces. The input are the cross ratio invariants of the $S^2$ Moebius transformations as six color charges which are perspective projections. The output as eletromagnetic interaction and force is generated in the evolution late.

Concerning the Heegard decompositions, There are $S^3$ decompositions in two brezels of genus k where $S^3$ splits into two energy systems: heat volumes with entropy inside for k = 0, two leptons for k = 1, two (anti-)quarks for k = 2, two nucleons (proton, neutron) for k = 3, two k roll mills for k = 4,6,8. The vibrating srings nodes are setting the Lissajous generated accoustic tone for this.

## II. EARTHWORM

There is a 10-dimensional model string theory which is not the earthworm. The author uses her presentation of *rgb*-gravitons as neutral color charge of nucleons for gravitational waves as the earthworm model. As experimentally measured stretching squeezing this is done by the earthworm

cylinder in a 6 cycle, as also postulated for nucleons pulsation. Accoustic energy, phonons are exchanged with the environment in time. This is experimentally also observed for electromagnetic waves EMI having redshift. Periodically or stochastically phonons are emitted decreasing the EMI frequency. The accoustic release is observed as noise or radiowaves in the universe. For pulsars as neutron stars it is observed as such periodic emissions where also graviton waves can be emitted.

If the cross ratios for r,g,b are composed as functions, the *rgb*-graviton is a rotational cross ratio, as complex Moebius transformation (z-1)/z. The coefficient matrix G is of order 6. How a Moebius transformation like G is constructed from a triple like r,g,b is discusssed later.

For nucleons, the *rgb*-graviton are presented in the tetrahedron configuration as a spin-like base triple Γ with vectors pointing in the color charged quark directions 1 r x, 2 g y, 6 b (-z). The numbers stand for octonian coordinates 1,2,6, having energies associated as electrical force (charge 1), heat (phonons 2), kinetic energy (frequency 6). In contrary to spin, the weights are color charges, as cross ratios, not a length measure as for spin coordinates $s = (s_x, s_y, s_z)$.

The action of the matrix G is for a dynamical 6 cycle, called SI rotor. For its six states the gluon exchange between two quarks makes the changes, presented as $D_3$ symmetry of the quark triangle.

The gluon action in six steps is used for the earthworm.

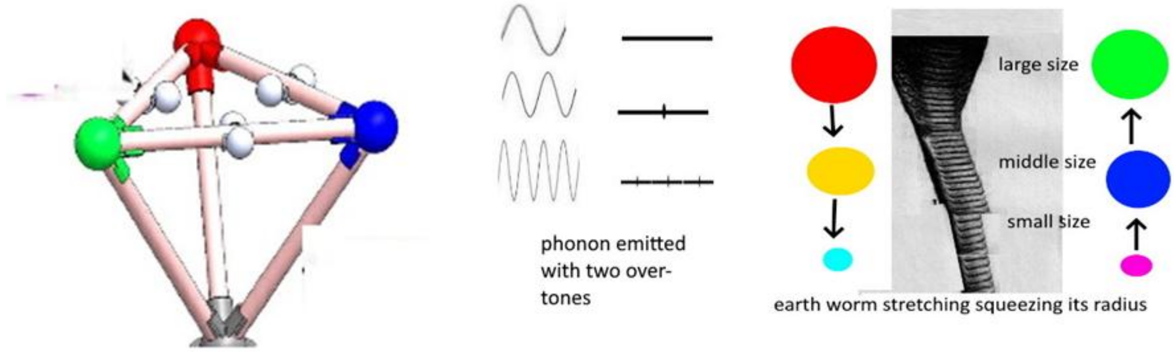



**Figure 1** rgb-graviton tetrahedron, gluons are marked on the rgb quark triangle sides; radial streching (or squeezing) of the earthworm in a time pulsation

If spectral series (photons) are emitted from atoms, the geometry is a circular U(1) cylinder for its helix expansion as exponential function and helix line on the cylinder surface. In the rgb-graviton case, observed as wave, it is neccessary to let two linked exponential helix waves for rgb and for their dual color charges c(u), u = r,g,b, expand in time on a cylinder. The surface is for a earthworm. There are six periodic in time exchanges between paired uc(u) color charges for six earth worm states. The pairing has associated the Heisenberg uncertainty pairings, in octonians 15 position-momentum, 23 angle-angular momentum, 46 time-energy. From the $D_3$ group a degenerate orbit for basic spin lengths, in proportion ½:1:2 is used for the radius of the cylinder, gravitational stretched or squeezed. In time intervals it means that the cylinder has the large radius 2 (graviton spin) in state 1, the middle radius 1 (the HU coupling) in state 2, the small radius ½ (helix distance as interval I) in state 3. The radius retraction is reversed in states 4,5,6 from small to middle to large. In time the cylindrical earthwurm radius stretched or squeezed during its time expansion is not an exponential differential equations solution as for EMI. Neither has spacetime as a vacuum stretching or sqeezing. It is the scaling of Minkowski to Schwarzschild metric which the earthworm generates for measures. Measures are quadric defined, mostly as projective incidences of a point with its dual hyperplane space. Minkowski metric is obtained this way. The nonlinear Schwarzschild rescaling has another source. A nonlinear Moebius transformation as a perspective projection is applied which provides the Schwarzschild factor for rescaling Minkowski metric.

The *rgb*-graviton $D_3$ dynamics of the SI rotor is added to the earthworm with the HU replacing gluon exchanges. The radius change is interpreted as an energy emition or absorption of accoustic generated phonons. At the small radius the tone c is generated as a vibrating string-like I between two HU members. The string length is 1. At the middle radius the string has a node at ½ and generates the overtone c'. At the large radius the string has three nodes as k/4, k = 1,2,3 and generates the overtone c'"'. Then in state 6 a phonon as accoustic tone c with two overtones c',c'"'

is emitted. The cycle for the expanding radii is then reversed to contraction. At state 1 a phonon c.c',c'"' is absorbed form the environment and the cycle starts again expanding.

For the sudden changes, there is a cusp potential for the length contraction, expansion. This catastrophe has three surface levels, maximum potential for a c'"' vibrating I, middle potential for c' and minimum potetnial for c. The cusp equation has two variables for a transversal radius circle of the earthworm $x^2 + y^2 = r^2$, two parameters as constants a,b and a control space having the equation $4a^3 - 27b^2 = 0$. The parameter a can be inerpreted as a revolution time T for the helix rotation as one winding about the circle. The parameter r = b is a length measure with value 2. It is the proportion between the changing radii ½:1 and 1:2. In this form, the third Kepler law appears as $T^3/r^2$ = constant. The radius is taken as proportion between two radii, a parameter. For the Kepler ellipse it is the length a of the main axis as diagonal 2b.

For the HUs 15, 23, 46 as uc(u) the gravitons color charges 1r, 2g, 6b have exchange paired 1-5 potential gravitational (mass related) energy, 2-3 rotation, 6-4 a time interval Δt, inverted to frequency 6 as f = 1/Δt. The *rgb*-graviton is for mass the replacement of a magnetic momentum/field quantum for the electrical force as charge. Higgs attributes the scalar mass to systems, electrical charge has the same form of - e/r, r radius, e constant, for its potential as mass. 15 is for their unification λp = h, h the Planck constant. For 23 the unification is for heat 2 (accoustic phonons) with rotational energy 3 φJ = h and for 46 time as interval the unification is E = hf. In the first case a wave length r = $\lambda$ is for the common potential inverted to -e/r. For the last case, time as interval is inverted to frequency. An inversion of phonons energy 2 as polar angle $\varphi$ to rotation is using angular speed $\omega$ = d$\varphi$/dt in the angular momentum 3.

It is observed that this is not 3-dimensional since the third space coordinate is not used. The action of spin is 3-dimensional presented by the real cross product and Pauli matrices. This cannot be transformed to the *rgb*-gravitons use of 2-dimensional $S^2$ cross ratios as perspective projections of a Riemannian complex sphere. The earthworm is not a U(1) rolled octonian coordinate 7 as universal cover of a circle, presented by an exponential function as wave. The *rgb*-earthworm is using color charges octonian space 123456 and has six not one or 3 coordinates, part of them are treated as energy parameters, not as variables. 7 carries one electromagnetic interaction energy (photons). 123456 carries six color charge energies with associated energies electrical force (charge) 1, heat (phonon) 2, rotation (angular speed, angular momentum) 3, magnetic energy (magnetic momentum) 4, mass (Higgs charge) 5, frequency (linear speed, momentum) 6. The exp wave function of 7 is projected into spacetime (as observable) to its real cosine part. The earthworm is observed as a length stretching squeeezing gravitational wave, not as a simple cosine wave. The metrical measure of the circles radius is not kept constant. Cross ratios are not preserving length, angles or time. Only geometrical incidence and the cross ratios of four perspective projected points on transversal lines is kept as invariant.

## III. $S_4$ subgroups

The cross ratios use quadruples for their permutations. The symmetry is $S_4$. The list of S4subgroups has in addition S3 and a derived quaternion group from Z4 ,an id group, S4 itself, A4 , Z2 x Z2, Z2, D4.

$S_3$ is also a factor group by the normal CPT subgroup Z2 x Z2. The id does scalings, $S_4$ is the tetrahedron symmetry, $D_3$ occurs at different places as factor group and for the SI rotor.

(a) The $A_4$ group contains no reflections.There are four rotations like that of an *rgb*-graviton tetrahedron which b rotate a (quark) triangle, keeping the tip of the tetrahedron fixed, three a rotations through the middle points of opposite edges. With an id added that makes 8 + 3 +1 = 12 members of $A_4$ . The (quark) triangle rotations b keep one element z in a cross ratio fixed and the

other three elements u,v,w are permuted with $Z_3$. The rotations a permute the four elements in pairs (12)(34), (13)(24), (14)(23).

For the four b rotations, one is for the nucleons *rgb*-graviton 126 with 3 quarks. Other useful permutations are for the degenerate $D_3$ orbits 0,1,∞; ½,1,2; 1,$p_1$, $p_2$, $p_j$ the third roots of unity.

In the first case the rgb are replaced by the cross ratios z, 1/z, (1-z) 145 with z = 0 for an electrical measuring triple as Gleason operator frame. In the second case the three basic spin values for fermions, bosons, graviton are 234 rotated with z = 2 and z/(z-1), (z-1)/z, (1-z). In the third case the cubic roots are 125 rotated with z, 1/z, z/(z-1) and z = $p_1$. The 234 tetrahedron tip for an earthworm pulsation is accoustic, heat with phonons. The 125 tetrahedron tip is for a Fibonacci series like difference equation with characteristic polynomial $z^3$ -1. The coefficients are x,y,z space coordinates for a space presentation as x + $yp_1$ + $zp_2$ with conjugation interchanging the $p_j$. The quaternions i,j,k are replaced by cubic roots for 3 vertices set of a tetrahedron triangle, opposite to the tetrahedron tip.
The three a rotations permute its elements with a cylindrical axis and two opposite sides of the tetrahedron as diameters of the cylinder. If the numbers are replaced by 0,1 in the $Z_2$ x $Z_2$ group
[(0,1); (1,0)], [(0,1); (1,1)], [(1,1); (1,0)] elements are permuted. For an xy-planes coordinates it means that base vectors are exchanged to [(1,0); (0,1)], [(1,0); (1,1)], [(1,1); (0,1)]. The first interchange of 1,i can be interpreted for exponential wave functions as differentiation. The real part is projected into spacetime as observable cosine wave, the derivative is projected as observable speed of the wave, the second deritative as its force action. An imaginary not observable part of an
complex function is made observabe by an i multiplication or reversely an observable part is made not observable by the i multiplication. If in the Heisenberg uncertainties (as examples) 15 1 is real and 5 imaginary, then an observable electrical potential is changed to a an observable gravity/mass potential (use 145). In 23, if 2 is real and 3 imaginary, observable heat is changed to observable rotational energy (use 123); similarly in 46, if 4 is real and 6 imaginary, observable magnetic energy is changed to observable kinetic energy (use 246).
(b) The Klein group Z2 x Z2 is for the CPT symmetry and the factoring of $S_4$ to $D_3$. The factor classes are described in the table: every class contains a color charge, an octonian coordiante 1,2,...,6, an energy and a $D_3$ symmetry.

( c) Z2 is for a dihedral $D_1$ with one point and one reflection. This can be used at many occasions for 0,1, for +1,-1, for clockwise, counterclockwise rotations, complex conjugation,...

(d) $D_4$ is a quadrangles symmetry with the $Z_4$ rotation as subgroup. The vertices can be the imaginary numbers +1,i,-1,-i as solutions of the $z^4$ -1 roots for $Z_4$. Beside introducing complex numbers $D_4$ can present the magnetic symmetry and the octonians as number system for 8 energies. Quaternions are the noncommutative version of Z2 x Z2 and octonians are doubling the quaternions.

Quaternions present spacetime coordinates.

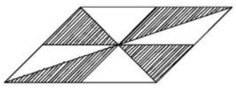


Figure 2 magnetic symmetry, also used for an 8 roll mill as catastrophe

***Quadruples, triples and Couplings***

The nth roots of unity have special applications for the cases n = 2,3,4, discussed now. They are not the source for them, but are replaced by useful combinatorial combinations. Some examples are discussed. The theme is open for research and experiments. The number n can be chosen also for the cases 5,6,8 with examples listed in articles of the author (6 or 8 roll mills for instance).

The cross ratio factor quadruples are listed above for the tetrahedron and its $S_4$ subroups. A cross ratio can map any quadruple of a $S_4$ factor to another one. In composing cross ratios as maps, it means that for instance 1/z applied can map as pairs (1-z) to 1/(1-z) and z/(z-1) to (z-1)/z and z to 1/z for the Heisenberg uncertainties. A postulate is that a reference triple consisting of three complex numbers (p,q,r) can be extended by

In the first line of the table are the spherical SI coordinates, possibly 7- (not 8-)dimensional extended with exponential/polar coordinates. In the second line are the linear Pauli/Euclidean coordinates, in the third line a distribution of color charges to the SI coordinates. The fourth (fifth) line contains the $D_3$ (SU(2)/Pauli) MTs as cross ratios. Their matrix names are in the sixth line, together with the Einstein matrices. The following line is a numbering for a strong 6-fold integration series (not the Fano figures numbers which are for octonians). The next line contains the Planck numbers. Energy vectors are in the second to last line and the last line contains natural constants and three more operators, C (conjugation for quantum numbers), T (time reversal) and P (space parity) of physics.

| | | | | | |
|---|---|---|---|---|---|
| $r$ or $re^{i\varphi_1}$ | $\varphi$ | $\theta$ | $ict$ | $iu$ | $iw$ |
| $x \in \mathbb{R}$ | $iy \in i\mathbb{R}$ | $z \in \mathbb{R}$ | | | |
| $r$ | $g$ | $\overline{g}$ | $\overline{b}$ | $b$ | $\overline{r}$ |
| $z$ | $\frac{z}{-z-1}$ | $\frac{-z-1}{z}$ | $(-z-1)$ | $\frac{1}{-z-1}$ | $\frac{1}{z}$ |
| $\frac{1}{z}$ | $-\frac{1}{z}$ | $-z$ | $z$ | | |
| $id; \sigma_1$ | $\alpha\sigma_1; \sigma_2$ | $\alpha^2; \sigma_3$ | $\alpha^2\sigma_1; id$ | $\alpha; \delta$ | $\sigma_1; id; \beta$ |
| 1 | 6 | 4 | 2 | 5 | 3 |
| length $\lambda_P$ | temp. $T_P$ | dens. $\rho_P$ | time $t_P$ | ener. $E_P$ | mass $m_P$ |
| $EM_{pot}$ | $E_{heat}$ | $E_{rot}$ | $E_{magn}$ | $E_{kin}$ | $E_{pot}$ |
| $c, e_0, \epsilon_0$ | $k$,C | $N_A$,T | $\mu_0$ | $h$ | $\gamma_G, R_S$,P |

Table 1 The line 1 6 4 2 5 3 has to be changed to 1 2 3 4 6 5.

the complex variable z to a cross ratio quadruple (z,p;q,r). This can be seen as a projective extension where p,q,r are interpreted as parameters and the extended space (p,q,r,1) is projective [p,q,r,w] with a variable w. The projective duality means that a point (p,q,r) has associated a hyperplane px +qy+rz = 0 in a 4-dimensional projective space [x,y,z,w]. - The examples show how quadruples, triples, pairings can arise.

An exercise in [13] is: find a Moebius transformation M = (aw+b)/(cw+d) with M(0) = p, M(1) = q, M(∞) = r. A solution is for instance given by defining a,b,c,d through p = c/d, q = (a+c)/(b+d) and

r = a/b, computing the cross ratio of (z,c/d;(a+c)/(b+d) ,a/b) and applying the map w = -1/z. This computation associates with a triple of points p,q,r a Moebius transformation M, having four scalars a,b,c,d with ad-bc ≠ 0 and one variable.
Another quintique example is the butterfly catastrophe with one variable and four parameters. It is
used for instance for the buoyancy and stability of an elliptic ship whose cusps have a butterfly configuration.

Pairings are know for instance from Cooper pairs, Heisenberg uncertainties, gluons, weak bosons and basic physical equations. The vector space dimensions for number systems show pairings. First, real numbers are paired extended complex numbers using the imaginary number i. In 2x2-matrix extension, the id matrix for x is paired with the second Pauli matrix having first row (0 1), second row (-1 0) which muliplies like the root of unity i. The spin pairing of complex numbers to space coordinates in form of $z_1$ = ct + iz, $z_2$ = x + iy has the 2x2-matrix with first row ($z_1$ $z_2$) and second row (-c($z_2$) c($z_1$)) where the third Pauli matrix is for z, the first Pauli matrix for y and the second Pauli matix for x. The id map is for time. In a similar way the Cayley-Dickson construction pairs two quaternions to an octonian 2x2-matrix with first row ($q_1$ $q_2$). The octonian coordinates are listed by their indices 01234567 since subspaces can be easier read. 1234 is quaternionic spacetime, 56 an energy space for mass. Frequency, 123456 is for color charges, 0 for them is an input force (6th roots of unity, the G-compass tool), 7 for the electromagnetic interaction as output force (- the polar angle φ is mapped 27 to the exponential function φ → exp(iφ)). The Heisenberg HU pairings 15 position, momentum 15 mean that radius is inverted to potential in r → -e/r. The time, energy 45 HU is for inverting a time interval to frequency Δt → 1/Δt. The angle, angular momentum uncertainty 23 invertes their Moebius transformations z/(z-1) → (z-1)/z by applying 1/z, heat as energy 2 to rotational energy 3. For the color charge force 0 can be used 03 as a new uncertainty which sets for speeds the bound c between systems speed (use 3 as angular speed and θ angle and substitute 2 in φJ = h by c) in the universe v < c and dark energy speeds v' > c. Another such inversion like v'v = $c^2$ is observed for dark matter as r'r = $Rs^2$ for the Schwarzschild radius of a black hole, dark matter. In the 15 pairing λp = h, potential is substituted by Rs. In both cases, the factors v/c or $r/R_s$ for the universe measures are inverted to the dark energy or dark matter measures

c/v' or Rs/r' and set equal. The associated surface geometries have different metrical quadrics. For speeds the Minkowski cone quadric applies, for radii the Euclidean quadric. If color charges (as force 0) are presented as vectors they rotate like magnetic field quantums and generate a cone surface $r^2 - c^2t^2 = 0$ (substitute for a variable speed v = Δx/Δt radius by a x-coordiante interval Δx and t by Δt; for the EMI 7 speed v = c holds). For the Schwarzschild radius as constant r = Rs, the second cosmic speed was set to c, as Euclidean quadric can be taken $r^2 = x^2 + y^2$ for a Bohr shell of a complex Riemannian sphere $S^2$ as surface geometry of a black hole with coordinates z = x + iy and a stereographic point ∞ or number added to the complex plane. It has the stereographic (Hopf) mapassociated.

The former list is repeated at other places in the article in another connection not related to pairings.

The last pairings for inverting dark energy or dark matter to universes energy or matter introduce surface locations. Dark matter is located 2-dimensional on a Riemannian sphere with color charges as geometrical invariants, dark energy has as projective closed pinched torus $T_0$ 3-dimensional dark energy inside with speeds large than c. $T_0$ is obtained from a torus by retracting one transversal circle to a critical Morse point; as surface invariant can be taken U(1) for a rotational frequency of EMI, using photons as helix line on a cylinder, expanding in time. If the Hopf map with the three Pauli matrices invariants of quaternions is used for presenting the lepton $h(S^3) = S^2$, defining three space coordinate quadrics by using four spacetime coordinates (projecting time to a constant like c) then dark energy can have a leptonic $S^2$ surface location moving inside the pinched torus with speed v' > c and momentum p = hfv'/$c^2$ where mass is replaced by m = hf/$c^2$. The leptonic $S^2$ surface contains the rotating energy on three possible radii in frequency f proportion ½:1:2 of the basic three spin values for fermions, bosons, graviton with associated momenta for dark energies $S^2$ in proportion $p_{1/2} < p_1 < p_2$. In sixth roots of unity the G-compass generates the signed six electrical charge values for leptons and quarks in proportion 1:2:3 as 1/3:2/3:1. A Moebius transformation can be computed which transforms the numbers of frequency proportions as 1:2:4 to the electrical charge proportions 1:2:3 numbers. In 1:2:4 the first proportion is used for a vibrating string as tone

c with two overtones c', c'''. The second proportion 1:2:3 lets the vibrating string generate c with overtone c' (1:2) and quint g' (2:3 or (1/3):(3/2)) as third overtone. Heat, phonons, can be exchanged from dark energy pinched tori with its environment. For black holes this energy exchange is described by experimental findings of physics, using the accretion disk.

From 14. is quoted a computation: *We investigate the luminosity of the accretion disk of a static black hole surrounded by dark matter with anisotropic pressure. We calculate all basic orbital parameters of test particles in the accretion disk, such as angular velocity, angular momentum, energy, and radius of the innermost circular stable orbit as functions of the dark matter density, radial pressure, and anisotropic parameter, which establishes the relationship between the radial and tangential pressures. We show that the presence of dark matter with anisotropic pressure makes a noticeable difference in the geometry around a Schwarzschild black hole, affecting the radiative flux, differential luminosity, and spectral luminosity of the accretion disk.*

A similar paper has not been detected by the author for dark energy (open research). Metrics and potential are mentioned. From two arxiv.org publications 15. is quoted: *A geometrical interpretation for dark energy as warp in the universe given by the extrinsic curvature. In particular, we study the phenomenological implications of the extrinsic curvature of a Friedman-Robertson-Walker universe in a five-dimensional constant curvature bulk, with signatures (4,1) or (3,2), as compared with the X-matter (XCDM) model.*
and
The *Wheeler-DeWitt (WDW) quantum potentialQ[gjk]* is *(as) the natural candidate for the dark energy*

For the general relativistic scaling factor $\cos^2\beta$ with $\sin^2\beta$ = Rs/r, r radius, Rs Schwarzschild radius of a central system like a sun, another triple transformation can map the reference points for cross ratios 0,1,∞ to 0,∞,Rs and the Moebius transformation is w = (r,0;∞,Rs) = (r-Rs)/r. For w = r the Schwarzschild scaling factor is obtained. It is interpreted as an unsymmetrical distance measure. A sun Q measures its distance to a rotating planet as |QP| = r, the planet P measures its distance up to the suns Schwarzschild radius as |PQ| = r – Rs.

As well the Schwarzschild case and Minkowski metric have an orthogonal watch projection associated (see the section on measuring tools). Two intersecting rays carry points for the coordinate or energy measures in one observed system and either down squeezing or up stretching the observer rescales the units by the cosine of an angle δ = β or = θ between the rays. The angle for Rs has $\sin^2\beta$ = Rs/r, for Minkowski sin θ = v/c. For the electromagnetic waves change of frequencies the angle for the two cyclinders axes carries such a mirror angle. Also Moebius transformations can be used for the rescalings.

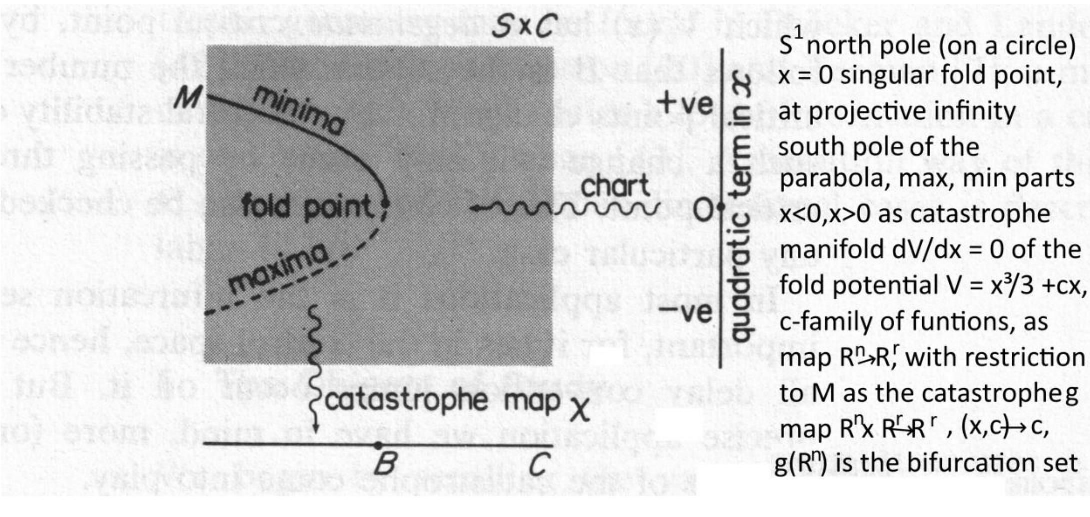
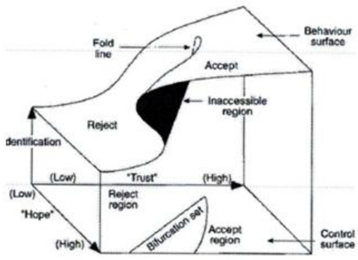



Figure 3 fold from catastrophe theory

A fold catastrophe can be associated with a critical point and the two bifurcation curves containing the minimum potential point at distance r, the maximum potential point at r-Rs. The different lengths arise from the different surface levels of the fold potential, radial measured.



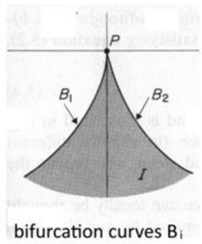




Figure 4 cusp, control space for parameters at right

For the two cosmic speeds of Q a cusp catastrophe can be associated which has three surface levels, due to a transversal computed inflection point. On the bifurcation curves $B_j$ in figure 4 at right the two cosmic speeds are in proportion 1:√2 measured for the maximum, minimum potentials. On the

cusp surfaces, the maximum potential lower part is for the free fall of P to Q, the middle part for a rosette rotation of P about Q and escape of P from Q is on the upper minimum potential level. Higgs sets in this case no common barycenter for Q,P. For the nonlinear rescaling of Minkowski metric to the Schwarzschild metric it means that P and Q act as observer and observed system when measuring their energy or spacetime units. The critical Morse function has for the Minkowski metric the diagonal form diag[1,1,1,-1], measuring for instance time as last spacetime coordinate imaginary. Geometrically, in reduced radius time coordinates, the projective [r,ict,w] plane has the cone quadric $r^2 - c^2t^2 = 0$ where the two lines intersect at the origin (0,0). The metric is $ds^2 = dr^2 - c^2dt^2$.

The radius is dimensional extended to space coordinates and the Euclidean measure $r^2 = x^2 + y^2 + z^2$. The former computation of the rescaling Schwarzschild factor measures time differentials as $dt' = \sqrt{(\cos \beta)}\cdot dt$, keeping the product drdt invariant, the radius differentials are transformed as $dr' = dr/\sqrt{(\cos \beta)}$. The Schwarzschild metric is $ds^2 = dr^2/(\cos \beta)^2 - (\cos \beta)^2c^2dt^2$.

The earthworm model shows how gravitons in a kind of wave expansion in time changes the distance measure about a sun Q. An underlying spacetime vacuum needs no spacetime curvature for this. It is the metrical measure about Q which is changed by the earthworm. The earthworm has nothing to do with the wormholes of string theory.

The tetrahedron model for a nucleon has as base triangle r,g,b color charged quarks. They can present the triple p,q,r in the former computation. In factoring the $S_4$ symmetry of the tetrahedron by the normal Klein group, the quark triangle symmetry $D_3$ is obtained. The reference triple 0,1,∞ was taken for the cross ratio [w,0;1,∞] to obtain for the six $D_3$ elements a cross ratio by permuting the reference triple, keeping w as *rgb*-graviton tip of the tetrahedron fixed. The conjugate c(u) color charges of u = r,g,b are used for the six series of $D_3$. In table 1 the six columns contain in the third line a color charge, in the fourth line an associated Moebius transformation, in the sixth line the associated $D_3$ elements as three reflections and a rotation α of order 3, composed with the first Pauli reflection matrix $\sigma_1$. In a theorem on quadrangular sets in [9], the tetrahedron configuration appears as part of a Koenig's cube.

Some examples of quadrics in dimensions 2 or 3, mentioned in figure 5 are the incidence of a point with

1. *its dual line* $x^2 + y^2 = 1$, a circle and for a distance measured

1a. conic sections for planets orbits: as in 1. or parabola, hyperbola

1b. Minkowski metric in reduced (x,y)≡(r,ict) radius time coordinates $x^2 - y^2 = 0$

2 *its dual space* Euclidean metric for space $x^2 + y^2 + z^2 = 1$

2a. 1-sheeted hyperboloid for its projective closure to a torus $x^2 + y^2 - z^2 = 1$

2b. elliptic cylinder $x^2 + y^2 = 1$

2c. cone $x^2 + y^2 - z^2 = 0$

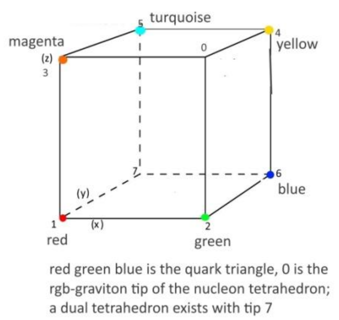



Figure 5 Koenig's cube; seven projective incidences determine the eight 0: a point incident with its projective dual hyperplane detrmines a measuring quadric

As a theorem on tetrahedrons is proved that the flat tetrahedron version is: 0126 are in a plane iff 3457 are in plane. This can be interpreted by mapping the tip of the tetrahedrons onto the barycenters of the triangles. The barycenter of a nucleon was determined as intersection of its barycentrical coordinates. They are set by the SI rotor.

Discussed is now the action of Moebius transformations which can map three complex numbers of a quadruple to an arbitrary chosen other complex triple. Independent of this, useful triples are listed.

Examples for triples are:

w = 2(1-z)/(z-4) maps 1,2,4 to 0,1,∞. The three basic spin values for fermions, bosons, graviton are mapped to the points of a projective real line $P^1$ as [0,1], x·[1,1] ≡ [x,1], [1,0], x a real variable. $P^1$ can be used for the circle of U(1) and means that its diametrical opposite points are identified. For spin it can mean that its up down changes of the spin vector can be done on a Moebius strip with a central circle $P^1$ where it changes as normal to the Moebius strip its direction after one revolution.

There are two clockwise and counterclockwise orientations on the circle for rotations. In a geometrical interpretation of the 0,1,∞ triple, 0 is for a 0.dimensional barycenter where Higgs sets a mass scalar, 1 is for a Minkowki cone, closed at projective infinity by a circle to a pinched torus. ∞ is for a cylinder, closed at projective infinity to a pinched torus where one transversal circle is retracted to a critical point of the manifold and where the manifold function has all derivates 0. The normal quadrical forms are in a 3-dimensional real projective space and normal forms in two dimensions for a circle, for a cylinder or a 2-dimensional sphere, a hyperbola $x^2 - y^2 = 1$ for a torus, parabola $x^2 - y = 0$ for a fold catastrophe, $x^2 - y^2 = 0$ for the cone ($r^2 - c^2t^2 = 0$), $x^2 + y^2 = 0$ for a barycenter or a rotation axis, - plus degenerate cases.

w = (8-2z)/(z+2) maps 1,2,4 to 2,1,0. This can be interpreted as a earthworm curved helix winding which includes the radii changes as parallel circles on a cone. The function is
f(r,t) = rt·exp(iπ((8-2t)/(t+2)), for t = 4 as f(r,4) = r/4, for t = 2 as f(r,2) = -r/2, for t = 1 as f(r,1) = r. The deformed helix winding includes the radial expansions for its three points on the r/4, r/2, r circles. This time generated curve is for the vibrating string phonons as overtone series c,c',c''. replaces for an earthworm a gluon as vibrating string in a nucleon.

The CPT operators as $Z_2$ x $Z_2$ Klein group and as conjugation, point reflection p→ -p and time reversal has associated a cross ratio w = 1/z for cubic roots substituted for C,P,T. Then $1,p_1,p_2$ is mapped to $1,p_2,p_1$. If conjugation is taken for multiplication then ab = ba = c for every triple as used in the Klein group.

For the quaternions i,j,k the multiplication is not commutative uv = -vu. The cross ratio w = -z is for conjugation is (-i,-j,-k). This is used for the noncommutative multiplication as uv = w(v)u.

For the r,g,b triple the gluon exchange and the $D_3$ SIrotor interchanged r,b with an added counterclockwise conic rotation and g,b with an added clockwise conic rotation. This is a $D_3$ representation. Interchanging the members r,g as cubic roots keeping b as number 1 fixed is as above done by w = 1/z. Interchanging orientation as +1, -1 can use w = -z. The $D_3$ rotation permutes 123→231→312. The cross ratio w = (z-1)/z has for $w^2 = 1/(1-z)$ as rotation α by $120^0$ of the quark triangle an application for the permutations 123, α(123) = 231 and $\alpha^2(123)$ = 312. For the orientations as +1, -1, using the $D_3$ Pauli matrix $\sigma_1$ (w = 1/z substituted) for -1 in multiplying it with the rotational elements for the three reflections of $D_3$. The six elements of $D_3$ present the six color charges (table 1).

The triple generating the Schwarzschild scaling factor is listed near the fold figure. For computing Rs, the Einstein energy-momentum tensor is used. Here we mention that the value $Rs = 2Gm/c^2$ is for a de Broglie wave length of graviton waves chosen. The second cosmic speed is set equal to c.
The triple Rs, first and second cosmic speed has several applications.
(a) Rs: In Planck numbers the four basic natural constants c,G,h,k are used. Setting $G^2m^2/c^4 = Gh/c^3$ the Planck mass (left part of the equation) is transformed to Planck length (right part of the equation); Rs/2 is a radial (measured wave) length. The triple c,G,h is used for this. In different powers they define in addition Planck energy and Planck time. Taking in the above construction of a Moebius transformation for a triple the values p = c/d, r = a/b and q = (a+c)/(b+d) (substituting p,q,r for c,h,g) a Moebius transformation M = (aw+b)/(cw+d) can be constructed which contains beside the three constants also heat measure, for instance as (cw+G)/(hw+k). Kelvin k is a rescaling of Planck energy. The perspective transformation M sets after a big bang with its coefficients the four basic natural numbers for the Planck numbers in the octonian subspace 12456, deleting k the c,G,h define them in

1456. In octonians 145 is a measuring Fano triple for electromagnetism (electrical charge, magnetic momentum, induction) and 6 is added for a circular speed ω rotation of
the charge. This can be set after Planck time together with the Hopf geometry for the weak interaction.
(b) The first cosmic speed squared (and normed by c) as gravitational potential - Gm/r keeps the radius as variable, not as constant Rs. Without sign it is doubled to 2GM/r for the second cosmic speed. Above a cusp catastrophe was mentioned for this. For a harmonic quadruple 0,Gm/r,2Gm/r,∞ the gravitational potential is constructed as the middle between 0,2Gm/r. This computation is in a real projective plane. A Moebius transformation for normed 0,1,2 values can use a = -1, b =1, c = 0, b+d = 2 in M = (1-w) which belongs inverted to kinetic energy 1/(1 - w) . If this energy is integrated, speeds for the momentum of systems are obtained. For orbits of planets about a central sun, their speed is compared with the two cosmic speeds of the sun.

Concerning the three problems, Einstein solved by his general relativity: the redshift was explained above in the section earthworm. The double lensing was explained in [2]. It means that a change of frequency for an electromagnetic wave P can occur by emitting a part or absorbing as in the case of double lensing energy which decreases or increases its frequency. In both cases the wave cylinder
has to break its axis for the future wave expansion. The mirror reflection θ describes the axis' breaking, the change of frequency depends on an interaction with another medium Q as (matter) energy source. The angle θ is measured when the P,Q interaction occurs at a point in space. A plane presents the interaction and its normal is used for measuring the leaning angle to the plane. In catastrophe theory such θ bucklings are used for elastic strings or structures. The lenght replaces a
wave length, it can be fixed or variable. For the sudden change of frequency the wave length λ as a substitute of the electrons jumps between Bohr shell radii is used. There are two parameters for θ, λ involved. The only catastrophe with 2 parameters is the cusp with the potential $V = x^4/4 + ax^2/2 +$
bx. If θ, λ are substituted for a,b, the control space has the equation $4\theta^3 + 27\lambda^2 = 0$. Disregarding the coefficient, the proportion satisfies $\theta^3/\lambda^2$ = constant. The research can be to see whether or not this is applicable in experiments for the P,Q interaction.
For the rosette motion of a planet P about a central sun Q, the general relativistic computation determines a periodic angle for the wave presentation. This is explained in [2] as an accelerating action of gravity as potential and force. Accelerated is the speed $v_1$ of P, its momentum. After one revolution the axis of the Kepler ellipse is changed by a fixed angle $\varphi_1$. The main diameter of the Kepler ellipse has its endpoints on two concentric circles. There are $v_1,\varphi_1$ as parameters, again a cusp catastrophe can be added this case. For the redshift the same setting applies. The parameters have been phonons energy and a changing wave length.

***Projective black hole***

A black hole is geometrically a Riemannian 2-dimensional sphere and has the six cross ratios as force with six values. They arise as invariants of the spheres Moebius transformations and replace quaternions for spacetime. Cross ratios preserve incidence and the cross ratio of four points as described earlier. From the Hopf map h is taken that spacetime coordinates are first projected down to space coordinates for the Hopf sphere $S^2$.Then the z-coordinate of space together with the sphericla angle are projected down to a tangent plane of $S^2$ having complex w = x+iy coordinates. In complex numbers w is noted mostly as z which is then not the third space coordinate. A black hole is observed having such a plane as accretion disk where matter or other energies are absorbed, killed forever in dimensions. The black hole as $S^2$ is not observable. A point or number ∞ is deleted. It is for a stereographic map st: $S^2 \rightarrow C$, C the complex z = x + iy tangent plane. In the Hopf notation, the spacetime coordinates are $z_1$ = z +ict, $z_2$ = x +iy. The composed maps $st(h(S^3))$ = C are described for C coordinates by $w = z_2/z_1$ for $z_1 \neq 0$ and $z_1 = 0$ is the point ∞ on $S^2$.

Simplifying the notation for the accretion disk, its complex coordinates are written as z = x + iy.

In a decay of a black hole such as a big bang, the missing coordinates z,ict have to be generated.

The spin as $s = (s_x,s_y,s_z)$ is adding though the use of the real cross product the missing z-coordinate and sets it as the line through the points 0 in C and ∞. The sphere $S^2$ has then a spin ½ available for

emitted leptons, generated after a big bang. For the cross ratios is this an extension of the xy-plane coordinates presented by the Pauli matrices $\sigma_1$ for x, $\sigma_2$ for y to z with $\sigma_3 = \sigma_2\sigma_1$ where the complex notation for $\sigma_2$ is not listed for the cross product. The coordinates are for a $S_4$ in the factor classes

(r x), (g y), (c(g) z) where c(g) is the conjugate magenta color charge, g is green, r is red. The space metric is Euclidean $r^2 = x^2 + y^2 + z^2$, r spherical radius. The spherical angle θ is also generated measuring for vectorial rotations about the z-axis an angle of a conic rotating vector with initial point ∞ towards the z-axis. The magnetic momentum of an electrical charged lepton is in superposition with spin along the z-axis. As magnetic field quantum it is presented by a rotating vector which traces out in a leaning $45^0$ angle towards the z-axis a cone. The projective incidence

for the Euclidean quadric is obtained by a point (a,b,c) being incident with its hyperplane

$ax + by + cz + dw = 0$ in $a^2 + b^2 + c^2 = 1$, norming the last projective coordinate. The parameters a,b,c are replaced by variables x,y,z for the metric and 1 is repleced by the variable r for the above metric. The magnetic cone has another incidence quadric associated as $r^2 - c^2t^2 = 0$, t time, c speed of light. It arises through the inverse Hopf map $h^{-1}$. The Heisenberg uncertainty angle-angular momentum requires form spin plus magnetic momentum as vector in the $S^3$ Hopf geometry that it rotates in a $45^0$ angle about a new time-rotation axis A. In the extension of coordinates an electrical charge as point is extended to a $45^0$ leaning circle on a torus with A as rotation axis and fills out the circle in rotation. Magnetic momentum is orthogonal to this circle. There are three critical points p,q,r on the circle, computed by the inverse Hopf map. For A can be taken a generated time axis

scaled by ic. In projecting A down to $S^2 = h(S^3)$ space coordinates, the Pauli matrices act by mulitplying them for quadrics with the complex coordinates $c(z_1)z_2$. These quadrics are for the complex dot and cross products on first, second space coordinates and a quadric for two circles as

$(x_1^2 + x_2^2) - (x_3^2 + x_3^2)$ with a suitable permutation of the spacetime coordinates. For the Hopf tori as location of the electrical charge in $S^3$ it shows up as topological product $S^1 \times S^1$. The Hopf map is for the weak interaction generated after a big bang. It has three field quantums, weak bosons, related to the three spin coordinates or Pauli matrices.Their geometry $S^3$ allows leptons and weak

bosons carrying mass. For the color charges it means that the turquoise color charge gets an octonian coordinate 5 associated. Spacetime 1234 coordinates are extended by a real cross product to 12345, a 5-dimensional octonian subspace. 5 is for measuring mass as energy in kg, not an additional spacetime like coordinate measured in meter or second. To 4 is added as energy magnetic force or vectors. To 1 x or r is added the electrical force as energy, to 2 y or φ heat as energy, to 3 z or θ rotational energy. The Heisenberg inversions position momentum 15 and 23 are complemented by the third HU time energy in the form $E = hf$ with frequency f satisfying the equations $hf = mc^2$ for m mass, h the Planck constant and $f = 1/\Delta t$ for a time interval Δt. The octonian cross product extension is 123456 with 6 for measuring f as kinetic energy in Hz.

In these coordinate extensions, the $D_3$ permutation symmetry members are added to the triples of the $S_4$ factor classes. Every class contains a color charge, an octonian coordinate, an energy and a $D_3$ symmetry for measuring with its eigenvalue the energy or xyzt-coordinate. The Pauli matrices are not used for this. As described earlier, $S_4$ as tetrahedron symmetry is factored by the Klein group to $D_3$. The second fermionic series of quarks can be generated in form of nucleon triples like r,g,b color charged quarks. This requires a radius inversion at the Schwarzschild radius Rs of the decaying black hole $r'r = Rs^2$. The quark radius in the universe is measured by r > Rs. It determines also the nucleon radius in spacetime. It has however not a fixed length measured in meter. The gluon exchange between quarks allows length stretching squeezing as property belonging to gravity. Quarks can only have a stable configuration in a nucleon as tetrahedron. Above the *rgb*-graviton was used for the tetrahedron. It means that this perspective projection maps the center of a nucleons bounding sphere down to the barycenter of the rgb quark vertices triangle. The $D_3$ symmetry is acting for generating barycentrical triangle coordinates which intersect in this barycenter. At this point Higgs sets a huge mass scalar for the nucleon compared with the quarks mass of about 10 percent. This new mass is computed in QCD. It relates to transferring inner kinetic or rotational energy to added mass and to rescaling mass by the special relativistic factor. Not only radius of a

black hole is inverted $v'v = c^2$, but also speeds at the Minkowski cone. Speeds v' in the black hole are larger than speed of light. The speed v with which nucleons move in the universe on their world line is special relativistic mass rescaling computed through using their formula in optics. The measuring quadric for spacetime as Minkowski metric is generated. The stretching squeezing property for the nucleon through the use of *rgb*-gravitons adds to this a nonlinear Schwarzschild radius scaling factor as described earlier. It is in time a cylic pulsation, contraction-expansion in three states where also phonons as energy are involved (see the section earthworm). The fermions quark are not energetically stable without being confined by gluon exchanges and an attched projective projection as *rgb*-graviton. They decay like the black hole, generate weak boson which itself decays into leptons. For instance a d-quark decays into a u-quark and reversely. Also in mesons as paired quarks the meson decays.

Since a black hole has six color charges as invariant cross ratios the space for nucleons is not spacetime. As noted earlier it is an octonian subspace 2356 which is projected into spacetime. For its projection a spacetime sphere bounding a complex inner $CP^2$ 2356 space. As mentioned, its radius is not a fixed constant, but depends on the nucleons energy level. In the 2356 projection to this $S^2$ sphere Bohr shells for color charged polar caps are generated. The black holes color charges cover in hemipshperes the nucleon volume inside $S^2$. The energies belonging to them act as a central vector in direction of the xyz-space axes. They are arranged as in the HU as 15 on x(-x), 23 on y(-y) and 46 on z(-z). A valve for an energy exchange of a nucleon with its environment can be a catastrophe cusp which allows the energy vector from a normal to $S^2$ up out-direction to change to a down in-direction for absorbing energy. Up is for energy emitted. The hedgehog was described in other articles of the author (see 2.-8.).

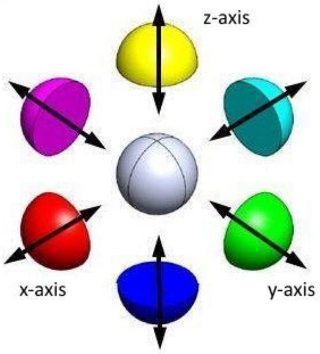



Figure 6 hedgehog

The universes matter can develop after the hedgehog is generated. This is after the Planck numbers are generated and physics exists with its formulas. An exception is for a long time period the dark universe where spectral series cannot be emitted from atomic kernels. Electromagnetic waves are not existing until the universe gets light.

*Measuring Tools*

In the former sections different tools are presented for measures. In particular, the real *cross poduct* for spin like measures and the *cross ratios* for color charge measures. They cannot be transformed into one another. Pauli matrices multiplied permute Pauli matrices noncommutative. Cross ratios as perspective projections composed give cross ratios, also noncommutative as composed functions. The cross product measures for instance with the third generated vectors length the area spanned by the composed vectors. Cross products are not preserving length or angles. They preserve cross

ratios of four collinear points and projective incidence of a suspace with its dual space. The incidence of a point with its hyperplane in a real vector space provides quadrics for measurements.

All normal forms for quadrics in 2 or 3 dimensions can be used for measures. The real cross product provides in componentwise multiplication for the real vector space a Euclidean metric. Minkowski metric is not generated by it. Since cross ratios contain a complex variable they are essentially nonlinear as observed for the Schwarzschild metric. Also Minkowski metric has such an instance. This is due to the fact that the variable of the Moebius transformation is in a functional relation with the measured systems variables. In the special relativistic renorming of energy measures, frequency is an example since the special relativistic speed as $v = \lambda f$ and frequency f belong to kinetic energy.

The transformation uses not the norming by the relativistic factor. Instead, an accoustic travelling wave character of the measured system is used and the Doppler effect. For z = v as variable and relativistic speed the Moebius transformations M (-v +c)/(v+c ) or (v +c)/(-v+c ) are used when the observer moves away or towards the transmitter as source of the accoustic wave. No medium is needed for this. In case the observer moves orthogonal to the wave, the special relativistic factor is renorming the observed wave frequency $f' = f \cdot \cos \varphi$, $\sin \varphi = v/c$.

For the cross ratios renorming effect it is important that such a Moebius transformation M containing four constants as parameters and one variable can be computed for the rescaling of measuring units from a reference triple p,q,r of complex numbers. After the table 1 is found how this is done. It is used above for the earthworm stretching and squeezing. Other examples are: take

for the Doppler effect p = c/d = 1/c or -1/c. R = a/b = -1/c or 1/c and q = 0/2c; for the Schwarzschild factor the matrix M = (r-Rs)/r was computed earlier, using as reference triple p = ∞,

r = 1/Rs, q = 2/Rs for the constants a = 1, b = Rs, c = 1, d = 0.

Beside these two tools the octonians seven spin-like frames, the measuring triples of the Fano memo are generating measuring Gleason operators. The weights attached to three paiwise orthogonal vectors can have real, complex or quatrnionic numbers as valued. For color charges, their vectors are attached instead of numbers a cross ratios. Since to them in table 1 are associated energies, they can be replaced by numbers for the measuring units of the energy. For instance mass as energy 5 is associated with the octonian subspace triple 257 and can carry for the fermionic series 3 complex numbers for their six masses of the fermionic series. This is a kg measure. For quarks and electrical charged leptons the weights are not changed. The neutrino oscillation shows that the kg frame can change in a base rotation the observable weight. This is an effect of the Heisenberg uncertainty HU 15 where a cross ratio interchanges the $S^2$ base vectors in a time interval. For the HU 23 this was for spin which gets as vector a leaning angle towards a rotation axis and traces out a cone by rotating about the axis. The frame has an additional fourth vector attached, for instance of spin which makes a $45^0$ angle towards the z-rotation axis. Generating from three four vectors is similar to the computation of a cross ratios four scalars from three reference numbers. The third HU 46 uses the M = 1/z inversion of a time interval to a kinetic frequency and is for differentiating time functions in the formula dg(t)/dt. As well time as frequency in 246 frame are presented by their $D_3$ matrices (1-z), 1/(1-z) and inverted by applying 1/z. For 2 the octonian 1 with the 1/z $D_3$ matrix is substituted.

The *Fano Gleason frames* are used for setting the measuring units of energies. The units can be resacled by cross ratios and more general by Moebius transformations.They can carry beside numbers as weights also vectors attached or cross ratio, Moebius transformations. The SI rotor is such a presentation for the cross ratios $D_3$ group. Beside the Fano frames 123 (space), 145 (electromagnetism), 167 (electromagnetic interaction), 245 (heat as force, generating space volumes with entropy inside), 257 (mass with Higgs), 347 (rotational energy with rotation axis generated), 356 (SI rotor) was introduced the strong interaction frame 126 for *rgb-gravitons*. The stretching squeezing property as pulsation was generated by them as mentioned above for the earthworm.

As another measuring tool for sudden energy related changes are the catastrophe potentials. The potentials use 1 or 2 variables and 1,2,3,4,5 parameters and an added critical Morse function with a

normed quadratic diagonal form diag[1,1,...,1,-1....,-1] such as the Minkowski or Euclidean metric. Above the fold and cusp is mentioned. The 6 roll mill model (see figure 8) uses the elliptic umbilic.

The roots of unities measure as dihedrals with the dihedral symmetry groups numerical orbits. Beside the six kg values for the members of the fermionic series, the sixth roots measure with their exponential the electrical charges of electrical charged particles and weak bosons. With two values of $z^2$-1 they measure besid signs as +1, -1 clockwise or counterclockwise orientations which arise in space as left or right hand screws. The use of triples and quadruples in this article is another example which can have dihedral symmetries. The Heegard decompositions of the Hopf sphere $S^3$ have for generated systems surfaces available as manifolds of genus n wich have nontrivial homology groups, circles which cannot be retracted to a point. Dihedrals $D_n$ generate the genus n

their points can be interpreted as poles inside a nonretractible homology circle for residual complex functional integrations about a closed contour.

## IV. CONCLUSION

New mathematical tools for physics are suggested, such as dihedrals, another presentation of graviton waves, the subgroups of the permutation symmetry of 4 elements, the transformations for triples and added quadruples with many examples, for instance gravitational problems are treated.

Cross ratios and Moebius transformations are important tools. A possible black hole big bang is described. Measuring physical events is not as simple as listing Lie algebras with their fiber bundle geometry. A 2-dimensional Riemannian sphere as geometry with six cross ratios as invariants is not transformable to them. Neither are the mentioned cubic roots measures for homology generators or the catstrophes for sudden changes of states. The Gleason measures are for the quantum mechanical measuring process in the Copenhagen interpretation: one system is the observer, the second the measured system. As observable outcome of a measure only one of the weights is obtained, the other ones remain undetermined. Possibly the sum of the weights can be simultaneously measured.

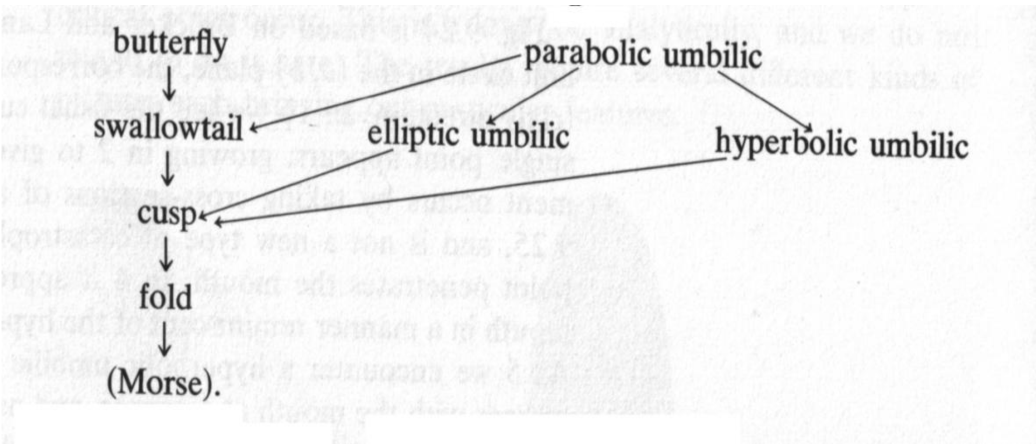



**Figure 7** the seven catastrophes and their inclusions

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Figure 8 MINT-Wigris models in the Emmy Noether Memorial museum, lower left the SI rotor, 2nd box the leptons and weak bosons, 3rd box sterteching squeezing, fusion, handcrafts by a template, 4th box 6 roll mill, gluon exchange, barycentrical coordinates, 5th box g-compass for color charges, dark matter, dark energy, hedgehog