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Matrix Algorithm for Finding Minimum Dominating Set

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ABSTRACT
In a given graph G = (V;E), a dominating set D is a subset of V such that any vertex not in D is adjacent to at least one vertex in D. Matrix algorithms for computing the minimum connected dominating set(MCDS) are essential for solving many practical problems, such as finding a minimum size backbone in Ad Hoc networks. Wireless Ad Hocnetworks appear in a wide variety of applications, including mobile commerce, search and discovery, and military battle field. In this paper wegive a adjacent matrix algorithm that finds a solution to the minimum dominating set and minimum dominating set problems. The algorithm uses the idea of starting from a solution where all vertices of the graph are included in adjacent matrix. Then it works by starting with first vertex nominated by us rest we can see in algorithm. We prove that this algorithm gives a constant performance guarantee. The results show that, despiteits simplicity, the proposed algorithm gives very good solutions.

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I. INTRODUCTION

In many applications of wireless networks, such as mobile commerce, search and rescue, and military battlefield, one deals with communication systems having no fixed infrastructure, referred to as Ad Hoc wireless networks. An essential problem concerning Ad Hoc wireless networks is to design routing protocols allowing for communication between the hosts. The dynamic nature of Ad Hoc networks makes this problem especially challenging. However, in some cases the problem of computing an acceptable virtual backbone can be reduced to the well known minimum connected dominating set problem in unit-disk graphs [5]& unit ball graph. Given a graph G = (V;E) with the set of vertices V and the set of edges E, a dominating set (DS) is a set $D \subseteq V$ such that each vertex in V - D is adjacent to at least one vertex in D. A connected dominating set (CDS) is a DS which is also a connected sub graph of G. We note that computing the minimum CDS (MCDS) is equivalent to finding a spanning tree with the maximum number of leaves in G. In a unit disk graph, two vertices are connected whenever the Euclidean distance between them is at most one.

Ad Hoc networks can be modelled using unit-disk graphs as follows. The hosts in a wireless network are represented by vertices in the corresponding unit-disk graph, where the unit distance corresponds to the transmission range of a wireless device. It is known that both CDS and MCDS problems are NPhard [8]. This remains the case even when they are restricted to planar, unit disk graphs [4]. Following the increased interest in wireless Ad Hoc networks, many approaches have been proposed for the MCDS in the recent years [5, 7, 14, 2]. Most of the heuristics are based on the idea of creating a dominating set incrementally, using some greedy technique. Some other approaches try to construct a MCDS using an initial independent set [14]. An independent set (IS) in G is a set $I \subseteq V$ such that, for each pair of vertices u; $v \in I$, $(u; v) \notin E$. An independent set I is maximal if any vertex not in I has a neighbour in I. Obviously, any maximal independent set is also a dominating set.

There are several polynomial-time approximation algorithms for the dominating set problem, when the graph is restricted to be planar. For instance, in [9], Guha and Khuller propose an algorithm which gives a performance guarantee of $2H(\Delta)$, where Δ is the maximum degree of the graph and H(n) = 1 + 1/2 + _ _ _ + 1/n is the harmonic function. Other approximation algorithms are given in [4, 11]. A Polynomial Time Approximation Scheme (PTAS) for MCDS in unit-disk graphs is also possible, as shown in [10] and more recently in [6].

A common feature of the current techniques for solving the MCDS problem is that the algorithms create the CDS from scratch, at each iteration adding some vertices according to a greedy criterion. For the general dominating set problem, the only exception known to the authors is briefly explained in [13], where a solution is created by sequentially removing vertices. These algorithms share the disadvantage that a setup time must be defined in order to construct a complete MCDS. This becomes a problem when, for example, a reorganization of the network requires a new execution of the algorithm. Other weakness of the current techniques is that frequently complicated strategies must be applied, in order to achieve a good performance guarantee.

In this paper, we propose a new algorithm for computing approximate solutions to the minimum dominating set problem. The algorithm uses a new technique which starts with adjacent matrix,and then one by one removes vertices from this initial solution, until a minimal dominating set is found. Using this technique, the proposed algorithm is able to work with feasible solutions since the beginning, and therefore there is no required set up time for the algorithm. The approach has also the advantage of being simple to implement, with experimental results comparable to the best to existing algorithms.This paper uses standard graph and usual notations.

II. DOMINATION IN GRAPHS

We now introduce the concept of dominating sets in graphs. A set $S \subseteq V$ of vertices in a graph$G = (V,E)$ is a dominating set if every vertex $v \in V$ is an element of S or adjacent to an elementof S. Alternatively, we can say that $S \subseteq V$ is a dominating set of G if $N[S] = V(G)$. A dominatingset S is a minimal dominating set if no proper subset $S' \subset S$ is a dominating set. The dominationnumber $\gamma(G)$ of a graph G is the minimum cardinality of a dominating set of G. We call such a seta γ-set of G.

Theorem2.1:**[1]** A dominating set S of a graph G is a minimal dominating set if and only if forany $u \in S$,

1. u is an isolate of S, or
2. There is $v \in V - S$ for which $N[v] \cap S = \{u\}$.

Proof.
Let S be a γ-set of G. Then for every vertex $u \in S$, $S - \{u\}$ is not a dominating set ofG. Thus, there is a vertex $v \in (V - S) \cup \{u\}$ that is not dominated by any vertex in $S - \{u\}$. Now,either $v = u$, which implies u is an isolate of S; or $v \in V - S$, in which case v is not dominated by$S - \{u\}$, and is dominated by S. This shows that $N[v] \cap S = \{u\}$.

In order to prove the converse, we assume S is a dominating set and for all $u \in S$, either u is anisolate of S or there is $v \in V - S$ for which $N[v] \cap S = \{u\}$. We assume to the contrary that S isnot a γ-set of G. Thus, there is a vertex $u \in S$ such that $S - \{u\}$ is a dominating set of G. Hence,u is adjacent to at least one vertex in $S - \{u\}$, so condition (1) does not hold. Also, if $S - \{u\}$ is adominating set, then every vertex in $V - S$ is adjacent to at least one vertex in $S - \{u\}$, so condition(2) does not hold for u. Therefore, neither (1) nor (2) holds, contradicting our assumption.

Theorem2.2: **[1]** Let G be a graph with no isolated vertices. If D is a γ -set of G, then $V(G) - D$is also a dominating set.

Proof.
Let D be a γ -set of the graph G and assume $V(G) - D$ is not a dominating set of G.
This means that for some vertex $v \in D$, there is no edge from v to any vertex in $V(G) - D$. Butthen the set $D - v$ would be a dominating set, contradicting the minimality of D. We conclude that$V(G) - D$ is a dominating set of G.
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III. MATRIX ALGORITHM FOR FINDING MINIMUM CONNECTED DOMINATING SET

Create vertex adjacency matrix.
Write all vertexes in a row like: 1, 2, 3, 4, 5, 6...
Start with first vertex (numbering assigned by you).
Keep first vertex in box .
Now start crossing (×) all other vertex having 1 in first row of adjacency matrix i.e. adjacent to the vertex which is in box.
Now write remaining vertex in a row which are neither in box nor crossed.
Start with first ever vertex in new row and repeat the steps 4 & 5.
Make new row again and again till all the vertex are either in box or crossed. In case at last all remaining vertices cannot be crossed then put all in box.
Write all the box vertices in set this one is a dominating set.
Repeating the process 1-9 for all vertices first time with first vertex, then second, then third ....we will get dominating set.
Once we get all the dominating set now we will move to short minimum connected dominating set

that we will see in our next paper soon. Here we are only concern with minimum dominating set.

Example

Let we have a connected graph G(V,E) where V is the set of vertex and E is the set of edges
Here V (1, 2, 3, 4, 6, 7, 8, 9)

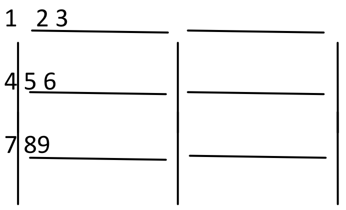

Fig 3.1: Graph G (V, E)

Now applying the above algorithm Step 1: first we create the adjacency matrix

	1	2	3	4	5	6	7	8	9
1	0	1	0	1	0	0	0	0	0
2	1	0	1	0	1	0	0	0	0
3	0	1	0	0	0	1	0	0	0
4	1	0	0	0	1	0	1	0	0
5	0	1	0	1	0	1	0	1	0
6	0	1	0	0	1	0	0	0	1
7	0	0	0	1	0	0	0	1	0
8	0	0	0	0	1	0	1	0	1
9	0	0	0	0	0	1	0	1	0

Step 2:
$1,2,3,4,5,6,7,8,9$
Step 3:
Start with vertex 1
Step 4:
$\boxed{1},2,3,4,5,6,7,8,9$
Step 5:
$\boxed{1},2(\times),3,4(\times),5,6,7,8,9$
Stpep6:
$3,5,6,7,8,9$
Step 7: Repeating step 4 & step 5
$\boxed{3},5,6(\times),7,8,9$
Step 8: since all vertices neither in box nor crossed therefore we repeat steps 4-7
$5,7,8,9$
$\boxed{5},7,8(\times),9$
$7,9$ (Cannot be crossed)
$\boxed{7}\boxed{9}$
Now finally all the vertices are either in box or crossed therefore we can make a dominating set by all the vertices in boxes: (1, 3, 5, 7, 9)
Now we again start with second vertex with the same procedure
$1,2,3,4,5,6,7,8,9$
$1(\times),\boxed{2},3(\times),4,5(\times),6,7,8,9$
$4,6,7,8,9$
$\boxed{4},6,7(\times),8,9$
$6,8,9$
$\boxed{6},8,9(\times)$
8
$\boxed{8}$
Therefore dominating set is (2, 4, 6, 8)
Now we again start with third vertex with the same procedure
$1,2,3,4,5,6,7,8,9$
$1,2(\times),\boxed{3},4,5,6(\times),7,8,9$
$1,4,5,7,8,9$
$\boxed{1},4(\times),5,7,8,9$
$5,7,8,9$
$\boxed{5},7,8(\times),9$
$7,9$
$\boxed{7},\boxed{9}$
Therefore dominating set is (3, 1, 5, 7, 9)
Now we again start with fourth vertex with the same procedure
$1,2,3,4,5,6,7,8,9$
$1(\times),2,3,\boxed{4},5(\times),6,7(\times),8,9$
$2,3,6,8,9$
$\boxed{2},3(\times),6,8,9$
$6,8,9$
$\boxed{6},8,9(\times)$
8
$\boxed{8}$
Therefore dominating set is (4, 2, 6, 8)
Now we again start with fifth vertex with the same procedure

$1, 2, 3, 4, 5, 6, 7, 8, 9$

$1, 2(\times), 3, 4(\times), \boxed{5}, 6(\times), 7, 8(\times), 9$

$1, 3, 7, 9$

$\boxed{1}, 3, 7, 9$

$\boxed{3}, 7, 9$

$\boxed{7}, \boxed{9}$

Therefore dominating set is (5, 1, 3, 7, 9)

Now we again start with sixth vertex with the same procedure

$1, 2, 3, 4, 5, 6, 7, 8, 9$

$1, 2, 3(\times), 4, 5(\times), \boxed{6}, 7, 8, 9(\times)$

$1, 2, 4, 7, 8,$

$\boxed{1}, 2(\times), 4(\times), 7, 8,$

$7, 8$

$\boxed{7}, 8(\times)$

Therefore dominating set is (6, 1, 8)

Now we again start with seventh vertex with the same procedure

$1, 2, 3, 4, 5, 6, 7, 8, 9$

$1, 2, 3, 4(\times), 5, 6, \boxed{7}, 8(\times), 9$

$1, 2, 3, 5, 6, 9$

$\boxed{1}, 2(\times), 3, 5, 6, 9$

$3, 5, 6, 9$

$\boxed{3}, 5, 6(\times), 9$

$5, 9$

$\boxed{5}, \boxed{9}$

Therefore dominating set is (7, 1, 3, 5, 9)

Now we again start with eighth vertex with the same procedure

$1, 2, 3, 4, 5, 6, 7, 8, 9$

$1, 2, 3, 4, 5(\times), 6, 7(\times), \boxed{8}, 9(\times)$

$1, 2, 3, 4, 6$

$\boxed{1}, 2(\times), 3, 4(\times), 6$

$3, 6$

$\boxed{3}, 6(\times)$

Therefore dominating set is (8, 1, 3)

Now we again start with ninth vertex with the same procedure

$1, 2, 3, 4, 5, 6, 7, 8, 9$

$1, 2, 3, 4, 5, 6(\times), 7, 8(\times), \boxed{9}$

$1, 2, 3, 4, 5, 7$

$\boxed{1}, 2(\times), 3, 4(\times), 5, 7$

$3, 5, 7$

$\boxed{3}, \boxed{5}, \boxed{7}$

Therefore dominating set is (9, 1, 3, 5, 7)

Finally we get following dominating sets

(1, 3, 5, 7, 9), (2, 4, 6, 8), (3, 1, 5, 7, 9), (4, 2, 6, 8), (5, 1, 3, 7, 9), (6, 1, 8), (7, 1, 3, 5, 9), (8, 1, 3), (9, 1, 3, 5, 7).

Ignoring the repeating sets four connected dominating sets are

(1, 3, 5, 7, 9), (2, 4, 6, 8), (6, 1, 8), (8, 1, 3)

Therefore minimum dominating sets are (6, 1, 8), (8, 1, 3).

Applying the process we learn we need to go for n times in n vertex graph.

IV. CONCLUSIONS

We proposed an adjacent matrix algorithm for minimum dominating set problem. Evaluation of MDS was carried out for any connected graphs.Here we see by an example to get smallest dominating sets obtained for a samples graph. A multi-start variant for the said algorithm is also proposed for the minimum weight dominating set (MWDS) problem.Example shows the results indicate that our performance is better than the some existing methods, classical greedy approximation algorithm for the problem, as well as hybrid heuristics basedon ant colony optimisation etc. The results obtainedby matrix algorithm indicate that it is a suitable approach to solve large-scale instancesof the problem.However, matrix algorithm is the algorithm, which produced the best results in our experimental studies.

Last but not least, illustration of the application of our approach in graphAn Order-based Algorithm for Minimum Dominating Setmining opens its possible further use in application areas. Partitioning of thenetwork around dominating set vertices leads to clusters, for which it holdsthat every vertex is in distance to a vertex of the dominating set, which is atmost 1. Therefore, matrix algorithm should be interesting for applications, which requirea fast and highly scalable technique and for which the corresponding networkexhibits small-world properties.

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