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On Somewhat $G_{(gs)*}$ Continuous Function In Grill Topological Spaces

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ABSTRACT
In this paper, we introduce and study a new class of somewhat continuous function. Its relation to various other somewhat continuous functions are investigated.

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I. INTRODUCTION

It is found from literature that during recent years many topologists are interested in the study of generalized types of closed sets. For instance, a certain form of generalized closed sets was initiated by Levine [9] whereas the notion of generalized *semi closed (g*s closed) set was studied by Veerakumar [12]. Following the trend, we have introduced and investigated a kind of generalized closed sets, the definition being formulated in terms of grills. The concept of grill was introduced by Choquet [3] in the year 1947. From subsequent investigations it is revealed that grills can be used as an extremely useful device for investigation of a number of topological problems.

II. PRELIMINARIES

Definition 2.1 : A nonempty collection G of non-empty subsets of a topological space X is called a grill [1] if

(i) $A \in G$ and $A \subseteq B \subseteq X \Rightarrow B \in G$ and

(ii) $A, B \subseteq X$ and $A \cup B \in G \Rightarrow A \in G$ or $B \in G$

Let G be a grill on a topological space (X, τ). In [7] an operator ϕ : P(X) $\rightarrow$ P(X) was defined by

$\phi(A) = \{x \in X / U \cap A \in G, \forall U \in \tau(x)\}$, $\tau(x)$ denotes the neighborhood of x.

Also the map ψ : P(x) $\rightarrow$ P(x), given by ψ (A) = $A \cup \phi(A)$ for all $A \in P(X)$.

Corresponding to a grill G, on a topological space (X, τ) there exists a unique topology τ_G on X given by

$\tau_G = \{U \subseteq X / \psi(X \setminus U) = X \setminus U\}$ where for any $A \subseteq X, \psi(A) = A \cup \phi(A) = \tau_G - cl(A)$.

Thus a subset A of X is τ_G-closed (resp. τ_G-dense in itself) if $\psi(A)$ = A or equivalently if $\phi(A) \subseteq A$ (resp. $A \subseteq \phi(A)$)

In this paper, we introduce and investiage a new class of continuous functions namely somewhat $G_{(gs)}$* continuous function. Also we see its relations to other somewhat continuous functions.

Throughout the paper, by a space X we always mean a topological space (X, τ) with no separation properties assumed. If $A \subseteq X$, we shall adopt the usual notations int(A) and cl(A) respectively for the interior and closure of A in (X, τ). Again τ_G – cl(A) and τ_G-int(A) will respectively denote the closure and interior of A in (X, τ_G). Similarly, whenever we say that a subset A of a space X is open (or closed) it will mean that A is open (or closed) in (X, τ). For open and closed sets with respect to any other topology on X, eg. τ_G, we shall write τ_G-open and τ_G-closed. The collection of all open neighborhoods of a point x in (X, τ) will be denoted by τ(x).

(X, τ, G) denotes a topological space (X, τ) with a grill G.

Definition 2.2: Let (X, τ) be a topological space. A subset A of X is said to be

(1) semiclosed if int cl(A) $\subseteq$ A

(2) generalized closed (g closed) if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in X.

(3) generalized semi closed (gs closed) of scl A $\subseteq$ U whenever A $\subseteq$ U and U is open in X.

(4) θ -closed if A = θ cl A where
θ cl A = {x $\in$X : cl(U) $\cap$ A $\neq \phi$, $\forall U \in \tau$ and $x \in U$}

(5) δ -closed if A = δ cl A where
δ cl A = {x $\in$X; int cl (U) $\cap$ A$\neq \phi$, $\forall U \in \tau$ and $x \in U$}

The complements of the above closed sets are respective open sets.

Definition 2.3:A subset A of a topological space (X, τ) is said to be (gs)* closed set if cl(A) $\subseteq$ U whenever $A \subseteq U$ and U is gs open.

The complement of the above closed set is respective open sets.

Definition 2.4: A function *f*: (X, τ) $\rightarrow$ (Y, σ) is said to be

(1) gs continuous if f^{-1}(U) is gs open in X, for every open set U of Y.

(2) θ continuous if f^{-1}(U) is θ open in X, for every open set U of Y.

(3) δ continuous if f^{-1}(U) is δ -open in X for every open set U of Y.

Definition 2.5: Let (X,τ) and (Y, σ) be topological spaces. A function *f* : (X, τ) $\rightarrow$ (Y, σ) is called

(1) Somewhat continuous if for every U $\in\sigma$ and f^{-1}(U) $\neq\phi$, there exists an open set V in X such that
V$\neq\phi$ and V $\subseteq f^{-1}$(U)

(2) Somewhat gs continuous if for every U $\in\sigma$ and U $\neq\phi$ there exists gs open set V in X such that V$\neq\phi$ and V $\subseteq f^{-1}$(U)

(3) Somewhat θ continuous if for every U $\in\sigma$ and U $\neq\phi$, there exists a θ open set V in X such that V$\neq\phi$ and V $\subseteq f^{-1}$(U)

(4) Somewhat δ continuous if for every U $\in\sigma$ and U $\neq\phi$, there exists a δ open set V in X such that V$\neq\phi$ and V $\subseteq f^{-1}$(U)

(5) Somewhat $_{(gs)}$* continuous if for every U $\in\sigma$ and U $\neq\phi$,there exists a $_{(gs)}$* open set V in X such that V$\neq\phi$ and V $\subseteq f^{-1}$(U).

III. SOMEWHAT $G_{(gs)*}$CONTINUOUS FUNCTION

Definition 3.1: Let (X, τ, G)and (Y, σ) be any two topological spaces. A function *f* : X $\rightarrow$ Y is said to be somewhat $G_{(gs)}$* continuous if for every U $\in\sigma$ and f^{-1}(U) $\neq\phi$, there exists $G_{(gs)}$* open set V in X such that
V$\neq\phi$and V $\subseteq f^{-1}$(U)

Example 3.2: Let X = {a, b} = Y
τ = {ϕ, X}, σ = {ϕ, {a}, Y}
G = {{a}, X}
Define *f* = (X, τ, G) $\rightarrow$ (Y, σ) to be the identity function.
f is somewhat $G_{(gs)}$* continuous.

Theorem 3.3:

(1) Every somewhat continuous function is somewhat $G_{(gs)}$* continuous.

(2) Every somewhat gs continuous function is somewhat $G_{(gs)}$* continuous.

(3) Every somewhat θ continuous function is somewhat $G_{(gs)}$* continuous.

(4) Every somewhat δ continuous function is somewhat $G_{(gs)}$* continuous.

(5) Every somewhat (gs)* continuous function is somewhat $G_{(gs)}$* continuous.

Proof: Obvious.

The converse of the above statements neednot be true can be seen from the following example.

Example 3.4: Let X = {a, b, c}, τ = {ϕ, {a}, {a, b}, X} G = {{a, b}, X}
Define *f* : (X, τ, G) $\rightarrow$ (X, τ) by *f* (a) = c, *f* (b) = b, *f* (c) = a.
f is somewhat $G_{(gs)}$* continuous but not somewhat continuous as there exists no open set V $\neq\phi$ in X such that V $\subseteq f^{-1}$ ({a}) = {c}

Example 3.5: Refer example 3.4
f is somewhat $G_{(gs)}$* continuous but not somewhat gs continuous as there exists no gs open set V $\neq\phi$ such that V $\subseteq f^{-1}$ ({a}) = {c}

Example 3.6: Let X = {a, b, c},τ = {ϕ, {a}, X},G = {{b, c}, X}
Let *f* : (X, τ, G) $\rightarrow$ (X, τ) be the identity function *f* is somewhat $G_{(gs)}$* continuous but not somewhat θ continuous as there exists as there exists no θ open set V $\neq\phi$ in X such that V $\subseteq f^{-1}$ ({a}) = {a}

Example3.7: Refer example 3.6
f is somewhat $G_{(gs)}$* continuous but not somewhat δ continuous as there exists to δ open set V $\neq\phi$ in X such that V $\subseteq f^{-1}$({a}) = {a}

Example 3.8: Refer example 3.6
Define f to be f (a) = b, f (b) = c, f(c) = a f is somewhat $G_{(gs)}$* continuous but not somewhat (gs)* continuous as there exists no (gs)* open set $V \neq \phi$ in X such that $V \subseteq f^{-1}(\{a\} = \{c\}$

Theorem 3.9: Let $f = (X, \tau, G) \rightarrow (Y, \sigma)$ and $g : (Y, \sigma) \rightarrow (Z, \eta)$ be any two functions. If f is somewhat $G_{(gs)}$* continuous and g is continuous function then $g \circ f$ is somewhat $G_{(gs)}$* continuous.

Proof: Let $U \in \eta$. Let $g^{-1}(U) \neq \phi$. As $U \in \eta$ and g is continuous, $g^{-1}(U) \in \sigma$.Suppose $f^{-1}g^{-1}(U) \neq \phi$. Since by hypothesis f is somewhat $G_{(gs)}$* continuous function, there exists a $G_{(gs)}$* open set V such that $V \neq \phi$ and $V \subseteq f^{-1}g^{-1}(U)$
That is $V \subseteq (g \circ f)^{-1}(U)$
This completes the proof.

Remark 3.10: In the above theorem, if f is continuous, and g is somewhat $G_{(gs)}$* continuous, then it is not necessary that $g \circ f$ is somewhat $G_{(gs)}$* continuous. The following example serves the purpose.

Example 3.11 : Let X = {a, b}, $\tau = \{\phi, X\}$, G = {{b}, X}, $\sigma = \{\phi, X\}$, G´ = {{a}, X}, $\eta = \{\phi, \{a\}, X\}$
Let $f : (X, \tau, G) \rightarrow (X, \sigma)$ be identity function $g : (X, \sigma, G') \rightarrow (X, \eta)$ be identity function.
f is continuous g is somewhat $G_{(gs)}$* continuous$g \circ f$ is not $G_{(gs)}$* continuous as there exists no $G_{(gs)}$* open set $V \neq \phi$ in (X, τ, G) such that $V \subseteq (g \circ f)^{-1}(\{a\}) = \{a\}$

Definition 3.12: Let M be a subset of a topological space (X, τ, G). Then M is said to be $G_{(gs)}$* dense in X, if there is no proper $G_{(gs)}$* closed set C in X such that $M \subseteq C \subseteq X$.

Theorem 3.13: Let $f : (X, \tau, G) \rightarrow (Y, \sigma)$ be a surjective function. Then the following are equivalent.
(i) f is somewhat $G_{(gs)}$* continuous function.
(ii) If C is closed subset of Y such that $f^{-1}(C) \neq X$, then there is a proper $G_{(gs)}$* closed subset D of X such that $D \supseteq f^{-1}(C)$
(iii) If M is $G_{(gs)}$* dense subset of X, then f(M) is a dense subset of Y.
Proof: (i) $\Rightarrow$ (ii)
Let C be a closed subset of Y such that $f^{-1}(C) \neq X$. Then Y – C is open in Y such that $f^{-1}(Y - C) = X - f^{-1}(C) \neq \phi$. By hypothesis (i) there exists a $G_{(gs)}$* open set V in X such that $V \neq \phi$ and $V \subseteq f^{-1}(Y - C) = X - f^{-1}(C)$. This implies $X - V \supseteq f^{-1}(C)$ and X – V = D is $G_{(gs)}$* closed in X. This proves (ii)
(ii) $\Rightarrow$ (iii)
Let Mbe $G_{(gs)}$* dense in X. We have to show f(M) is dense in Y. Suppose not, then there exists a proper closed set C in Y such that $f(M) \subseteq C \subseteq Y$. Clearly $f^{-1}(C) \neq X$. Hence by (ii), there exists a proper $G_{(gs)}$* closed set D such that $M \subseteq f^{-1}(C) \subseteq D \subseteq X$.
This contradicts the fact that M is $G_{(gs)}$* dense in X. Hence (iii)
(iii) $\Rightarrow$ (ii)
Suppose that (ii) is not true. This means there exists a closed set C in Y such that $f^{-1}(C) \neq X$. But there is no proper $G_{(gs)}$* closed set D in X such that $f^{-1}(C) \subseteq D$. This means $f^{-1}(C)$ is $G_{(gs)}$* dense in X. But in (iii)
$f(f^{-1}(C)) = C$ must be dense in Y, which is a contradiction to the choice of C
(ii) $\Rightarrow$ (i)
Let $U \in \sigma$ and $f^{-1}(U) \neq \phi$. Then Y – U is closed and $f^{-1}(Y - U) = X - f^{-1}(U) \neq X$.
By hypothesis of (ii), there exists a proper $G_{(gs)}$* closed set D such that $D \supseteq f^{-1}(Y - U)$. This implies $X - D \subseteq f^{-1}(U)$ and X – D is $G_{(gs)}$* open and $X - D \neq \phi$.

Theorem 3.14: Let (X, τ, G) and (Y, σ) be any two topological spaces, A be an open set in X and $f : (A, \tau/A) \rightarrow (Y, \sigma)$ be somewhat $G_{(gs)}$* continuous function such that f (A) is dense in Y. Then any extension F of f is somewhat $G_{(gs)}$* continuous function.
Proof: Let U be any open set in (Y, σ) such that $F^{-1}(U) \neq \phi$. Since $f(A) \subseteq Y$ is dense in Y and $U \cap f(A) \neq \phi$, it follows that $F^{-1}(U) \cap A \neq \phi$. That is $f^{-1}(U) \cap A \neq \phi$. Hence by hypothesis on f, there exists a $G_{(gs)}$* open set V in A such that $V \neq \phi$ and $V \subseteq f^{-1}(U) \subseteq F^{-1}(U)$. This implies F is somewhat $G_{(gs)}$* continuous function.
The intersection of two $G_{(gs)}$* open sets need not in general a $G_{(gs)}$* open set. But in the following theorem we assume the intersection of two $G_{(gs)}$* open sets is $G_{(gs)}$* open.

Theorem 3.15: Let (X, τ, G) and (Y, σ) be any two topological spaces. $X = A \cup B$, where A and B are open sets in X.
$f : (X, \tau, G) \rightarrow (Y, \sigma)$ be a function such that f/A and f/B are somewhat $G_{(gs)}$* continuous function. Then f is somewhat $G_{(gs)}$* continuous function.
Proof: Let U be any open set in (Y, σ) such that $f^{-1}(U) \neq \phi$. Then $(f/A^{-1})(U) \neq \phi$ or $(f/B)^{-1}(U) \neq \phi$ or both $(f/A)^{-1}(U) \neq \phi$ and $(f/B)^{-1}(U) \neq \phi$.
Case: 1 Let $(f/A)^{-1}(U) \neq \phi$
Since f is somewhat $G_{(gs)}$* continuous, there exists a $G_{(gs)}$* open set V in A such that $V \neq \phi$ and

$V \subseteq (f/A)^{-1}(U) \subseteq f^{-1}(U)$. Since V is $G_{(gs)}$* is open in A and A is open in X, V is $G_{(gs)}$* open in X. Then *f* is somewhat $G_{(gs)}$* continuous function.
Case 2 : Let $(f/B)^{-1}(U) \neq \phi$. The rest of the proof is similar to case : 1.
Case 3: Let $(f/A)^{-1}(U) \neq \phi$ and $(f/B)^{-1}(U) \neq \phi$. The proof follows from both the cases 1 and 2 then *f* is somewhat $G_{(gs)}$* continuous function.

Definition3.16 :A topological space X is to be $G_{(gs)}$* separable, if there exists a countable subset B of X which is $G_{(gs)}$* dense in X.

Theorem 3.17: If *f* is somewhat $G_{(gs)}$* continuous function from X onto Y and if X is $G_{(gs)}$* separable then Y is separable.
Proof: Let $f : X \rightarrow Y$ be somewhat $G_{(gs)}$* continuous function such that X is $G_{(gs)}$* separable. Then by definition, there exist a countable subset B of X which is $G_{(gs)}$* dense in X. Then by theorem 3.13, *f*(B) is dense in Y. Since B is countable, *f*(B) is also countable which is dense in Y. Hence Y is separable.

Definition 3.18 : If X is a set and τandσ are topologies for X, then τ is said to be weakly equivalent to σprovided if $U \in \tau$ and $U \neq \phi$, then there is an open set V in (X, σ) such that $V \neq \phi$ and $V \subseteq U$ and if $U \in \sigma$ and $U \neq \phi$, then there is an open set V in (X, τ, G) such that $V \neq \phi$ and $V \subseteq U$.

Definition3.19: If X is a set and τandσ are topologies for X, then τ is said to be $G_{(gs)}$* weakly equivalent to σprovided if $U \in \tau$ and $U \neq \phi$, then there is a $G_{(gs)}$* open set V in (X, τ, G) such that $V \neq \phi$ and $V \subseteq U$ and if $U \in \sigma$ and $U \neq \phi$ then there is a $G_{(gs)}$* open set V in (X, τ, G) such that $V \neq \phi$ and $V \subseteq U$.

Theorem 3.20: Let $f : (X, \tau, G) \rightarrow (Y, \sigma)$ be somewhat continuous function and let τ* be a topology for X, which is $G_{(gs)}$* weakly equivalent to τ. Then the function $f : (X, \tau^*, G) \rightarrow (Y, \sigma)$ is somewhat $G_{(gs)}$* continuous function.
Proof: Let U be any open sets in (Y, σ) such that $f^{-1}(U) \neq \phi$. Since by hypothesis $f : (X, \tau, G) \rightarrow (Y, \sigma)$ is somewhat continuous, by definition there exists an open set O in (x, τ, G) such that $O \neq \phi$ and $O \subseteq f^{-1}(U)$. Since O is an open set in (X, τ, G) such that $O \neq \phi$ and since by hypothesis, τ is $G_{(gs)}$* weakly equivalent to τ* by definition, there exists a $G_{(gs)}$* open set V in (X, τ*, G) such that $V \neq \phi$ and $V \subseteq O \subseteq f^{-1}(U)$. Thus for any open set U in (Y, σ) such that $f^{-1}(U) \neq \phi$, there exists a $G_{(gs)}$* open set V in (X, τ*, G) such that $V \neq \phi$ and $V \subseteq f^{-1}(U)$. So $f: (x, \tau^*, G) \rightarrow (Y, \sigma)$ is somewhat $G_{(gs)}$* continuous function.

Theorem 3.21 : Let $f : (X, \tau, G) \rightarrow (Y, \sigma)$ be somewhat continuous surjection and let σ* be a topology for Y, which is weakly equivalent to σ.
Then the function $f : (X, \tau, G) \rightarrow (Y, \sigma^*)$ is somewhat $G_{(gs)}$* continuous function.
Proof: Let U be any open set in (Y, σ*) such that $f^{-1}(U) \neq \phi$ which implies $U \neq \phi$.Since σ and σ* are weakly equivalent, there exist an open set W in (Y, σ) such that $W \neq \phi$ and $W \subseteq U$. Now, W is an open set such that $W \neq \phi$. This implies $f^{-1}(W) \neq \phi$ as *f* is a surjection. Now, by hypothesis, $f: (X, \tau, G) \rightarrow (Y, \sigma)$ is somewhat $G_{(gs)}$* continuous function. So, there exists a $G_{(gs)}$* open set V in X such that $V \neq \phi$ and $V \subseteq f^{-1}(W)$. Now, $W \subseteq U$ implies $f^{-1}(W) \subset f^{-1}(U)$. So, we have $V \subseteq f^{-1}(U)$, which implies $f: (X, \tau, G) \rightarrow (Y, \sigma^*)$ is somewhat $G_{(gs)}$* continuous function.

Definition 3.22:A function $f: (X, \tau) \rightarrow (Y, \sigma, G)$ is said to be somewhat $G_{(gs)}$* open function provided that for $U \in I$ and $U \neq \phi$, there exists a $G_{(gs)}$* open set V in Y such that $V \neq \phi$ and $V \subseteq f(U)$.

Example 3.23: Let X = {a, b, c}, τ = {ϕ, {a}, {a, b}, X} G = {{a},{a,b},{a,c}, X}
Define $f : (X, \tau) \rightarrow (X, \tau, G)$ to be identify function *f* is somewhat $G_{(gs)}$* open function.

Theorem 3.24:
(1) Every somewhat open function is somewhat $G_{(gs)}$* open.
(2) Every somewhat (gs) open function is somewhat $G_{(gs)}$* open.
(3) Every somewhat τ_G open function is somewhat $G_{(gs)}$* open.
(4) Every somewhat $_{(gs)}$* open function is somewhat $G_{(gs)}$* open.
(5) Every somewhat θ open function is somewhat $G_{(gs)}$* open.
(6) Every somewhat δ open function is somewhat $G_{(gs)}$* open.
Proof: Obvious.
Converse of the above statements need not be true can be seen from the following examples.

Example 3.25: Refer example 3.6
Define $f : (X, \tau) \rightarrow (X, \tau, G)$ by *f* (a) = b, *f* (b) = c, *f* (c) = a
f is somewhat $G_{(gs)}$* open but not somewhat open as, there exists no open $V \neq \phi$ in X such that
$V \subseteq f(\{a\}) = \{b\}$

Example 3.26: Let X = {a, b, c}, τ = {ϕ,{a, b}, X} G = {{c},{a,c},{b,c},X}
Define *f* : (X, τ) $\rightarrow$ (X, τ, G) by *f* (a) = c, *f* (b) = c, *f*(c) = b
f is somewhat $G_{(gs)}$* open but not somewhat gs open as there exists no gs open set V$\neq\phi$ in X such that V $\subseteq$*f*({a, b}) = {c}

Example 3.27: Let X = {a, b, c}
τ = {ϕ, {a, b}, X} G = {{a},{b},{c},{a,b},{a,c},{b,c}, X}
Define *f*: (X, τ) $\rightarrow$ (X, τ, G) by *f* (a) = a, *f* (b) = a, *f*(c) = b
f is somewhat $G_{(gs)}$* open but not somewhat τ_G open as there exists no τ_G open set V$\neq\phi$ in X such that V $\subseteq$*f*({a, b}) = {a}

Example 3.28: Refer example 3.6
Define *f* by *f* (a) = c, *f* (b) = b, *f*(c) = a
f is somewhat $G_{(gs)}$* continuous but not somewhat (gs)* continuous as there exists no (gs)* open set V$\neq\phi$ in X such that V $\subseteq$*f*({a}) = {c}

Example 3.29: Refer example 3.6
Define *f* : (X, τ) $\rightarrow$ (X, τ, G) to be the identity function.
f is somewhat $G_{(gs)}$* open but not somewhat θ open as there exists no θ open set V $\neq\phi$ such that
V $\subseteq$ *f* (a) = {a}

Example 3.30: Refer example 3.6
Define*f* :(X, τ) $\rightarrow$ (X, τ, G) to be the identity function *f* is somewhat $G_{(gs)}$* open but not somewhat δ open as there exists no δ open set V$\neq\phi$ such that V $\subseteq$*f*(a) = {a}

Theorem 3.31: If *f* : (X, τ) $\rightarrow$ (Y, σ) is an open map and *g* : (Y, σ) $\rightarrow$ (Z, η, G) is somewhat $G_{(gs)}$* open map,then *g* o *f* : (X, τ) $\rightarrow$ (Z, η, G) is somewhat $G_{(gs)}$* open map.
Proof: Let U $\in\tau$ Suppose that U $\neq\phi$.
Since *f* is an open map *f* (U) is open and *f* (U) $\neq\phi$. Since g is somewhat $G_{(gs)}$* open map and *f* (U) $\in\sigma$ such that *f* (U) $\neq\phi$, there exists $G_{(gs)}$* open set V $\in$ (Z, η, G) such that V $\subseteq$*g* (*f* (U)), which implies *g* o *f* is somewhat $G_{(gs)}$* open.

Theorem 3.32: If *f* : (X, τ) $\rightarrow$ (Y, σ, G) is a bijection, then the following are equivalent.
(1) *f* is somewhat $G_{(gs)}$* open map.
(2) If C is a closed subset of X such that *f* (C) $\neq$ Y, then there exists a $G_{(gs)}$* closed subset D of Y such that D $\neq$ Y and D $\supseteq$*f* (C)
Proof:(1) $\Rightarrow$ (2)
Let C be any closed subset of X such that *f* (C) $\neq$ Y. Then X – C is open in X and X – C $\neq\phi$. Since *f* is somewhat $G_{(gs)}$* open, there exists a $G_{(gs)}$* open set V $\neq\phi$ in Y such that V$\subseteq$*f*(X – C). Put D = Y – V. Clearly D is $G_{(gs)}$* closed in Y. Let us prove D $\neq$ Y. For if D = Y, then V = ϕ a contradiction.
Since V $\subseteq$*f* (X – C), D = Y – V $\supseteq$ Y – [*f*(X – C) = *f*(C)]
(2) $\Rightarrow$ (1)
Let U be any nonempty open set in X. Put C = X – U. Then C is a closed subset of X and *f*(X – U) = *f*(C) = Y – *f*(U) implies *f*(C) $\neq$ Y. So, by (2), there is a $G_{(gs)}$* closed subset D of Y such that D $\neq$ Y and *f*(C) $\subset$ D. Put V = Y – D.
Clearly V is $G_{(gs)}$* open and V $\neq\phi$. Further, V = Y – D $\subseteq$ Y – *f*(C) = Y – [Y – *f*(U)] = *f*(U).

Theorem 3.33: Let *f* : (X, τ) $\rightarrow$ (Y, σ, G) be somewhat $G_{(gs)}$* open function and A be any open subset of X. Then *f* /A : (A, τ/A) $\rightarrow$ (Y, σ, G) is also somewhat $G_{(gs)}$* open function.
Poof: Let U $\in\tau$/A such that U $\neq\phi$. Since U is open in A and A is open in (X, τ), U is open in (X, τ). Since by hypothesis *f* : (X, τ) $\rightarrow$ (Y, σ, G) is somewhat $G_{(gs)}$* open function, there exists a $G_{(gs)}$* open set V in Y such that V $\neq\phi$ and V $\subseteq$*f*(U). Thus for any open set U in (A, τ/A) with U $\neq\phi$, there exists a $G_{(gs)}$* open set V in Y such that V $\neq\phi$ and V $\subseteq$ (*f*/A) (U). This implies *f*/A is somewhat $G_{(gs)}$* open function.

Theorem 3.34: Let (X, τ) and (Y, σ, G) be any two topological spaces and X = A $\cup$ B, where A and B are open subsets of X and *f* : (X, τ) $\rightarrow$ (Y, σ, G) be a function such that *f*/A and *f*/B are somewhat $G_{(gs)}$* open. Then *f* is also somewhat $G_{(gs)}$* open function.
Proof: Let U be any open subset of (X, τ) such that U $\neq\phi$.Since X = A $\cup$ B, either A $\cap$ U $\neq\phi$ or B $\cap$ U $\neq\phi$ or both A $\cap$ U $\neq\phi$ andB $\cap$ U $\neq\phi$. Since U is open in (X, τ), U $\cap$ A is open is (X, τ/A) and U $\cap$ B is open in (X, τ/B)
Case : 1 Suppose that U $\cap$ A $\neq\phi$, U $\cap$ A is open in τ/A. Since by hypothesis, *f*/A is somewhat $G_{(gs)}$* open function, there exists a $G_{(gs)}$* open set V in (Y, σ, G) such that V $\neq\phi$ and V $\subseteq$*f* (U $\cap$ A) $\subseteq$*f*(U). This implies *f* is somewhat $G_{(gs)}$* open function.
Case : 2 Suppose that U $\cap$ B $\neq\phi$. The rest of the proof is same as case : 1.
Case : 3 Suppose that U $\cap$ A $\neq\phi$ and U $\cap$ B $\neq\phi$. Then *f* is obviously somewhat $G_{(gs)}$* open function from case 1 and case 2.

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