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Second-order elastic analysis for steel plane frames from multi-storey buildings: comparative study between Direct Stiffness Method and Moment Amplification Method

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ABSTRACT
Due to the saturation of the physical spaces intended for construction in large urban centers, it has been chosen to verticalise the buildings as an alternative for cities to support demographic growth. As a building gains height, the action of the wind, for example, causes significant effects, resulting in an increase in the requesting efforts when applied simultaneously with the other acting actions. These effects, called second-order effects, if not considered in the structural design, can compromise the overall stability of the structure and consequently the local stability of the structural elements. Therefore, this study aims to analyze the effect of the second-order geometric elastic in plane steel frames with rigid beam-column connections, components of the structural systems of buildings with multiple floors, using the Direct Stiffness Method (DSM) with use of the geometric stiffness matrix, in comparison with the Moment Amplification Method (MAM), proposed by same normative codes as an approximate method of second-order elastic analysis. For this purpose, computational codes based on the theories of the aforementioned methods were implemented. After observing the parametric study, the results using the MAM were satisfactory as long as the maximum horizontal displacement of the top of the columns relative to the base did not exceed the limit value (H/400).

Keywords- Degree of lateral displacement of the structure, DSM, MAM, geometric nonlinearity.

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I. INTRODUCTION

Due to population growth and the fact that the physical spaces available for the implementation of new commercial and residential enterprises in large metropolises are becoming increasingly saturated, the process of building verticalization has been chosen to meet the demand, giving rise to increasingly tall and slender buildings.

In the structural system of a building, composed of an association of frames, substructures exist that, due to their high lateral stiffness, are responsible for supporting the horizontal actions acting on the structure. These substructures that provide the building with greater stability are called bracing substructures and are classified according to the type of beam-column connection used. Three basic types of bracing substructures are identified: rigid frame, lattice frames and shear walls. Evidently, in addition to supporting horizontal stresses, the bracing substructures also transmit the gravitational loads to the foundations [1].

Therefore, it is known that in buildings with multiple floors, the action of the wind causes significant effects, resulting in an increase in the stresses applied when applied simultaneously with the other actions acting on the structure. Thus, to ensure a good performance of the structural system chosen for the building, it is essential to perform a very detailed structural analysis, which according to [1], provides responses of the structure, when requested by a set of actions, through equilibrium relationships, to verify the stability of the structure regarding the ultimate limit state and the service limit state.

In this analysis, it is verified whether the structure fulfill all the requirements of local and global stability, thus ensuring that throughout its useful life, its elements do not exceed its resistant capacity, which can be achieved with the increase in deformations as a result of the actions.

The structural analysis of the elastic nature can be processed in 1st-order theory, also known as geometric linear analysis, or in 2nd-order theory, known as nonlinear geometric analysis. The main characteristic of linear geometric analysis is to obtain stresses and displacements of the structure considering it in its undisturbed position. In contrast, the nonlinear geometric analysis establishes the equilibrium of the structure considering it in its deformed position, thus obtaining soliciting forces and displacements more consistent with the reality of slender structures. It is important to note that the study of the overall stability of structures should be done in 2nd order theory, as it is in this that the effects of excessive displacements are considered.

Research developed by [2], [3], [4], [5], [6] and [7] describe the using different approximate analytical methods for 2nd-order as the Fictitious lateral forces; Gamma-z coefficient (γ_z); Method of Amplification of Forces (factors B_1 and B_2); Iterative Gravity Loading Method and the P-Δ effect; two Iterative Cycles Method; Fictitious displacement; Negative Stiffness Method; Stability Functions and the Equivalent Column Method.

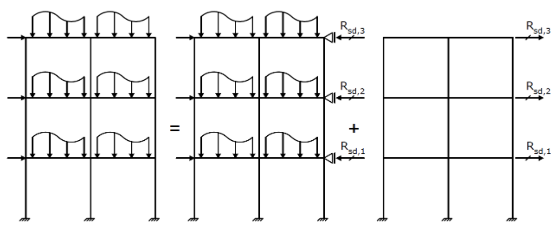
A precise way to evaluate the 2nd-order effects in structural systems composed of frame associations is by considering the axial force in the translational transverse strains of the structural element based on the direct stiffness method (DSM), considering the geometric stiffness matrix. On the other hand, the [8] allows that through two first-order analyses, the second-order effects are considered in an approximate manner through the Moment Amplification Method (MAM). Because of what has been mentioned, it is up to the design engineer to choose the method that best represents the real physical behavior of the structure, depending on its characteristics and sensitivity to 2nd-order effects, in order to obtain increasingly efficient and safe structures. Therefore, this study proposes to analyze the effect of the 2nd-order elasticity in plane steel frames with rigid beam-column connections formed by full web profiles through a comparative study between the direct stiffness method (DSM) and the amplification method (MAM).

I.1 DIRECT STIFFNESS METHOD (DSM)

The method of structural analysis used is the displacement method, in which the fundamental parameters of the solution are displacements (linear and angular) of the nodes of the verified structural model. The displacement method aims to approximate its methodology to the procedures usually adopted in computer programs. This version of the displacement method is known as the direct stiffness method or matrix analysis of structures.

I.2 MOMENT AMPLIFICATION METHOD (MAM)

According to [8], when using the Moment Amplification Method (MAM), the structure under analysis, subjected to a combination of calculation actions, called the Original Structure (Fig. 1a), is transformed into the sum of two other structures, as shown in Fig. 1. One of the structures considered contains the total load applied to the original structure, but with its nodes prevented from moving laterally, by means of fictitious horizontal retainers placed on each floor, this structure is called Structure nt (“no translation”, i.e., without lateral displacement) (Fig. 1b). Conversely, the other structure is only subjected to the effect of the reactions of the fictitious containments, applied in the opposite direction in which it acts in the nt structure, at the same points where such containments were placed.”, i.e., with lateral displacement) (Fig. 1c).



(a) Original Structure (b) Structure nt (c) Structure lt

Fig. 1: Representation of the model for analysis.

The nt and lt structures are used to obtain the amplification coefficientsB_1 andB_2. The coefficient B_1 considers the local effect P-δ in the bending moment value (single requesting force influenced by this effect), therefore applying only the nt structure. The coefficient B_2 considers the global effect P-Δ in the value of the bending moment and the axial force (the two requesting forces influenced by this effect), thus applying only the structure lt.

After obtaining the amplification factors, it becomes possible to calculate design forces/moments and use them as the result of a second-order analysis. Equations (1), (2) and (3), defined below, are used to calculate the 2nd-order axial calculation force, the 2nd-order calculation bending moment and the 2nd-order calculation shear force, respectively for analysis via MAM:

$$N_{sd,2} = N_{nt} + B_2 N_{lt} \tag{1}$$

where N_{nt} and N_{lt} are the axial forces for the calculation of the nt and lt structures obtained in the first-order elastic analysis, respectively.

$$M_{sd,2} = B_1 M_{nt} + B_2 M_{lt} \tag{2}$$

where M_{nt} and M_{lt} are the bending moments requesting the calculation of the nt and lt structures obtained in the first-order elastic analysis, respectively.

$$V_{sd,2} = V_{nt} + V_{lt} \tag{3}$$

where V_{nt} and V_{lt} are the shear forces requesting the calculation of the nt and lt structures obtained in the first-order elastic analysis, respectively.

As described in the literature [9] and [10], there are many practical cases where second-order moments may be calculated by amplifying the first-order moments, however, the total moments are not simply a direct amplification of the first-order moments for the second-order moments do not necessarily have the same distribution as the first-order moments.

I.2.1 Coefficient B_1

It is adopted/calculated as follows: if the requesting axial force in the bar is tensile, one should consider B_1 equal to 1.0 because it does not cause an increase in the bending moment value. Conversely, if the calculated axial force is of compression, the coefficient B_1 is calculated according to Equation (4):

$$B_1 = \frac{C_m}{1 - \left|\frac{N_{sd,1}}{N_e}\right|} \geq 1 \tag{4}$$

whereN_e is the axial force that causes the elastic buckling by bending the bar in the plane of action of the bending moment, calculated with the actual length of the bar, considering, if applicable, the initial imperfection of the material;$N_{sd,1}$ is the axial compressive force requesting calculation in the bar considered, in first order analysis$\left(N_{sd,1} = N_{nt,sd,1} + N_{lt,sd,1}\right)$; and C_m is a coefficient of equivalence of moments. A value of 1.0 is conservatively adopted if there are transverse forces between the ends of the bar in the flexion plane (Fig. 2a); otherwise, it should be calculated according to Equation (5), as shown below:

$$C_m = 0{,}60 - 0{,}40\frac{M_1}{M_2} \tag{5}$$

where M_1/M_2 is the relationship between the smallest and largest bending moments that require calculation in the nt structure in the flexion plane at the supported ends of the bar, taken as positive when the moments cause reverse curvature (Fig. 2b) and negative when they cause simple curvature (Fig. 2c). In Equation (5), we adopt $\left(M_1 = M_{1,nt,sd} \text{ e } M_2 = M_{2,nt,sd}\right)$, with $M_{2,nt,sd} \geq M_{1,nt,sd}$.

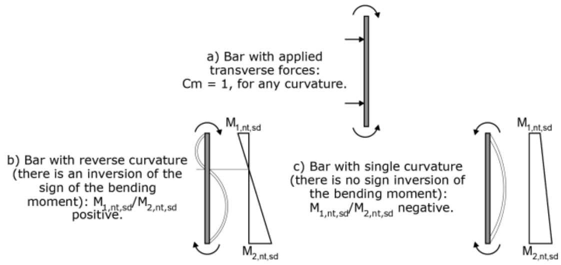


Fig.2: Representation of the case in which $C_m = 1$ and determination of the relationship $M_{1,nt,sd}/M_{2,nt,sd}$.

I.2.2 Coefficient B_2

It is calculated by Equation (6), shown below:

$$B_2 = \frac{1}{1 - \frac{1}{R_s}\frac{\Delta_h}{h}\frac{\sum N_{sd}}{\sum H_{sd}}} \tag{6}$$

whereR_s an adjustment coefficient that depends on the characteristics of the system resistant to horizontal actions, taken equal to 0.85 in the structures where all the bracing substructures are rigid frames, and equal to 1.0 for the other structures; Δ_h the relative horizontal displacement between the upper and lower floors (interfloor displacement) of the floor considered, obtained in the structure lt; $\sum N_{sd}$ the total gravitational load that acts on the floor considered, encompassing the loads acting on the bracing substructures and on the elements that do not belong to these substructures; and $\sum H_{sd}$ the shear force in the floor, produced by the acting horizontal calculating forces, used to determine Δ_h and obtained in the structure lt; h the height of the floor considered.

II. DEVELOPED COMPUTATIONAL CODES

In the code itself, the elastic analyses of the 1st and 2nd-order for plane steel frames are presented, and their design is based on the direct stiffness method (DSM), considering or not considering the geometric stiffness matrix. Through the aforementioned implementation, it is possible to obtain the internal forces in the members, the reactions in the supports and the displacements (linear and angular) in the nodes of reticulated structures.

The code was prepared in the MATHCAD 14 software, which, for its compilation, receives an input file containing some parameters of the discretized bars of the studied frames.

The necessary information of the structure bars are length (L), cross-sectional area (A), web area (A_{web}), inertia moment (I) (around the axis on which the flexion acts), longitudinal modulus of elasticity (E), transverse elasticity modulus (G) and the loads to which they are subjected (in the case of the bar being a column, its weight that acts in the longitudinal direction of the bar enters as an axial load in the calculations).

Fig. 3 shows a schematic of the calculation routine performed by the computational code that processes the elastic analysis with geometric nonlinearity in the plane steel frames.

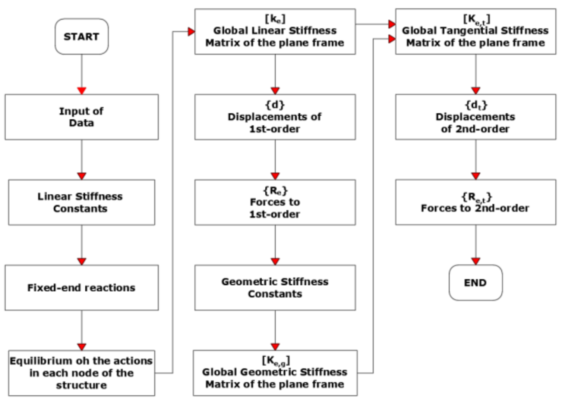


Fig. 3: Routine for calculating the code that performs the second-order elastic analysis.

In another specific code, the two first-order elastic analyses considered by the MAM are presented, and their conception is also based on the DSM, considering only the linear stiffness matrix. Through the aforementioned code, the ordering forces of the 2nd-order acting on the plane steel frames are obtained in an approximate manner. Its preparation was also performed in MATHCAD 14 software, which, for its compilation, receives the same input file.

A schematic of the calculation routine performed by this computational code is shown in Fig. 4.

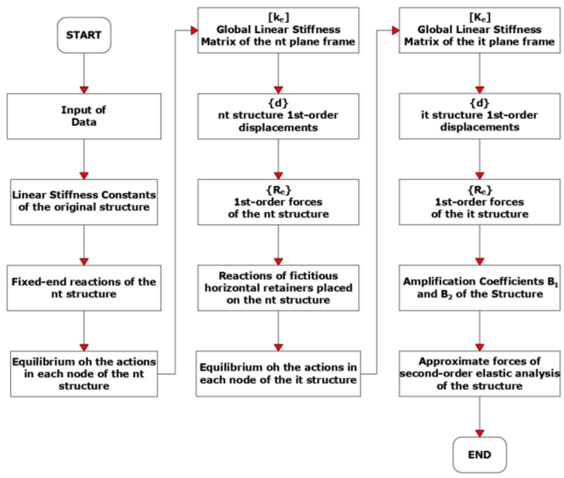


Fig. 4: Routine for calculating the code that performs the approximate 2nd-order elastic analysis.

Both computational codes developed to perform the 2nd-order similar elastic analyses use the technique of dividing the structure into finite elements, which considers each beam and each column as a discretized element and then determines the stiffness matrices of these elements and the entire structure [11].

II.1 Stiffness Matrix of the plane frame Finite Element

The possible displacements (angular and linear) at the ends of a typical plane frame element, also called the degrees of freedom of the finite element, are shown in Fig. 5, which correspond to the displacements of an element under axial deformation (u_i and u_f) together with the displacements of an element under flexion(v_i, v_f, θ_i, and θ_f).

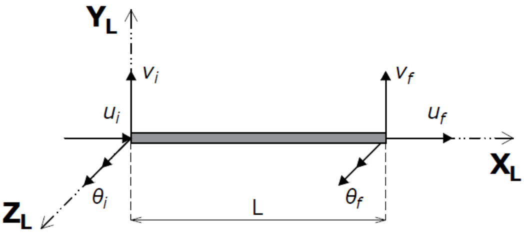


Fig. 5: Degrees of freedom of a plane frame element.

Considering a plane structure formed by straight bars, each of these can be analyzed as being elastically supported so that the forces can be evaluated from the displacements of the extremes. Then, for each member, an oriented orthogonal reference is defined so that one of its axes is along the member axis, that is, a Local Coordinate System (LCS). Thus, for the beam finite element established in Fig. 5, it is possible to define six degrees of freedom $\{\mathbf{d}\}$, which are related to six elastic reactions $\{\mathbf{R}_e\}$ (Fig. 6) in the LCS, through:

$$\{\mathbf{R}_e\} = [\mathbf{K}_e]\{\mathbf{d}\} + \{\mathbf{R}_e^F\} \tag{7}$$

where $\{\mathbf{R}_e\}$ is the vector of elastic reactions; $[\mathbf{K}_e]$ is the linear stiffness matrix; $\{\mathbf{d}\}$ is the vector of nodal displacements; and $\{\mathbf{R}_e^F\}$ is the vector of fixed-end reactions.

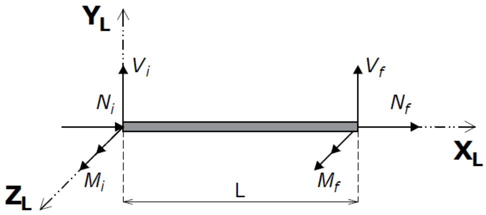


Fig. 6: Elastic reactions in a plane frame element.

The stiffness constants of the linear stiffness matrix of the finite element of the beam can be calculated from the flexibilities (considering the axial deformation energy, by shearing and by bending), which in turn can be calculated using the Principle of Virtual Work (PVW) [6].

The geometric stiffness matrix is also defined $[\mathbf{K}_{e,g}]$ of the beam finite element to consider the effect of the axial force on the deformations of the plane frame because the presence of the axial forces generally causes a reduction in the rigidity of the structure, causing an increase in its elastic deformations. From the PVW applied to the definition of the finite element method, the geometric stiffness matrix will be defined by the following index expression:

$$(\mathrm{K}_{e,g})_{ij} = \int_0^L \mathrm{NN}(x)\psi_i'(x)\psi_j'(x)\ dx \tag{8}$$

where $\mathrm{NN}(x)$ is the normal force function in the finite element analysis; L is the length of the bar; and $\psi_i(x)$ is the i-th function of shape, defined by:

$$\begin{aligned}
\psi_1(x) &= 1 - \frac{x}{h} \\
\psi_2(x) &= 1 - 3\left(\frac{x}{h}\right)^2 + 2\left(\frac{x}{h}\right)^3 \\
\psi_3(x) &= -x + 2\frac{x^2}{h} - \frac{x^3}{h^2} \\
\psi_4(x) &= \frac{x}{h} \\
\psi_5(x) &= 3\left(\frac{x}{h}\right)^2 - 2\left(\frac{x}{h}\right)^3 \\
\psi_6(x) &= \frac{x^2}{h} - \frac{x^3}{h^2}
\end{aligned} \tag{9}$$

being, however, $(K_{e,g})_{1,2}$, $(K_{e,g})_{1,3}$, $(K_{e,g})_{1,5}$, $(K_{e,g})_{1,6}$, $(K_{e,g})_{4,2}$, $(K_{e,g})_{4,3}$, $(K_{e,g})_{4,5}$, $(K_{e,g})_{4,6}$ and their respective symmetric NULL, because the case in which there is no interaction between the axial and flexional degrees of freedom is considered, due to the regime of small deformations.

The tangential stiffness matrix of the finite element of the bar used $[\mathbf{K}_{e,T}]$ is generically given by:

$$[\mathbf{K}_{e,T}] = [\mathbf{K}_e] - [\mathbf{K}_{e,g}] \tag{10}$$

II.2 GLOBAL STIFFNESS MATRIX OF THE PLANE FRAME

To assemble the global stiffness matrix of the plane frame, a systematic procedure is used to conveniently add the stiffness contributions of each member of the structure bar to obtain a stiffness matrix that relates the forces and nodal displacements of the entire structure. the structure. One way to assemble the global stiffness matrix is to establish the equilibrium conditions of the actions in each node of the structure with respect to each degree of freedom of the global coordinate system (GCS) due to the elastic reactions of the elements that compete in this system. node, as well as the possible external actions applied directly to the nodes. Each equilibrium equation results in a row of the global stiffness matrix, which is symmetric and has the number of rows and columns equal to the number of degrees of freedom of the structure.

In Fig. 7, is shown as an example, a plane frame with three floors, in which an arbitrary numbering is indicated, and the displacements (linear and angular) are considered positive in the direction that are indicated in Fig. 8.

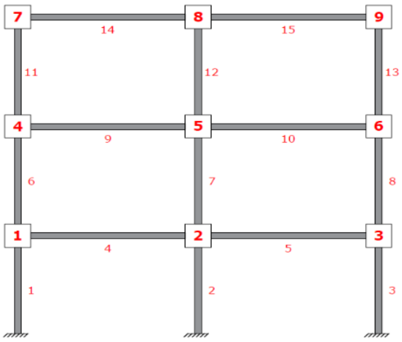


Fig. 7: Plane frame with three floors with arbitrary numbering of bars and nodes.

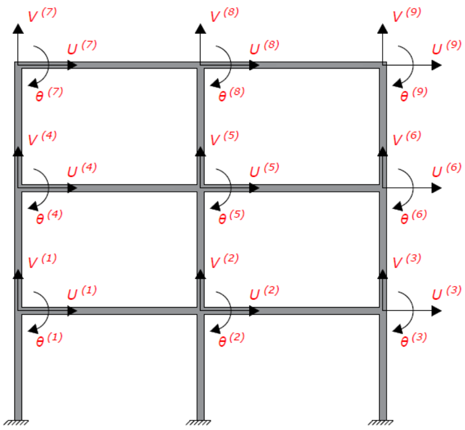


Fig. 8: Arbitrary numbering of the degrees of freedom of the frame of Fig. 7.

After defining the numbering, we analyze, for example, the actions related to node 1 (Fig. 9) to obtain their equilibrium equations, considering that there are no external loads applied to the node.

Therefore, the expression of equilibrium of vertical forces is given by:

$$\begin{aligned} &N_f^1 + V_i^4 + N_i^6 = 0 \Rightarrow \\ &\Rightarrow -r_1 u_i^1 + r_1 u_f^1 + N_f^{F1} + t_4 v_i^4 - b_4 \theta_i^4 \\ &\qquad - t_4 v_f^4 - b_4 \theta_f^4 + V_i^{F4} \\ &\qquad + r_6 u_i^6 - r_6 u_f^6 + N_i^{F6} \\ &\qquad = 0 \end{aligned} \tag{11}$$

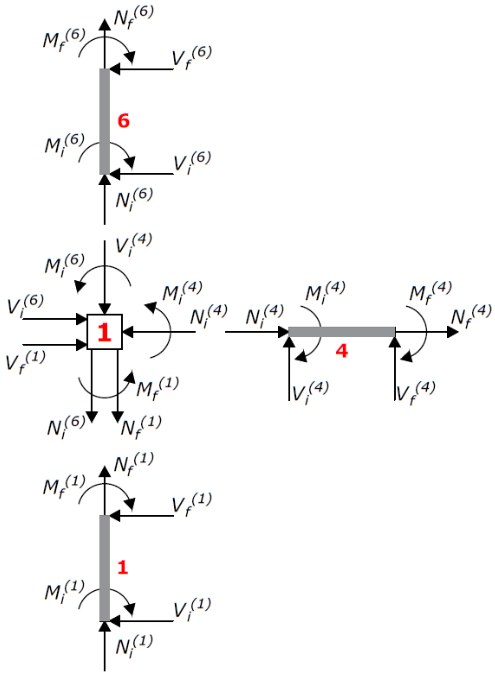


Fig. 9: Representation of the corresponding elastic reactions that balance node 1 of the plane frames of Fig. 7.

The equilibrium expression of bending moments is given by:

$$\begin{aligned} &M_f^1 + M_i^4 + M_i^6 = 0 \Rightarrow \\ &\Rightarrow -b_1 v_i^1 + a_1 \theta_i^1 + b_1 v_f^1 + k_1 \theta_f^1 + M_f^{F1} \\ &\qquad - b_4 v_i^4 + k_4 \theta_i^4 + b_4 v_f^4 \\ &\qquad + a_4 \theta_f^4 + M_i^{F4} - b_6 v_i^6 \\ &\qquad + k_6 \theta_i^6 + b_6 v_f^6 + a_6 \theta_f^6 \\ &\qquad + M_i^{F6} = 0 \end{aligned} \tag{12}$$

The expression of equilibrium of horizontal forces is given by:

$$\begin{aligned} &V_f^1 - N_i^4 + V_i^6 = 0 \Rightarrow \\ &\Rightarrow -t_1 v_i^1 + b_1 \theta_i^1 + t_1 v_f^1 + b_1 \theta_f^1 + V_f^{F1} \\ &\qquad - r_4 u_i^4 + r_4 u_f^4 - N_i^{F4} \\ &\qquad + t_6 v_i^6 - b_6 \theta_i^6 - t_6 v_f^6 \\ &\qquad - b_6 \theta_f^6 + V_i^{F6} = 0 \end{aligned} \tag{13}$$

Moving from LCS to GCS and ordering their terms, Equations (11), (12) and (13) are, respectively:

$$(r_1 + t_4 + r_6)V_1 - b_4\theta_1 - t_4V_2 - b_4\theta_2 - r_6V_4 = -\left(N_f^{F1} + V_i^{F4} + N_i^{F6}\right) \quad (14)$$

$$-b_4V_1 + (k_1 + k_4 + k_6)\theta_1 - (b_1 - b_6)U_1 + b_4V_2 + a_4\theta_2 + a_6\theta_3 - b_6U_3 = -\left(M_f^{F1} + M_i^{F4} + M_i^{F6}\right) \quad (15)$$

$$-(b_1 - b_6)\theta_1 + (t_1 + r_4 + t_6)U_1 - r_4U_2 + b_6\theta_4 - t_6U_3 = \left(V_f^{F1} - N_i^{F4} + V_i^{F6}\right) \quad (16)$$

Equations (14), (15) and (16) can be linked to the matrix form in the Global Coordinate System (GCS) of the frames, as follows:

$$\{\mathbf{F}\} = [\mathbf{K_G}]\{\mathbf{D}\} \quad (17)$$

where $\{\mathbf{F}\}$ is the vector of the nodal forces of the structure in the GCS; $[\mathbf{K_G}]$ is the tangential stiffness matrix of the structure in the GCS; and $\{\mathbf{D}\}$ is the vector of nodal displacements of the structure in the GCS.

Using the same procedure for the other degrees of freedom of the plane structure, the remainder of the global tangential stiffness matrix is obtained.

In the process of copying the developed computational codes, the structure is discretized as follows: the elements are numbered from left to right and from bottom to top, starting the numbering by element 1 (one) until the nth element of the structure. The same configuration is adopted for the numbering of the structure nodes, starting at node 1 (one) and continuing until the i-th node.

III. TECHNICAL SPECIFICATIONS OF THE STEEL USED FOR THE PROFILES

ASTM A572 Gr. 50 steel was considered for the laminated profiles used for the beams and columns of the frames, whose physical and mechanical properties are:

- Longitudinal modulus of elasticity (E): 200,000 MPa;
- Modulus of Elasticity Modulus (G): 77,000 MPa;
- Yelding stress (f_y): 345 MPa;
- Rupture stress(f_u): 450 MPa.

IV. ACTIVE ACTIONS

For the analyses performed in the parametric studies of the frames, the floor plan of a model building was used (Fig. 10), considering the variation in the number of columns and the number of floors.

The permanent loads considered, acting on the slabs of the model building, are described below (Table 1):

Table 1: Permanent loads in the slabs.

Own weight * (ribbed slab D = 31 cm) $g_{1,k} = 4.03\ \mathrm{kN/m^2}$
Coating (regularization) $g_{2,k} = 0.05 \cdot 21 = 1{,}05\ \mathrm{kN/m^2}$
Floor (finishing) $g_{3,k} = 1.00\ \mathrm{kN/m^2}$
Sum of the permanent loads in the slabs $\sum g_k = 6.08\ \mathrm{kN/m^2}$

* The self-weight of the slabs was taken from an ATEX BRAZIL catalog.

Regarding the variable loads, an overload was considered $q_k = 3\ \mathrm{kN/m^2}$. In a simplified way, we considered all the slabs of the standard pavements, supported on their four ends (Fig. 11).

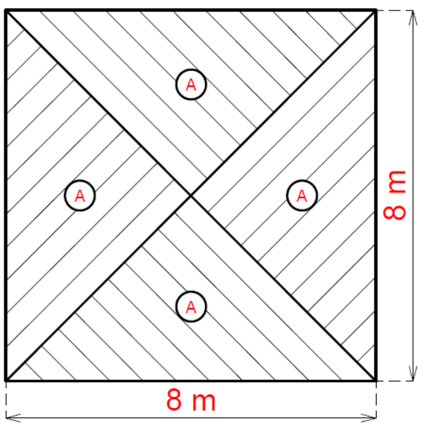


Fig. 11: Type representation of the slab areas of influence.

Therefore, the contribution area of the slab in the beams that support it, calculated using the plastic hinges method, will be:

$$A = \frac{8 \cdot 4}{2} = 16\ \mathrm{m^2} \quad (18)$$

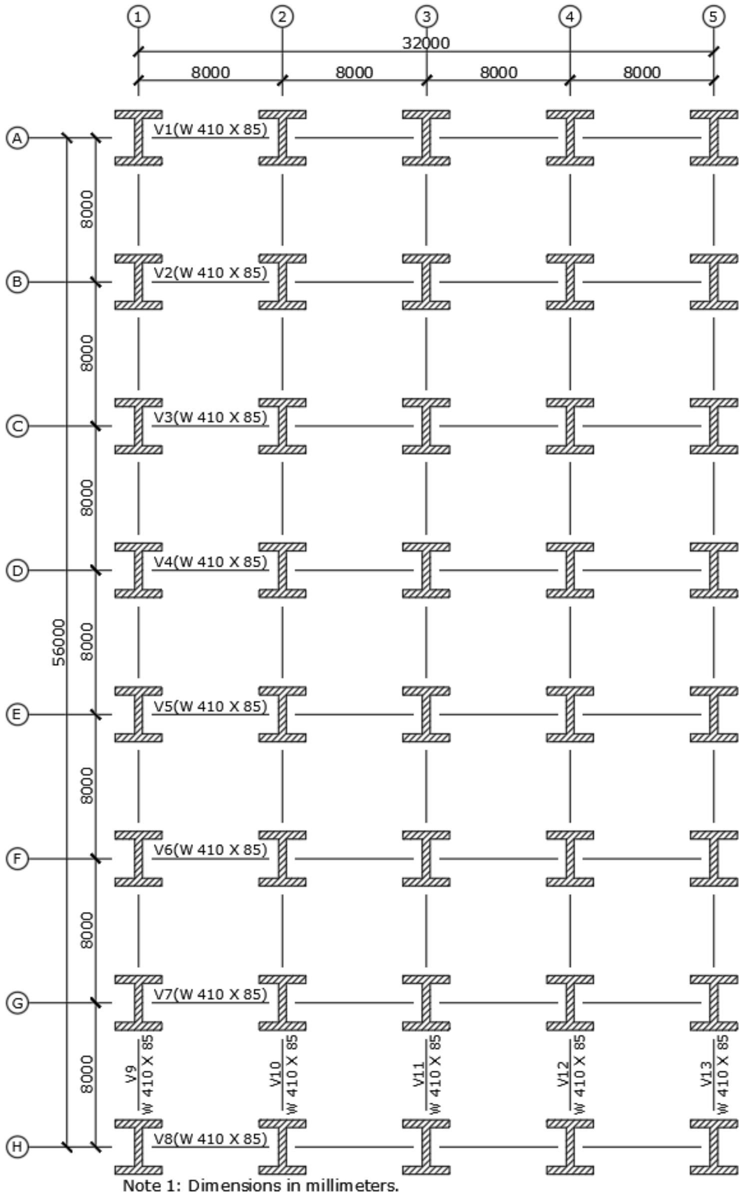


Fig. 10: Floor plan of the model building considered.

Then, the linear, permanent and variable loads, respectively, applied by the slab to the beams are:

$$gw_{1,k} = \frac{6{,}08 \cdot 16}{8} = 12.16 \text{ kN/m} \quad (19)$$

$$w_{1,q,k} = \frac{3 \cdot 16}{8} = 6.00 \text{ kN/m} \quad (20)$$

The beams of the building are made of steel, with a W 410 x 85 profile, and support, in addition to the loads imposed by the slabs and their own weight, 20 cm thick and 359 cm high wall loads. For the central beams of the standard pavement, for example, the distributed permanent load is shown in Table 2:

Table 2: Permanent load distributed in the beam.

Reaction of slabs
$gw_{1,k} = 2 \cdot 12.16 = 24.32$ kN/m
Wall
$gw_{2,k} = 0.20 \cdot 3.59 \cdot 13 = 9.33$ kN/m
Self -weight (I profile 410 x 85)
$gw_{3,k} = 0.85$ kN/m
Sum of the permanent loads in the beam
$\sum gw_k = 34.50$ kN/m

Then, the distributed calculation load ($w_{g,d}$) acting on the central beams, used to calculate the calculation effort, is:

$$gw_d = 1.4\,(34.50 + 6.00) = 56.70\ \text{kN/m} \qquad (21)$$

All the considerations established in this study were made with the purpose of adopting, in the parametric studies of the subsequent item, loads close to those that act on the beams of the real buildings. For practical reasons, it was adopted as the characteristic distributed load and calculation in the beams to calculate the displacements and forces of calculation of the 1st and 2nd-order acting on the frames, respectively, $gw_k = 35.00\ \text{kN/m}$ and $gw_d = 60.00\ \text{kN/m}$.

V. PARAMETRIC STUDY

The parametric study developed in this study consisted of analyzing the behavior of the forces and displacements of the 1st- and 2nd-order, in two families of distinct steel plane frames, with rigid beam-column connections, via DSM and MAM. For the studied structures, the number of columns (frame with 4 and 8 columns) was fixed, and the number of floors was varied (frame with 1, 2, 4, 8, 16 and 32 floors). As the number of floors of the frame varied, the profiles of its bars were altered to stiffen the structure. In Table 3, the profiles adopted for the beams and columns of the frames in each situation analyzed are shown.

Table 3: Structural configuration of the analyzed plane frames.

No. of columns	No. of floors	Profile	
		Column	Beam
4	1	CS 250x52	W 410x85
	2	CS 250x66	W 410x85
	4	CS 350x135	W 410x85
	8	CS 450x227	W 410x85
	16	CS 550x395	W 410x85
	32	CS 650x525	W 410x85
8	1	CS 250x52	W 410x85
	2	CS 250x66	W 410x85
	4	CS 350x135	W 410x85
	8	CS 450x227	W 410x85
	16	CS 550x395	W 410x85
	32	CS 650x525	W 410x85

V.1 PLANE STEEL FRAME WITH RIGID CONNECTIONS AND 4 COLUMNS

In Fig. 12, a plane frame model with 4 columns and n floors is shown, which represents the family of plane frames with 4 columns studied.

The order adopted for the numbering of its bars is shown in (Fig. 12a), and the external calculation load used in the calculation of the requesting forces acting on it are shown in (Fig. 12b).

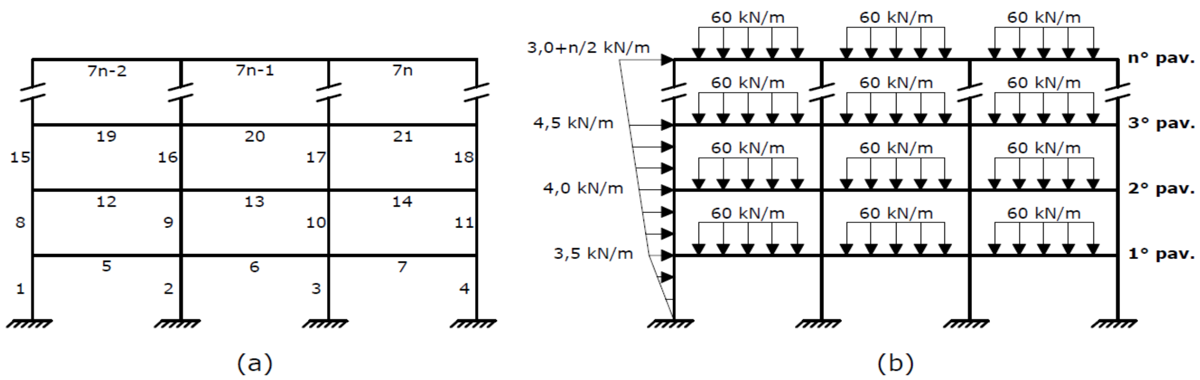


Fig. 12: Illustration of a plane frame model with 4 columns in pavement.

V.2 PLANE STEEL FRAME WITH RIGID CONNECTIONS AND 8 COLUMNS

In Fig. 13, a plane frame model with 8 columns and n floors is shown, which represents the family of plane frames with 8 columns studied.

The order adopted for the numbering of its bars is shown in (Fig. 13a), and the external calculation load used in the calculation of the requesting forces acting on it are shown in (Fig. 13b).

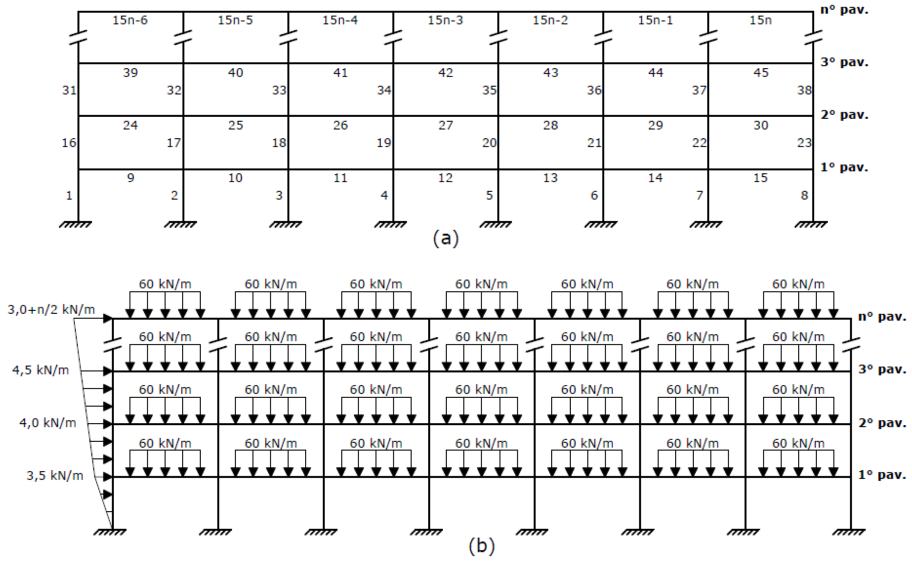


Fig. 13: Illustration of a plane frame model with 8 columns in pavements.

VI. RESULTS AND DISCUSSIONS

VI.1 PLANE STEEL FRAME WITH RIGID CONNECTIONS AND 4 COLUMNS

In this topic, the results obtained from the 1st- and 2nd-order elastic analyses via DSM and approximate 2nd-order elastic analysis via MAM will be presented in plane steel frames with rigid beam-column connections of 4 columns.

Table 4 shows the values of the 1st and 2nd-order displacements, the degree of lateral displacement of the structure and the amplifier factor B_2 (per floor) for frames with 1, 2, 4, 8, 16 and 32 floors.

It is observed that the structures are classified, according to the degree of displacement of the structure, as of small displacement (for the frames with 1, 2, 4 and 8 floors), medium displacement (for the frame with 16 floors) and great displacement (for the 32-storey frame). Conversely, the classification, according to the amplification factor B_2 (adopted by [8] as a parameter for the evaluation of the structures regarding the sensitivity to lateral displacements), is small displacement (for the frame with 1 floor), medium displacement (for the frames with 2, 4 and 8 floors) and great displacement (for the frames with 16 and 32 floors).

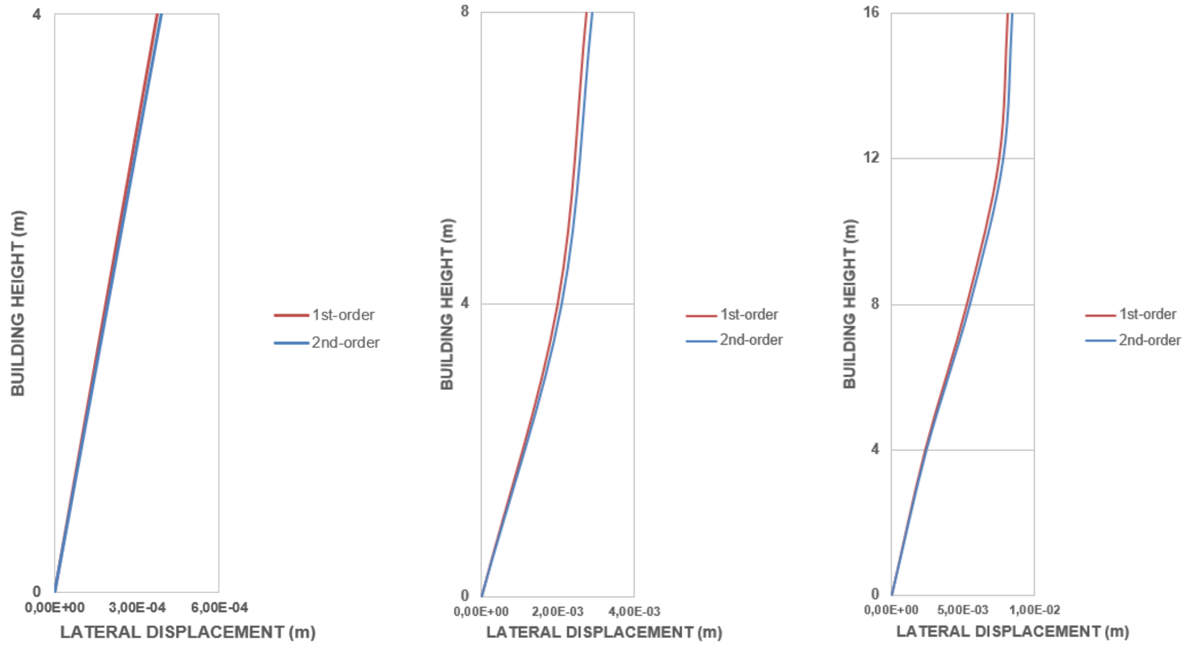
Table 4: Results of 1st- and 2nd-order displacements, degree of lateral displacement and amplifier factor B_2, obtained by pavement, of the plane frame with 4 columns.

Floor	$\Delta_{h,1}$(M)	$\Delta_{h,2}$(M)	$\Delta_{h,1}/\Delta_{h,2}$	B_2
		1 Floor		
0	0.00E+00	0.00E+00	-	-
1	3.74E-04	3.88E-04	1.0380	1.0503
	Maximum Value		1.0380	1.0503
		2 floors		
0	0.00E+00	0.00E+00	-	-
1	0.0020	0.0021	1.0450	1.0856
	0.0028	0.0029	1.0480	1.1083
	Maximum Value		1.0480	1.1083
		4 floors		
0	0.00E+00	0.00E+00	-	-
1	0.0023	0.0024	1.0410	1.0720
2	0.0053	0.0055	1.0440	1.1063
3	0.0075	0.0078	1.0420	1.0577
4	0.0081	0.0085	1.0430	0.9759
	Maximum Value		1.0440	1.1063

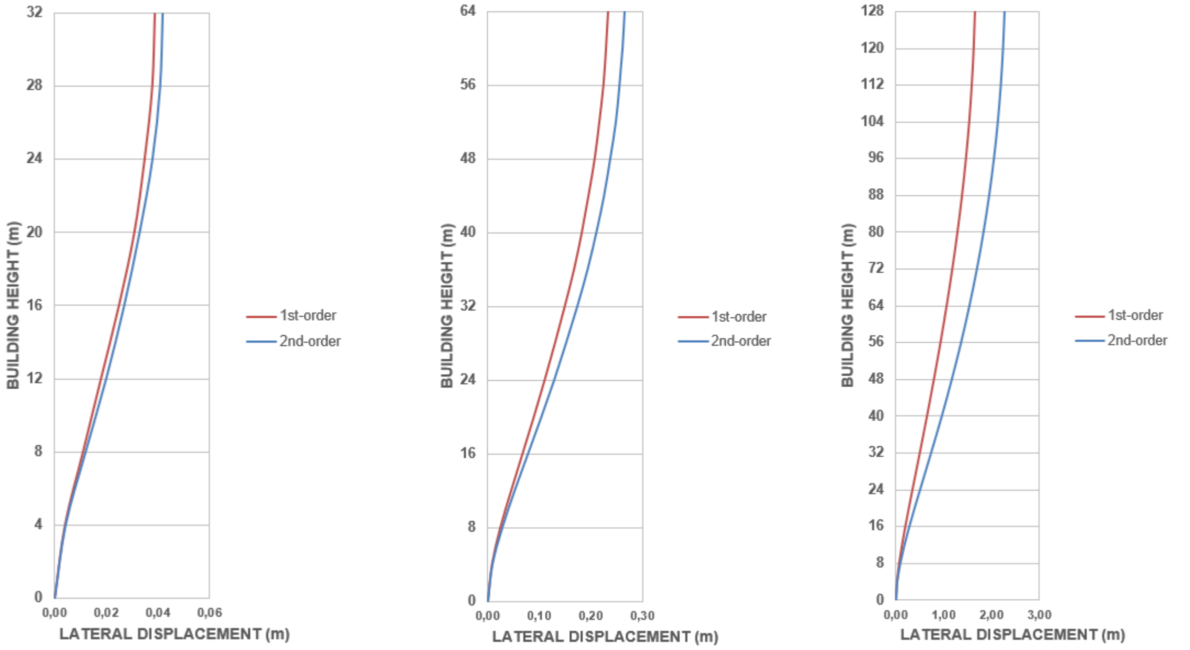
		8 floors		
0	0.00E+00	0.00E+00	-	-
1	0.0040	0.0042	1.0650	1.0927
2	0.0110	0.0120	1.0750	1.1860
3	0.0180	0.0200	1.0770	1.1784
4	0.0250	0.0270	1.0760	1.1578
5	0.0310	0.0330	1.0740	1.1259
6	0.0350	0.0280	1.0720	1.0964
7	0.0380	0.0410	1.0700	1.0491
8	0.0390	0.0420	1.0690	0.9742
	Maximum Value		1.0770	1.1860
		16 Floors		
0	0.00E+00	0.00E+00	-	-
1	0.0077	0.0087	1.1290	1.1320
2	0.0240	0.0280	1.1490	1.3338
3	0.0450	0.0520	1.1600	1.4177
4	0.0670	0.0780	1.1660	1.4430
5	0.0890	0.1040	1.1690	1.4286
6	0.1100	0.1290	1.1690	1.3955
7	0.1300	0.1520	1.1680	1.3550
8	0.1490	0.1740	1.1650	1.3127
9	0.1670	0.1940	1.1620	1.2710
10	0.1820	0.2110	1.1580	1.2312
11	0.1950	0.2260	1.1550	1.1937
12	0.2070	0.2380	1.1510	1.1590
13	0.2160	0.2490	1.1490	1.1269
14	0.2240	0.2560	1.1460	1.0968
15	0.2290	0.2620	1.1440	1.0567
16	0.2330	0.2660	1.1430	0.9338
	Maximum Value		1.1690	1.4430
		32 floors		
0	0.00E+00	0.00E+00	-	-
1	0.0190	0.0250	1.3250	1.2246
2	0.0640	0.0880	1.3740	1.7498
3	0.1240	0.1750	1.4080	2.2561
4	0.1930	0.2770	1.4330	2.6632
5	0.2670	0.3870	1.4510	2.8965
6	0.3430	0.5020	1.4640	2.9665
7	0.4200	0.6180	1.4720	2.9228
8	0.4970	0.7330	1.4770	2.8171
9	0.5730	0.8470	1.4790	2.6850
10	0.6480	0.9580	1.4780	2.5464
11	0.7210	1.0650	1.4760	2.4114
12	0.7930	1.1680	1.4720	2.2843
13	0.8630	1.2670	1.4680	2.1666
14	0.9310	1.3620	1.4620	2.0585
15	0.9970	1.4520	1.4560	1.9594
16	1.0600	1.5370	1.4500	1.8685
17	1.1210	1.6180	1.4430	1.7851
18	1.1790	1.6940	1.4360	1.7084
19	1.2350	1.7660	1.4300	1.6377
20	1.2870	1.8320	1.4230	1.5724
21	1.3370	1.8940	1.4170	1.5119
22	1.3830	1.9510	1.4110	1.4559
23	1.4260	2.0030	1.4050	1.4039
24	1.4650	2.0510	1.4000	1.3556
25	1.5010	2.0940	1.3950	1.3108
26	1.5340	2.1320	1.3900	1.2694
27	1.5620	2.1660	1.3860	1.2315
28	1.5870	2.1950	1.3830	1.1972
29	1.6090	2.2200	1.3800	1.1656

30	1.6270	2.2410	1.3770	1.1359
31	1.6410	2.2580	1.3760	1.0953
32	1.6540	2.2730	1.3740	0.8234
	Maximum Value		1.4790	2.9665

Fig. 14 shows the behavior of the structure regarding lateral displacements along its entire height for the elastic analyses in the 1st and 2nd order. It can be seen that the frames with 1, 2, 4 and 8 floors had a maximum horizontal displacement at the top of the columns lower than the maximum allowed by [8], while the frames with 16 and 32 floors had a maximum horizontal displacement at the top of the columns higher than the maximum allowed by the standard.



(a) 1 Floor (b) 2 floors (c) 4 floors



(d) 8 floors (e) 16 floors (f) 32 floors

Fig. 14: Graphs of lateral displacement versus height of the plane frame with 4 columns.

The calculation forces acting on the bars of the 1st floor of each of the structures studied with 4 columns were also analyzed after performing the 1st- and 2nd-order elastic analysis via DSM and the approximate 2nd-order elastic analysis via MAM. It was observed that the results obtained from the analyses (rigorous and approximate) of the 2nd order are very close to each other when compared to the frames with 1, 2, 4 and 8 floors, which was expected, since these structures displace very little laterally. The 16-storey frame, which had lateral displacement close to twice as allowed by the standard, showed coherence in the results of the shear and normal forces, while the bending moments acting on its bars showed a difference in their results of less than 10% for the analyses (rigorous and approximately) of the 2nd-order. The 32-storey frame, which had a second-order lateral displacement greater than seven times that allowed by the standard, showed discrepancies in all results.

VI.2 Plane steel frame with rigid connections and 8 columns

In this topic, the results obtained from the 1st- and 2nd-order elastic analyses via DSM and approximate 2nd-order elastic analysis via MAM will be presented in plane steel frames with rigid beam-column connections of 8 columns.

Table 5 shows the values of the 1st- and 2nd-order displacements, the degree of lateral displacement of the structure and the amplification factor B_2 (per floor) for frames with 1, 2, 4, 8, 16 and 32 floors.

It is observed that the structures are classified, according to the degree of displacement of the structure, as of small displacement (for the frames with 1, 2, 4 and 8 floors), medium displacement (for the frame with 16 floors) and great displacement (for the 32-storey frame). Conversely, the classification, according to the amplification factor B_2 (adopted by [8] as a parameter for the evaluation of the structures regarding the sensitivity to lateral displacements), is small displacement (for the frame with 1 floor), medium displacement (for the frames with 2, 4 and 8 floors) and great displacement (for the frames with 16 and 32 floors).

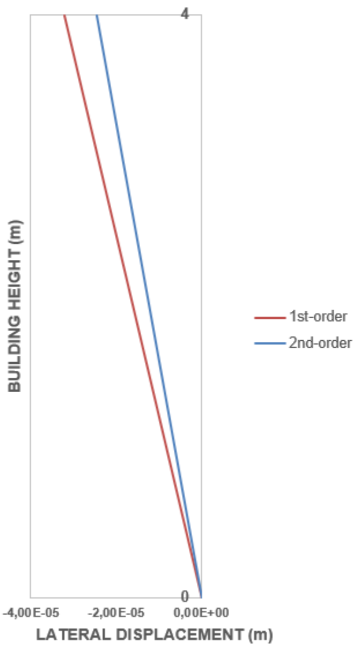
Fig. 15 shows the behavior of the structure regarding lateral displacements along its entire height for the elastic analyses in the 1st and 2nd-order. The frames with floors 1, 2, 4, 8 and 16 had a maximum horizontal displacement at the top of the columns lower than the maximum allowed by [8], while the 32-storey frame had a maximum horizontal displacement at the top of the columns higher than the maximum allowed by the standard

Table 5: Results of 1st- and 2nd-order displacements, degree of lateral displacement and amplifier factor B_2, obtained per floor, of the plane frame with 8 columns.

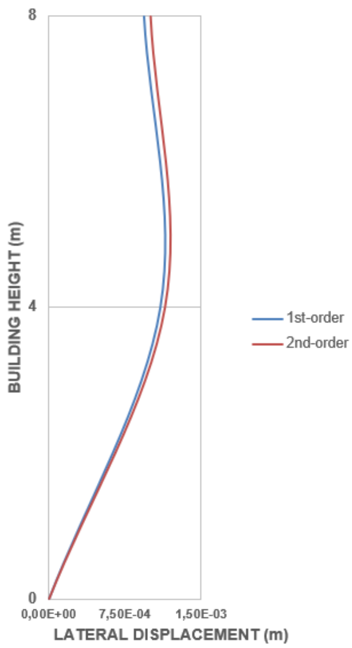
Floor	$\Delta_{h,1}$(M)	$\Delta_{h,2}$(M)	$\Delta_{h,1}/\Delta_{h,2}$	B_2
		1 Floor		
0	0.00E+00	0.00E+00	-	-
1	-3.20E-04	-2.46E-05	0.7680	1.0645
	Maximum Value		0.7680	1.0645
		2 floors		
0	0.00E+00	0.00E+00	-	-
1	0.0011	0.0011	1.0460	1.1117
2	0.0009	0.0010	1.0740	1.0706
	Maximum Value		1.0740	1.1117
		4 floors		
0	0.00E+00	0.00E+00	-	-
1	0.0012	0.0012	1.0410	1.0873
2	0.0024	0.0025	1.0470	1.1092
3	0.0034	0.0036	1.0440	1.0672
4	0.0031	0.0033	1.0530	1.0328
	Maximum Value		1.0530	1.1092
		8 floors		
0	0.00E+00	0.00E+00	-	-

1	0.0040	0.0020	1.0660	1.1072
2	0.0019	0.0054	1.0790	1.1961
3	0.0050	0.0089	1.0810	1.1880
4	0.0083	0.0120	1.0800	1.1622
5	0.0110	0.0150	1.0770	1.1268
6	0.0140	0.0170	1.0740	1.0949
7	0.0160	0.0180	1.0720	1.0524
8	0.0170	0.0180	1.0740	1.0189
	Maximum Value		1.0810	1.1961
		16 Floors		
0	0.00E+00	0.00E+00	-	-
1	0.0036	0.0041	1.1320	1.1472
2	0.0110	0.0130	1.1540	1.3575
3	0.0200	0.0230	1.1650	1.4425
4	0.0300	0.0350	1.1710	1.4596
5	0.0390	0.0460	1.1730	1.4378
6	0.0490	0.0570	1.1730	1.4003
7	0.0570	0.0670	1.1710	1.3572
8	0.0650	0.0760	1.1680	1.3132
9	0.0730	0.0850	1.1640	1.2705
10	0.0790	0.0920	1.1610	1.2298
11	0.0850	0.0980	1.1570	1.1913
12	0.0900	0.1030	1.1530	1.1548
13	0.0940	0.1080	1.1500	1.1201
14	0.0970	0.1110	1.1470	1.0895
15	0.0990	0.1130	1.1460	1.0552
16	0.0990	0.1140	1.1460	1.0007
	Maximum Value		1.1730	1.4596
		32 floors		
0	0.00E+00	0.00E+00	-	-
1	0.0088	0.0120	1.3270	1.2454
2	0.0290	0.0400	1.3760	1.8178
3	0.0560	0.0780	1.4100	2.3633
4	0.0860	0.1230	1.4350	2.7662
5	0.1180	0.1710	1.4530	2.9598
6	0.1500	0.2200	1.4650	2.9840
7	0.1830	0.2700	1.4720	2.9039
8	0.2160	0.3180	1.4760	2.7742
9	0.2480	0.3660	1.4770	2.6282
10	0.2800	0.4120	1.4750	2.4828
11	0.3100	0.4570	1.4720	2.3454
12	0.3410	0.5000	1.4670	2.2187
13	0.3700	0.5400	1.4610	2.1029
14	0.3980	0.5790	1.4550	1.9975
15	0.4250	0.6160	1.4480	1.9015
16	0.4520	0.6510	1.4410	1.8139
17	0.4770	0.6840	1.4340	1.7337
18	0.5000	0.7140	1.4270	1.6601
19	0.5230	0.7430	1.4200	1.5924
20	0.5450	0.7700	1.4130	1.5299
21	0.5650	0.7940	1.4070	1.4721
22	0.5830	0.8170	1.4000	1.4185
23	0.6000	0.8370	1.3940	1.3687
24	0.6160	0.8560	1.3890	1.3225

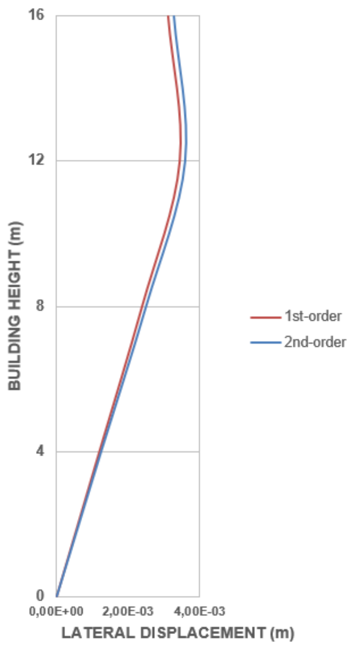
25	0.6300	0.8720	1.3840	1.2794
26	0.6430	0.8860	1.3790	1.2392
27	0.6540	0.8990	1.3750	1.2015
28	0.6630	0.9090	1.3710	1.1660
29	0.6710	0.9180	1.3680	1.1327
30	0.6770	0.9250	1.3660	1.1046
31	0.6820	0.9310	1.3640	1.0772
32	0.6850	0.9340	1.3640	0.9718
	Maximum Value		1.4770	2.9840



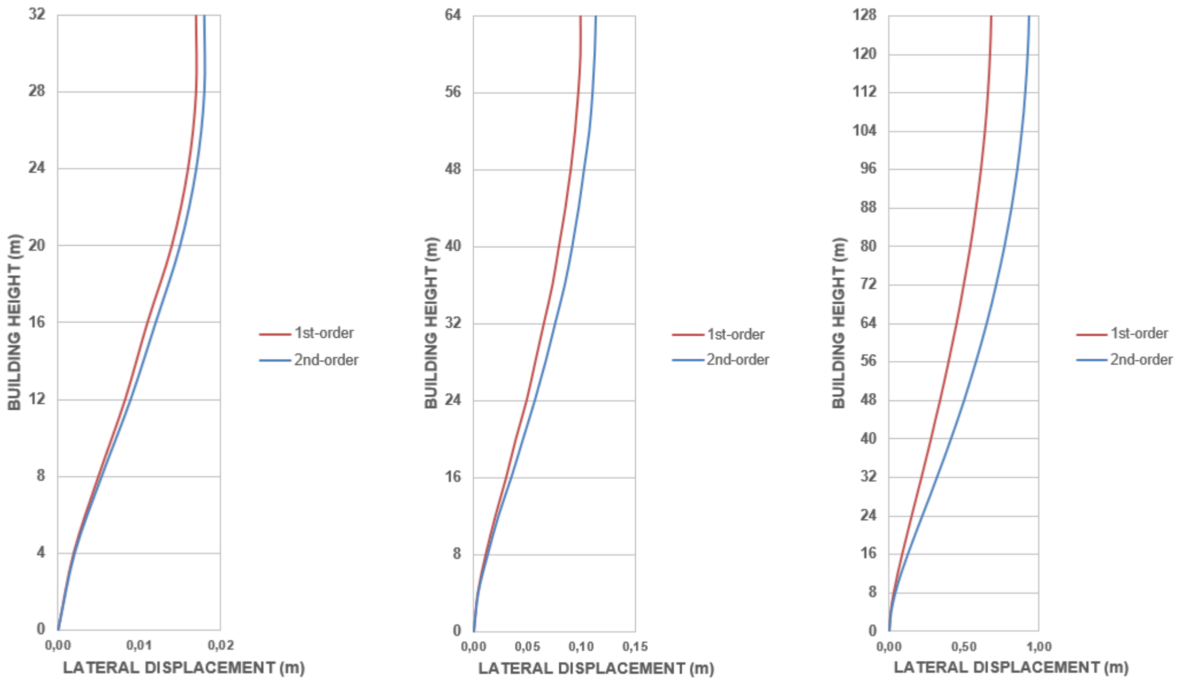
(a) 1 Floor



(b) 2 floors



(c) 4 floors



(d) 8 floors (e) 16 floors (f) 32 floors

Fig. 15: Graphs of lateral displacement versus height of the plane frame with 8 columns.

The calculation forces acting on the bars of the 1st floor of each of the structures studied with 8 columns were also analyzed after performing the 1st- and 2nd-order elastic analyses via DSM and the approximate 2nd-order elastic analysis via MAM.

It was observed that the results obtained from the second-order (rigorous and approximate) analyses are very close to each other, with the values of the approximate elastic analysis sometimes showing values higher than those of the rigorous analysis when compared to the frames with 1, 2, 4, 8 and 16 floors, which was expected, since these structures moved very little laterally. The 32-storey frame, which had a second-order lateral displacement greater than three times that allowed by the standard, showed a discrepancy in all results, as also seen in the 4-columnand 32-storey frame analyzed in item 6.1 of this study.

VI.3 DETAILED ANALYSIS OF THE STRESSES ACTING ON THE BOTTOM OF THE MOST COMPRESSED COLUMN OF THE ANALYZED FRAMES

VI.3.1 Plane steel frame with rigid connections and 4 columns

Table 6 shows the results of the bending moment acting on the bezel of the most compressed column (bar 4) of the column with 4 columns for situations with 1, 2, 4, 8, 16 and 32 floors.

Table 6: Comparison of the bending moment of the calculation obtained by first and second-order elastic analysis and by the MAM acting in the bottom of bar 4 (most compressed column) of the frame with 4 columns for the various situations studied.

Floor	1st Order	2nd Order	
		DSM	MAM
	$M_{sd,1}$(kN.m)	$M_{sd,2}$(kN.m)	$M_{sd,2}$(kN.m)
1	-5.448E+01	-5.504E+01	-5.455E+01
2	-4.765E+01	-4.883E+01	-4.896E+01
4	-9.127E+01	-9.408E+01	-9.505E+01
8	-2.039E+02	-2.191E+02	-2.222E+02
16	-7.024E+02	-8.446E+02	-8.151E+02
32	-2.799E+03	-4.728E+03	-3.618E+03

In Figs. 16 and 17, the graphical comparison between the bending moment values shown in Table 6 is shown as a function of the number of floors that the frame has, obtained by the 1st and 2nd-order elastic analysis via DSM and approximate elastic analysis via MAM.

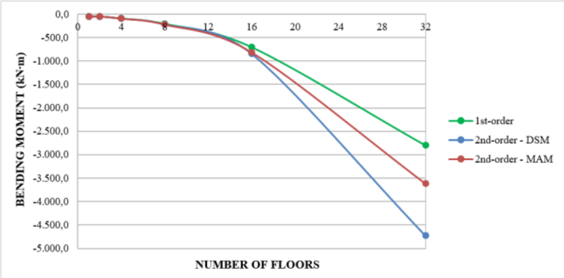


Fig. 16: Calculation bending moment curves obtained by 1st and 2nd-order elastic analysis and by the MAM, acting on the bottom of bar 4 (most compressed column) of the frame with 4 columns, for the various situations studied.

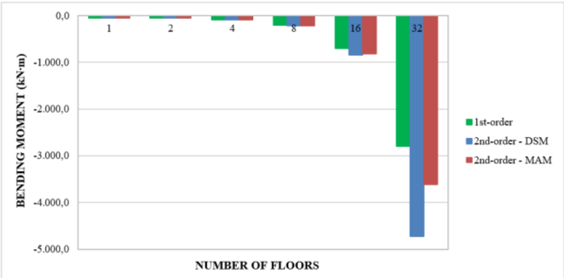


Fig. 17: Comparative graph of the results shown in Table 6.

Table 7 shows the results of the shear force acting on the bottom of the most compressed column (bar 4) of the column with 4 columns for situations with 1, 2, 4, 8, 16 and 32 floors.

Table 7: Comparison of the shear force obtained by 1st and 2nd-order elastic analysis and by the MAM, acting in the bottom of bar 4 (most compressed column) of the frame with 4 columns, for the various situations studied.

Floor	1st Order	2nd Order	
		DSM	MAM
	$V_{sd,1}$(kN)	$V_{sd,2}$(kN)	$V_{sd,2}$(kN)
1	4.216E+01	4.221E+01	4.216E+01
2	3.343E+01	3.347E+01	3.343E+01
4	5.607E+01	5.627E+01	5.607E+01
8	8.357E+01	8.405E+01	8.357E+01
16	1.532E+02	1.535E+02	1.532E+02
32	3.789E+02	3.345E+02	3.789E+02

In Figs. 18 and 19, the graphical comparison between the shear force values shown in Table 7 is shown as a function of the number of floors that the frames have, obtained by the 1st and 2nd-order elastic analysis via DSM and approximate elastic analysis via MAM.

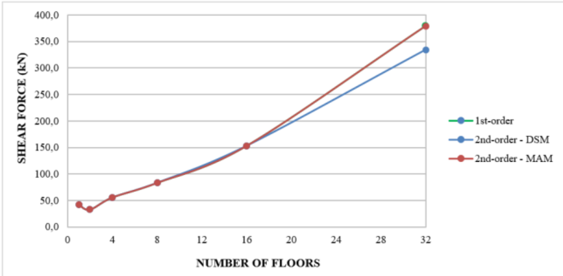


Fig. 18: Calculation shear curves obtained by 1st and 2nd-order elastic analysis and by the MAM acting on the bottom of bar 4 (most compressed column) of the frame with 4 columns for the various situations studied.

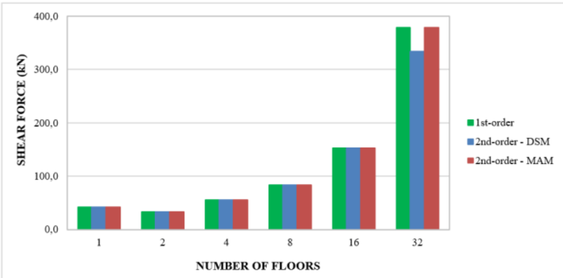


Fig. 19: Comparative graph of the results shown in Table 7.

Table 8 shows the results of the normal force acting on the bottom of the most compressed column (bar 4) of the column with 4 columns for situations with 1, 2, 4, 8, 16 and 32 floors.

Table 8: Comparison of the normal calculation effort obtained by the 1st- and 2nd-order elastic analysis and by the MAM, acting in the crimping of bar 4 (most compressed column) of the frame with 4 columns, for the various situations studied.

Floor	1st Order	2nd Order	
		DSM	MAM
	$N_{sd,1}$(kN)	$N_{sd,2}$(kN)	$N_{sd,2}$(kN)
1	-2.104E+02	-2.104E+02	-2.104E+02
2	-4.443E+02	-4.444E+02	-4.445E+02
4	-9.783E+02	-9.794E+02	-9.796E+02

8	-2.128E+03	-2.140E+03	-2.139E+03
16	-4.938E+03	-5.100E+03	-5.039E+03
32	-1.354E+04	-1.725E+04	-1.476E+04

In Figs. 20 and 21, the graphical comparison between the normal force values shown in Table 8 is shown as a function of the number of floors that the porch has, obtained by the 1st and 2nd-order elastic analyses via DSM and approximate elastic analysis via MAM.

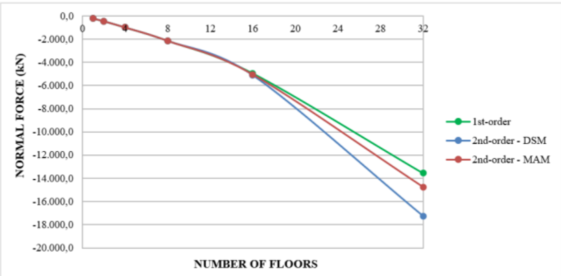


Fig. 20: Curves of the normal force obtained by 1st and 2nd-order elastic analysis and by the MAM, acting in the bottom of bar 4 (most compressed column) of the frame with 4 columns, for the various situations studied.

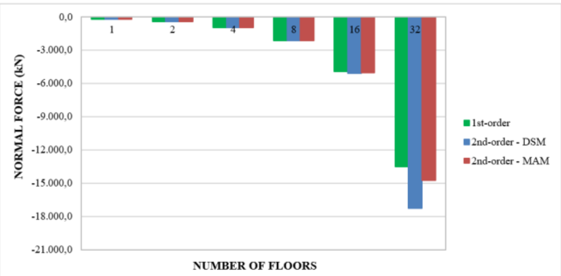


Fig. 21: Comparative graph of the results shown in Table 8.

VI.3.2 Plane steel frame with rigid connections and 8 columns

Table 9 shows the results of the bending moment acting on the bottom of the most compressed column (bar 8) of the column with 8 columns for situations with 1, 2, 4, 8, 16 and 32 floors.

Table 9: Comparison of the bending moment of calculation obtained by 1st and 2nd-order elastic analysis and by the MAM, acting in the bottom of bar 8 (most compressed column) of the frame with 8 columns, for the various situations studied.

Floor	1st Order	2nd Order	
		DSM	MAM
	$M_{sd,1}$(kN.m)	$M_{sd,2}$(kN.m)	$M_{sd,2}$(kN.m)
1	-5.149E+01	-5.199E+01	-5.143E+01
2	-4.351E+01	-4.436E+01	-4.464E+01
4	-7.425E+01	-7.596E+01	-7.695E+01
8	-1.309E+02	-1.387E+02	-1.420E+02
16	-3.608E+02	-4.297E+02	-4.218E+02
32	-1.326E+03	-2.227E+03	-1.745E+03

In Figs. 22 and 23, the graphical comparison between the bending moment values shown in Table 9 is shown as a function of the number of floors that the frame has, obtained by the 1st and 2nd-order elastic analysis via DSM and approximate elastic analysis via MAM.

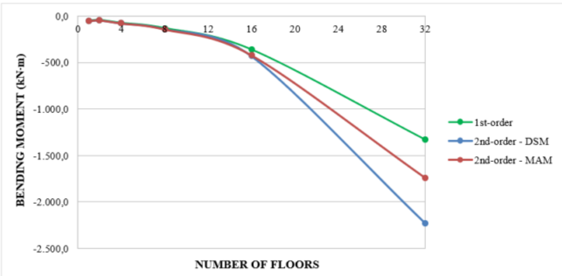


Fig. 22: Calculation bending moment curves obtained by 1st and 2nd-order elastic analysis and by the MAM, acting on the bottom of bar 8 (most compressed column) of the frame with 8 columns, for the various situations studied.

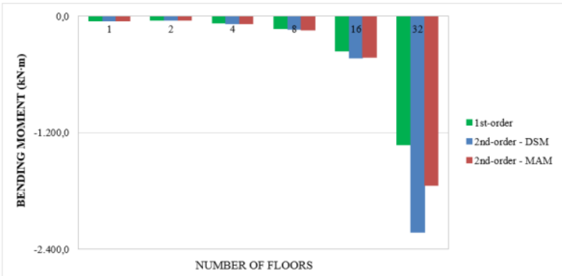


Fig. 23: Comparative graph of the results shown in Table 9.

Table 10 shows the results of the shear force acting on the bottom of the most compressed column (bar 8) of the column with 8 columns for situations with 1, 2, 4, 8, 16 and 32 floors.

Table 10: Comparison of the shear force obtained by 1st and 2nd-order elastic analysis and by the MAM, acting in the bottom of bar 8 (most compressed column) of the frame with 8 columns, for the various situations studied.

Floor	1st Order	2nd Order	
		DSM	MAM
	$V_{sd,1}$(kN)	$V_{sd,2}$(kN)	$V_{sd,2}$(kN)
1	4.058E+01	4.062E+01	4.058E+01
2	3.174E+01	3.174E+01	3.174E+01
4	5.091E+01	5.104E+01	5.091E+01
8	6.791E+01	6.831E+01	6.791E+01
16	1.032E+02	1.045E+02	1.032E+02
32	2.131E+02	2.094E+02	2.131E+02

In Figs. 24 and 25, the graphical comparison between the shear force values shown in Table 10 is shown as a function of the number of floors that the frames have, obtained by the 1st- and 2nd-order elastic analysis via DSM and approximate elastic analysis via MAM.

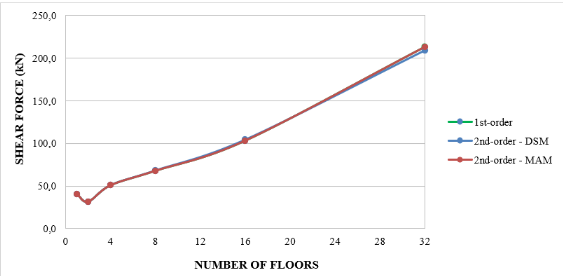


Fig. 24: Calculation shear curves obtained by 1st and 2nd-order elastic analysis and by the MAM, acting in the bottom of bar 8 (most compressed column) of the frame with 8 columns, for the various situations studied.

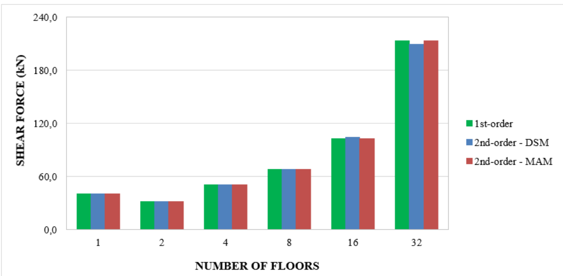


Fig. 25: Comparative graph of the results shown in Table 10.

Table 11 shows the results of the normal force acting on the bezel of the most compressed column (bar 8) of the column with 8 columns for situations with 1, 2, 4, 8, 16 and 32 floors.

Table 11: Comparison of the normal calculation effort obtained by 1st- and 2nd-order elastic analysis and by the MAM, acting in the bottom of bar 8 (most compressed column) of the frame with 8 columns, for the various situations studied.

Floor	1st Order	2nd Order	
		DSM	MAM
	$N_{sd,1}$(kN)	$N_{sd,2}$(kN)	$N_{sd,2}$(kN)
1	-2.089E+02	-2.088E+02	-2.088E+02
2	-4.409E+02	-4.409E+02	-4.409E+02
4	-9.671E+02	-9.675E+02	-9.675E+02
8	-2.071E+03	-2.077E+03	-2.077E+03
16	-4.595E+03	-4.665E+03	-4.643E+03
32	-1.116E+04	-1.265E+04	-1.171E+04

Figs. 26 and 27 show the graphical comparison between the values of normal force present in Table 11 as a function of the number of floors that the frame has, obtained by the 1st- and 2nd-order elastic analysis via DSM and approximate elastic analysis via MAM.

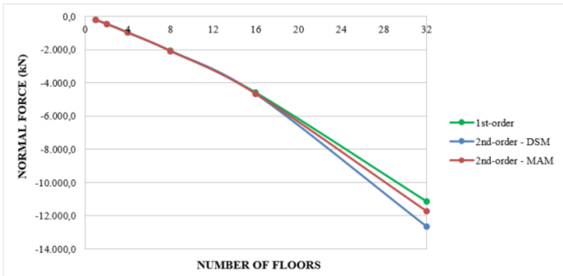


Fig. 26: Curves of the normal force obtained by 1st and 2nd-order elastic analysis and by the MAM acting in the bottom of bar 8 (most compressed column) of the frame with 8 columns for the various situations studied.

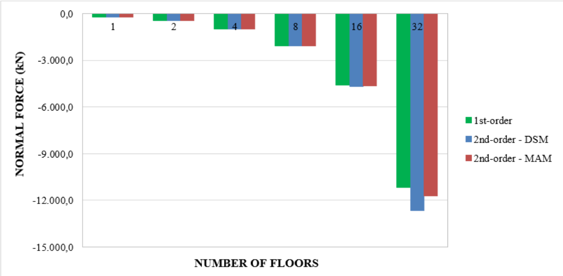


Fig. 27: Comparative graph of the results shown in Table 11.

VII. CONCLUSION

Considering the observed results of the previously studied plane frames, the method proposed by [8], which is based on amplifier factor B_1 and B_2, was satisfactory as long as the maximum horizontal displacement of the top of the column relative to the base, verified in the service limit state of excessive displacements of the structure, did not exceed the limit value (H/400). For lateral displacements of the top of the structure higher than the limit established by the Brazilian standard, the 2nd-order requesting forces obtained through the MAM become divergent from the same 2nd-order requesting forces obtained through the 2nd-order elastic analysis via DSM using the geometric stiffness matrix, which is the methodology used by the structural analysis software.

It is suggested for future studies that the same comparative analysis be performed in steel spatial frames of a real project.

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