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The algebra of saving

José Roberto Mercado Escalante*
**Independent Researcher, Mexico*

ABSTRACT
Wepresentthese as a single object with four facets: fractional fields, fractions, logarithmic rates of change, and the core of the Pareto distribution. Weidentify as fractional the fields linked to demand, supply, constant capital, surplus value, etc. We build with savings and other fractions a vortex that represents economic growth. Weforman algebra,defining a commutatorbetweenincome and consumption.

Keywords-Fractional diffusion equation, income-consumption commutator, Pareto distribution, Boticelli vortex.

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I. INTRODUCTION

We study some aspects of Economic Development, related to Population, Dynamics and Savings. Our approaches are enlightened with mathematical concepts aimed at illustrating fractions, fractional fields, logarithmic rates of variation and the nuclei provided by the Pareto distribution and that as a whole, we see them as a tetrahedron and its faces.

Throughout the document we return to the Principle of Duality which we observe multi-present in the mythology of the original peoples, in particular in Mitlán and its lord Mictlántecuhtli, lord of death and life, located in the cardinal north, with the black color as its chromatic appearance. In this work, it will reappear on multiple occasions as complementary indexes that reflect the reality of pairs of exclusive paths and especially, in the vortex that reflects economic growth.

From the point of view of conservation of mass, we approach population growth. Concept that refers us to the authors Lomonósov, 1748 and Lavoisier, 1785; and it is continued with Euler's statements in terms of probabilities and differential operators. In later stages and from the algebraic formulations, Noether Emmy establishes the relationship between the conservation principles and the symmetries of the differential equations.

For the analysis of population growth, with its mortality and birth rates, we start from Malthus's approaches, from which we rescue geometric and bounded growth, but we complement it with the qualities of concavity and proportionality to its local value,[1]. Changes in income relative to population and in consumption relative to population are linked through the propensity to save and thus affect the growth cycles by period, [2].

We consider different economic fields and their relationships, define some of their fractions as a special form of relationship, and mention their complementary dual paths. We recall the definition of the Pareto probability distribution, we return to the relations of the fields but now their relative variations define the exponents,[3]. Reciprocally, from the core of the Pareto distribution and its exponents, we construct logarithmic changes. With certain fields, we define nodes and we move from one to another through the fractions until we form the magic cycle. With the collection of successive cycles that reflect economic growth, we form the "Boticelli" vortex. We take up the relative variations and define the exponent called productivity. We recall the definition of profit rate and study its evolution to conclude in its trend decrease, [4].

We recall from the topic of fluids the so-called Darcy flow and the temporal evolution of a field such as the anti-gradient of the Darcy flow. However, now we consider it proportional to the fractional derivative of the same field and the proportionality coefficient with the possibility of assuming the two signs. We see that the fields, as the quantities consumed and supplied, satisfy this fractional diffusion equation, which is why we call them fractional fields. An analogous scheme is available for the other fields in this study.

For both the representative businessperson and the worker, we consider the sequence of income and consumption and introduce a commutator that gives the algebra structure. By an intermediate fractal volume, we estimate the ideal case, where the height is the interest and the fractal base is proportional to the inverse of the probability of income.

II. POPULATION GROWTH

Of course, the population is the agent, subject and object, of economic development and its growth affects the different stages that economic cycles contain. We imagine a country as an enclosure and its population as the content. Through the birth rate and the death rate, we see the evolution of the number of population. We recall that Malthus Thomas Robert stated that growth is geometric but that it is limited by the amount of food that the society under consideration is capable of producing, [1].

The most elementary form of growth that we propose is linear. Then a geometric growth of the quadratic type can arise as the combination of a probability of success with a probability of failure and each one to the unit power, from which ascends the logistic description as a special case of the Beta distribution of L. Euler. Finally, we consider a more general geometric growth of the concave type but with saturation.

In the case of the linear model, we assume the difference between the birth and death rates as apositive constant that expresses the population growth rate:

$$\Delta(n_t - m_t) = \Delta P_b = cte > 0, \quad (1)$$

Linear growth is cut off or ends when it reaches a certain level due to food availability and becomes a reference value for time, $P_b(t) = P_0 + ct,\ t \le \tilde{t}$.The next growth considered is the quadratic $P_b = P_0 + ct - at^2$ and its logistic form:

$$P_b = P_0 + P_l(t/\tilde{t})(1 - t/\tilde{t}), (2a)$$

$$\tilde{t} = c/a, P_l = c^2/a, (2b)$$

We express the probability of success by the fraction $p = t/\tilde{t}$ and the probability of failure by its complement: $q = 1 - p = 1 - t/\tilde{t}$. We observe that the maximum is at $t_c = (1/2)\tilde{t}$ an instant, which we will call critical or saturation time because it would locate the depletion of food.

Finally, the superposition of the different powers places us in a population variation proportional to the present value of that population. You get exponential growth. The logistic curvature is incorporated, which is concave, so the second derivative must be negative so that all its strings are below the curve, so the exponent of the exponential is negative and the constant of proportionality is also is negative. Therefore, there is an increasing, concave and saturated curve that approximates the asymptotic value, P_a,

$$\frac{d}{dt}(P_b - P_a) = -\alpha(P_b - P_a), P_b(0) = 0, \quad (3a),$$

$$P_b = P_a\left(1 - e^{-\alpha t}\right), \quad (3b).$$

Although we handle the same time for the two phenomena, it must be specified that the decrease in birth rate Δn_t occurs first and then, with a delay, the decrease in mortality occurs Δm_t. This delay is important, for example, in the phenomenon of retirees. The decrease in Δm_t impacts on the increase in the number of retirees and their social burden; however, as the decrease occurs first in Δn_t, the proportion of retirees in the population grows and with the aforementioned delay or later, the decrease occurs in Δm_t, [1].

We find at least one analog in electrical circuits. It is about visualizing the variable P_b as the electric charge that satisfies the differential equation of a circuit with self-induction, resistance and capacitance: $\frac{d^2}{dt^2}P_b + \frac{R}{L}\frac{d}{dt}P_b + \frac{P_b}{LC} = 0$ and appropriate initial conditions. For simplicity, we suppress the capacitance and compare the critical time with $t_c \leftrightarrow \frac{L}{2R}$ as a parameter or attenuation coefficient. Within this analogy, the stage of old age, more connected with the mortality rate, is represented by the resistance that the energy draws from the system in contrast to childhood, closer to the birth rate, which would be represented by the auto - inductance.

Furthermore, seeing the population growth rate as the difference between birth and death rates, the most important thing is its relationship with economic development. Although Malthus focused his attention more on the birth rate Δn_t, he must have done it on the relationship between the death rate Δm_t and economic growth. At present, we know that the three main factors that affect Δm_t are: public health, food and communications, [1], and that the rate of population growth, with respect to income and consumption, affects the rate of propensity to save in each cycle of the period.

On the other hand,decrease of Δn_t was partially due to the different opportunities that arose, for example, outside the home for women; because development manifests itself in the progressive creation of a diversity of occupations.As well as a change in attitude towards the birth rate, which revealed prior to birth control methods and the workload involved in caring for and raising children. As an illustrative data we cite that in Europe and in a century the decrease of Δn_t was from 30°/₀₀ to 15°/₀₀, [1].

Another manifestation of Economic Development is the displacement of the proportion of the population from agriculture to industry and services. In the underdeveloped, the occupation in agriculture can be 75%, while in the developed; this proportion would be between 12 and 15%, [1].

We will see later that both the rate of consumption with respect to the population, and the rate of income with respect to the population affect the transit from Profit to Savings.

III. FRACTIONAL FIELDS

The classical economists elaborated the conceptual fields among which it is pertinent to highlight Adam Smith, David Ricardo and Karl Marx. These fields are Investment and its level is Savings, while National Income is the conjunction of National Savings and Consumption, with National Savings in turn made up of Public Investment, Balance of Payments and Private Investment. In the subsequent paragraphs, we explain why they are fractional.

The elasticity coefficient in its special connotationcan be seen as the fraction or index that is allocated to the necessary Consumption relative to population growth, usually represented by the percentage of income that is consumed. The complement defines Savings,

$$\Delta C_n\big|_0 = e\Delta Y\big|_0, \quad \Delta P_b\big|_0 = 0, \quad (4)$$

Making the increase in population explicit is $\Delta C_n - \Delta P_b = e(\Delta Y - \Delta P_b)$ either $\Delta C_n = e\Delta Y + (1-e)\Delta P_b$ where the variation in consumption is a convex combination of the increase in income and population. Stated in another way, the complementary rates of consumption with respect to the population and of income with respect to the population are e-proportional, [2].

Within the populationthere are two classes that we highlight: that of businesspersons and that of workers. For the case of a hypothetical average representative worker, we have: $\Delta C_{nT} = e_T \Delta S_a$ for what the savings would be $A_{hT} = \Delta S_a - \Delta C_{nT}$, we could call it the self-tip that the worker pays, although it usually is $e_T \to 1$, which means that the individual savings is very small, but could reach be very large collectively, if we remember that mathematically the innumerable sum of infinitesimals is infinite.

In the case of the representative average businessperson, its elasticity can be represented as the fraction of its profits that is allocated to necessary consumption $\Delta C_{nE} = e_E \Delta U_t$. Something similar happens with the import rate and its coefficient of elasticity with respect to its income, exports (including so-called remittances), [2].

We introduce a coefficient that divides the Savings and determines the size of the investment: $\Delta I_v = \theta \Delta A_h$ and its complement: $\Delta R_t = (1-\theta)\Delta A_h$ the hoarding fraction. So the investment in terms of income would remain $\Delta I_v = \theta(1-e)\Delta Y$.

It is important to note that in the analysis, population growth must be discounted from the growth of national income, therefore, in an appropriate economic growth, both the growth rate of national income, as well as that of consumption, must exceed the growth of the population.

It is interesting to compare it with the physical notion of elasticity and Hooke's law: the stress is proportional to the deformation, while the potential energy is proportional to the square of the deformation. As for the growth of national income, we would judge it as deformation and the increase in consumption as effort. Therefore, the necessary consumption provides the "force" required to obtain the income. Moreover, the potential energy is an energy in latency; it is "vislatens" in contrast to the "vis viva" of kinetic energy. We compare saving with potencial energy whose use is delayed with respect to time, while the pair of inverse curves of demand and supply would be analogous to energy "vis viva".

3.1 Evolution of earnings

The distribution of wealth or income is one of the most important elements for the population. In this issue,the Pareto (Vilfredo) distribution covers a prominent role. In particular, it is about the behavior of the averages, which occurs through a power-type nucleus and thatwith the operation of the logarithm, outcomes a decreasing straight line, with the slope given by the Pareto index.

The Pareto distribution for a random variable of value x is null in $x \le x_0$, as long as its complement assumes the values:

$$P(x; x_0, \alpha) = \frac{\alpha}{x_0}\left(\frac{x_0}{x}\right)^{\alpha+1}, (5a)$$

$$x > x_0 > 0, \alpha > 0, (5b)$$

For averages, we must deal with the density expressed by the nucleus $(x/\alpha)P(x > x_0) = (x_0/x)^{\alpha}$, while its logarithms are represented by a decreasing line of slope $-\alpha$.

By means of the Pareto distribution we return to the issue of the elasticity coefficient in a more general way, assuming that the quantity demanded of a product follows the Pareto distribution, its index being the price elasticity coefficient:

$$P_r(P; P_m, e) = \frac{e}{P_m}\left(\frac{P_m}{P}\right)^{e+1}, (6a)$$

$$P > P_m > 0, e > 0, (6b)$$

The averages behave as said by means of the core of the price relative to an enhanced minimum Price $Q_d = (P_m/P)^{e_d}, \quad e_d > 0$. The slope of the quantity demanded with respect to its price is $\frac{dQ_d}{d(P/P_m)} = -e_d \frac{Q_d}{P/P_m}$, then: $e_d = -\frac{dQ_d/Q_d}{dP/P}$ the price elasticity coefficient is the negative of the relative change in demand with respect to the change in price or the logarithmic change in quantity with respect to the logarithm of the price, [3].

On the horizontal axis, we represent the random variable of price, higher than the minimum, and on the vertical the quantity demanded or consumed, which is proportional to the probability provided by the Pareto distribution.

On the other hand, the curvature of the demand is positive, so it corresponds to a convex curve where all its chords are above the curve. The appearance of the curve depends on the value of e_d, if it is almost straight then it e_d is small, almost zero; if it is parabolic, the curvature is 2 y $e_d = 1$; if it is quite curved or rounded $e_d > 1$, and is said to be elastic; in particular if $e_d < 1$, the curvature is less than 2, and it is said inelastic, [3].

The counterpart of the demand, the supply of the product, then follows a curve with a positive slope, negative curvature of the form $Q_o = (P_m/P)^{-e_o}, \quad e_o > 0$, so the power must be positive. Now in the vertical, we represent the quantity-supplied proportional to the complementary probability. Moreover, for the same reason, the appearance of the curve follows the same guidelines as the demand curve, [3].

Furthermore, the income elasticity of demand is defined by $Q = (Y_m/Y)^{-e_Y}, \quad e_Y > 0$, which translates into $e_Y = +\frac{dQ/Q}{dY/Y}$. Thus the income elasticity of consumption is $C = (Y/Y_m)^{e_Y}, \quad e_Y > 0$. Note that its shape is similar to the supply curve, while the random variable linked to income is located on the horizontal axis. The logarithmic rate of the logarithm of the quantity of product demanded is the income elasticity.

The special version is $C = (1/Y_m)Y, \quad e_Y = 1$ and the elasticity is given by $C = e_1 Y, \quad e_1 = (1/Y_m) < 1,$, thus it is the Pareto index $\alpha = 1$.

In successive periods, the exponent of the amount consumed can change $Q_d(t) = (P/P_m)^{-e_d(t)}$, $e_d > 0$ the logarithmic change is $d \ln Q_d(t) = -e'_d(t) \ln(P/P_m) dt$. In addition, a similar formulation outcomes for the supply curve but with the respective change of the sign of the exponent.

Although the shifts in the demand and supply curves can be positive or negative, the phenomenon known as inflation can be represented by a positive shift in the demand curve parallel to the price axis, so that the minimum price and all the others increase in a certain proportion. Moreover, inflation can appear due to a negative shift in the supply curve, in the direction parallel to the quantities of product.

On the contrary, the positive shift in supply leads to a decrease in the minimum or reference price. This is the well-known drop in prices due to the abundance of the product offered in contrast to the increase in prices due to its scarcity.

3.2 The Magic Cycle

In the Trinitarian formula, the surplus value or surplus rate is composed of the rates of: profit, income and interest, [4], [5]. The first two,we can group as a profit rate, although there is also the option of composing the last two as a new interest rate, depending on the ownership of the land. Therefore, the addition of utility and interest rates determine the surplus-value rate, and from them calculated per unit of surplusvalue, then result two complementary indices, one associated with Profit and the other with Interest.

$$\Delta pV = \Delta G + \Delta R_t + \Delta I_t, \quad (7a),$$
$$\Delta pV = \Delta U_t + \Delta I_t, \quad (7b),$$
$$\Delta U_t = \gamma \Delta pV, \Delta I_t / \Delta pV = 1-\gamma, \quad (7c).$$

Therefore, for a given rate of surplus value, a decrease in the interest rate produces an increase in profits and therefore, is a stimulus to investment.

Then we form a cycle of at least four nodes. We start from the node that we call Utility and carry out a fraction through the complement $1-e$ until we reach the node that we call Savings, while its complement is Consumption (necessary). Then, we carry out another fraction θ and reach the Investment node, its complement is Treasury o Hoarding.Later we carry out an expansion ρ and obtain the node that we call surplus value or surplus. Finally, we carry out the fraction γ and close the cycle arriving at the new Profit node; its complement is Interest. The repetition of this cycle, constituted for three contractions and a dilation, could take an economy from the underdeveloped level to the developed level and that would transform a relatively low level of National Savings to a level 3 times higher. Quantitatively, there is a sequence of three contractions and a dilation, whose intensity must be high enough to produce growth in each cycle. The intensity criterion is $\rho \cdot (1-e) \cdot \theta \cdot \gamma = r > 1$.

For example, we could start at the plus-value node, in a period marked as zero, then we go through the cycle until we return to the plus-value node but in the following period, one; therefore:

$$pV_0 \cdot \gamma \cdot (1-e) \cdot \theta \cdot \rho = pV_1 = r \cdot pV_0, (8)$$

Thus in successive periods we would have similar sequences. However, the different factors can be reinforced with a view to increasing the desired capital gains and the economic growth that is sought, although they can also be weakened and we must remember that each of the factors are linked to random phenomena and with dual paths.

One of the main options is to strengthen savings, and therefore weaken hoarding, with the aim of increasing investment and continuing the cycle. Therefore, another manifestation of economic development is the greater proportion in the participation of savings, and therefore of profits, in national income.

As an example, we illustrate with what we will call the “Boticelli vortex”. We start from certain data provided by [2]. For the propensity to consume $e = 1/2$, for the rate of utility with respect to surplus-value $\gamma = 9/10$, for the saving rate $4/5$, and that of hoarding $1/5$, for the dilation towards surplus-value $\rho = 5$, thus the growth factor of the period $r = 1.8$. We associate a fraction of the plane angle with each one of the nodes until completing the first cycle, the first corresponds to the growth factor, the second to the utility, until the last one is the plus-value; so the first angle is $\varphi_1 = 2\pi / r = (10/9)\pi$; the second $\varphi_2 = \varphi_1 \cdot \gamma = \pi$; and so on $\varphi_3 = \varphi_2 \cdot (1-e) = (1/2)\pi$; and $\varphi_4 = \varphi_3 \cdot \theta = (2/5)\pi$; up to $\varphi_5 = \rho \cdot \varphi_4 = 2\pi$, which completes the plane angle. Then we define the radii of the successive cycles $r_{ai} = 3(\ln 3.4)^i, \quad i \in \{0,8\}$, being the number of turns or cycles 9, finally we define the vortex by the spatial curve, (9), $0 \le t \le 16\pi$,

$$\left(3(1.23)^{t/2\pi} \cos t, 3(1.23)^{t/2\pi} \sin t, 5(t/2\pi)\right), (9)$$

We start with the 3% savings rate, typical of an underdeveloped country, until it becomes somewhat higher than the 15% value of some developed country. We call it Boticelli because it represents the theoretical possibility of leaving the "hell" of underdevelopment in 9 cycles, with which we allude to the nine levels of Dante's "hell", (Fig.1).

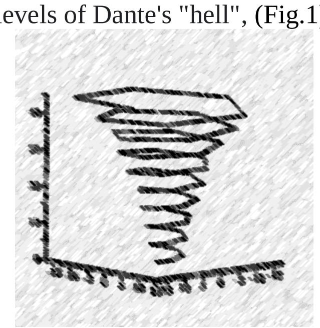

Figure 1:“Boticelli vortex”.

In other words, each cycle consists of three steps backward and one forward that surpasses them, in that lies the "magic" of the sequence of cycles, it is about the "value that values", [4]. We also compare it with an elastic body that undergoes three successive contractions until it accumulates enough potential energy that allows it to later, display the expected expansion.

In addition, economic development can also,to describe a transition process where National Savings grow from 1/20, or 5% to 3/20, or 15% or more per year. In the social sphere, it is about the formation of a new social class, called the "businesspersons", whose goal is to obtain and grow their profits and who progressively obtain a greater proportion of the National Income. By contrast, the class of workers, sectors of the peasants, landowners and aristocrats have little inclination to Savings. Thus, the States, unions and other social organizations should intensely promote the practice of Savings until it could become part of the national culture.

We define Productivity, in a similar way to a density, given by the quotient between the "mass" of surplus value, produced by workers and the "volume" of jobs that workers occupy.A density satisfies the condition, that if the measure of "volume" of a certain enclosure is zero, then the measurement of the "mass" it contains is also zero, which means the obvious statement: if the number of workers is zero in a facility, the surplusvalue produced will also be zero, [2], [6].

The amount of jobs offered at salaries, which exceed the minimum salary, behave as $Z / Z_0 = (S_a / S_m)^{\varepsilon \cdot t}$ $a = S_a / S_m > 1$. The logarithmic change of the quantity in context is expressed by $d \ln Z(t) = +\varepsilon \cdot \ln a dt$, and $\hat{\varepsilon} = \varepsilon \ln a$; or in the form $dZ(t) / Z(t) = \hat{\varepsilon} \cdot dt$. Analogously, for the plus-value $pV / pV_0 = \tilde{p}^{\tilde{r} \cdot t}, \quad \hat{r} = \tilde{r} \ln \tilde{p}$ which results in $dpV(t) / pV(t) = \hat{r} \cdot dt$. Therefore, productivity will be $dpV / pV = p_d dZ / Z = (\hat{r} / \hat{\varepsilon}) dZ / Z$, or we observe that productivity enhances the surplus-value in:

$$pV / pV_0 = (Z / Z_0)^{\hat{r} / \hat{\varepsilon}}, (10a)$$

$$pV / pV_0 \approx 1 + p_d \ln(Z / Z_0) + \ldots, (10b)$$

The logarithmic rate of surplus value should also, be related to the number of workers. Productivity enhances the surplus value $\frac{d \log pV}{d \log Z} = p_d = \frac{\hat{r}}{\hat{\varepsilon}}$, but the growth in the rate of new jobs weakens productivity, assuming the permanence of the other factors, and therefore attenuates the change in surplus value. In fact, at present shows the tendency to decrease in this number, known as an increase in unemployment. However, the increase in the number of jobs, boosted by productivity, contributes to an increase in the added value, which has repercussions, through the cycle, with an investment growth and finally, strengthens the consumption of the two mentioned and outstanding classes.

It can be seen that the relationship between the growth factor and the logarithmic rate of surplus-value, or its Pareto index, is expressed by $r \approx 1 + \hat{r}, \quad \hat{r} = \tilde{r} \ln \tilde{p}$.

On the other hand, the profit rate defines the fraction of the surplus value obtained over the total capital invested in labor, materials and constant capital in machinery and others, [4], [5]:

$$T_g(t) = \frac{pV}{S_a + K_c} = \frac{S_a + U_t}{S_a + K_c} < 1, (11)$$

Labor outcomes from the confrontation between demand and supply, so it is governed by the nucleus derived from a Pareto distribution. We assume that the salary, including raw materials, or prime cost, behaves as a random variable that follows a Pareto distribution, with α -index, which is achieved in a similar way to income-dependent consumption, whose salaries exceed S_m is $(S_a / S_m)^{-\alpha \cdot t}$. But we also assume that constant capital follows a supply curve $K_c / K_0 = (K / K_m)^{+e \cdot t} = c^{e \cdot t}$ and an analogous one for utility $U_t / U_0 = (U_t / U_m)^{+\mu \cdot t} = b^{\mu \cdot t}$; but also and for simplicity, we assume the initial values equal to unity, so the relative values are $K_c / S_a = (a^{\alpha / e} c)^{e \cdot t}$, $K_0 / S_0 = 1$ and $U_t / S_a = (a^{\alpha / \mu} b)^{\mu \cdot t}$, $U_0 / S_0 = 1$. The variation with time or evolution of the rate of profit depends on the derivative of a quotient, so the sign of this evolution depends in turn on the expression in the numerator; then if the quotient is: u / v, then $\frac{1}{u} \frac{du}{dt} > \frac{1}{v} \frac{dv}{dt}$ either $\frac{d \log u}{dt} > \frac{d \log v}{dt}$, if we assume that the derivative is positive. If this is the case then $\frac{d}{dt} \log\left(1 + (a^{\alpha} b^{\mu})^t\right) > \frac{d}{dt} \log\left(1 + (a^{\alpha} c^{e})^t\right)$.

However, for a sufficiently long period $t > t_c$, it turns out $\mu \ln b \geq e \ln c$, where in particular we observe its independence from salary. So, $b^{\mu \cdot t} \geq c^{e \cdot t} \quad \Leftrightarrow \quad U_t \geq K_c,\ t > t_c$ but this contains a contradiction, because $T_g(t) = \frac{1 + U_t / S_a}{1 + K_c / S_a} < 1$ it implies that $T_g(t > t_c) = \frac{U_t}{K_c} \leq 1$. In conclusión $\frac{d}{dt} T_g(t > t_c) \leq 0$ and it expresses the trend decrease of the profit rate, [7].

In economic dynamics, there are price changes, as in inflation, or changes in exponents, as in product supply changes. We know that the anti-gradient of a flow represents the change in time of a field, as in the case of Darcy flow in fluids. A more general notion is the Darcy fractional flux where it is proportional to the fractional anti-gradient of the same field. We consider the field of prices, relative to a minimum value, be the Darcy flow proportional to the fractional anti-gradient, of order beta, of the quantity of product offered, as a function of the period, [8], [9].

$$\left(\frac{\partial}{\partial t} \right) Q_o = \left(-\frac{\partial}{\partial p} \right) q_D, \quad q_D = D_{o\beta} \left(-\frac{\partial^\beta}{\partial p^\beta} \right) Q_o, \ (12)$$

Compatibility with $Q_o(t) = p^{+e_o(t)}, \quad p = P / P_m$, is achieved if the proportionality of the diffusion coefficient satisfies $e_o'(t) = D_{o\beta} \frac{\Gamma(1 + \beta + e_o)}{\Gamma(1 + e_o)}$. In the limit $D_{o2} = \frac{e_o}{1 + e_o}, \quad \beta \to 1$, the inverse diffusion problem is given and solved in terms of the elasticity of supply. Note that all variables are dimensionless. We will have similar results for the other fields with a positive exponent such as income elasticity or with productivity. This is the reason for calling them fractional because they are a solution of a fractional diffusion equation, [8].

For the field associated with demand, the change of sign of the exponent is import, then $e_d'(t) = D_{d\beta} \frac{\Gamma(1 + \beta - e_d)}{\Gamma(1 - e_d)}$ and in the limit we have $D_{d2} = \frac{e_d}{e_d - 1}, \quad \beta \to 1$ and it also solve the inverse problem of diffusion in terms of the elasticity of demand, but now we must limit ourselves to the case inelastic. Thus, the growth or decrease of the exponent is linked to the sign of the coefficient, as is also observed in the logarithmic change $d \log Q_d(t) = -e_d'(t) \ln pdt$. The commercial flow of products from places with low prices to places with high prices is known, that is, in the opposite direction to normal diffusion. Similar results are obtained for fields with a negative exponent. For example, for salary evolution $S(t) = (S_a / S_m)^{-\alpha(t)}$, the logarithmic change in salary is given by $d \log S(t) = -\alpha'(t) \cdot \ln adt,$ and $a = S_a / S_m > 1$.

On the contrary, in the evolution from one period to another of constant capital $K_c(t) = (K / K_m)^{+e \cdot t} = c^{e \cdot t}$, as well as for utility $U_t(t) = (U_t / U_m)^{\mu \cdot t} = b^{\mu \cdot t}$; the growth of the exponent is equivalent to the growth of the relative variation of both constant capital and utility, thus the coefficient has the positive sign of normal diffusion. Similarly, the quotient of constant capital with salaries, known as the composition of capital $K_c / S_a = \left(a^{\alpha / e} c \right)^{e \cdot t}$, $K_0 / S_0 = 1$ and the quotient of utility over salary $U_t / S_a = \left(a^{\alpha / \mu} b \right)^{\mu \cdot t}$, $U_0 / S_0 = 1$ the exponents are increasing and the diffusion is normal, that is, in the sense of the anti-gradient.

The relative salary rate $\frac{dS(t)}{S} = -\alpha'(t) \ln adt$ can be attenuated by lowering the value of the logarithm, which is achieved by lowering the salaries or increasing the minimum salary. On the other hand for constant capital and utility, the relative rate is $dK_c(t) / K_c = e'(t) \ln cdt$. An increase in the logarithm slightly intensify that rate relative, which can be achieved by means of a de-hoarding, or reinforcement of savings, with a view to intensifying investment, something similar to what happened with the integration of the aristocracy and landowners to capitalism during its progressive phase.

A partial or total migration of the countries of different production processes to places where salaries are lower is observed, which is reflected in the growth of the proportion $K_c / S_a = \left(a^{\alpha / e} c \right)^{e \cdot t}$, as long as the other factors remain given during the period.

One among many manifestations of economic development is the displacement of the proportion of the population from agriculture to industry and services, but also due to the increase in the productivity of agriculture; but the increase in the proportion $K_c / S_a = \left(a^{\alpha/e}c\right)^{e \cdot t}$ of capital invested on the salary paid, impulses this increase in productivity, for the other factors given. Nevertheless, in the face of a significant increase in the aforementioned proportion, this gives rise to the possibility of a reduced growth of the Salary, as well as of the interest payment expenses.

The Pareto index, as already mentioned, reflects the degree of participation of the population in the wealth generated by society. The growth of the index or exponent is similar to normal diffusion; it is in accordance with the historical evolution of capitalism in its progressive or expansive phase. Meanwhile, the decreasing exponent is similar to the anti-diffusion or diffusion in the sense of the gradient of income or wealth. We observe this decreasing exponent in the current era, because during neoliberalism we perceive the tendency to a greater concentration of wealth in the wealthiest portions of society, which we interpret as a historical regression. We say that the sign of the derivative of the index reflects the class struggle.

In inflation, in its most common form, there is a positive shift in the demand curve $\dfrac{Q_d\left(p_0 + dp_0\right)}{Q_d\left(p_0\right)} = 1 - e_d d \ln p_0,\ p_0 = P_0 / P_{m0}$, then there is growth in the logarithmic rate of price, then there is a reduction in the rate of consumption of the product, but in turn the value of the minimum price equilibrium with supply has increased to the new value $\hat{P}_m$, or $P_{m0} \mapsto \hat{P}_m$. If we now consider a negative shift in the supply curve $\dfrac{Q_o\left(\hat{p} - d\hat{p}\right)}{Q_o\left(\hat{p}\right)} = 1 - e_o d \ln \hat{p},\ \hat{p} = P / \hat{P}_m$ so $-\dfrac{\Delta p}{p} \approx -\Delta \ln \hat{P}_m$ turns; or else the equilibrium point follows the sequence $\left(P_{m0}, Q_m\right) \to \left(\hat{P}_m, \hat{Q}_m\right) \to \left(P_{m0}, \hat{Q}_m\right),\quad \hat{Q}_m > Q_m$.Then, it will eventually return to its previous value, but now on a supply-curve that exceeds the original one;or, the pair of demand-supply curves undergoes a positive displacement, parallel to the axis of the quantity of product demanded or supplied.

In the case of oligopolies, these artificially agree on the price increase of certain products, and the answer may be to increase the supply with new companies or with the increase in imports of the product. Similarly, in the case of excess demand, there has been a positive shift in the demand-curve and can be answered by a negative shift in the supply-curve. The logarithmic change of different entities, such as: the value of the random variable, price, income, jobs, constant capital, etc., as cause, have their effect on the logarithmic change of its related entities, such as: the probability value, the quantity consumed or offered, the quantity demanded, the surplus-value obtained, where the quotients are the different exponents associated with each couple.Thus, we have the Pareto index, the price elasticity, income elasticity, productivity, etc. In addition, these entities are representable by the thermodynamic phenomenon of adiabatic expansion, where the logarithmic change in pressure, with respect to the logarithmic change in volume, is the adiabatic exponent, a constant greater than unity. Furthermore, it is interesting to see that the inverse of the diffusion coefficient is analogous to the pressure exponent for the expression of temperature in the case of adiabatic expansion, an expansion that manifests itself in cooling. Isothermal expansion represents the special case of index 1.

IV. THE ALGEBRA OF SAVING

It is usual,that advertising induces a representative average workerto increase his consumption by means of credit cards or consumer loans. In a similar way to how the average businessperson obtains bank credits that allow him to increase constant capital, which strengthens his profits, although a fraction is shared it with banks or the financial sector, and then expand your savings with a view to future investments. However, the entities occur at different times for these two representative individuals and with different intensity. This presents us with the sequence of obtaining income and then consumption, but in two different ways, for the businessperson first obtains income and at a later moment carries out consumption, while the representative worker the sequence is the other way around, first he consumes with his credit card and then you get your income, the salary. Thus, we can define a commutator that forms an algebra structure, [2], [6], with the necessary clarification that our notation is local to this section of the document:

$$\left[C, Y\right] = CY - Y'C', \ (13)$$

Let us study this difference, the first is $CY = eY^2$. For the second, the income contains the monthly payment of the credit card so,this must be discounted as in $Y'C' = e\left(Y - I_t\right)^2$. So, the difference is $\left[C, Y\right] = eY^2 - e\left(Y - I_t\right)^2$, which leads to $\left[C, Y\right] = 4eY^2\left(1 - I_t / 2Y\right)\left(I_t / 2Y\right)$, doing

$p = I_t / 2Y < 1$, being interpreted as half the interest rate. It is rewritten as $[C,Y] = \frac{2}{3} eY^2 \cdot \rho(p,2,2)$, $\rho = 6p(1-p)$ and recognized as a Beta Euler structure, the density of the logistic probability distribution. On the other hand, we observe that the expression $CY = eY^2$ remembers the elastic potential energy, while $Y'C' = e(Y - I_t)^2$ it will be the diminished potential energy.

The logistics graph has two minimums at zero and a maximum in between, exactly equidistant or in the middle. Therefore, any value that its image assumes will be intermediate between zero and the maximum in the vertical with value ¼ and $[C,Y] \le eY^2$, in turn this maximum is intensified with a relatively high elastic coefficient and attenuated, in the opposite case. Let us see the location of the two classes. The worker has a relatively low income so the probability of success is higher then, his horizontal line would be closer to the maximum ¼; however, you might get a slight attenuation if the interest is lower. Abusinessperson has a relatively high income so his probability of success is low, so his horizontal line would be closer to the minimum and hecould commit to a somewhat higher interest. In conclusion, the alteration of the algebra sequence harms the worker in a greater extent, with respect to his income.

A more general treatment of the commutator is as follows $CY = (Y/Y_m)^e Y = Y_m (Y/Y_m)^{e+1}$, $Y'C' = Y_m((Y - I_t)/Y_m)^{e+1}$, then $[C,Y] = Y_m (Y/Y_m)^{e+1}\left(1 - (1 - I_t/Y)^{e+1}\right)$. We highlight the factor dependent on the interest rate as the function $f(c_r) = 1 - c_r^{\gamma}$, $\gamma = e + 1$, $c_r = 1 - r$, $r = I_t / Y$, the function is decreasing and concave, so all its chords are below the curve, the chord of lower heights passes through (0,1) and (1,0) and has slope -1, so $f(c_r) \ge r$. Also, its tangent line has slope -1 and passes through the value $f(c_r) = 1 - (1/\gamma)^{\gamma/\gamma - 1}$, (intermediate value theorem). Then,the two parallel lines of slope -1, mentioned, bound the curve between one. Therefore, the commutator has a lower bound, and the interest rate determines one,

$$[C,Y] \ge rY_m (Y/Y_m)^{e+1}, \ (14a)$$
$$[C,Y] \ge r \cdot (e / P_r(Y > Y_m, e > 0)), \ (14b)$$

The commutator evaluated per unit of minimum income depends on two factors: interest and the power of income relative to the minimum. The latter can be seen as a fractal area because its dimension is greater than 1 but less than 2, which we take as a base and the other factor, interest, as height, thus a fractal volume is formed. If the area is low, the height may be higher and vice versa.

However, the second factor is proportional to the inverse of the probability of a person's income, higher than the minimum, with a Pareto index equal to the income elasticity coefficient. If the income is relatively high, as is, for example, the representative businessperson, the probability is low, and then its inverse, the fractal area is high so the interest, the height, can be low to reach an intermediate predetermined value of the commutator. On the contrary, if the income of the representative worker is low, the probability is relatively high, its inverse or fractal area is low, and then its height, the interest, is high to reach the preset value of the commutator. It seems that the ideal is to keep the value of the fractal volume intermediate.

The situation is similar for relations between pairs of countries. We compare the National Debt of an underdeveloped country with another developed one. The first could be classifiedwith the expression "junk bonds" which translates into a low relative national income and therefore, little possibility of assuming a debt, and then the intermediate level is achieved with a higher interest rate. On the contrary, the latter has a high national income and can reach the intermediate value with a low interest rate. In addition, the former additionally could be subjected to pressure from funds that manage credit and subject it to undesirable conditions for the majority of its population and that has the possibility of unleashing intense social conflicts. In both health and education, we can illustrate similar phenomena.

V. CONCLUSIONS

Both demand and supply are random phenomena that behave according to the Pareto distribution; and their respective curves and exponents are according to the nucleus linked to the averages of that distribution.

Public investment acts as the determining field of economic development while private investment provides the random field; and it does so on a multiplicity of scales.

The fields studied are fractional because they arise as solution to a fractional diffusion equation, with the novelty that the coefficient admits the two signs and the phenomenon occurs in a space of periods and prices.

The logarithmic change in the value of the random variable, represented by different economic entities as a cause, has its effect on the logarithmic change in the probability value, and other economic fields, its quotient being the Pareto index and different elasticity's, with signs included.

The sequence of the cycle is a chain of sensitive links that could eventually break it, stagnate it, slow it down or make it more dynamic. In particular, frugality should be encouraged and on the contrary, corruption, luxury, negligence and other forms of wastefulness should be condemned; and all with the essential objective of strengthening savings.

For both the representative worker and the underdeveloped country, the ideal is to keep the value of the fractal volume intermediate, where the height is the interest and the base is the fractal area proportional to the inverse of the probability of income.

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