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Future and Current Scenario on Partial Differential Equations

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ABSTRACT

Background: Partial differential equations are equations that consist of a function with multiple unknown variables and their partial derivatives.

Method and material: We conducted this research paper by observing the different types of reviews, as well as conducting and evaluating literature review papers.

Result: Partial differential equations are equations that consist of a function with multiple unknown variables and their partial derivatives. They are useful in studying various phenomena such as sound, heat, fluid flow, and waves.

Conclusions: Partial differential equations will consist of equations that consist of a function with multiple unknown variables and their partial derivatives. There will be no general theory for solving all partial differential equations (PDEs).

Keywords: Partial Differential, Equations, Derivation, Classification, Application etc

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I. INTRODUCTION

Partial differential equations are equations that consist of a function with multiple unknown variables and their partial derivatives. In other words, partial differential equations help to relate a function containing several variables to their partial derivatives. These equations fall under the category of differential equations.

Partial differential equations are very useful in studying various phenomena that occur in nature such as sound, heat, fluid flow, and waves. In this article, we will take an in-depth look at the meaning of partial differential equations, their types, formulas, and important applications.

Partial differential equations are abbreviated as PDE. These equations are used to represent problems that consist of an unknown function with several variables, both dependent and independent, as well as the partial derivatives of this function with respect to the independent variables.

Partial differential equations can be defined as a class of differential equations that introduce relations between the various partial derivatives of an unknown multivariable function. Such a multivariable function can consist of several dependent and independent variables. An equation that can solve a given partial differential equation is known as a partial solution.

Partial Differential Equations Example

An example of a partial differential equation is$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$. This is a one dimensional wave equation.

- Hest conduction equation: $\frac{\partial T}{\partial t} = C \frac{\partial^2 T}{\partial x^2}$
- Laplace equation: $\Delta^2 \phi = \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial Y^2} = 0$
- Wave equation of a vibrating membrane: $\frac{\partial^2 u}{\partial t^2} = C(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial Y^2})$

Partial Differential Equations Formula

Partial differential equations can prove to be difficult to solve. Hence, there are certain techniques such as the separation method, change of variables, etc. that can be used to get a solution to these equations. The general formulas for partial differential equations are given below:

- First-Order Partial Differential Equations:
  $$F(x_1, x_2, \ldots, x_n, \quad w, \quad \frac{\partial w}{\partial x_1}, \frac{\partial w}{\partial x_2}, \ldots, \frac{\partial w}{\partial x_n}) = 0$$
  Here, $w = (x_1, x_2, \ldots, x_n)$ is the unknown function and F is the given function.
- Second-Order Partial Differential Equations:
  The general formula of a second-order PDE in two variables is given as $a_1(x,y)u_{xx} + a_2(x,y)u_{xy} + a_3(x,y)u_{yx} + a_4(x,y)u_{yy} + a_5(x,y)u_x + a_6(x,y)u_y + a_7(x,y)u = f(x, y)$.

Typical PDEs

As there is no general theory known for solving all partial differential equations and given the variety of phenomena modeled by such equations, research focuses on particular PDEs that are important for theory or applications. Following is a list of partial differential equations commonly found in mathematical applications. The objective of the enumeration is to illustrate the different categories of equations that are studied by mathematicians; here, all variables are dimensionless, all constants have been set to one.

a. Linear equations.

1. Laplace's equations: $\Delta u = 0$
2. Helmholtz's equation (involves eigenvalues): $-\Delta u = \lambda u$
3. First-order linear transport equation: $ut + c\, u_x = 0$
4. Heat or diffusion equation: $u_t - \Delta u = 0$
5. Schr¨odinger's equation: $i\, u_t + \Delta u = 0$
6. Wave equation: $utt - c^2 \Delta u = 0$
7. Telegraph equation: $u_{tt} + d\, u_t - u_{xx} = 0$

b. Nonlinear equations.

1. Eikonal equation: $|Du| = 1$
2. Nonlinear Poisson equation: $-\Delta u = f(u)$
3. Burgers' equation: $u_t + u\, u_x = 0$
4. Minimal surface equation: div " $\left(\frac{Du}{(1 + |Du|\, DuI^2)1/2}\right)$ # = 0
5. Monge-Amp`ere equation: $\det(D^2u) = f$
6. Korteweg-deVries equation (KdV): $u_t + u\, u_x + u_{xxx} = 0$
7. Reaction-diffusion equation: $u_t - \Delta u = f(u)$

c. System of partial differential equations.

1. Evolution equation of linear elasticity: $u_{tt} - \mu\Delta u - (\lambda + \mu)D(\text{div } u) = 0$
2. System of conservation laws: $u_t + \text{div } F(u) = 0$
3. Maxwell's equations in vaccum:

curl $E = -B_t$
curl $B = \mu_0\varepsilon_0 E_t$
div B = div E = 0

4. Reaction-diffusion system: $u_t - \Delta u = f(u)$
5. Euler's equations for incompressible, inviscid fluid: ' $u_t + u \cdot Du = -Dp$ div u = 0
6. Navier-Stokes equations for incompressible viscous fluid: ' $u_t + u \cdot Du - \Delta u = -Dp$ div u = 0

In the following section, we will learn more about the types of partial differential equations

Order and Degree of Partial Differential Equations

Order and degree of partial differential equations are used to categorize partial differential equations. The most commonly used partial differential equations are of the first-order and the second-order.

Order of Partial Differential Equations

Order of a partial differential equation can be defined as the order of the highest derivative term that occurs in the PDE. Suppose a partial differential equation is given as

$$\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = z + xy \qquad \frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = z + xy.$$

As the order of the highest derivative is 1, hence, this is a first-order partial differential equation.

Degree of Partial Differential Equations

The degree of a partial differential equation is the degree of the highest derivative in the PDE. The partial differential equation $\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = z + x$ will have the degree 1 as the highest derivative is of the first degree.

Partial Differential Equations Types

Partial differential equations can be broadly divided into 4 types based on the order of the partial derivatives as well as the nature of the equation. These are given below:

First-Order Partial Differential Equations

Partial differential equations where the highest partial derivatives of the unknown function are of the first order are known as first-order partial differential equations. If the equation has n number of variables then we can express a first-order partial differential equation as F (x1, x_2..x_n, kx_1 ...kx_n). First-order PDEs can be both linear and non-linear. A linear partial differential equation is one where the derivatives are neither squared nor multiplied.

Second-Order Partial Differential Equations

Second-order partial differential equations are those where the highest partial derivatives are of the second order. Second-order PDEs can be linear, semi-linear, and non-linear. Linear second-order partial differential equations are easier to solve as compared to the non-linear and semi-linear second-order PDEs.

The general formula for a second-order partial differential equation is given as $au_{xx}+bu_{xy}+cu_{yy}+du_x+eu_y+fu = g(x,y)$. Here, a, b, c, d, e, f, and g are either real-valued functions of x and/or y or they are real constants.

Quasi Linear Partial Differential Equations

In quasilinear partial differential equations, the highest order of partial derivatives occurs, only as linear terms. First-order quasi-linear partial differential equations are widely used for the formulation of various problems in physics and engineering.

Homogeneous Partial Differential Equations

A partial differential equation can be referred to as homogeneous or non-homogeneous depending on the nature of the variables in terms. The partial differential equation with all terms containing the dependent variable and its partial derivatives is called a non-homogeneous PDE or non-homogeneous otherwise.

Partial Differential Equations Classification

Suppose we have a linear second-order PDE of the form $Au_{xx} + 2Bu_{xy} + Cu_{yy}$ + other lower-order terms = 0. Then the discriminate of such an equation will be given by B^2 - AC. Using this discriminate, second-order partial differential equations can be classified as follows:

- **Parabolic Partial Differential Equations:** If $B^2 - AC = 0$, it results in a parabolic partial differential equation. An example of a parabolic partial differential equation is the heat conduction equation.
- **Hyperbolic Partial Differential Equations:** Such an equation is obtained when $B^2 - AC > 0$. The wave equation is an example of a hyperbolic partial differential equation as wave propagation can be described by such equations.
- **Elliptic Partial Differential Equations:** $B^2 - AC < 0$ are elliptic partial differential equations. The Laplace equation is an example of an elliptic partial differential equation.

Classification	Canonical Form	Type	Example
$b^2 - ac > 0$	$\frac{\partial^2 u}{\partial\xi\partial\eta...}$ +………=0	Hyperbolic Partial Differential Equation	Wave propagation equation
$b^2 - ac = 0$	$\frac{\partial 2u}{}$ +……..=0	Parabolic Partial Differential	Heat conduction equation

	∂η2	Equation	
b^2 - ac < 0	$\frac{\partial^2 u}{\partial \alpha^2} + \frac{\partial^2 u}{\partial \beta^2} + \ldots\ldots\ldots = 0$	Elliptic Partial Differential Equation	Laplace equation

Solving Partial Differential Equations

There can be many methods that can be used to solve a partial differential equation. Suppose a partial differential equation has to be obtained by eliminating the arbitrary functions from an equation z = yf(x) + xg(y). The steps to do so are as follows:

Step 1: Differentiate both sides with respect to x and y.

$$\frac{\partial z}{\partial x} = yf(x) + g(y) \text{ --- } (1)$$

$$\frac{\partial z}{\partial y} = f(x) + xg'(y) \text{ --- } (2)$$

Step 2: Now differentiate (1) w.r.t to y and (2) w.r.t x.

$$\frac{\partial 2z}{\partial x \partial y} = f(x) + g'(y)$$

Step 3: Multiply the first equation by x and the second equation by y then add the resultant.

$$\frac{x\partial z}{\partial x} + \frac{y\partial z}{\partial y} = xg(y) + yf(x) + xy(f'(x) + g'(y)) = z + xy(f'(x) + g'(y))$$

Substituting from step 2 we get,

$$\frac{x\partial z}{\partial x} + \frac{y\partial z}{\partial y} = z + xy \left(\frac{\partial^2 z}{\partial x \partial y}\right)$$

The general, particular or singular solution can be determined for this equation by using various methods such as change of variables, substitution, etc.

Partial Differential Equations Applications

Partial differential equations are widely used in scientific fields such as physics and engineering. Some applications of partial differential equations are given below:

- Partial differential equations are used to model equations to describe heat propagation. The equation is given by $u_{xx} = u_t$
- Propagation of light and sound is given by the wave equation. This equation is a second-order partial differential equation and is given by $u_{xx} - u_{yy} = 0$.
- The Black-Scholes equation is another important second-order partial differential equation that is used to construct financial models.

Important Notes on Partial Differential Equations

- A partial differential equation is an equation consisting of an unknown multivariable function along with its partial derivatives.
- There a broadly 4 types of partial differential equations. These are first-order, second-order, quasi-linear partial differential equations, and homogeneous partial differential equations
- Second-order partial differential equations can be classified into three types - parabolic, hyperbolic, and elliptic.

Method and material: We conducted this research paper by observing the different types of reviews, as well as conducting and evaluating literature review papers.

II. RESULT.

Partial differential equations are equations that consist of a function with multiple unknown variables and their partial derivatives. They are useful in studying various phenomena such as sound, heat, fluid flow, and waves. A partial solution is an equation that can solve a given partial differential equation. There is no general theory for solving all partial differential equations (PDEs). Research focuses on particular PDEs that are important for theory or applications. In this section, we will examine the different types of partial differential equations. Partial differential equations can be broadly divided into 4 types based on the order of the partial derivatives as well as the nature of the equation. The degree of a partial differential equation is the degree of the highest derivative in the PDE, denoted by degree 1. Parabolic Partial Differential Equations If B2–AC = 0, it results in a parabolic partial differential equation. Hyperbolic and elliptic partial differential equations are examples of hyperbolic equations. The Laplace equation is an example of an elliptical partial equation. Step-by-step instructions for solving the equation yf (x) + g(y) with respect to x and y. multiply the first equation by x, then add the second equation by y, then multiply them together. A partial differential equation is an equation consisting of an unknown multivariable function along with its partial derivatives. There are four types of partial differential equations, including first-order, second-order, and quasi-linear. Partial differential equations are widely used in scientific fields such as physics and engineering.

III. FUTURE SCOPE & CONCLUSION

Partial differential equations will consist of equations that consist of a function with multiple unknown variables and their partial derivatives. There will be no general theory for solving all partial differential equations (PDEs). In this section, we will examine the different types of partial differential equations. There are four types of partial differential equations, including first-order, second-order, and quasi-linear. Partial differential equations will be widely used in scientific fields such as physics and engineering.

Applications of Partial Differential Equations Partial differential equations will be widely used in scientific fields such as physics and engineering. Some applications of partial differential equations are given below: Partial differential equations are used to model equations to describe heat propagation. This equation will be a second-order partial differential equation and is given by u_{xx}-u_{yy} = 0.

The Black-Scholes equation will be another important second-order partial differential equation that will be used to construct financial models. Important Notes on Partial Differential Equations A partial differential equation is an equation consisting of an unknown multivariable function along with its partial derivatives. There are broadly four types of partial differential equations. These will be first-order, second-order, quasi-linear partial differential equations, and homogeneous partial differential equations. Second-order partial differential equations can be classified into three types: parabolic, hyperbolic, and elliptic.

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