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Common fixed point theorems of Geraghty type contraction maps for digital metric spaces.

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Abstract: In this paper, we prove the existence of common fixed point theorems of Geraghty type contraction for digital metric spaces. Our results extend the Banach fixed point theorem for digital metric spaces and common fixed point theorems in digital metric spaces.

Keywords: Fixed points; Banach Contraction Principle, Geraghty contraction, Digital Image space, commutative maps, common fixed point theorems.
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I. INTRODUCTION AND PRELIMINARIES

Fixed point theory play a very important role Functional Analysis and general topology. Digital topology is the tool to study of digital images which studies features of 2D and 3D digital images.

The Banach Contraction Principle is one of most theorem in fixed point theory. In1976, Jungck proved the existence of common fixed points for compatibility and weakly compatibility maps. Jungck [14] proved common fixed point results.

In 1973, [14] Geraghty extended Banach contraction theorem by replacing the contraction constant by a function with specific properties.

$S = \{\beta : [0, \infty) \to [0, 1) / \beta(t_n) \to 1 \Rightarrow t_n \to 0\}$.

Definition1.1 [15] Let (X, d) be a metric space. A selfmap $f : X \to X$ is said to be a *Geraghty contraction* if there exists $\beta \in S$ such that $d(f(x), f(y)) \leq \beta(d(x, y))d(x, y)$ *for all* $x, y \in X$.

Theorem 1.2[15] Let (X, d) be a complete metric space. Let $f : X \to X$ be a *Geraghty contraction*. Then for any choice of initial point $x_0 \in X$, the iteration $\{x_n\}$ defined by $x_n = f(x_{n-1})$ for $n = 1, 2, 3, ...$ converges to the unique fixed point z of f in X.

Let X be a subset of Z^n for a positive integer n where Z^n is the set of lattice points in the n- Dimensional Euclidean Space and ꝃ be represent an adjacency relation for the members of X.

A digital image consists (X, ꝃ).

Definition 1.3. [5] Let l, m be positive integers, $1 \leq l \leq m$ and two distinct points $(p = p_1, p_2, p_3, \ldots p_m)$, $(q = q_1, q_2, q, \ldots q_m) \in Z^n$

p and q are k_l- adjacent if there are at most l indices i such that $|p_i - q_i| = 1$, and for all other indices j such that $|p_j - q_j| \neq 1$, $p_j = q_j$.

The following are the consequences above definition.

Two points p and q in Z are 2- adjacent if $|p - q| = 1$.

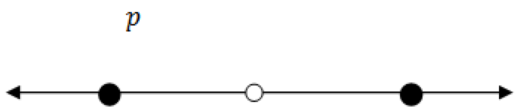

Fig: 1:2 - adjacent

Two points p and q in Z^2 are 8- adjacent if they are if they are distinct and differ by at most 1 in each coordinate.

Two points p and q in Z^2 are 4- adjacent if they are if they are 8- adjacent and differ in exactly one coordinate.

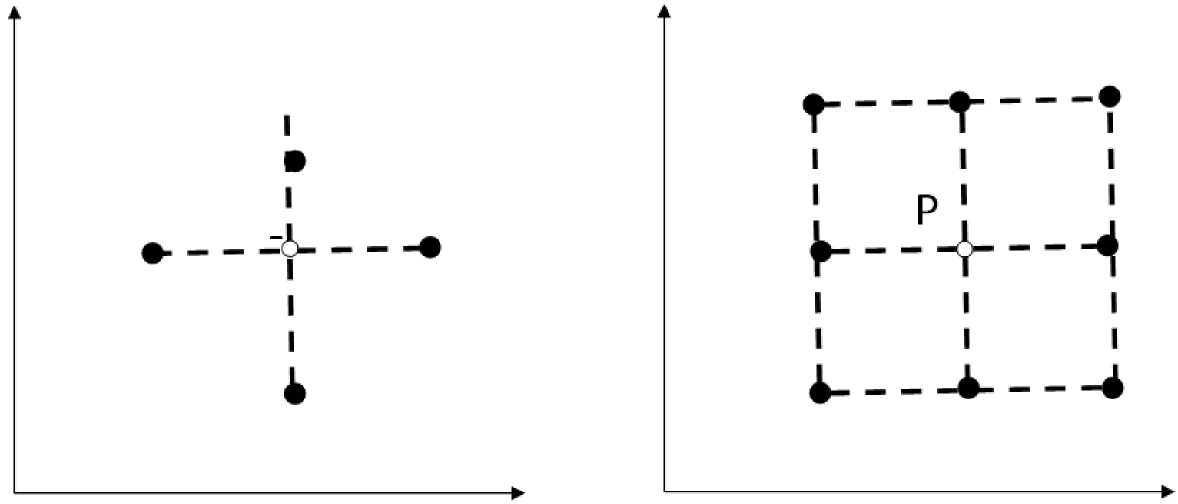

Fig:1.3

Two points p and q in Z^3 are 26- adjacent if they are if they are distinct and differ by at most 1 in each coordinate.

Two points p and $q$in Z^2 are 18- adjacent if they are if they are 26- adjacent and differ at most two coordinates.

Two points p and $q$in Z^2 are 6- adjacent if they are if they are 18- adjacent and differ at most two coordinates.

Definition 1.4 [5] A k – *neighbor* of p $\in Z^n$ is a point of Z^n that is k – adjacent to p where k $\in$ {2,4,6,8,18,26} and n $\in$ 1,2,3.

The set $N_k(p) = \{q | q \text{ } is \text{ } k - adjecent \text{ } to \text{ } p\}$ is called the k- neighborhood of p.

A digital interval is defined by $[a, b]_Z = \{ z \in Z | \, a \le z \le b\}$, where a, b $\in Z$ *and* $a < b$.

Definition 1.5 [5] A digital image X $\subset Z^n$is *k-connected* if and only if for every pair of different points $x, y \in X$, there is a set $\{x_0, x_1, x_2, ... x_r\}$ of points of a digital image X such that $x = x_0, y = x_r$ and x_i and x_{i+1} are k- neighbors where i= 0,1,2,… r-1.

Definition 1.6 [7]. Let$(X, k_0) \subset Z^{n0}$, $(Y, k_1) \subset Z^{n1}$ be digital images and $f: X \to Y$ be a function.

If for every $k_0 -$ connected subset U of X, $f(U)$ is a $k_1 - connected$ subset of Y, then f is said to be $(k_0, k_1) - continuous$.

f is$(k_0, k_1) - continuous$ if and only if for every $k_0 - adjacent$ points $\{x_0, x_1\}$ *of X*, either $f(x_0) = f(x_1)$ or $f(x_0)$ *and* $f(x_1)$ are a $k_1 -$ adjacent in Y.

f is$(k_0, k_1) - continuous$, bijective andf^{-1} is (k_1, k_0)- continuous, then f is called$(k_0, k_1) - isomorphism$ and denoted by $X \cong (k_0, k_1)^Y$.

A (2- k) – continuous function $f: [0, m]_Z \to X$ such that $f(0) = x$, and $f(m) = y$is called a digital k- path from x to y in a digital image X. In a digital image (X, k) , for every two points, if there is a k-path, then X is called k-path connected.

A simple closed k-curve of m$\ge$ 4 points in a digital image X is a sequence $\{ f(0), f(1), f(2),,, f(m-1)\}$ of images of the k-path $f: [0, m-1]_Z \to X$ such that $f(i)$ and $f(j)$ are k-adjacent if and only if $j = i \pm mod \, m$.

A point $x \in X$ is called k-corner if x is k- adjacent to two and only two points y,z $\in X$. Such that y and z are k-adjacent to each other.

If y, z are not k-corners and if x is the only point k-adjacent to both y,z then we say that the k- corner is simple.

X is called a generalized simple closed k-curve if what is obtained by removing all simple k-corners of X is a simple closed k-curve.

For a k-connected digital image (X, k) in Z^n, there is a following statement

$$|X|^x = N_{3^n-1}(x) \cap X$$

$$k \in \{2n(n \ge 1), 3^n - 1(n \ge 2), 3^n - \sum_{t=0}^{r-2} C_t^n 2^{n-t} - 1(2 \le r \le n-1, \; n \ge 3)\},$$

where $C_t^n = \frac{n!}{(n-t)!t!}$.

Definition :1.7 [19] Let (X, k) be a digital image in Z^n, n$\ge$ 3 and $\bar{X} = Z^n$- X. Then X is called a closed k- surface if it satisfies the following.

If (k, $\bar{k}$) $\in \{(k, 2n), (2n, 3^n - 1)\}$, where the $\hat{k}$-adjacency is taken from (2.1) with $k \ne 3^n - 2^n - 1$ and $\bar{k}$ is the adjacency on X, then

a) For each point $x \in X$, $|X|^x$ has exactly one k-component k-adjacent to x;

b) $|\bar{X}|^x$ has exactly two $\bar{k}$- adjacent to xx; we denote by C^{xx} and D^{xx} these two components; and

c) For any point $y \in N_k(x) \cap X$, $N_{\bar{k}}(y) \cap C^{xx} \neq 0$ and $N_{\bar{k}}(y) \cap D^{xx} \neq 0$, where $N_k(x)$ means the k- neighbors of x.

Further, if a closed k –surface X does not have a simple k-point, then X is called simple.

If (k, $\bar{k}) = (3^n - 2^n - 1, 2n)$ then X is connected, for each point $x \in X$, $|X|^x$ is a generalized simple closed k- curve. If the image $|X|^x$ is a simple closed k-curve and the closed k-surface X is called simple.

Let (X, k) bea digital image and its subset be (A,k). (X, A) is called a digital image pair with k-adjacency and when A is a singleton set $\{x_0\}$, then (X,x_0) *is called a point digital image* .

Definition: 1.8 [26] Let (X, k) be a digital image and $f: (X, k) \to (X, k)$ be any (k, k) – continuous function. We say the digital image (X, k) has the fixed point property.

If for every (ƙ, ƙ) – continuous map $: f\ X \to X$ there exists $x \in X$ such that $f(x) = x$.

The fixed point property is preserved by any digital isomorphism. It is a topological invariant.

Let (X, d, k) be denote the digital metric space with k-adjacency where d is usual Euclidean metric for Z^n.

Definition:1.9 A sequence $\{x_n\}$ of points of digital metric space (X, d, k) is said to be a Cauchy sequence if for all ϵ>0, there exists $\alpha \in N$ such that for all $n, m > 0$ then $d(x_n, x_m)$<ϵ.

Definition: 1.10 A sequence $\{x_n\}$ of points of digital metric space (X, d, k) converges to a limit $a \in X$ if for all ϵ>0, there exists $\alpha \in N$ such that for all $n > \alpha$ then $d(x_n, a)$<ϵ.

Definition: 1.11 A digital metric space (X, d, k) is said be a complete digital metric space if any Cauchy sequence $\{x_n\}$ of points (X, d, k) converges to a point a of(X, d, k).

Definition: 1.12 Let (X, k) be any digital image. A function $f: (X, k) \to (X, k)$ is called right-continuous if $f(a) = \lim_{x \to a^+} f(x)$ where $a \in X$.

Definition: 1.13 [12] Let (X, d, k) be any digital metric space and $f: (X, d, k) \to (X, d, k)$ be a self digital map. If there exists $\lambda \in (0,1)$ such that for all $x, y \in X$,

$d(f(x), f(y)) \leq \lambda\, d(x, y)$, then f is called a digital contraction map.

Proposition: 1.14 Every digital contraction map is digitally continuous.

Theorem 1.15 [12]Let(X, d, k) be a complete digital metric space which has a usual Euclidean metric in Z^n. Let $f: X \to X$be a digital contraction map. Then f has a unique fixed point, i.e., there exists a unique $u \in X$ such that $f(u) = u$.

In 2015 OzgurEge, IsmetKaraca generalized Banach contraction Principle as follows.

Theorem:1.16 [12] Let (X, d, k) be a complete digital metric space which has a usual Euclidean metric d in Z^n and let $f: X \to X$ be a digital self map. Assume that there exists a right- continuous real function $\gamma: [0, u] \to [0, u]$ where u is sufficiently large real number such that $\gamma(a) < a$ if a>0, and let f satisfies

$d\left(f(x_1), f(x_2)\right) \leq \gamma\left(d(x_1, x_2)\right)$forall $x_1, x_2 \in (X, d, k)$.

Then f has a unique fixed point u $\in (X, d, k)$ *a*nd the sequence $f^n(x)$converges to$u$ for every $x \in X$.

In 2019 [27] P. H. Krishna and D A Tatajee defined digital Geraghty contraction map and proved the existence of fixed points of Geraghty contraction theorem in digital metric spaces.

Definition :1.17 [27] Let (X, d, k) be any digital metric space and $f: (X, d, k) \to (X, d, k)$ be a self digital map is said to be a digital Geraghty contraction map if there exists $\beta \in S$ such that , $d(f(x), f(y)) \leq \beta(\, d(x, y))d(x, y)$ forall $x, y \in X$.

Here we observe that every digital contraction map is a digital Geraghty contraction map but its converse need not be true.

Theorem [:1.18] **[27]** Let (X, d, k) be a complete digital metric space with Euclidean metric d in $Z^n$and let $f: X \to X$ be a digital Geraghty contraction map. Then f has a unique fixed point u$\in (X, d, k)$.

Definition 1.19 [25] : Suppose that (X, d , k) be a complete digital metric spaces and S, T: $X \to X$ be a map defined on X. Then S and T are said to be commutative if S(T(x))= T(S(x)) for all x in X.

Proposition 1.20 [25]: Let T be a selfmap. Then T has a fixed point if and only if there is a constant selfmap S which commutes with T.

Theorem 1.21 [25] Let T be a continuous selfmap of a complete digital metric space (X, d, k) . Then T has a fixed point in X if and only if there exists an $\alpha \in (0,1)$ and a mapping **S**: $X \to X$ which commutes with T and satisfies

S(X) $\subset T(X)$ and $d(S(x), S(y)) \leq \alpha d(T(x), T(y))$ for all x,y in X .

Then T and S have a unique common fixed point.

Definition: 1.22 [25] : Suppose that (X, d , k) be a complete digital metric spaces and S, T: $X \to X$ be a map defined on X. Then S and T are said to be weakly commutative iff $d(S(T(x)), T(S(x))) \leq$

$d(S(x),T(x))$ for all x in X .
Here observe that every pair of commutative maps is weakly commutative but the converse need not be true, and the weakly commutative maps commute on the coincidence points.

Theorem : 1.23 [25] Let T be a continuous selfmap of a complete digital metric space (X,d,k) . Then T has a fixed point in X if and only if there exists an $\alpha \in (0,1)$ and a mapping **S**: $X \to X$ which commutes weakly with T and satisfies
S(X) ⊂ $T(X)$ and
$d(S(x),S(y)) \le \alpha d(T(x),T(y))$ for all x,y in X .
Then T and S have a unique common fixed point.

Definition 1.24. [25] Two self mappings S and T of a digital metric space (X,d,k) are said to be *compatible* if $d(S(T(x_n)), T(S(x_n)))= 0$, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} S(x_n) = \lim_{n\to\infty} T(x_n) = u$ for some $u \in X$.

Theorem 1.25. :[25] Let S and T be a continuous compatible selfmaps of a complete digital metric space (X,d,k) . Then S and T have a unique common fixed point in X if there exists an $\alpha \in (0,1)$ and a mapping and they satisfy
S(X) ⊂ $T(X)$ and
$d(S(x),S(y)) \le \alpha d(T(x),T(y))$ for all x,y in X .

II. MAIN RESULTS

Now we define digital Geraghty contraction for a a pair of maps in digital metric spaces.

Definition: 2.1 Let (X,d,k) be any digital metric space. Let T and S be self maps on X.
If there exists $\beta \in S$ such that , $d(S(x),S(y)) \le \beta(\, d(T(x),T(y))(d(T(x),T(y))$
for all $x,y \in X$, then we say that (T,S) is a pair of Geraghty contraction maps in digital metric spaces.
Now we prove the existence of common fixed points of Geraghty contraction maps in pair of maps in digital metric spaces.

Theorem : 2.2 Let T be a self of a complete digital metric space (X,d,k) and let (T,S) is a pair of Geraghty contraction maps in digital metric spaces such that
(i) S(X) ⊂ $T(X)$
(ii) T is continuous
(iii) S and T are commutes.
Then T and S have a unique common fixed point.

Proof: Let $x_0 \in (X,d,k)$,
Since S(X) ⊂ $T(X)$, we define the sequence $Tx_n = S(x_{n-1})$ for each $n \ge 1$.
If $Tx_{n+1} = Tx_{n+2}$ for some n then $Tx_{n+1} = Sx_{n+1}$ $x_{n+1}$is a coincident point of T and S.
Without loss of generality, we assume that if $Tx_{n+1} \ne Tx_{n+2}$ for each n, then we have $d(Tx_{n+2},\, Tx_{n+1}) > 0$

We consider
$d(Tx_{n+2},\, Tx_{n+1}) = d(Sx_{n+1},\, Sx_n) \le \beta(d(Tx_{n+1},\, Tx_n))(d(Tx_{n+1},\, Tx_n))$ (2.2.1)
Since $\beta \in S$,
$d(Tx_{n+2},\, Tx_{n+1}) < d(Tx_{n+1},\, Tx_n)$, Which follows that $\{ d(Tx_{n+2},\, Tx_{n+1})\}$ is a decreasing sequence of non-negative reals and so
$\lim_{n\to\infty}(d(Tx_{n+2},\, Tx_{n+1}))$ exists and it is r (say).
Now we show that r=0.
If r>0 then from (2.21) we have
$d(Tx_{n+2},\, Tx_{n+1}) = d(Sx_{n+1},\, Sx_n) \le \beta(d(Tx_{n+1},\, Tx_n))d(Tx_{n+1},\, Tx_n)$

$\frac{d(Tx_{n+2},Tx_{n+1})}{d(Tx_{n+1},Tx_n)} \le \beta(\, d(Tx_{n+1},\, Tx_n)) < 1$ for each $n \ge 1$.
On letting $n \to \infty$, we get
$1= \lim_{n\to\infty} \frac{d(Tx_{n+2},Tx_{n+1})}{d(Tx_{n+1},Tx_n)} \le$
$\lim_{n\to\infty}\beta\, d(Tx_{n+1},\, Tx_n) \le 1.$
So that $\beta(\, d(Tx_{n+1},\, Tx_n)) \to 1$ as $n \to \infty$, that implies
$\lim_{n\to\infty} d(Tx_{n+1},\, Tx_n) = 0.$
Hence r = 0.
Let $\{x_n\}$ be a sequence in X such that $d(Tx_{n+1},\, Tx_n) \to 0$ as $n \to \infty$.
Suppose that$\{x_n\}$ not a Cauchy sequence.
Then there exists $\epsilon > 0$ integers $m(k)$ and $n(k)$ with $m(k) > n(k) > k$
such that $d(Tx_{m(k)},\, Tx_{n(k)}) \ge \epsilon$ (2.2.2)
We choose $m(k)$, the least positive integer satisfying$d(Tx_{m(k)},\, Tx_{n(k)}) \ge \epsilon$, then we have $m(k) > n(k) > k$ with
$d(Tx_{m(k)},\, Tx_{n(k)}) \ge \epsilon, d(Tx_{m(k)-1},\, Tx_{n(k)}) < \epsilon$

$$\epsilon \le d(Tx_{m(k)},\, T x_{n(k)}) \le d(Tx_{m(k)},T x_{m(k)-1}) + d(Tx_{m(k)-1},Tx_{n(k)})$$

$< d(Tx_{m(k)},\, Tx_{m(k)-1}) + \epsilon.$
Since $d(Tx_{n(k)},Tx_{n(k)+1}) \to 0$ as $k \to \infty$, we have $\epsilon \le d(Tx_{m(k)},\, Tx_{n(k)}) < \epsilon$.
This implies $d(Tx_{m(k)},\, Tx_{n(k)}) \to \epsilon$ as k $\to \infty$.

$$\epsilon \le d(Tx_{m(k)},\, Tx_{n(k)}) \le d(Tx_{m(k)},\, Tx_{m(k)-1}) + d(Tx_{m(k)-1},\, Tx_{n(k)+1}) + d(Tx_{n(k)+1},\, Tx_{n(k)}) < d(Tx_{m(k)},\, Tx_{m(k)-1}) + \epsilon + d(Tx_{n(k)+1},\, Tx_{n(k)}).$$

Since $d(Tx_{n(k)},Tx_{n(k)+1}) \to 0$ as $k \to \infty$, we have $\epsilon \le d(Tx_{m(k)},\, Tx_{n(k)}) < \epsilon$
This implies $d(Tx_{m(k)-1},\, Tx_{n(k)+1}) \to \epsilon$ as k $\to \infty$.

$$\epsilon \le d(Tx_{m(k)}, T\,x_{n(k)}) \le d(Tx_{m(k)}, Tx_{m(k)+1}) + d(Tx_{m(k)+1}, Tx_{n(k)-1}) + + d(Tx_{n(k)-1}, Tx_{n(k)}) < d(Tx_{m(k)},\ Tx_{m(k)+1}) + \epsilon + d(Tx_{n(k)+1},\ Tx_{n(k)}).$$

Since $d(Tx_{m(k)}, Tx_{m(k)+1}) \to 0$ as $k \to \infty$, we have $\epsilon \le d(Tx_{m(k)},\ Tx_{n(k)}) < \epsilon$

This implies $d(Tx_{m(k)+1},\ Tx_{n(k)-1}) \to \epsilon$ as k $\to \infty$.

$d(Tx_{m(k)+1}\ , Tx_{n(k)}) = d(S(x_{m(k)}\ ),\ S(x_{n(k)-1}))$
$\le \beta\left(\ T(x_{m(k)}, x_{n(k)-1})\right) T(x_{m(k)}, x_{n(k)-1})$

$\frac{T(x_{m(k)+1}\ ,Tx_{n(k)})}{T(x_{m(k)},Tx_{n(k)-1})} \le \beta\left(\ T(x_{m(k)}, x_{\,n(k)-1})\right) < 1$

On letting $k \to \infty$, we get

$$1=\frac{\epsilon}{\epsilon} \le \lim_{k\to\infty} \beta\left(\ T(x_{m(k)}, x_{n(k)-1})\right) \le 1.$$

So that $\beta\left(\ T(x_{m(k)}, x_{n(k)-1})\right) \to 1$ as $k \to \infty$,

since $\beta \in S, T(x_{m(k)}, x_{n(k)-1}) \to 0$ as $k \to \infty$.

Hence it follows that $\epsilon = 0$, a contradiction.

Therefore $\{Tx_n\}$ is a Cauchy sequence in X, and since (X, d, k) is a complete digital metric space, $S^n(x)$ converges in (X, d, k).

Since T(X) is complete,

there exists $u \in T(X)$ such that $\lim_{n\to\infty} Tx_{n+1} = \lim_{n\to\infty} Sx_n = Tu = u$ for some $y \in X$.

Suppose that if $Sy \ne Ty$ i.e., $d(Sy\,, Ty\ ) > 0$.

Since $\{Tx_n\}$ is a decreasing sequence of non-negative reals and $\{Tx_n\}$ converges to Tu for some $u \in X$.

$$d(Sx_n, Su) \le \beta(d(Tx_n, Tu))d(Tx_n, Tuu)$$

On letting $n \to \infty$.

$\lim n \to \infty \quad d(Sx_n, Su) \le \lim n \to \infty \quad \beta(d(Tx_n, Tu))d(Tx_n, Tu)$

$\lim n \to \infty \quad d(Sx_n, Sy) \le \lim n \to \infty \quad \beta(d(Tx_n, Tu))d(Tu, Tu)$

$$\lim n \to \infty \quad d(Sx_n, Su) = 0$$

$d(Tu, Su)$ =0

Therefore Tu=Su and hence u is a coincidence point of T and S.

Since S and T commutes so that S(T(x_n)= T (S(x_n)) for all n.

Thus S(u)= T(u), and consequently by commutativity , T(T(u))= T(S(u))= S(S(u)).

So that T(S(u))= S(T(u)) = S(S(u)).

Suppose that $d(S(u), S(S(u))$ >0

$d(S(u), S(S(u))$
$\le \beta(\ d(T(u), T(S(u))d(T(u), T(S(u)))$

$Since\ \beta\ in\ S$, it follows that

$d(S(u), S(S(u)) \ < \ d(T(u), T(S(u)))$

$d(S(u), S(S(u)) \ < \ d(S(u), S(S(u)))$, which is contradiction

$so\ that\ \ d(S(u), S(S(u))$ =0

$and\ hence\ \ S(u))$
$= S(S(u)\ so\ thas\ S(u) is\ a\ fixed\ point\ of\ S.$

S(u) is common fixed point of S and T.

Theorem 2.3. Let T be a self of a complete digital metric space (X, d, k) and let (T,S) is a pair of Geraghty contraction maps in digital metric spaces such that

(i) S(X) ⊂ $T(X)$
(ii) T is continuous
(iii) S and T are weakly commutative.

Then T and S have a unique common fixed point.

Proof :

As in the proof of the theorem $\{Tx_n\}$ is a Cauchy sequence in X, and since (X, d, k) is a complete digital metric space, $S^n(x)$ converges in (X, d, k).

Since T(X) is complete,

there exists $u \in T(X)$ such that $\lim_{n\to\infty} Tx_{n+1} = \lim n\to\infty Sxn= Tu=u$ for some $y \in X$.

Suppose that if $Sy \ne Ty$ i.e., $d(Sy\,, Ty\ ) > 0$

we get Since S and are weakly commutative

$d(T(S(x_n), S(T(x_n)) \ \le d(T(x_n), S(x_n)$

Which implies d(T(u), S(u)) $\le d(u, u)$

Therefore T(u)= S(u) and hence u is a coincidence point of S and T.

So that T(S(u))= S(T(u))= S(S(u))

Suppose that $d(S(u), S(S(u))$ >0

$d(S(u), S(S(u))$
$\le \beta(\ d(T(u), T(S(u))d(T(u), T(S(u)))$

$Since\ \beta\ in\ S$, it follows that

$d(S(u), S(S(u)) \ < \ d(T(u), T(S(u)))$

$d(S(u), S(S(u)) \ < \ d(S(u), S(S(u)))$, which is contradiction

$so\ that\ \ d(S(u), S(S(u))$ =0

$and\ hence\ \ S(u))$
$= S(S(u)\ so\ that\ S(u) is\ a\ fixed\ point\ of\ S.$

S(u) is common fixed point of S and T.

Application of Geraghty Common fixed point theorems in Digital metric spaces.

The complex issue of storage of images involving limited memory can be resolved by resorting to digital contractions. In this connection the common fixed point theorem can be of immense help in reconfiguring the image storage. To preserve a specific image in a memory slot which has been already utilized, the image has to be substituted by a contraction constant with a function of specific property. This is enabled by the imposition of compatibility or commutative among the pair of maps.

III. CONCLUSION:

In this paper we introduced common fixed point theorems with Geraghty type contractions for the digital metric spaces , using commutative and weakly commutative mappings. This concept may be of immense help in reconfiguring the image storage **.**

REFERENCES

[1]. S. Banach, Sur les operations dans les ensembles abstraitsetleurs applications aux equations integrales, Fund.Math., 3 (1922), 133-181.
[2]. G. Bertrand, Simple points, topological numbers and geodesic neighborhoods in cubic grids, Pattern RecognitionLetters, 15 (1994), 1003-1011.
[3]. G. Bertrand, R. Malgouyres, Some topological properties of discrete surfaces, J. Math. Imaging Vis., 20 (1999),207-221.
[4]. L. Boxer, Digitally continuous functions, Pattern Recognition Letters, 15 (1994), 833-839.
[5]. L. Boxer, A classical construction for the digital fundamental group, J. Math. Imaging Vis., 10 (1999), 51-62.
[6]. L. Boxer, Properties of digital homotopy, J. Math. Imaging Vis., 22 (2005), 19-26.
[7]. L. Boxer, Digital products, wedges and covering spaces, J. Math. Imaging Vis., 25 (2006), 159-171.
[8]. L. Boxer, Continuous maps on digital simple closed curves, Appl. Math., 1 (2010), 377-386.
[9]. O. Ege, I. Karaca, Fundamental properties of simplicial homology groups for digital images, American Journal ofComputer Technology and Application, 1 (2013), 25-42.
[10]. O. Ege, I. Karaca, Lefschetz Fixed Point Theorem for Digital Images, Fixed Point Theory Appl., 2013 (2013),13 pages.
[11]. O. Ege, I. Karaca, Applications of the Lefschetz Number to Digital Images, Bull. Belg. Math. Soc. Simon Stevin,21 (2014), 823-839.
[12]. OzgurEge, IsmetKaraca , Banach fixed point theorem for digital images, Journal of Non linear Science and Applications, 8(2015), 237-245.
[13]. G. V. R. Babu, K.K. M. Sarma and P. H. Krishna. Fixed points of ψ – weakGeraghty contractions in partially ordered metric spaces. Journal of Advancedresearch in pure mathematics. 6(4), 2014, 9 - 23.
[14]. Jungck G, Commuting mappings and fixed point , Amer. Math , Monthly 1976, (83), 261-263.
[15]. M. A. Geraghty. On contractive maps. Proc. of Amer. Math. Soc., 1973, 40:604-608.
[16]. S. E. Han, An extended digital (k0; k1)-continuity, J. Appl. Math. Comput., 16 (2004), 445-452.
[17]. S. E. Han, Minimal simple closed 18-surfaces and a topological preservation of 3D surfaces, Inform. Sci., 176(2006), 120{134. 2.4O. Ege, I. Karaca, J. Nonlinear Sci. Appl. 8 (2015), 237-245
[18]. S. E. Han, Connected sum of digital closed surfaces, Inform. Sci., 176 (2006), 332-348.
[19]. G. T. Herman, Oriented surfaces in digital spaces, CVGIP: Graphical Models and Image Processing, 55 (1993), 381-396.
[20]. M. Jleli, B. Samet, A new generalization of the Banach contraction principle, J. Inequal. Appl., 2014 (2014), 8pages.
[21]. I. Karaca, O. Ege, Some results on simplicial homology groups of 2D digital images, Int. J. Inform. Computer Sci., 1 (2012), 198-203.
[22]. T. Y. Kong, A digital fundamental group, Computers and Graphics, 13 (1989), 159-166.
[23]. R. Malgouyres, G. Bertrand, A new local property of strong n-surfaces, Pattern Recognition Letters, 20 (1999),417-428.
[24]. A. Rosenfeld, Digital topology, Amer. Math. Monthly, 86 (1979), 76-87.
[25]. W. Shatanawi, H. K. Nashine, A generalization of Banach's contraction principle for nonlinear contraction in apartial metric space, J. Nonlinear Sci. Appl., 5 (2012), 37{43. 1tegrals, Fund. Math., 3 (1922), 133-181.
[26]. ASha Rani, K Kumari Jyothi, Common fixed point therems in digital metric spaces. International Journal of Scientific & Engineering Research Volume 7, Issue 12, 2016.(1704-1716).
[27]. P. H. Krishna and D A Tatajee, Geraghty Contraction principle for digital Images. . International Research Journal of Pure Algebra-9(9), 2019, 1-6.