

## Potential of Phasor Estimation Algorithm using Tensor Product - A Review

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### ABSTRACT

In this paper a dynamic phasor estimation algorithm is discussed with the help of tensor product, the proposed algorithm can have many qualities for dynamic conditions, it can have great simplicity as well as great robustness for dynamic as well for pure sine waves, phasor estimation process follows some simple procedure based on tensor product and linear algebra. The potential of proposed phasor estimation algorithm is tested for different dynamic/noisy events as per IEEE C37.118.1-2011 standards, by observing the results it can be said that the algorithm can perform well for dynamic/noisy conditions, the great advantages of algorithm can be its simple procedure of implementation, it can also be based on simple equations, which can make it easy to realize. For signal conditions like ramp event, modulation event, step event, as well noisy event the propose algorithm can have tremendous robustness and performance. Its simplicity and robustness can make the algorithm best suited for Wide Area Monitoring for measuring current as well voltage signal having various disturbances.

**Index Terms:** Tensor product, phasor measurement units, wide area monitoring, smart grids.

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### I. INTRODUCTION

Now a days smart grid has become very important need for ensuring greater stability and reliability of power system, the performance of smart grid depends on WAM (Wide area measurement), as WAMS are important stages of smart grid, hence more focus is needed to get better efficiency and reliability of smart grids. Phasor measurement units are vital part of any WAM, PMU'S are used to get fundamental phasors from distorted as well pure sinusoidal waves, that means PMU'S are able to give fundamental magnitude, phase, frequency as well as rate of change of frequency from a input signal. The input signal may be distorted from modulation event, frequency ramp event, noise event, and step events also. These all disturbances have been taken into account, and potential of the proposed algorithm is tested, as per IEEE C37.118.1-2011 standards.

### II. LITERATURE SURVEY

There are numerous literatures [1], [2], [3], [4], [5], [6], [7], [8] present to estimate phasors for dynamic conditions, There are significant differences among them. The phasor estimation based on DFT and least error square algorithm are very old techniques and best suited for pure sinusoidal signal, but for dynamic events, the algorithm fails to get fundamental phasor, for dynamic events DFT and least square algorithms can be used with filters, then it will lead to huge cost requirement, all these demerits made above algorithms unsuitable for

estimation of dynamic phasors.

In [1], algorithm based on taylor series expansion is discussed, the dynamic phasor within an observation data window is approximated by 2<sup>nd</sup> order taylor expansion.

In [2], a phasor estimation algorithm based on Hilbert transform and convolution is discussed, the algorithm is suitable for P-class PMU in protection application. In [3], dynamic phasor estimator based on subspace technique is proposed and high sampling rate and few modifications in the subspace-based techniques are suggested to estimate the voltage phasor with a fundamental frequency component without using antialiasing filter to the input signal. In [4], two fast and precise dynamic phasor estimation algorithms under oscillations and off nominal conditions are discussed, The methods use the signal model under these dynamic conditions, linearize them by using Taylor's series expansion, and estimate the phasor using least squares technique. Frequency and its rate of change are also calculated using adjacent phasors with minimum complexity. The above discussed literatures introduce a model based algorithm, and that contains various complex steps to find out phasor of the signal, with requiring large no. of samples which makes them difficult to use practically and would cost heavy price.

Hence keeping all these requirements in consideration a novel phasor estimation technique can be introduced, which would not require any filter for dynamic phasor estimation, also it will give very good performance and will be based on simple procedures.

### III. DYNAMIC PHASOR ESTIMATOR

Tensors were introduced in 1940s and 1950s by G.kron for use in circuit theory only, tensors can also be used in areas of signal processing, image processing[1].

Here one of the properties of tensors is used for phasor estimation in PMU, normally tensors are multidimensional matrix having various informations. That property of tensor is used here to get fundamental informations from a signal, Here tensor product or kroncker product is discussed.

Assume a sinusoidal wave

$$X(t) = X_m \sin(2\pi ft + \theta) \quad (1)$$

Where  $X(t)$ =input signal,  $X_m$ =peak magnitude of the signal,  $\theta$ =phase angle,  $f$ =fundamental frequency,  $t$ =time,

To prove the algorithm Taking  $N=4$  samples

Assume  $2\pi f = w$

In matrix form of above sine wave after sampling can be written as

$$x(n) = \begin{bmatrix} \sin(\theta) \\ \sin(w n_1 + \theta) \\ \sin(w n_2 + \theta) \\ \sin(w n_3 + \theta) \\ \sin(w n_4 + \theta) \end{bmatrix} \quad (2)$$

finding out Tensor product of equation (2) with itself

$$x(n) \otimes x(n) = \begin{bmatrix} \sin(\theta) \\ \sin(w n_1 + \theta) \\ \sin(w n_2 + \theta) \\ \sin(w n_3 + \theta) \\ \sin(w n_4 + \theta) \end{bmatrix} \otimes \begin{bmatrix} \sin(\theta) \\ \sin(w n_1 + \theta) \\ \sin(w n_2 + \theta) \\ \sin(w n_3 + \theta) \\ \sin(w n_4 + \theta) \end{bmatrix} \quad (3)$$

Size of tensor spectrum depends on number of samples taken like

Signal has  $1 \times (N + 1)$  size

Size of Tensor spectrum =  $1 \times 25$

$$T(n) = \begin{bmatrix} \sin(\theta) \sin(\theta) \\ \sin(\theta) \sin(w n_1 + \theta) \\ \sin(\theta) \sin(w n_2 + \theta) \\ \sin(\theta) \sin(w n_3 + \theta) \\ \sin(\theta) \sin(w n_4 + \theta) \\ \sin(w n_1 + \theta) \sin(\theta) \\ \sin(w n_1 + \theta) \sin(w n_1 + \theta) \\ \sin(w n_1 + \theta) \sin(w n_2 + \theta) \\ \sin(w n_1 + \theta) \sin(w n_3 + \theta) \\ \sin(w n_1 + \theta) \sin(w n_4 + \theta) \\ \sin(w n_2 + \theta) \sin(\theta) \\ \sin(w n_2 + \theta) \sin(w n_1 + \theta) \\ \sin(w n_2 + \theta) \sin(w n_2 + \theta) \\ \sin(w n_2 + \theta) \sin(w n_3 + \theta) \\ \sin(w n_2 + \theta) \sin(w n_4 + \theta) \\ \sin(w n_3 + \theta) \sin(\theta) \\ \sin(w n_3 + \theta) \sin(w n_1 + \theta) \\ \sin(w n_3 + \theta) \sin(w n_2 + \theta) \\ \sin(w n_3 + \theta) \sin(w n_3 + \theta) \\ \sin(w n_3 + \theta) \sin(w n_4 + \theta) \\ \sin(w n_4 + \theta) \sin(\theta) \\ \sin(w n_4 + \theta) \sin(w n_1 + \theta) \\ \sin(w n_4 + \theta) \sin(w n_2 + \theta) \\ \sin(w n_4 + \theta) \sin(w n_3 + \theta) \\ \sin(w n_4 + \theta) \sin(w n_4 + \theta) \end{bmatrix} \quad (4)$$

And we are finding Tensor product of signal with itself, hence Tensor spectrum will have the size of  $1 \times N^2$

Generalized equation to find out samples containing fundamental phase in Tensor spectrum of sine wave with itself -Assume  $N$  samples are taken hence

$$\text{starting sample number} = \left( (N + 1) \times \frac{N}{2} \right) + 1$$

$$\text{Ending sample number} = \left( (N + 1) \times \frac{N}{2} \right) + 1 + N,$$

Hence samples between starting sample and ending sample will contain fundamental signal

$$\text{Starting sample number} = \left( (4 + 1) \times \frac{4}{2} \right) + 1 = 11$$

$$\text{Ending sample number} = \left( (4 + 1) \times \frac{4}{2} \right) + 1 + 4 = 15$$

Samples containing fundamental = 11<sup>th</sup>, 12<sup>th</sup>, 13<sup>th</sup>, 14<sup>th</sup>, and 15<sup>th</sup> of  $T(n)$ , Now adding the samples containing fundamental

$$F(n) = \sin(w n_2 + \theta) \sin(\theta) + \sin(w n_2 + \theta) \sin(w n_1 + \theta) + \sin(w n_2 + \theta) \sin(w n_2 + \theta) + \sin(w n_2 + \theta) \sin(w n_3 + \theta) + \sin(w n_2 + \theta) \sin(w n_4 + \theta) \quad (5)$$

Taking out  $\sin(w n_2 + \theta)$  common from equation (5) we can re-write the equation (5)

$$F(n) = \sin(\omega n_2 + \theta) [\sin(\theta) + \sin(\omega n_1 + \theta) + \sin(\omega n_2 + \theta) + \sin(\omega n_3 + \theta) + \sin(\omega n_4 + \theta)] \quad (6)$$

From equation (2) we can write equation (6) as  $F(n) = Kx(n)$  (7)  
 Where  $x(n)$  is fundamental sine wave, and  $K = \sin(\omega n_2 + \theta)$  will have some constant value

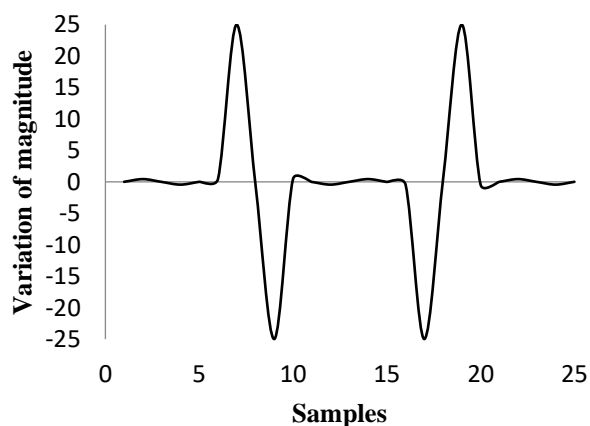


Fig.1: Tensor spectrum with 4 samples

From figure (1), it can be seen that the fundamental wave is contained within 11 to 15 samples, the minimum value of that small spectrum is to be found out and location of that minimum value is to be tracked which gives relationship between phase angle and location of small spectrum.

Like this if  $N=8$  samples fundamental sine wave can be found in tensor spectrum by adding samples from

$$\text{Starting sample number} = \left( (8 + 1) \times \frac{8}{2} \right) + 1 = 37$$

$$\text{Ending sample number} = \left( (8 + 1) \times \frac{8}{2} \right) + 1 + 8 = 45$$

Here tensor spectrum will have size of  $1 \times (9 \times 9) = 1 \times 81$

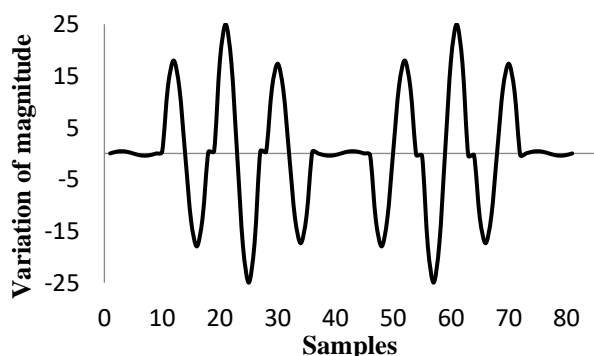


Fig.2: Tensor spectrum with 8 samples

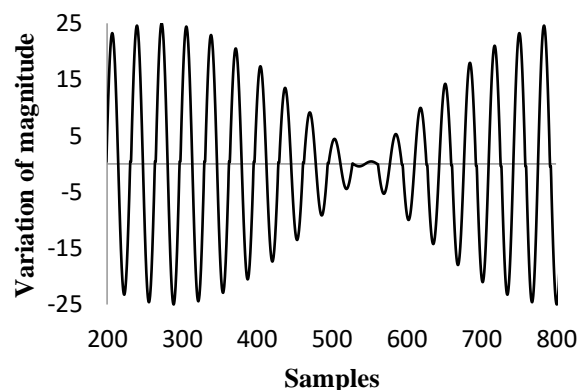


Fig.3: Tensor spectrum with 32 samples

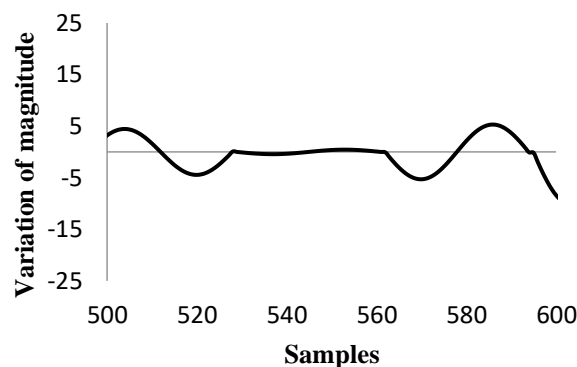


Fig.4: Fundamental view of tensor spectrum with 32 samples

In this paper  $N=256$  is taken hence

$$\text{Starting sample number} = \left( (256 + 1) \times \frac{256}{2} \right) + 1 = 32897$$

$$\text{Ending sample number} = \left( (256 + 1) \times \frac{256}{2} \right) + 1 + 256 = 33153$$

Here tensor spectrum will have size of

$$1 \times (257 \times 257) = 1 \times 66049$$

Here  $F(n)$  can be found by adding samples from sample number 32897 to sample number 33153

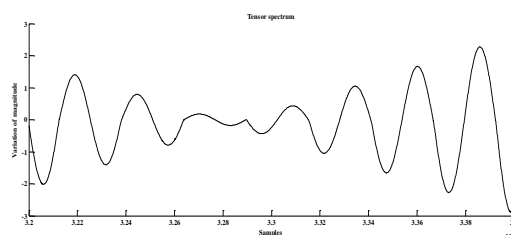
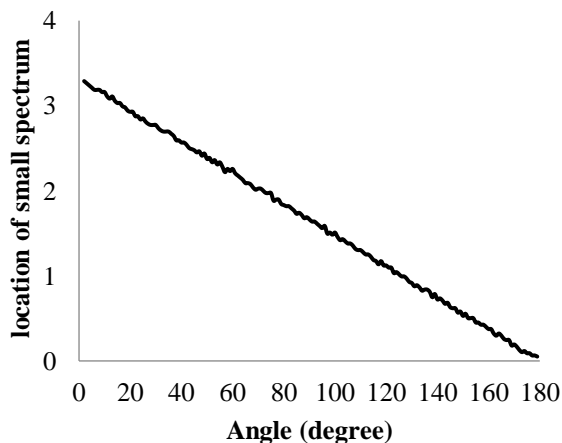


Fig.5: Fundamental view of tensor spectrum with 256 samples

So it can be witnessed from above tensor

spectrum that peak value of the spectrum is constant which is square of fundamental magnitude taken, hence the magnitude can be found by taking square root of peak value of tensor spectrum and also by taking minimum value of  $F(n)$  and finding out its location in X-axis with respect to phase change, relation between phase change and location of small value of fundamental spectrum in X-axis can be obtained as it can be seen it gives linear relation.



**Fig.6:** Location of small spectrum for pure sine wave

Fig (6) shows the relation between change of location of fundamental spectrum with respect to phase angle variation for pure sinusoidal wave. It can be observed from fig (7),(8),(9),(10) that for dynamic signal also the algorithm gives linear relation with phase change which is desirable.

**TABLE-1** signals used

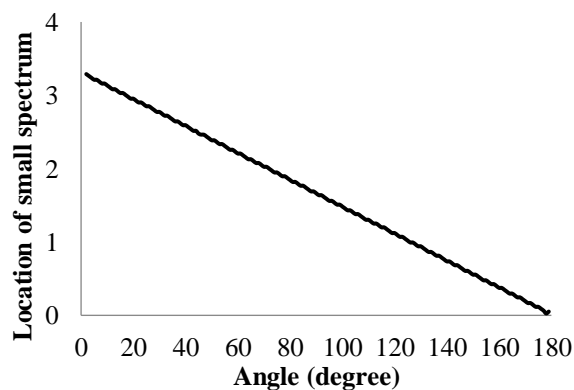
Signals	Equations
Sine wave	$x(t) = X_m \sin(2\pi ft + \theta)$
Step change event	$x(t) = X_m (1 + K_{xs} U_1(t)) \sin(2\pi ft + K_{as} U_1(t) + \theta)$
Frequency ramp event	$x(t) = X_m \sin(2\pi ft^2 + \theta)$
Modulation event	$x(t) = X_m (1 + K_{xm} \sin(2\pi f_m t + \theta)) \sin(2\pi ft + K_{am} \sin(2\pi f_m t) + \theta)$
Noise event	$x(t) = X \sin(2\pi ft + \theta) + \epsilon$

In table (1)  $x(t)$ =input signal,  $X_m$ =peak magnitude of the signal,  $\theta$ =phase angle,  $f$ =fundamental frequency,  $t$ =time,  $K_{xm}$ =modulation index,  $K_{am}$ =phase sensitivity,  $f_m$ =modulation frequency,  $R_f$ =frequency ramp rate,  $K_{xs}$ =magnitude step size,  $K_{as}$ = phase step size,  $U_1(t)$ =unit step signal,  $\epsilon$ =Gaussian noise present in the signal

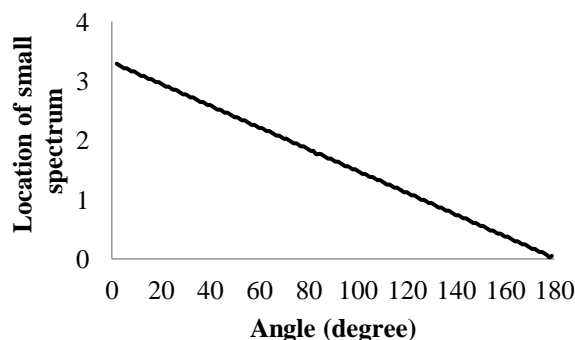
**TABLE-2** specifications used

Parameter	Notation	Specifications
Nominal magnitude	$X_m$	5 volts
Nominal frequency	$f$	50Hz
Phase angle	$\theta$	30 Degree
Phase angle sensitivity	$K_{am}$	0.1
Modulation frequency	$f_m$	0.2 to 2 Hz
Step change size	$K_{xs}$	0.1
Phase step size	$K_{as}$	0.1
Noise	$\epsilon$	15 db to 50 db SNR

In this work, the following specifications as shown in Table 2 are taken to test proposed phasor estimation algorithms. The proposed algorithm is able to estimate one phasor per cycle at a sampling rate of 256 samples per cycle.



**Fig.7:** Change of location of small spectrum for frequency ramp event



**Fig.8:** Change of location of small spectrum for noise event

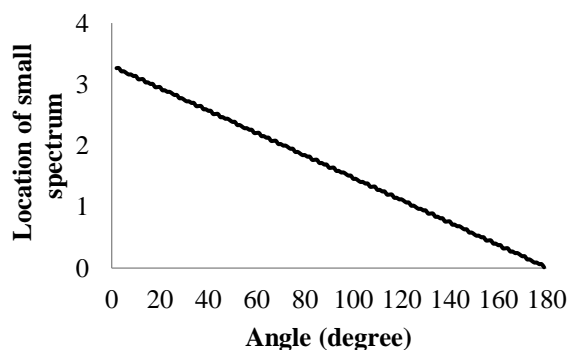


Fig.9: Change of location of small spectrum for modulation event

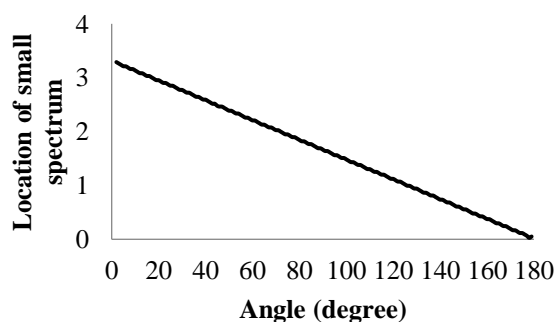


Fig.10: Change of location of small spectrum for step event

#### IV.CONCLUSION

This paper presents potential of simplest and robust dynamic phasor estimation algorithm based on tensor product of signal with itself, the potential of algorithm has been successfully tested under compliance test recommended by IEEE C37.118.1-2011 standards. results shows that the algorithm can be suitable for dynamic phasor estimation and can also be suitable for pure sinusoidal wave, also this algorithm can avoid use of model based and complex algorithms still in use.

#### REFERENCES

- [1] Tianshu Bi, Hao Liu, Qian Feng, Cheng Qian, and Yilu Liu, "Dynamic Phasor Model-Based Synchrophasor Estimation Algorithm for M-Class PMU," *IEEE Transactions on power delivery*, vol.30, no. 3, june 2015
- [2] Shaik Affijulla, and Praveen Tripathy, "Development of Phasor Estimation Algorithm for P-class PMU suitable in Protection Applications," *IEEE Transactions on Smart Grid*, vol.
- [3] P. Banerjee, S. C. Srivastava, "A Subspace-Based Dynamic Phasor Estimator for Synchrophasor Application," *IEEE Transactions on instrumentation and measurement*, vol 61, no. 9, september 2012
- [4] Sadegh Vejdani, Majid Sanaye-Pasand, Om P. Malik, "Accurate Dynamic Phasor Estimation Based on the Signal Model Under Off-Nominal Frequency and Oscillations," *IEEE Transactions on Smart Grid*, vol. 8, no. 2, march 2017
- [5] Andrzej Borys, "On Using Tensors in the Areas of Circuit Theory, Signal Processing, and Telecommunications," in the IEEE conference on Signal Processing: Algorithms, Architectures, Arrangements, and Applications (SPA), 20-22<sup>nd</sup> September 2017, Poznan, Poland
- [6] J A de la O Serna, "Dynamic phasor estimates for power system oscillations," *IEEE Transactions on Instrumentation and Measurement*, vol. 56, no. 5, pp. 1648-1657, Oct. 2007.
- [7] W Premerlani, B Kasztenny and M Adamiak, "Development and implementation of a synchrophasor estimator capable of measurements under dynamic conditions," *IEEE Transactions on Power Delivery*, vol.23, no. 1, pp. 109-123, Jan. 2008.
- [8] A G Phadke and B Kasztenny, "Synchronized phasor and frequency measurement under transient conditions," *IEEE Transactions on Power Delivery*, vol. 24, no. 1, pp. 89-95, Jan. 2009.
- [9] R K Mai, Z Y He, L Fu, B Kirby and Z Q Bo, "A dynamic synchrophasor estimation for online application," *IEEE Transactions Power Delivery*, vol. 25, no. 2, pp. 570-578, Apr. 2010.
- [10] M A Platas Garza and J A de la O Serna, "Dynamic phasor and frequency estimates through maximally flat differentiators," *IEEE Transactions on Instrumentation and Measurement*, vol. 59, no. 7, pp.1803-1811, Jul. 2010.
- [11] Karimi Ghatemani, Masoud, Boon Teck Ooi and Alireza Bakhshai, "Application of enhanced phase-locked loop system to the computation of synchrophasors," *IEEE Transactions on Power Delivery*, vol. 26, no.1, pp. 22-32, Jan. 2011.
- [12] D Belega and D Petri, "Accuracy of the multi-cycle synchrophasor estimator provided by the interpolated DFT algorithm," *IEEE Transactions on Instrumentation and Measurement*, vol. 62, no. 5, pp.942-953, May 2013.
- [13] J A de la O Serna, "Synchrophasor estimation using Prony's method," *IEEE Transactions on Instrumentation and Measurement*, vol. 62, no. 8, pp. 2119-2128, Aug. 2013.