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# Identification of external force acting on a machine or a structure in the case of unknown excitation points

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**ABSTRACT**
Identification of any external forces acting on a machine or a structure is important for the diagnosis of potential failures or distress. One of the force identification methods is based on the frequency response function. This method is very useful but requires knowledge of the location of the acting force. In this study, an identification method is proposed for the case of unknown excitation points. In the method, the distributed external force is approximately expressed using the same number of unknown parameters as the sensors. The optimum identification condition is determined by two objective functions of the reconstruction accuracy of measured data and the spatial variation of the identified external force. The identification method proposed is checked using the actual measured data. A beam structure elastically supported at both ends is considered. The beam is excited by using an electromagnet. This excitation method is non-contact way so the magnitude of the external force cannot be directly measured by the load cell. The external force is identified using the measured data with an excitation frequency. It was shown that the identified external force was feasible using the proposed method.
**Keywords-**Force identification, Frequency response function, Inverse problem, Non-contact excitation, Singular value decomposition

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## I. INTRODUCTION

A machine or a structure often vibrates due to an external force acting on it. To design a machine or a structure and predict its potential failures, it is necessary to know the external force, even when the external force cannot be measured directly. Identifying the external force is established as an inverse problem, and many studies of this problem have been performed in a variety of research fields [1]-[4].

We consider the identification of force location and magnitude acting on a structure with a certain frequency. For example, when the acting force of the engine mounts from engine is identified, the location of acting force is known and the acting force is concentrating. The other hand, for example, when the flexible panel is excited by noise generated from a machine, the distributed force acts on the panel and the location of acting force is unknown. We are interested in the latter problem.

Here some researches are introduced through the review papers, because there are many studies about the force identification. K.K.Stevens [5] presented an overview for the discrete systems. The procedure was based on the frequency response function (FRF) matrix where the locations of external forces had to be known, so FRF was not applicable if the location of the acting force was unknown. And the inverse of coefficient matrix of FRF was discussed from the viewpoint of the number of measuring points and force locations. C.Y.Shih, Q.Zhang, and R.J.Allemang [6] proposed two force identification methods based on the FRF matrix. One was the principal coordinate transformation method where the singular value decomposition method was used. And the other was the modal coordinate transformation method where the response was expanded by using the reciprocal modal vectors and the relationship between the numbers of measuring points and the adopted modal vectors was discussed. B.J Dobson, and E.Rider [7] have reviewed some papers based on the FRF matrix. Here the relationship of the numbers of measuring points and force locations was also discussed. And the response was expressed by the superposition of modal vectors and the force was reconstructed directly using the equation of motion. S.E.S.Karlsson [8] showed that a prior information was effective to identify the distributed force and the distributed external force was expanded with the shape functions and the coefficients of functions were identified based on the reconstruction accuracy of the responses. The concrete shape functions or any examples, however, have not been shown. As the recent review paper, J.Sanchez, and H.Benaroya [9] showed a direct method of the expansion of the external force by the shape functions [8]. And some regularization methods were introduced, such as Tikhonov method [10] and truncation of singular value method. A.S.Sekhar [11] carried out a diagnosis of rotating machinery using the method which was similar with the one [7]. In the research, the dynamic response for all degrees of freedom was estimated using the modal analysis method from a set of measurement data, and the external force was identified by substituting the response into the equation of motion. Since the exact response and the estimated response were not identical, this method was effective only for systems with concentrated mass. As shown above, we cannot find any method which can be applied when we have no prior information with respect to the distribution of the external force.

The difficulty of force identification in the case of unknown force location is due to a number of unknown parameters. The unknown parameters are the magnitudes of external forces at all degrees of freedom; however, the number of sensors is typically much less than the number of unknown parameters.

The present author proposed a method [12]. In the method, the number of external force was set according to the number of sensors. Then, the magnitudes of the external forces were identified for all combinations of force location. Finally, the locations of the external forces were determined by checking the number of combinations for every location pattern. In the numerical example, the validity and the applicability of the proposed method were checked using the data without and with measurement error. In the experiment, we showed that the stepwise identification approach was feasible in actual applications.

In this paper, we propose a new force identification method which is applicable when we have no information about the location of external force. In the method, the distributed force is approximated by unknown parameters whose number was the same as or less than the number of sensors in the physical coordinate system; then the distributed force is discretized by using the shape function of Finite Element Method (FEM). The external force is identified by changing the number of singular values, and the optimum external force is determined from the objective functions based on the reconstruction accuracy of the measured data and the spatial variation of the identified external force. To determine the order of the singular values, the Tikhonov method [10] or L-curve method [13] has been proposed. Despite their known effectiveness, it is difficult to apply these methods to the inverse problem in the present study because the number of singular values is very few.

The validity of the proposed method is checked using actual measurement data. In the experiment, a simple beam structure elastically supported at both ends is used and the distributed force acts on the beam by using an electromagnet. The mathematical model is constructed in advance, and the distributed external force is identified by the proposed method.

## II. OUTLINE OF FORCE IDENTIFICATION USING FRF

For simplicity of explanation, we ignore damping here. The equation of motion of a machine or a structure is obtained using FEM as follows:

$$[M]\{\ddot{w}(t)\} + [K]\{w(t)\} = \{f(t)\}, \quad (1)$$

where $[M]$ and $[K]$ are the mass and stiffness matrices, respectively, and $\{w(t)\}$ and $\{f(t)\}$ are the response and external force vectors, respectively.

In this system, when the harmonic excitation

$$\{f(t)\} = \{F\}\exp(j\omega t), \quad (2)$$

acts on the structure, the response can be assumed as follows:

$$\{w(t)\} = \{W\}\exp(j\omega t). \quad (3)$$

Substituting Eqs. (2) and (3) into Eq. (1), the equation of motion is obtained as follows:

$$([K] - \omega^2[M])\{W\} = \{F\}. \quad (4)$$

Using this equation, the FRF matrix $[H(\omega)]$ can be obtained as follows:

$$\{W\} = [H(\omega)]\{F\}. \quad (5)$$

Here the response $\{W_p\}$ can be measured with $p$ sensors as follows:

$$\{W_p\} = [C]\{W\}, \quad (6)$$

where matrix $[C]$ is the $(p \times n)$ matrix that expresses the locations of sensors. When the external force acts at $q$ points on the structure whose locations are known, the vector $\{F_q\}$ is determined from $\{F\}$ by extraction of the location of the acting force. The relationship between the response $\{W_p\}$ and the external force $\{F_q\}$ can be written as follows:

$$\{W_p\} = [H_{pq}(\omega)]\{F_q\}. \quad (7)$$

The result depends on the numbers of $p$ and $q$. This method can be applied when the location of the acting force is known in advance.

## III. IDENTIFICATION OF THE DISTRIBUTED EXTERNAL FORCE

The identification method is explained for a simple beam structure of length $l$, but this method can be applied to general structures or machines. It is assumed here that the dynamic responses are measured at 4 points ( $p$ =4).

### 3.1 Approximation of the distributed external force

The distributed external force $\{f(t)\}$ is expressed using 4 unknown parameters, whose number is the same as the number of sensors, as shown in Fig.1. In the figure, $f_a(t)$ at node number $n_a$ (coordinate $x_a$ ), $f_b(t)$ at $n_b$ ($x_b$) , $f_c(t)$ at $n_c$ ($x_c$) and $f_d(t)$ at $n_d$ ($x_d$) are the unknown parameters, and the distributed external force between two unknown parameters is linearly approximated as follows:

$$f(x,t) = \begin{cases} 0 & (0 \le x < x_a) \\ \Phi_{ab1}(x) f_a(t) + \Phi_{ab2}(x) f_b(t) & (x_a \le x < x_b) \\ \Phi_{bc1}(x) f_b(t) + \Phi_{bc2}(x) f_c(t) & (x_b \le x < x_c) \\ \Phi_{cd1}(x) f_c(t) + \Phi_{cd2}(x) f_d(t) & (x_c \le x < x_d) \\ 0 & (x_d \le x \le l) \end{cases} , \quad (8)$$

where, for example, $\Phi_{ab1}(x)$ and $\Phi_{ab2}(x)$ are expressed as follows:

$$\Phi_{ab1}(x) = \frac{x - x_b}{x_a - x_b}, \quad \Phi_{ab2}(x) = \frac{x_a - x}{x_a - x_b} \quad (9)$$

In Fig.1, there is one nodal point between two unknown parameters ( $m$ =1), but the value of $m$ may be changed. The distributed external force in Eq. (8) is discretized for the area of node number $n_a$ to $n_d$ as follows:

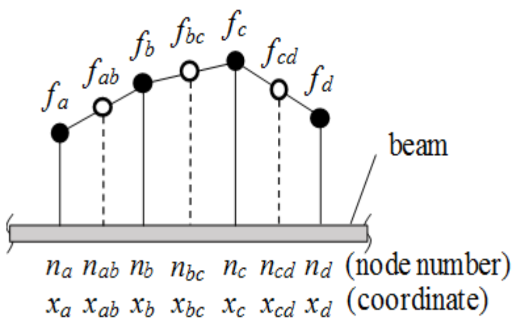



**Figure 1** approximation of distributed force

$$\begin{Bmatrix} f_a(t) \\ f_{ab}(t) \\ f_b(t) \\ f_{bc}(t) \\ f_c(t) \\ f_{cd}(t) \\ f_d(t) \end{Bmatrix} = \begin{bmatrix} 1 & & & \\ 1/2 & 1/2 & & \\ & 1 & & \\ & 1/2 & 1/2 & \\ & & 1 & \\ & & 1/2 & 1/2 \\ & & & 1 \end{bmatrix} \begin{Bmatrix} f_a(t) \\ f_b(t) \\ f_c(t) \\ f_d(t) \end{Bmatrix}$$

(10)

To discretize the distributed external force acting on the $e$ -th element, the next equation is used:

$$\{f(t)\}_e = \int_0^h f(x,t)\{L(x)\}_e \, dx , \quad (11)$$

where $h$ is the length of the element and $\{L(x)\}_e$ is the shape function. The distributed external force at both ends of the $e$ -th element is set as $f_i(t)$ and $f_j(t)$ , and the distributed external force is linearly approximated by Eq.(11) as follows:

$$\{f(t)\}_e = \int_0^h \left( \left(1 - \frac{x}{h}\right) f_i(t) + \left(\frac{x}{h}\right) f_j(t) \right) \begin{Bmatrix} L_1(x) \\ L_2(x) \\ L_3(x) \\ L_4(x) \end{Bmatrix}_e dx ,$$

(12)

and as the result of calculation, we can obtain the next equation:

$$\begin{Bmatrix} f_{ti}(t) \\ f_{ri}(t) \\ f_{tj}(t) \\ f_{rj}(t) \end{Bmatrix} = \begin{bmatrix} (7/20)h & (3/20)h \\ (1/20)h & (1/30)h \\ (3/20)h & (7/20)h \\ -(1/30)h & -(1/20)h \end{bmatrix} \begin{Bmatrix} f_i(t) \\ f_j(t) \end{Bmatrix} ,$$

(13)

where $f_{ti}(t)$ and $f_{ri}(t)$ are the transverse force and the moment/ $h$ , respectively, and $f_i(t)$ and $f_j(t)$ are the distributed external force at both ends of $e$ -th element like $f_a$ , $f_{ab}$ or $f_{ab}$ , $f_b$ , and so on. Then, Eq. (13) can be expanded for the area of Fig.1 as follows:

$$\{f_{ta} \ f_{ra} \ f_{tab} \ f_{rab} \ \cdots \ f_{td-1} \ f_{rd-1} \ f_{td} \ f_{rd}\}^T = [T_A]\{f_a \ f_{ab} \ f_b \ f_{bc} \ f_c \ f_{cd} \ f_d\}^T ,$$

(14)

where matrix $[T_A]$ is the transformation matrix of the distributed external force on the nodal points to the discretized external force using the shape function, and its size is 14× 7. Equation (14) can be rewritten using Eq. (10) as follows:

$$\{f_{ta} \ f_{ra} \ f_{tab} \ f_{rab} \ \cdots \ f_{td-1} \ f_{rd-1} \ f_{td} \ f_{rd}\}^T = [T_B]\{f_a \ f_b \ f_c \ f_d\}^T ,$$

(15)

This equation means that, in the case of Fig.1, by approximating the distributed external force using 4 unknown parameters $f_a(t)$ , …, $f_d(t)$ , 14 elements in the vector $\{f(t)\}$ can be determined. By changing the number of internal nodal points, $m$ , various types of external force can be approximately expressed. In the case of $m$ =0, for example, the external force concentrates on three elements.

The external force is considered a harmonic one with a certain frequency $\omega$ , and the magnitudes of the external forces in Eq. (15) become unknown parameters. Then, $\{F\}$ in Eq. (5) is expressed as follows:

$$\{F\} = \{0 \ \cdots \ 0 \ F_{ta} \ \cdots \ F_{rd} \ 0 \ \cdots \ 0\}^T = \begin{bmatrix} [0] \\ [T_B] \\ [0] \end{bmatrix} \begin{Bmatrix} F_a \\ F_b \\ F_c \\ F_d \end{Bmatrix} \equiv [T_f]\{F_p\}. \quad (16)$$

Using Eqs. (5) and (16), Eq. (6) can be rewritten as follows:

$$\{W_p\} = [C]\{W\} = [C][H(\omega)]\{F\} = [C][H(\omega)][T_f]\{F_p\} \equiv [\tilde{H}(\omega)]\{F_p\} \quad (17)$$

The feature of this method is that the various types of external force, including both concentrated and distributed forces, can be expressed using a few unknown parameters whose number is the same as the number of sensors. When the number of unknown parameters is less than that of the sensors, the same procedure can be applied.

**3.2 Identification method of the distributed external force**

The matrix $[\tilde{H}(\omega)]$ can be constructed by setting the location of variables $F_a$ , $F_b$ , $F_c$ and $F_d$ ; then, the external force will be identified as follows.

Four unknown parameters are identified from Eq. (17) by singular value decomposition. First, the matrix $[\tilde{H}(\omega)]$ is expressed as follows:

$$[\tilde{H}(\omega)] = [U][B][V]^T , \quad (18)$$

where $[B]$ is the diagonal matrix composed of the singular values $\lambda_i$ ( $i = 1, \cdots, 4$ ). Using Eqs. (17) and (18), the external force $\{\overline{F}_p\}$ can be obtained as follows:

$$\{\overline{F}_p\} = [V][\overline{B}]^{-1}[U]^T\{W_p\} , \quad (19)$$

where $[\overline{B}]^{-1}$ is the diagonal matrix composed of the reciprocals of the singular values in which the very small singular values are truncated.

To search for the optimum condition of force identification, two types of objective functions are considered. One is the reconstruction accuracy of measured data $J_1$ , which is usually used. The other is the spatial variation of identified external force $J_2$ . They are expressed as follows:

$$J_1 = \frac{1}{4}\sqrt{\sum_{j=1}^{4}\left(W_{p,j} - \overline{W}_{p,j}\right)^2} , \quad (20)$$

$$J_2 = \sqrt{\sum_{j=1}^{3}\left\{\frac{\overline{F}_{p,j+1} - \overline{F}_{p,j}}{(m+1)h}\right\}^2} . \quad (21)$$

It is often observed that when the measured data is reconstructed too accurately, the identified external force varies too greatly and the identified force is not feasible. In this study, therefore, the extreme variation of external force is reduced using the objective function $J_2$ .

These two objective functions have no trade-off relation and it is desired that both objective functions become to be minimum at the same analysis condition. The analysis condition consists of the number of internal nodal points $m$ , the one of the adopted singular values and the location of distributed external force $n_a$ .

**3.3 Identification procedure of the distributed external force**

The identification of external force is carried out as follows:

(1) The mathematical model is constructed by FEM, and the matrix $[H(\omega)]$ is obtained.

(2) The number of variables is set to be equal to or less than the number of sensors, and the number of internal nodal points $m$ is set.

(3) The adopted singular values and the location of distributed external force $n_a$ in Fig.1 are set; then the external force $\{\overline{F}_p\}$ is identified by Eq. (19). The response $\{\overline{W}_p\}$ is calculated by Eq. (17) in which $\{F_p\}$ is replaced by $\{\overline{F}_p\}$.

(4) Two objective functions $J_1$ and $J_2$ are calculated.

(5) Check the $J_1$ and $J_2$ at a certain analysis condition, and determine the analysis condition where $J_1$ and $J_2$ show the minimum values at the same analysis condition.

## IV. APPLICABILITY CHECK OF THE PROPOSED METHOD

The identification method proposed in this paper was checked using the actual measured data.

**4.1 Experimental setup and mathematical model**

The experimental setup is illustrated in Fig.2, which is a beam structure elastically supported at both ends. The properties of the beam and the boundary conditions are shown in Table 1. The density was calculated from the total mass and the volume of the beam, and Young's modulus was determined from the first natural frequency in the case of freely supported beam. The mathematical model was constructed by 58 beam elements, and the beam was assumed to be fixed in the transverse direction and elastically supported in the rotating direction at both ends. The coefficients of the rotating stiffness are determined such as that the measured natural frequencies by the impact test agree with the simulated ones. The accelerations of the beam are measured by the accelerometers (PCB 353B15) at 4 points, S1, S2, S3 and S4, situated 160, 240, 340 and 420 mm from the left end, respectively. The mass of each accelerometer ($2.12\times 10^{-3}$kg) is considered as a concentrated mass in the mathematical model.

The accelerances against an impact at 270 mm are shown in Fig.3. In the mathematical model, the damping property was ignored. It is shown that the natural frequencies calculated from the mathematical model agree well with the measured ones, though the magnitude of accelerance has a little difference between the experimental data and simulated result. Here the updated mathematical modal was constructed.

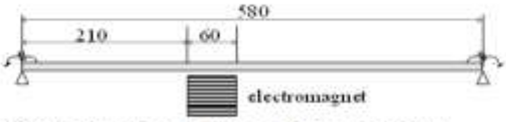



**Figure 2** elastically supported beam structure excited by an electromagnet

**Table 1** specifications of beam structure

| | | | |
|---|---|---|---|
| length | 580.0 mm | density | 7619.0 kg/m³ |
| width | 35.0 mm | Young's modulus | 200.0 GPa |
| thickness | 1.6 mm | spring constant | 93.0 Nm/rad |

The beam is excited by using an electromagnet located from 210 mm to 270 mm with 3.0 mm clearance between the electromagnet and the beam. The area of the electromagnet corresponds to 6 elements, which are the node numbers 22, 23, …, 28. Here the left end of the beam is node number 1, so that the right end is node number 59. The beam is excited by non-contact way, so the magnitude of the external force acting on the beam cannot be directly measured by the load cell. The accelerations are measured as the real and imaginary parts of Fourier spectrum against the reference signal in the data analyzer (ONO SOKKI DataStation DS2000). As an example, the excitation frequency is set as 90.0 Hz.

In the previous study [14], it was recognized that the diagnosis accuracy improved when the measured data was modified to adjust to the simulated result by using the ratio between the measured data and simulated result in the case of model update. Because the actual structure may have an uncertain property which cannot be modeled yet. So in this study, the forced responses were modified by using the ratios calculated from Fig.3. And the ratios are shown in Table 2.

### 4.2 Measured acceleration data

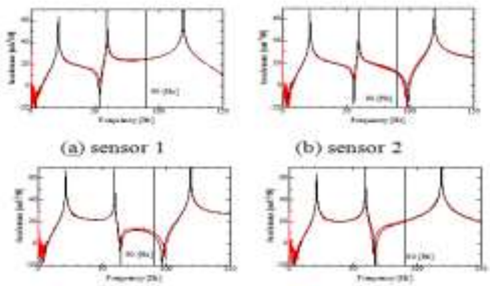
The measured data are shown in Table 3. It is shown that the accelerations at 4 points are in phase and the phases are almost same. The magnitude of each data is considered as the acceleration.

### 4.3 Identification results

The identification results are shown for various $m$ . There are 4 singular values in Eq. (18), and usually all singular values are not adopted from the viewpoint of expanding error. Three cases of adopted singular values are considered: Case 1 ( $\lambda_1$ ), Case 2 ( $\lambda_1$ , $\lambda_2$ ) and Case 3 ( $\lambda_1$ , $\lambda_2$ , $\lambda_3$ ).

#### 4.3.1 The case of $m$ =0

In this case, 4 unknown parameters are in series without interval. The corresponding area of the electromagnet is 7 nodal points, so that the area in this case is smaller than the actual one. Moving the assumed area of the external force from the left end to right, the objective functions $J_1$ and $J_2$ are calculated and shown in Fig.4. The horizontal axis indicates the node number $n_a$ , which can be set from 2 to 55. From Fig.4, in Case 2 (2 singular values are adopted), the node number where the objective function shows the minimum value is same for $J_1$ and $J_2$, though in Case 1 (1 singular value is adopted) and Case 3 (3 singular valuesare adopted), it is not so. The best condition is Case 2 with $n_a$ =24(the area of the external force is 24, 25,26, 27). The identified external force and the reconstructed response are shown in Fig.5. The responses are reconstructed accurately, and the identified external function is feasible.



(a) sensor 1 (b) sensor 2
(c) sensor 3 (d) sensor 4

**Figure 3** accelerances at 4 sensor points
red line: measured data, black line: simulated result)

**Table 3** Fourier spectrum of response excited by electromagnet

| | Real part (m/s²) | Magnitude (m/s²) |
|---|---|---|
| | Imaginary part (m/s²) | Phase (rad) |
| Sensor 1 | -0.898 | 1.26 |
| | 0.881 | -0.776 |
| Sensor 2 | -0.352 | 0.472 |
| | 0.315 | -0.730 |
| Sensor 3 | -0.113 | 0.151 |
| | 0.100 | -0.725 |
| Sensor 4 | -0.157 | 0.226 |
| | 0.163 | -0.803 |

**4.3.2 The case of $m$ =1**

In this case, 4 unknown parameters are in series with one nodal point, so that the external force is assumed at 7 nodal points. The area of the electromagnet corresponds to 6 elements that is composed of 7 nodes. The number of nodes of the actual area is equal to the number of assumed nodes. The objective functions $J_1$ and $J_2$ are calculated and shown in Fig.6. The horizontal axis, $n_a$ , is set from 2 to 52. From the figure, the best condition is Case 2 with $n_a$ =22, and the identified external force and the reconstructed response are shown in Fig.7. From the figure, it is shown that the responses are reconstructed accurately and the identified external function is feasible.

The area of assumed excitation force is the same as the one of the electromagnet, but the external force acts on the beam by non-contact way, so the acting area of excitation force might be larger than the size of the electromagnet.

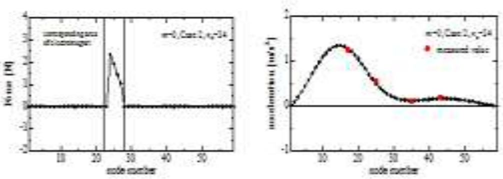
**4.3.3 The case of $m$ =2**

In this case, 4 unknown parameters are in a series with two nodal points, so that the external force is assumed at 10 nodal points. The area of assumed excitation force is a little larger than the size of the electromagnet.

The objective functions $J_1$ and $J_2$ are calculated and shown in Fig. 8. The horizontal axis, $n_a$ , is set from 2 to 49. From the figure, the best condition is Case 2 with $n_a$ =22, the identified external force and the reconstructed response are shown in Fig.9. From the result of Fig. 9, the responses are reconstructed accurately and the identified external function is feasible.

(a) Objective function $J_1$ (b) Objective function $J_2$

**Figure 4** Behavior of objective function ( $m$ =0)

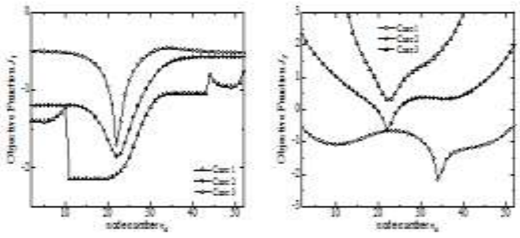


(a) Identified force (b) Reconstructed response

**Figure 5** Results in Case 2 with $n_a$=24

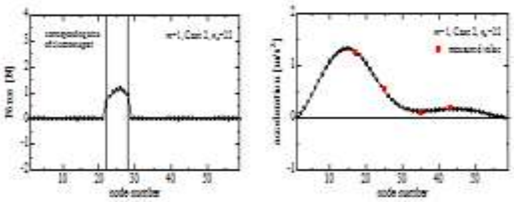
**4.3.4 The case of $m$ =3**

In this case, 4 unknown parameters are in a series with three nodal points, so that the external force is assumed at 13 nodal points. The area of assumed excitation force is much larger than the size of the electromagnet.



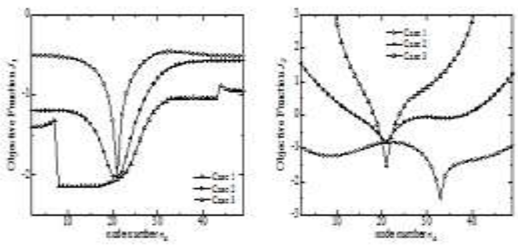
(a) Objective function $J_1$ (b) Objective function $J_2$

**Figure 6** Behavior of objective function ( $m$ =1)



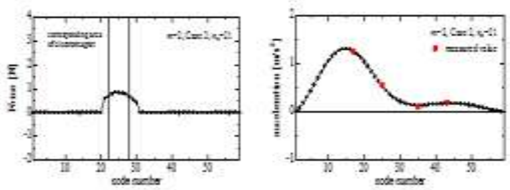
(a) Identified force (b) Reconstructed response

**Figure 7** Results in Case 2 with $n_a$=22



(a) Objective function $J_1$ (b) Objective function $J_2$

**Figure 8** Behavior of objective function ( $m$ =2)



(a) Identified force (b) Reconstructed response

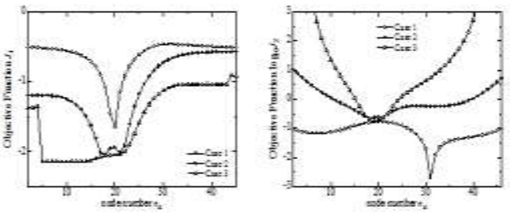
**Figure 9** Results in Case 2 with $n_a$=21

The objective functions $J_1$ and $J_2$ are calculated and shown in Fig. 10. The horizontal axis, $n_a$, is set from 2 to 46. From the figure, we cannot find the best condition, where $J_1$ and $J_2$ show the minimum values at the same analysis condition. In Fig.10(a), small values of $J_1$ can be observed at $n_a$ =18 and 21, and in Fig.10(b), the values of $J_2$ are small at $n_a$ =19, 20. Therefore, considering the behaviors of $J_1$ and $J_2$, we may determine the best analysis condition as $n_a$ =19 or 20. In the case of $n_a$ =19, the corresponding area of the electromagnet is the center in the assumed area of external force. But in general, the consideration cannot be applied because we don't know the size and location of excitation source.

**4.3.4 Discussions**

The best analysis condition can be obtained for each value of $m$ , which is the number of the internal nodes between two unknown parameters. Here the values of $J_1$ and $J_2$ of the best analysis condition are shown in Table 4. It is recognized that the optimum analysis condition is $m$ =2 and Case 2 with $n_a$ =21, because $J_1$ and $J_2$ show the smallest values.

To determine the area of external force more precisely, it is necessary to check the accuracy of reconstruction of the response data not used for identification.



(a) Objective function $J_1$ (b) Objective function $J_2$

**Figure 10** Behavior of objective function ( $m$ =3)

**Table 4** The best analysis condition for each $m$

| $m$ | | $J_1$ | $J_2$ |
|---|---|---|---|
| 0 | Case 2, $n_a$=24 | $1.91\times10^{-2}$ | $7.68\times10^{-1}$ |
| 1 | Case 2, $n_a$=22 | $1.41\times10^{-2}$ | $2.38\times10^{-1}$ |

## V. CONCLUSIONS

We propose a force identification method that is applicable to the case of unknown force locations. In the method, the distributed external force is approximately expressed using several unknown parameters whose number matches that of the sensors. The optimum identification condition is determined by the objective functions of the reconstruction accuracy of measured data and the variation of the identified external force. To check the applicability of the method, the actual distributed external force was indeed identified by the method proposed in this study. It was shown that a sufficient result could be obtained, and method was effective.

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