

Analysis of the Capacity of Massive MIMO in Vehicular Networks

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ABSTRACT

In this paper, the effects of outdated channel state information and pilot contamination on the capacity of mMIMO in VANET are analyzed. Mathematical expressions for the signal to interference plus noise ratio (SINR) and mMIMO capacity are derived and compared with numerical results. The results show that the capacity of mMIMO drops considerably after a short interval in fast fading scenarios. Interference due to pilot contamination from nearby clusters considerably reduces the capacity of the system if the power from these cluster is approximately equal or greater than the transmitted power.

Keywords-channel estimation, mMIMO, pilot contamination, VANET

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I. INTRODUCTION

Vehicular Ad hoc Network (VANET) aim to provide safety as well as commercial applications to users on the road. VANET should provide two modes of communication, Vehicle to Vehicle (V2V) and Vehicle to Roadside (V2R). The communication time in both modes can be as short as a few seconds due to the high speeds of the nodes. Hence, high data rates are required to send data between the nodes before losing the link. Massive Multiple Input Multiple Output (mMIMO) systems can provide massive capacity without increasing the bandwidth or transmit power provided channel state information (CSI) is known. Unlike mobile terminals, space and power constraints are not strict in VANET terminals, hence, mMIMO systems can be used at the node to provide high point to point data rates [1] or support multiple users in a cluster-based VANET network [2]. However, due to the high speeds encountered in VANET, the channel varies quickly, and CSI may become invalid very quickly thus reducing the capacity of mMIMO. The aim of this paper is to investigate the capacity of mMIMO in the fast-fading VANET environment for single and multiple-cluster scenarios. The rest of the paper is organized as follows, the next section reviews related work, section III introduces the system model, section VI includes the results and discussion, and section V concludes the paper.

II. RELATED WORK

In cellular mMIMO, channel estimation is obtained via an orthogonal training sequence whose length must be at least equal to the number of transmitters [3]. The effects of outdated CSI due to fast fading were studied in [4, 5, 6] for users in a single cell. It was found that the user throughput drops at high Doppler shifts. In multicell scenarios, the frequency is reused, and the estimated CSI are contaminated by interference from nearby cells using the same frequencies. This is known as pilot contamination. It was shown in [7, 8] that as the number of antennas approaches infinity, pilot contamination becomes the limiting factor for mMIMO. Several methods to reduce pilot contamination were proposed in the literature [9, 10, 11, 12]. These studies considered fixed cells with moderate user speeds. In VANET, however, cars move at high relative speeds, hence the communication time can be very small especially in V2R scenarios. Moreover, cars can form clusters and these clusters move and may merge with other clusters or split into smaller ones, hence there are no fixed boundaries between clusters as in fixed cells.

III. SYSTEM MODEL

In this section we derive the system model. Throughout this paper, upper-case bold symbols are used to indicate matrices, bold lower-case symbols for vectors. $(.)^*$ denotes Hermitian, $E[.]$ denotes expectation and \mathbf{I} is the identity matrix.

3.1 Single Cluster Scenario

In massive MIMO system with M transmit antennas and N receive antennas ($N > M$), the received signal vector \mathbf{r}_k at time index k is given by:

$$\mathbf{r}_k = \mathbf{H}_k \mathbf{s}_k + \mathbf{n}_k \quad (1)$$

Where \mathbf{H}_k is the $N \times M$ matrix of channel coefficients s_k is the $1 \times M$ vector of transmitted symbols whereas \mathbf{n}_k is the $1 \times N$ vector of additive white Gaussian noise. We define the elements of \mathbf{H}_k as $h_{ij} = \sigma_j g_{ijk}$ where σ_j is the attenuation and shadowing factor for user/transmit antenna j to the receiver, and g_{ijk} is the complex fast fading coefficient from user j to receive antenna i at time index k . Without loss of generality, g_{ijk} is assumed to have zero mean and unity variance.

Decoding at the receiver, is performed by multiplying the received signal by an $M \times N$ decoding matrix (\mathbf{D}) to yield an estimate (\mathbf{y}_k) of the transmitted symbols:

$$\mathbf{y}_k = \mathbf{D} \mathbf{r}_k = \mathbf{D} \mathbf{H}_k \mathbf{s}_k + \mathbf{D} \mathbf{n}_k \quad (2)$$

The decoding matrix is calculated from an estimate of the channel matrix $\hat{\mathbf{H}}_0$. This estimate may not be accurate, hence we have $\mathbf{H}_0 = \hat{\mathbf{H}}_0 + \mathbf{Z}$, where \mathbf{Z} is the $N \times M$ matrix of estimation error. The channel estimate is usually obtained after a training period prior to transmission. Several methods to calculate \mathbf{D} exist such as MRC, ZF and MMSE methods [8, 13, 14]. These methods calculate the decoding matrix to obtain:

$$\mathbf{D} \hat{\mathbf{H}}_0 \approx \mathbf{\Sigma} \quad (3)$$

where $\mathbf{\Sigma}$ is a diagonal matrix whose elements depends on the algorithm used. Typically, $\mathbf{\Sigma} = \text{diag}\{\sigma_1^2, \sigma_2^2, \dots, \sigma_M^2\}$ [7]. In slow fading scenarios, it is usually assumed that $\mathbf{H}_k = \hat{\mathbf{H}}_0$ for the duration of transmission, however, in fast fading channels, the channel matrix changes rapidly and hence (3) may become invalid as k increases.

Let $\mathbf{A}_k = \mathbf{H}_k - \mathbf{H}_0$, (1) can be written as:

$$\mathbf{r}_k = (\hat{\mathbf{H}}_0 + \mathbf{Z}) \mathbf{s}_k + \mathbf{A}_k \mathbf{s}_k + \mathbf{n}_k \quad (4)$$

The receiver obtains an estimate \mathbf{y}_k of the transmitted symbols as:

$$\mathbf{y}_k = \mathbf{D} \mathbf{r}_k = \mathbf{\Sigma} \mathbf{s}_k + \mathbf{D} \mathbf{Z} \mathbf{s}_k + \mathbf{D} \mathbf{A}_k \mathbf{s}_k + \mathbf{D} \mathbf{n}_k \quad (5)$$

Let the variance of $\mathbf{D} \mathbf{n}_k$ be σ_n^2 . The variation in the channel responses causes the interference given by the third term in (5). Let $\mathbf{e}_k = \mathbf{D} \mathbf{A}_k \mathbf{s}_k$, then:

$$\mathbf{E}[\mathbf{e}_k \mathbf{e}_k^*] = \mathbf{E}[\mathbf{D} \mathbf{A}_k \mathbf{s}_k \mathbf{s}_k^* \mathbf{A}_k^* \mathbf{D}^*] = \mathbf{D} \mathbf{E}[\mathbf{A}_k \mathbf{s}_k \mathbf{s}_k^* \mathbf{A}_k^*] \mathbf{D}^* \quad (6)$$

Assuming white data and equal power (P) for all symbols, (6) reduces to:

$$\mathbf{E}[\mathbf{e}_k \mathbf{e}_k^*] = \mathbf{D} \mathbf{E}[\mathbf{A}_k \mathbf{P} \mathbf{I} \mathbf{A}_k^*] \mathbf{D}^* = P \mathbf{D} \mathbf{E}[\mathbf{A}_k \mathbf{A}_k^*] \mathbf{D}^* \quad (7)$$

The expected value can be calculated as:

$$\mathbf{E}[\mathbf{A}_k \mathbf{A}_k^*] = \mathbf{E}[(\mathbf{H}_k - \hat{\mathbf{H}}_0)(\mathbf{H}_k - \hat{\mathbf{H}}_0)^*] = \mathbf{E}[\mathbf{H}_k \mathbf{H}_k^*] + \mathbf{E}[\hat{\mathbf{H}}_0 \hat{\mathbf{H}}_0^*] - \mathbf{E}[\mathbf{H}_k \hat{\mathbf{H}}_0^*] - \mathbf{E}[\hat{\mathbf{H}}_0 \mathbf{H}_k^*] \quad (8)$$

Assuming the channels have equal variance σ_H^2 , we can write:

$$\mathbf{E}[\mathbf{H}_p \mathbf{H}_q^*] = \sigma_H^2 R(p - q) \mathbf{\Gamma} \quad (9)$$

Where $R(p - q)$ is the temporal autocorrelation function and $\mathbf{\Gamma}$ is the $N \times N$ matrix of spatial correlation coefficients.

Using Jake's model [15], the temporal correlation is given by:

$$R(p - q) = J_0(2\pi f_d(p - q)T) \quad (10)$$

Where J_0 is the zeroth order Bessel's function of the first kind, f_d is the maximum Doppler shift and T is the symbol duration. Several models exist for the spatial correlation ($\mathbf{\Gamma}$), the exponential correlation is widely used for linear arrays [16]:

$$\gamma_{pq} = \begin{cases} 1, & p = q \\ \alpha^{|p-q|}, & p \neq q \end{cases} \quad (11)$$

Where γ_{pq} is the element at row p and column q of $\mathbf{\Gamma}$, and α is the spatial correlation between any two adjacent antennas, $0 \leq \alpha \leq 1$.

Using (8) to (11) equation (6) reduces to:

$$\mathbf{E}[\mathbf{e}_k \mathbf{e}_k^*] = 2P \sigma_H^2 (1 - J_0(2\pi f_d kT)) \mathbf{D} \mathbf{\Gamma} \mathbf{D}^* \quad (12)$$

Since $\mathbf{\Gamma}$ is a left hand circulant matrix, we have $\mathbf{\Gamma} = \mathbf{W}^* \mathbf{B} \mathbf{W}$ [17], where \mathbf{W} is the Fourier matrix and \mathbf{B} is a diagonal matrix, whose diagonal elements are the eigen values of $\mathbf{\Gamma}$ given by:

$$\lambda_m = \sum_{p=0}^{N-1} \alpha^p e^{j2\pi \frac{pm}{N}} = \frac{1 - \alpha^N}{1 - \alpha e^{j2\pi \frac{m}{N}}} \quad (13)$$

$m = 0, 1, \dots, N-1$. Equation (12) then becomes:

$$\mathbf{E}[\mathbf{e}_k \mathbf{e}_k^*] = 2P \sigma_H^2 (1 - J_0(2\pi f_d kT)) \mathbf{D} \mathbf{W}^* \mathbf{B} \mathbf{W} \mathbf{D}^* \quad (14)$$

Fig 1 shows a plot of the eigen values for $N = 100$. Note that most of the eigen values are approximately

equal to 1. This is true for all large values of N , hence, we can approximate $\mathbf{B} \approx \mathbf{I}$ and since $\mathbf{W}^* \mathbf{W} = \mathbf{I}$, and for large N , $\mathbf{D} \mathbf{D}^* \approx \mathbf{\Sigma} = \sigma_H^2 \mathbf{I}$ [1, 7], (14) reduces to:

$$E[\mathbf{e}_k \mathbf{e}_k^*] = 2P\sigma_H^4 (1 - J_0(2\pi f_d kT)) \mathbf{I} \quad (15)$$

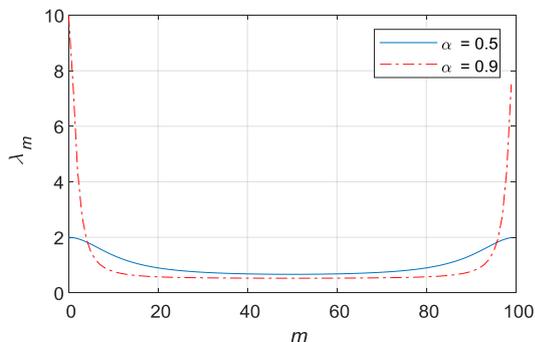


Fig 1. Eigen values of Γ for $N = 100$.

We now turn our attention to the second term in (5). This term is due to the error in the channel estimation. Let $\mathbf{e}_s = \mathbf{D} \mathbf{Z} \mathbf{s}_k$. If the data is white, then:

$$E[\mathbf{e}_s \mathbf{e}_s^*] = E[\mathbf{D} \mathbf{Z} \mathbf{s}_k \mathbf{s}_k^* \mathbf{Z}^* \mathbf{D}^*] = P D E[\mathbf{Z} \mathbf{Z}^*] \mathbf{D}^* \quad (16)$$

When the estimation is mainly due to noise, $E[\mathbf{Z} \mathbf{Z}^*] = \text{diag}(\beta_1^2, \beta_2^2, \dots, \beta_N^2)$. Where β_i^2 is the variance of the channel estimation error for antenna i . Typically, the variance is equal for all antennas ($\beta_i^2 = \sigma_Z^2$ for all i), thus:

$$E[\mathbf{e}_s \mathbf{e}_s^*] = P D \sigma_Z^2 \mathbf{I} \mathbf{D}^* = P \sigma_H^2 \sigma_Z^2 \mathbf{I} \quad (17)$$

The value of σ_Z^2 depends on the channel estimation process used. Typically, during the channel estimation, the users transmit pilot symbols for channel estimation. The minimum number of pilots required is equal to the number of users M . The optimum pilot sequence Φ should satisfy $\Phi \Phi^* = P M \mathbf{I}$ [4]. After sending the pilot signals, the receiver constructs a matrix of received vectors as:

$$\mathbf{Y} = \mathbf{H} \Phi + \mathbf{n} \quad (18)$$

where \mathbf{n} is the matrix of M noise vectors. The channel estimate is then calculated as:

$$\hat{\mathbf{H}} = \frac{1}{PM} \mathbf{Y} \Phi^* = \mathbf{H} + \frac{1}{PM} \mathbf{n} \Phi^* \quad (19)$$

Assuming the channel does not vary during the estimation process (see [4, 5]), the error in estimation is given by:

$$\sigma_Z^2 = E[\mathbf{Z} \mathbf{Z}^*] = \frac{1}{p^2 M^2} E[\mathbf{n} \Phi^* \Phi \mathbf{n}^*] = \frac{\sigma_n^2}{PM} \mathbf{I} \quad (20)$$

For the last term in (5), assuming the noise has a variance of σ_n^2 , it is straightforward to obtain:

$$E[\mathbf{D} \mathbf{n}_k \mathbf{n}_k^* \mathbf{D}^*] = \sigma_H^2 \sigma_n^2 \mathbf{I} \quad (21)$$

The capacity (C) of mMIMO for an SNR (ρ) is bounded by [1]:

$$\log_2(1 + \rho N) \leq C \leq \min\{\min(M, N) \log_2(1 + \rho \max\{\lambda_i\}(M, N))\} M \quad (22)$$

The lower bound occurs when the channel is highly correlated ($\alpha \approx 1$), whereas the upper bound is achieved when the channels are independent ($\alpha = 0$, $\Gamma = \mathbf{I}$).

Using (5), (15), (17), and (21), we redefine ρ to include the interference after k symbols as:

$$\rho = \frac{P \sigma_H^2}{2P \sigma_H^2 [1 - J_0(2\pi f_d kT)] + P \sigma_Z^2 + \sigma_n^2} \quad (23)$$

3.2 Precoding

We now investigate a downlink channel with precoding. Let there be N transmit antennas and M receive/user antennas ($N \gg M$). In this case the transmitter multiplies the symbols by a precoding matrix \mathbf{H}_p to produce the pre-coded symbols \mathbf{x}_k at time index k given by:

$$\mathbf{x}_k = \mathbf{H}_p \mathbf{s}_k \quad (24)$$

Several methods to calculate the pre-coding matrix exist (see [1] and references within) to achieve:

$$\mathbf{H}_0 \mathbf{H}_p \approx \mathbf{\Sigma} \quad (25)$$

The received signal is then:

$$\mathbf{r}_k = \mathbf{H}_k \mathbf{x}_k + \mathbf{n}_k = (\mathbf{H}_0 + \mathbf{Z}) \mathbf{H}_p \mathbf{s}_k + \mathbf{A}_k \mathbf{H}_p \mathbf{s}_k + \mathbf{n}_k \quad (26)$$

Let $\mathbf{e}_k = \mathbf{A}_k \mathbf{H}_p \mathbf{s}_k$ and assuming white data, it can be easily shown that:

$$E[\mathbf{e}_k \mathbf{e}_k^*] = P E[\mathbf{A}_k \mathbf{H}_p \mathbf{H}_p^* \mathbf{A}_k^*] \quad (27)$$

For a large number of antennas, $\mathbf{H}_p \mathbf{H}_p^* \approx \mathbf{\Sigma}$ and (27) reduces to the same expression as (12). Similarly, the channel estimation error will be identical to (17).

3.3 Multi-Cluster Scenario

The analysis presented in the previous sections assumed only one cluster exists. When there are several clusters, transmissions in different clusters

interfere thus reducing the capacity. Assuming L adjacent clusters, the received signal vector (\mathbf{x}_{jk}) for cluster j at time index k is given by[7]:

$$\mathbf{x}_{jk} = \sum_{l=1}^L \mathbf{H}_{lk} \mathbf{s}_{lk} + \mathbf{n}_{jk} = \mathbf{H}_{jk} \mathbf{s}_{jk} + \sum_{l \neq j}^L \mathbf{H}_{lk} \mathbf{s}_{lk} + \mathbf{n}_{jk} \quad (28)$$

Where \mathbf{H}_{jk} , \mathbf{s}_{jk} , and \mathbf{n}_{jk} are the channel matrix, transmitted signal vector and noise vector respectively for cluster j at time index k .

During the channel estimation, the received signal will be contaminated by interference from adjacent clusters. The cluster head creates a matrix \mathbf{X}_j from the signals \mathbf{x}_{jk} , $k = 1, 2, \dots, M$, received for each transmitted vector from Φ . \mathbf{X}_j is given by:

$$\mathbf{X}_j = \mathbf{H}_j \Phi_j + \sum_{l \neq j}^L \mathbf{H}_l \mathbf{S}_l + \mathbf{V}_j \quad (29)$$

Where \mathbf{S}_l is the matrix of M transmitted vectors in cluster l and \mathbf{V}_j is a matrix of M noise vectors in cluster j . Variations in the channel during the channel estimation were ignored and hence we dropped the subscript k for the channel matrix. An estimate $\hat{\mathbf{H}}_j$ of the channel is then calculated as:

$$\hat{\mathbf{H}}_j = \frac{1}{PM} \mathbf{X}_j \Phi_j^* = \mathbf{H}_j + \frac{1}{PM} \sum_{l \neq j}^L \mathbf{H}_l \mathbf{S}_l \Phi_j^* + \frac{1}{PM} \mathbf{V}_j \Phi_j^* \quad (30)$$

The worst-case scenario occurs when the \mathbf{S}_l and Φ_j are equal since this results in the highest correlation and highest interference. Assuming $\mathbf{S}_l = \Phi_j$, we get:

$$\hat{\mathbf{H}}_j = \mathbf{H}_j + \sum_{l \neq j}^L \mathbf{H}_l + \frac{1}{PM} \mathbf{V}_j \Phi_j^* \quad (31)$$

The receiver in cluster j uses the channel estimate $\hat{\mathbf{H}}_j$ to decode the signal. For simplicity, we will assume MRC is used and hence, the decoding matrix is $\mathbf{D} = \hat{\mathbf{H}}_j^*$. The decoded signal is then given by:

$$\mathbf{y}_{jk} = \hat{\mathbf{H}}_j^* \mathbf{x}_{jk} = \sum_{m=1}^L \sum_{l=1}^L \mathbf{H}_m^* \mathbf{H}_{lk} \mathbf{s}_{lk} + \sum_{m=1}^L \mathbf{H}_m^* \mathbf{n}_{jk} + \frac{1}{PM} \sum_{l=1}^L \Phi_j^* \mathbf{V}_j \mathbf{H}_{lk} \mathbf{s}_{lk} + \frac{1}{PM} \Phi_j^* \mathbf{V}_j \mathbf{n}_{jk} \quad (32)$$

Where the subscript k is the time index. For large number of antennas and i.i.d channel matrices we have $\mathbf{H}_m^* \mathbf{H}_{lk} = \mathbf{0}, \forall m \neq l$. Also, the last two term in (32) are negligible compared to the second term for practical signal to noise ratios (SNR) and M values. Hence, (32) reduces to:

$$\mathbf{y}_{jk} = \sum_{l=1}^L \mathbf{H}_l^* \mathbf{H}_{lk} \mathbf{s}_{lk} + \sum_{m=1}^L \mathbf{H}_m^* \mathbf{n}_{jk} \quad (33)$$

Let $\mathbf{A}_{lk} = \mathbf{H}_{lk} - \mathbf{H}_l$, and using (3), we get:

$$\mathbf{y}_{jk} = \sigma_{jj}^2 \mathbf{s}_{jk} + \sum_{l \neq j}^L \sigma_{lj}^2 \mathbf{s}_{lk} + \sum_{l \neq j}^L \mathbf{H}_l^* \mathbf{A}_{lk} \mathbf{s}_{lk} + \sum_{l=1}^L \mathbf{H}_l^* \mathbf{n}_{jk} \quad (34)$$

Where σ_{lj} is the shadowing and path loss factor from the transmitter in cluster l to the receiver in cluster j .

The first term in (34) is the desired signal, the second term is due to the interference from adjacent clusters, the third term is the interference due to channel variation, whereas the last term I due to the noise at the receiver. For the interference due to channel variations, (12) can be applied to yield:

$$E[\mathbf{H}_l^* \mathbf{A}_{lk} \mathbf{s}_{lk} \mathbf{s}_{lk}^* \mathbf{A}_{lk}^* \mathbf{H}_l] = 2P \sigma_{lj}^4 (1 - J_0(2\pi f_d k T)) \quad (35)$$

And for the last term:

$$E[\mathbf{H}_l^* \mathbf{n}_{jk} \mathbf{n}_{jk}^* \mathbf{H}_l] = \sigma_{lj}^2 \sigma_n^2 \mathbf{I} \quad (36)$$

Hence the SINR is given by:

$$\rho = \frac{P \sigma_{jj}^4}{\sum_{l \neq j}^L P \sigma_{lj}^4 + \sum_{l=1}^L 2P \sigma_{lj}^4 [1 - J_0(2\pi f_d k T)] + \sum_{l=1}^L \sigma_{lj}^2 \sigma_n^2} \quad (37)$$

The ratio $\sigma_{jl} / \sigma_{jj}$ is critical for the performance of the system. In cellular systems, the basestations, and thus cells, are fixed and the user is usually associated with the cell that has the highest signal, hence $\sigma_{jj} \geq \sigma_{lj}$. However, in VANET, since cluster heads can be moving cars, it may become possible, even if momentarily, that $\sigma_{jj} \leq \sigma_{lj}$. This will considerably reduce the performance.

IV. RESULTS

In this section theoretical and numerical simulation results are presented. For the results, we set $M = 10$, $N = 100$ and SNR = 20dB. Initially, it is assumed perfect channel knowledge is available and hence $\sigma_z = 0$.

Fig. 2 compares the mean square error of the channel estimation vs symbol index (k) from (15) with simulations for various Doppler shifts. As k increases the channel changes and hence the error between the initial estimate and the actual channel increases. For high Doppler shifts, the initial estimate is almost useless after only 10 symbols. The results from simulation and (15) are identical.

Fig. 3 shows the capacity of mMIMO vs Doppler shift. The capacity dropped from approximately 140bits/s/Hz at 0 Doppler shift to approximately 60bits/s/Hz when $f_d T > 0.01$. Correlation between the antennas further reduce the capacity of the system. The theoretical and numerical results match very well.

Fig 4. presents the capacity of mMIMO when several clusters exist. When $\sigma_{jl}/\sigma_{jj} < 0.2$, the interference from adjacent clusters is negligible. However as σ_{jl}/σ_{jj} increases, the interference increases causing considerable drop in the capacity. Nodes at the edge of the cluster have the lowest σ_{jj} since they are the farthest from the cluster head. When nearby users in adjacent clusters transmit, it is very likely that $\sigma_{jl}/\sigma_{jj} > 1$ for edge nodes and, hence the capacity will be very low.

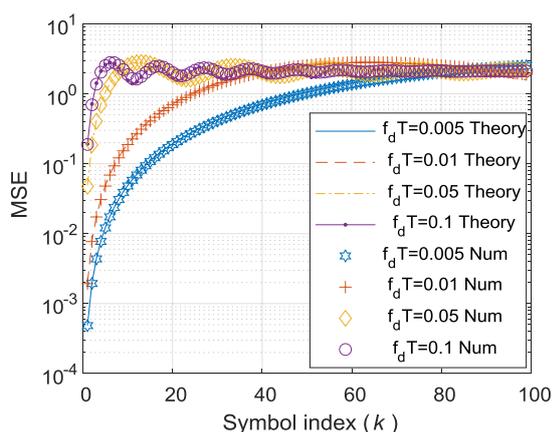


Fig 2. MSE of channel estimation for 1 cluster

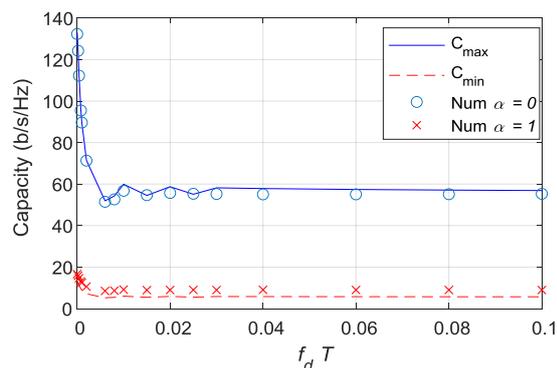


Fig 3. Capacity vs Doppler Shift for 1 cluster

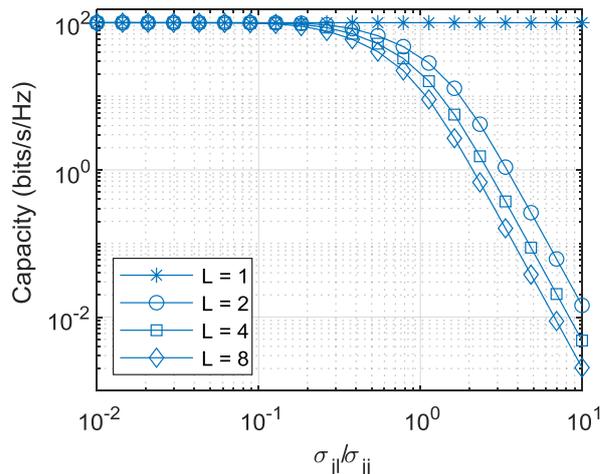


Fig 4. Capacity variation due to interference from multiple clusters

Fig 5. shows the effects of channel aging and interference from multiple cells on capacity. Clearly channel estimation errors have the dominant effect when σ_{jl}/σ_{jj} is small, however for high σ_{jl}/σ_{jj} , interference from adjacent clusters causes large drops in system capacity.

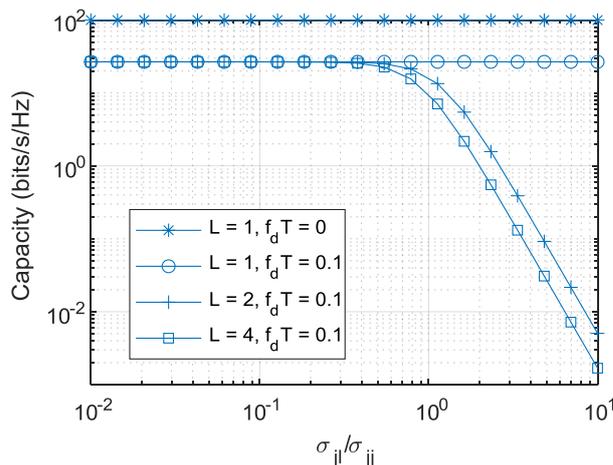


Fig 5. Capacity variation due to interference and channel aging

V. CONCLUSION

In this paper the effects of outdated CSI and interference due to pilot contamination were analyzed and expressions for the capacity of mMIMO in VANET were obtained. It was found that in fast fading scenarios, the capacity of mMIMO drops considerably after a very short period due to outdated CSI. The effects of pilot contamination were limited when the clusters were far, but as the clusters approach each other, the interference power increases leading to a large drop in the capacity.

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