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Peformace Evaluation of Least Square Algorithm for Phasor Estimation and Comparison with DFT

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ABSTRACT
As the population is increasing rapidly, Energy demands also needs to be fulfilled efficiently, for that supervision and monitoring of power system in various dynamic events have to be done precisely, Such tasks are normally done by Supervisory control and Data Acquisition System(SCADA) and Phasor Measurement Unit(PMU) also. PMU is best among all in terms of accuracy and time delaying as it uses Global Positioning System for synchronization with other PMU’s, and strong phasor estimation algorithm with less number of samples giving accurate and real time tracking of power system in normal events as well as dynamic events. This paper evaluates and compares Least square method and Discrete Fourier Transform method for various dynamic events as per IEEE standard of synchrophasor measurement. Morever dynamic events taken here are not model based signals, in earlier literatures model based signals were used, that problem is eliminated in this paper.

***Index Terms*—Tensor product, phasor measurement units, wide area monitoring, smart grids.**

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I. INTRODUCTION

Phasor measurement units are vital part of any WAM, PMU’S are used to get fundamental phasors from distorted as well pure sinusoidal waves, that means PMU’S are able to give fundamental magnitude, phase, frequency as well as rate of change of frequency from a input signal. The input signal may be distorted from modulation event, frequency ramp event, noise event, and step events also. These all disturbances have been taken into account, and algorithm is tested, as per IEEE **C37.118.1-2011** standards. The phasor estimation based on DFT and least error square algorithm are very old techniques and best suited for pure sinusoidal signal, but for dynamic events, the algorithm fails to get fundamental phasor, for dynamic events DFT and least square algorithms can be used with filters, then it will lead to huge cost requirement, all these demerits made above algorithms unsuitable for estimation of dynamic phasors. In [1], taylor series based algorithm is discussed, the dynamic phasor of an observation data window is imprecised by 2nd order taylor series, algorithm here is models based algorithm but PMU must be capable of estimating phasor for every signal, so it can be unsuitable for some other dynamics. In [2], a phasor estimation technique , with Hilbert transform and convolution is discussed, the algorithm here is little complex and not based on simple procedures. In [3], dynamic phasor estimator based on subspace technique is proposed which is based on large sampling rate and some changes in the subspace-based techniques are taken into account to find the fundamental phasor without anti-aliasing filter to the input signal. In [4], two precise and fast dynamic phasor estimation techniques subjected oscillations and off nominal conditions are discussed, These methods utilizes the signal model under some dynamic conditions, then linearize them using Taylor’s coeffcient expansion, and then least square technique is used to find the phasor. Frequency and its rate of change are also calculated using adjacent phasors with minimum complexity. In [5], phasor estimation algorithm based on conventional

Discrete Fourier Transform is discussed, normally DFT gives very good performance for static signals but it is unsuitable for dynamic signals. In [6] phasor estimation algorithm based on taylor series and least square algorithm is discussed which has been implemented and has been tested for pure sinusoidal wave as well some dynamic signals, We have replaced model based signal from some dynamic signals as per IEEE standard of synchrophasor measurement. In [7], a phasor estimation technique based on modified least square algorithm is considered to find the dynamic phasor of a fundamentamental component of frequency with time-changing amplitude. The fault current is supposed to be the mixture of a decaying dc offset, decaying fundamental frequency component and harmonic having constant amplitude. the decaying dc offset exponential function and fundamental frequency component are changed by Taylor series and coefficients. Then, the Least Square method is utilized to find the time constants and magnitudes of decaying components. In [8], DFT is used, a fault current is taken into account as it contains decaying dc component, normally DFT has inaccuracy in phasor estimation. The algorithm can be implemented in four steps- Generation of auxiliary signal by high frequency modulation of fault then DFT of summation of auxiliary signal and fault current is found out. There are some more literatures also[9-11] to estimate phasors for dynamic conditions, There are significant differences among them. The above discussed literatures have many advantages and disadvantages too, all of them have different performances for different test signals. The main problem while implementing the algorithm, the algorithm must have simple procedures for implementations. The above discussed algorithms contains complex and large equations, The paper has following sections- Section II describes modified least square algorithm. Section III describes simulation results for various dynamic signals. Section IV contains conclusion of the work.

II. METHODOLOGY

Phasor Estimation by DFT

The commonly used method for phasor estimation is the DFT. DFT is a digital filter that can extract the phasor of the frequency components inside the input signal. For realtime DFT, it is always desirable to compute the phasor with lower samples per cycle, which is usually 24 sample per cycle in power system. In power system, DFT is implemented using rectangular window . It is tuned to work under known frequency and has no leakage effect for the signal of that frequency or integer multiples of it. Since the exact grid frequency is non-stationary and often unknown due to continuous disturbances, the leakage effect of DFT could be dominant at off-nominal frequency conditions. Therefore, DFT has to be tuned to the system frequency such that holds true all the time[4].

Consider a sinusoidal input signal of frequency , given by

$$x(t) = Xsin(w_o t + \varphi)$$

(1)

Assume that is sampled times per cycle such that T_o=N*T_s . Then,

$$x(t) = Xsin(\frac{2\pi k}{N} + \varphi)$$

(2)

and $f_s = Nf_0$. In the transform domain, transformed components are separated by . Thus, choice m = 1 corresponds to extracting the fundamental frequency component. The Discrete Fourier Transform of contains the fundamental frequency component given by

$$X_1 = \frac{2}{N}\sum_{k=0}^{N-1} x_k e^{-j\frac{2\pi k}{N}}$$

$$X_1 = \frac{2}{N}\sum_{k=0}^{N-1} x_k \cos(\frac{2\pi k}{N}) - j\frac{2}{N}\sum_{k=0}^{N-1} x_k \sin(\frac{2\pi k}{N})$$

$$X_1 = X_C - jX_S$$

(3)

Above equations(2),(3),(4) in [4],where in equation

(4)

X_1 is the calculated phasor; N is the total number of samples in one cycleand X_k is the k_{th} sample of waveform.And in equation (5) phase is given by

$$\varphi = \tan^{-1}(\frac{X_S}{X_C})$$

(4)

Equation(5) in[4] Where in equation (6) X_S is imaginary part of FFT of X_1 and X_C is real part of X_1 As the phasor representation is only possible for a pure sinusoid. In practice a waveform is often corrupted with other signals of different frequencies. It then becomes necessary to extract a single frequency component of the signal and then represent it by a phasor. Extracting a single frequency component is often done with a “Fourier transform” calculation. In sampled data systems, this becomes the “discrete Fourier transform” (DFT) or the “fast fourier transform” (FFT). It is only possible to consider a portion of time span over which the phasor representation is considered. This time span, also known as the “data window”, is very important in phasor estimation of practical waveforms.

Phasor Estimation by Least square algorithm

Consider a single phase voltage signal corrupted by Gaussian noise $\varepsilon(t)$

$$x(t) = X_m \cos(wt + \varphi) + \varepsilon(t)$$

(5)

And $x(t)$ is uniformly sampled at N times per cycle of the signal to obtain:

$$x_n = X_m \cos\left(\frac{2\pi n}{N} + \varphi\right) + \varepsilon_n \quad (6)$$

Where Xm is the peak voltage magnitude and is the nominal frequency in radians, φ is the phase angle and ε_n is a zero mean Gaussian noise Now phasor of the signal is:

$$X = \frac{X_m}{2} e^{j\varphi} = X_r + X_i$$

$$X_n = X_m \cos(\varphi)\cos(n\theta) - X_m \sin(\varphi)\sin(n\theta)$$

Where $\theta = \frac{2\pi}{N}$

Hence $X_n = X_r \cos(n\theta) - X_i \sin(n\theta)$ (7)

The unknown phasor can be estimate from the sampled data using data window of M samples[

$$\begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ . \\ . \\ . \\ . \\ x_{M-1} \end{bmatrix} = \begin{bmatrix} \cos(0) & \sin(0) \\ \cos(\theta) & \sin(\theta) \\ \cos(2\theta) & \sin(2\theta) \\ . & . \\ . & . \\ . & . \\ . & . \\ . & . \\ \cos((M-1)\theta) & \sin((M-1)\theta) \end{bmatrix} \begin{bmatrix} X_r \\ X_j \end{bmatrix} + \begin{bmatrix} \varepsilon_0 \\ \varepsilon_1 \\ \varepsilon_2 \\ . \\ . \\ . \\ . \\ \varepsilon_{M-1} \end{bmatrix} \quad (8)$$

In matrix notation

$$[x] = [B][X] + [\varepsilon] \quad (9)$$

the matrix $[X]$ composed of the unknown variables, can be determined by using the least square technique as follows:

$$X = \left[\frac{1}{[B]^T[B]}\right][B]^T[x] \quad (10)$$

Above equations (6),(7),(8),(9),(10),(11) in[5]

Finally the dynamic phasor of the fundamental frequency component can be obtained as follows:

$$X = X_1 + jX_2$$

X gives fundamental phasor and X_1, X_2 are real and imaginary parts of fundamental phasor Here phase angle $\varphi = \tan^{-1}(\frac{X_2}{X_1})$ and magnitude M= $\sqrt{X_1^2 + X_2^2}$ Unlike discrete Fourier transform one of the advantage of the least square technique is that it can be used for calculating phasors from fractional cycle data window which are often used in developing high speed relaying applications. If M samples are used for estimating phasor of sinusoid input signal sampled at rate of N samples per cycle such that M is less than N then does not forms a simple matrix and thereby increasing the computational burden.

Signals taken for testing with their equations

Signals	Equations
Sine wave	$x(t) = A_m sin(2\pi ft + \theta)$
Step change event	$x(t) = A_m(1 + K_{xs}U_1(t))sin(2\pi ft + K_{as}U_1(t) + \theta)$
Frequency ramp event	$x(t) = A_m sin(2\pi ft + \pi R_f t^2 + \theta)$
Modulation event	$x(t) = A_m(1 + K_{xm}sin(2\pi f_m t + \theta))sin(2\pi ft + K_{am}sin(2\pi f_m t) + \theta)$
Noise event	$x(t) = A_m sin(2\pi ft + \theta) + N$

In the above equations
$x(t)$=input signal A_m=peak magnitude of the signal θ=phase angle f=fundamental frequency
t=time K_{xm}=modulation index K_{am}=phase sensitivity f_m=modulation frequency
R_f=frequency ramp rate K_{xs}=magnitude step size K_{as}=phase step size $U_1(t)$=unit step signal
N=Gaussian noise present in the signal

Specifications used in test signals

Parameter	Notation	Specifications
Nominal magnitude	A_m	**230 volts**
Nominal frequency	f	**50Hz**
Phase angle	θ	**30 Degree**

Phase angle sensitivity	K_{am}	**0.1**
Harmonic level		**3rd ,5th and 7th**
Modulation frequency	f_m	**0.2 to 2 Hz**
Step change size	K_{xs}	**0.1**
Phase step size	K_{as}	**0.1**
Noise	N	

SECTION III – SIMULATION RESULTS

The results of Discrete fourier transform and Least square algorithm have been obtained for different test signal and both the algorithms have been compared:

For pure sinusoidal wave:

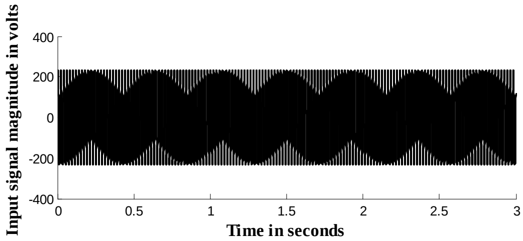


Fig. 4: Pure sine wave as input.

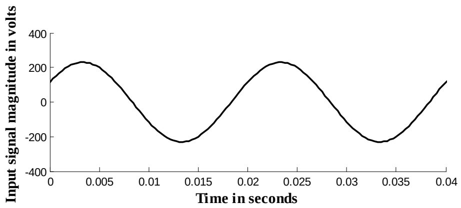


Fig. 5: Cyclic representation of fig.4.

Comparison of two algorithms

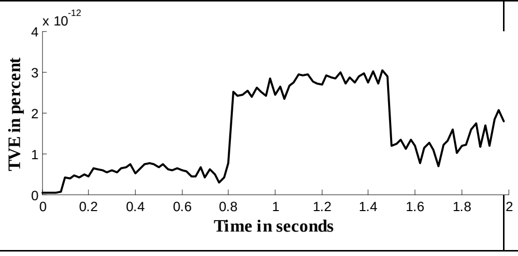
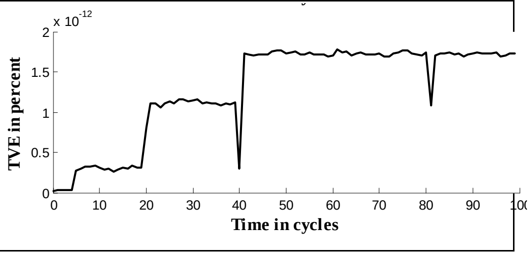
Parameter	**Discrete Fourier Transform**	**Least Square Algorithm**
TVE		

Table.1: Comparsion of two algorithms for pure sine wave.

Both the algorithms have been tested for pure sine waves and we can observe For pure sine wave DFT has more oscillations in phase and magnitudes have approximately same waveshapes but LSA has less total vector error.

For Sinusoidal wave with 3rd harmonic:

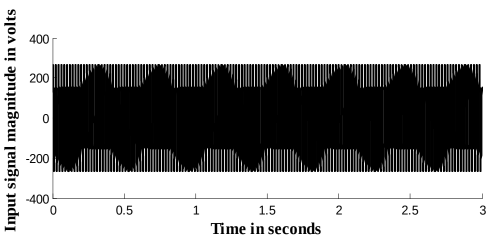


Fig. 6: Sine wave with 3rd harmonic as input .

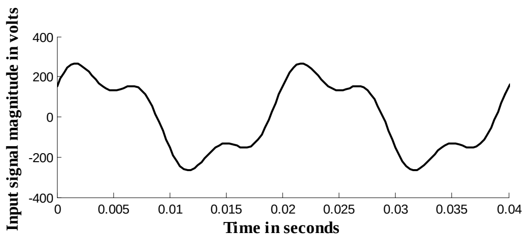


Fig. 7: Cyclic representation of fig.6.

Comparison of two algorithms

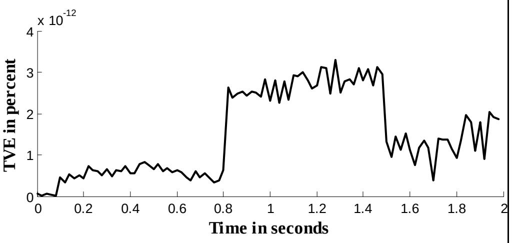
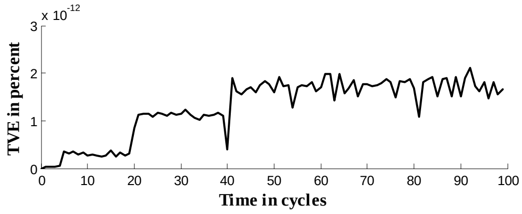
Parameter	Discrete Fourier Transform	Least Square Algorithm
TVE		

Table.2: Comparison of two algorithms for sine wave with 3rd harmonic.

On testing both the algorithms for sine wave with 3rd harmonic ,we can see DFT has more oscillations in estimated magnitude and phase and LSA has less total vector error.

For Sinusoidal wave with 5th harmonic:

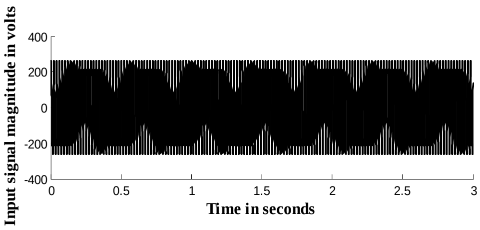


Fig. 8: Sine wave with 5th harmonic as input.

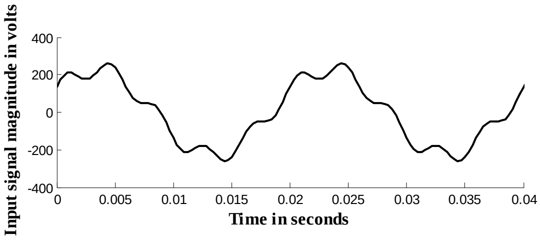


Fig. 9: Cyclic representation of fig.8.

Comparison of two algorithms

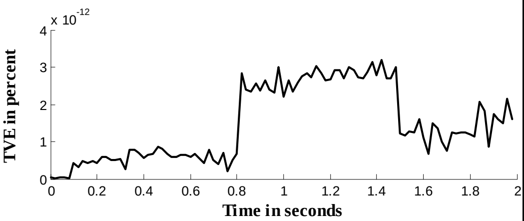
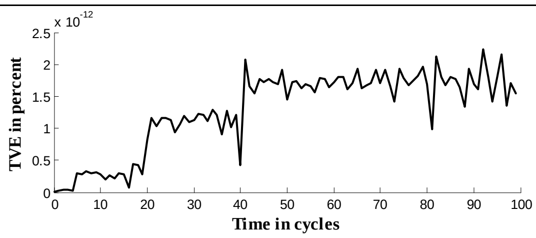
Parameter	Discrete Fourier Transform	Least Square Algorithm
TVE		

Table.3: Comparion of two algorithms for sine wave with 5th harmonic.

Here also we can say that for sine wave with 5th harmonic LSA performs better than DFT.

For Sinusoidal wave with 7th harmonic:

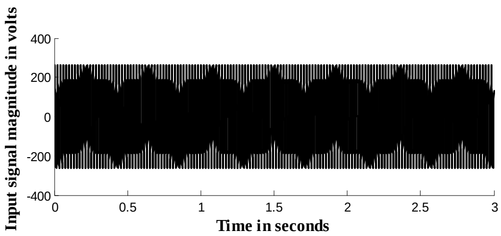


Fig. 9: Sine wave with 7th harmonic as input.

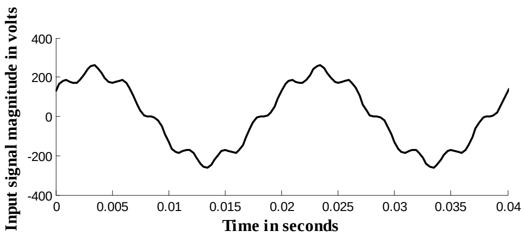


Fig. 10: Cyclic representation of fig.9.

Comparison of two algorithms

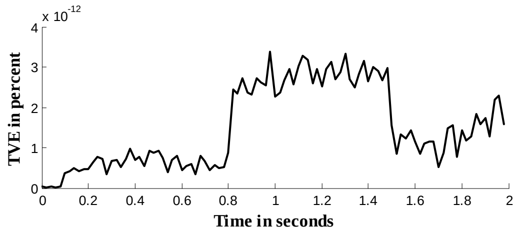
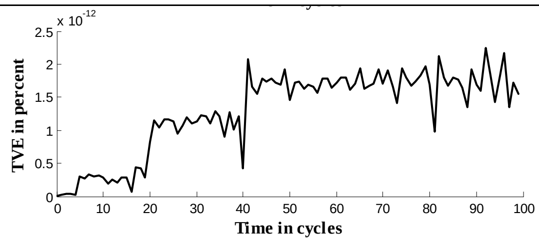
Parameter	Discrete Fourier Transform	Least Square Algorithm
TVE		

Table.4: Comparion of two algorithms for pure sine wave with 7th harmonic

As we can see that DFT has more oscillations for estimated magnitude and phase as compared to LSA and LSA has less TVE error as compared to DFT.

For Sinusoidal wave with harmonics (3rd to 7th):

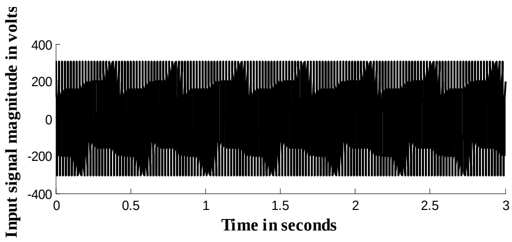


Fig. 11: Sine wave with 3rd to 7th harmonic as input.

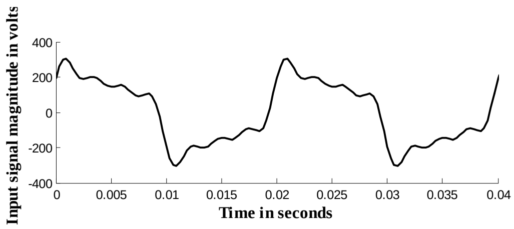


Fig. 12: Cyclic representation of fig.11.

Comparison of two algorithms

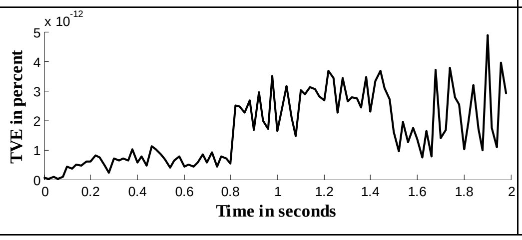
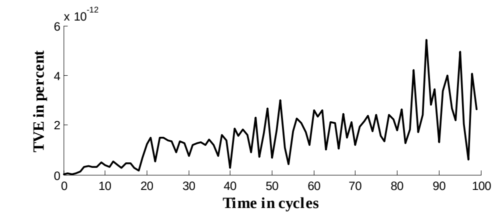
Parameter	Discrete Fourier Transform	Least Square Algorithm
TVE		

Table.5: Comparion of two algorithms for sine wave with 3rd to 7th harmonic

If sine wave is tested for 3rd to 7th harmonic we can see we are getting similar performances but for LSA total vector error is less.

For Sinusoidal wave with modulation event fm = 2 Hz modulation index = 0.1

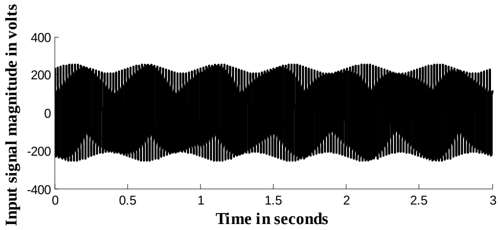


Fig. 12: Sine wave with modulation event.

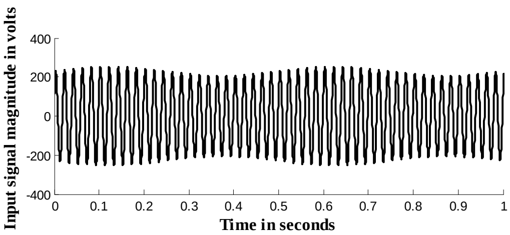


Fig. 13: Cyclic representation of fig.12.

Comparison of two algorithms

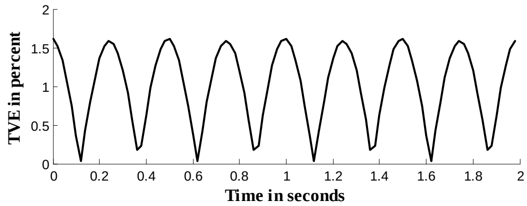
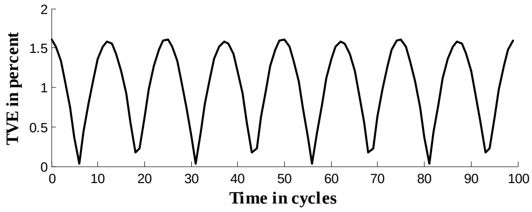
Parameter	Discrete Fourier Transform	Least Square Algorithm
TVE		

Table.6: Comparion of two algorithms for sine with modulation event.

Modualation event is tested here with modulation index=0.1 and modulation frequency as 2Hz,modulation frequency lies between 0.2 Hz to 2 Hz.
For modulation event also the algorithms have similar performances and is limited to 3% according to IEEE standards.

For Sinusoidal wave with random noise event With random noise of 12 magnitude

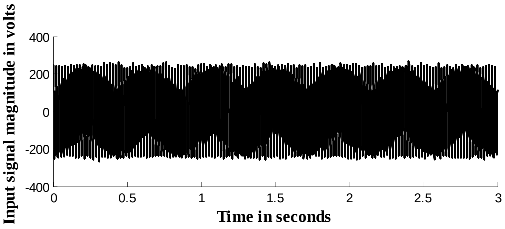


Fig. 14: Sine wave with random noise.

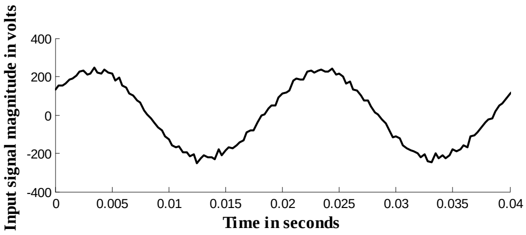


Fig. 15: Cyclic representation of fig.14.

Comparison of two algorithms

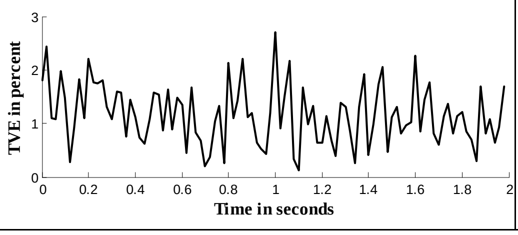
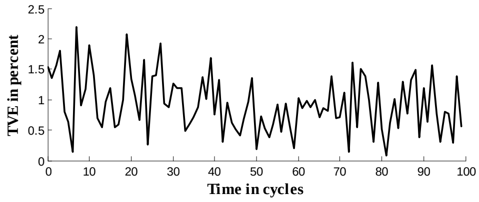
Parameter	Discrete Fourier Transform	Least Square Algorithm
TVE		

Table.7: Comparion of two algorithms for sine with random noise.

On compairing both the algorithms for noise event with mahnitude of 12, we can observe LSA is performing better than DFT and both the algoritms are TVE with respect to IEEE standard.

For Sinusoidal wave with frequency ramp rate event:
Rf=ramp constant
(Rf=1 for 0.2 sec to 1 sec)
(Rf=-1 for 1.2 sec to 1.5 sec)

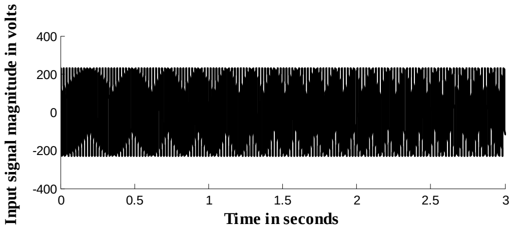


Fig. 16: Sine wave with random noise.

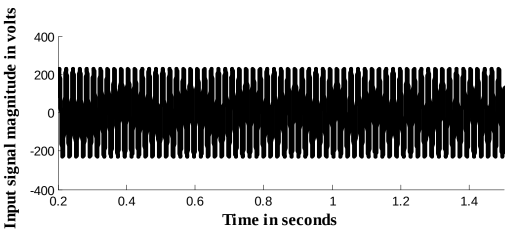


Fig. 17: Cyclic representation of fig.16.

Comparison of two algorithms

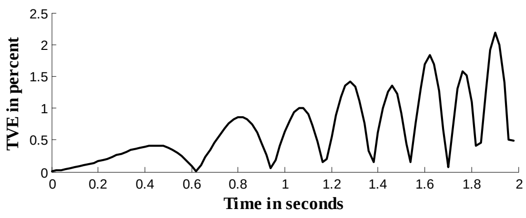
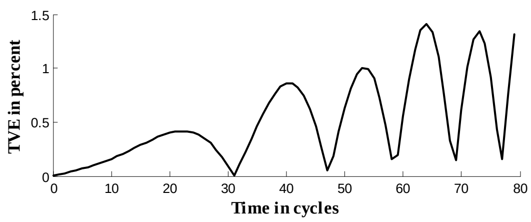
Parameter	Discrete Fourier Transform	Least Square Algorithm
TVE		

Table.8: Comparion of two algorithms for sine with ramp event.

Here after applying frequency ramp event the phase angle rapidly changes.
For 0.2 seconds to 1 seconds positive change is taken and for 1.2 to 1.5 seconds negative change is taken After compairing the algortithms for ramp change event we can say that both the algorithms have same performances for magnitude and phase detection in case of DFT total vector error changes rapidly.

For Sinusoidal wave with step change event:
(step wave applied for 0.6 sec to 1.6 sec)

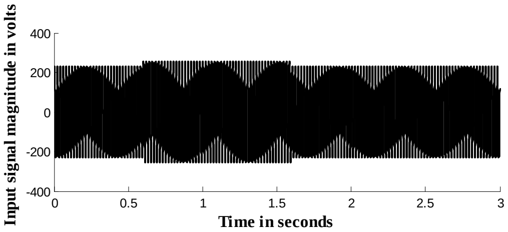


Fig. 18: Sine wave with step event .

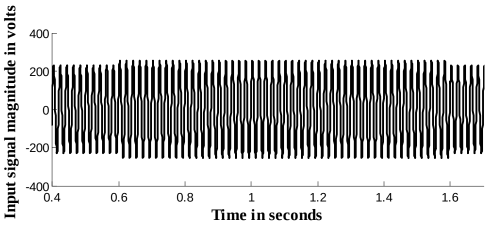


Fig. 19: Cyclic representation of fig.16.

Comparison of two algorithms

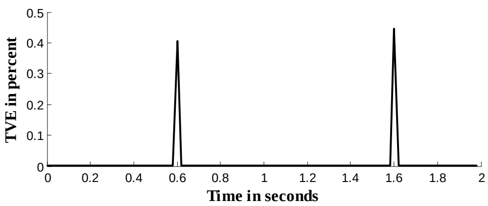
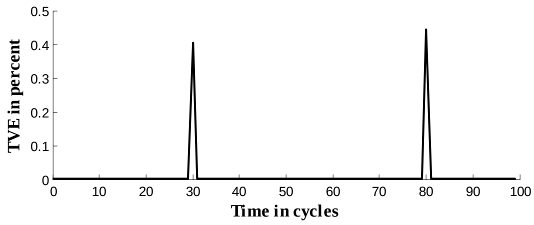
Parameter	Discrete Fourier Transfrom	Least Square Algorithm
TVE		

Table.9: Comparion of two algorithms for sine with step change event.

After compairing the algortithms for step change event from 0.6 sec to 1.6 sec we can say that both the algorithms have same performances.

III. CONCLUSIONS

Discrete fourier transform algorithm and Least square algorithm for phasor estimation have been successfully developed and compared with each other, It can be seen from the ouput plots that algorithms are meeting IEEE standards as for dynamic events it is mainting 3% total vector error

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