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Measuring of phase shift under polyharmonic signals condition

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RESUME

Measuring of phase shift under polyharmonic signals conditions.

The tasks measuring of phase shift of electrical signal in polyharmonic signals conditions considered, analyzed an error arising during measuring and ways of decreasing it. The new algorithm based on Walsh functions and the results of analysis are presented, The simulation model on Simulink program environment developed, key factors to impact to error value are defined.

Keywords: harmonic signal, phase shift, electric power, sampling, signal processing, error, Walsh function, sign change, simulation model.

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INTRODUCTION

Measurement of the power factor and phase shift under polyharmonic signals conditions is a rather complex problem, which urgently requires properly its solution.

In the case of the presence of the main and additional harmonic current components, for electric power, the following is true:

$$P(t) = P_1(t) + P_a(t) \ , \qquad (1)$$

$P_1(t)$ - power of the main harmonics (voltage and current); *$P_a(t)$* - power of the additional harmonic. Accordingly, for the instantaneous value of electric power, we obtain the following expressions:

$$P_1(t) = P - \left[P\cos 2\omega t + Q\sin 2\omega t\right] \qquad (2)$$

$$P_a(t) = U_m \sin\omega t \sum_{i=1}^{l} I_{mi} \cdot \sin(\omega_i t - \varphi_i)$$

In the first approximation, we assume that the voltage is sine signal and in the current signal existence only the third harmonic except the fundamental harmonic and we determine the power produced by the additional harmonic:

$$P_a(t) = T_3^p \cos 2\omega t - T_3^p \cos 4\omega t + T_3^q \sin 2\omega t - T_3^q \sin 4\omega t \qquad (3)$$

If we take into account expressions (2) and (3) in (1), for the instantaneous value of electric power *$P(t)$* obtain the following expression:

$$P(t) = P - \left[P\cos 2\omega t + Q\sin 2\omega t\right] + T_3^p \cos 2\omega t - T_3^p \cos 4\omega t + T_3^q \sin 2\omega t - T_3^q \sin 4\omega t$$

In this expression, the distortion power spectra due to the pairs P$\cos 2\omega t$, $T_3^p \cos 2\omega t$ and Q$\sin 2\omega t$, $T_3^q \sin 2\omega t$ coincide with the spectra of the alternating components P$\cos 2\omega t$ and $\sin 2\omega t$ of the fundamental harmonic. Grouping the components with the same frequency, we get:

$$P(t) = P + (T_3^p - P)\cos 2\omega t + (T_3^q - Q)\sin 2\omega t - T_3^p \cos 4\omega t - T_3^q \sin 2\omega t \ .$$

Where $T_s^p = (T_3^p - P)\cos 2\omega t$ and $T_s^q = (T_3^q - Q)\sin 2\omega t$ are the active (T_s^p) and reactive components of the distortion power whose spectra coincide. To analyze the coincident spectra for *l* existing cases, we write the expression for the total distortions power in the following form:

$$T_n(t)=\sum_{n=1}^{l}\left[T_n^p\cos(\omega-\omega_n)t-T_n^p\cos(\omega-\omega_n)t-T_n^q\sin(\omega-\omega_n)t-T_3^q\sin(\omega-\omega_n)t\right]$$

$$T_n(t)=\sum_{n=1}^{l}\left[T_n^p\cos(1-n)\omega t-T_n^p\cos(1+n)\omega t-T_n^q\sin(1+n)t-T_3^q\sin(1-n)\omega t\right]$$

For the third harmonic, the frequencies of the components *(n=3)* $T_3^p\cos 2\omega t$ and $T_3^q\sin 2\omega t$ E2 and the components of $P\cos 2\omega t$ and $Q\sin 2\omega t$ arising due main harmonic coincide, and so a measurement error of the distortion power arises due to influence of this effect. The method is proposed to measure the power, also makes it possible to estimate the phase shift. To confirm this possibility, we use expression (2)

$$P_1(t)=P-\left[P\cos 2\omega t+Q\sin 2\omega t\right]$$

Applying the third and fourth Walsh transformations, we get:

$$S(3)=\frac{1}{N}\sum_{n=0}^{N-1}P_1(n),W(3,\beta_k)$$

(4a)

$$S(4)=\frac{1}{N}\sum_{n=0}^{N-1}P_1(n),Wal(4,\beta_k)$$

(4b)

Where $P_1(n)$ - discrete values of the instantaneous power signal

$$P_1(n)=P-\left[P\cos(2\frac{2\pi}{N}n\right)+Q\sin(2\frac{2\pi}{N}n)$$

In accordance with the third and fourth Walsh transformations, we write the following

$$Wal(3,\beta_k)=(-1)^{\beta_2(N)}\text{ and }$$

$$Wal(4,\beta_k)=(-1)^{\beta_2(N)+\beta_3(N)}$$

Substituting the expressions for the electric power $P_1(n)$ and the Walsh transformations in (4) for $S(3)$ and $S(4)$, we obtain:

$$S(3)=\frac{1}{N}\sum_{n=o}^{N-1}Q\sin(2\frac{2\pi}{N}n)(-1)^{\beta_2(N)}$$

$$S(4)=\frac{1}{N}\sum_{n=o}^{N-1}P\cos(2\frac{2\pi}{N}n)(-1)^{\beta_2(N)+\beta_3(N)}$$

Or taking into account the peculiarities of the Walsh transform, we obtain the following expressions

$$S(3)=\frac{1}{N}\sum_{n=o}^{N-1}\left|Q\sin(2\frac{2\pi}{N}n)\right|$$

$$S(4)=\frac{1}{N}\sum_{n=o}^{N-1}\left|P\cos(2\frac{2\pi}{N}n)\right|.$$

Using these formulas, we can determine the average values of the active and reactive components of electrical power and the tangent of the phase angle.

$S(3)=Q_{ort}$ and $S(4)=P_{ort}$

$$tg\,\varphi=\frac{S(3)}{S(4)}$$

$$\sum_{m=1}^{5}(\omega_{5-m+1}\oplus\omega_{5-m})\beta_m=(1\oplus 1)\beta_1+(1\oplus 0)\beta_2+(0\oplus 0)\beta_3+(0\oplus 0)\beta_4+(0\oplus 0)\beta_5+\beta_2$$

In this case, when evaluating the reactive components of electric power, we use expression (5)

$$S(Q)=\frac{1}{N}\sum_{n=0}^{N-1}S(k)(-1)^{\sum_{m=1}^{n}(\omega_{n-m+1}\oplus\omega_{n-m})\beta_{QM}}$$

(5)

Assuming in the repetition interval $n = 32$ for determining the reactive components of the signal $P(t)$ from the relation $N=2n$, we obtain $n=5$. Then, taking into account the given value of n, we write from expression (5):

$$S(Q)=\frac{1}{32}=\sum_{k=0}^{31}S(k)(-1)^{\sum_{m=1}^{5}(\omega_{5-m+1}\oplus\omega_{5-m})\beta_{QM}}$$

(6)

Let us analyze the power of the sign function

$$\sum_{m=1}^{5}(\omega_{5-m+1}\oplus\omega_{5-m})\beta_m=\sum_{m=1}=(\omega_5\oplus\omega_4)\beta_1+(\omega_4\oplus\omega_3)\beta_2+(\omega_3\oplus\omega_2)\beta_3+(\omega_2\oplus\omega_1)\beta_4+(\omega_1\oplus\omega_0)\beta_5$$

Taking into account that $\omega_0=\omega_1=\omega_1=\omega_3=0$ and $\omega_4=\omega_4=1$ for $S(Q)$ from the last expression we get

$$\sum_{m=1}^{5}(\omega_{5-m+1}\oplus\omega_{5-m})\beta_m=(1\oplus 1)\beta_1+(1\oplus 0)\beta_2+(0\oplus 0)\beta_3+(0\oplus 0)\beta_4+(0\oplus 0)\beta_5+\beta_2$$

Where the bit factor β_2 is defined as follows

$\beta_2=Q;[0,N/4]v$ on interval $(N/2,3N/4)$

$\beta_2=1;[0,N/4,N/2]v$ on interval $(N/4,N-1)$

Then, taking this into account, we get for (6):

$$S(Q) = \frac{1}{32} = \left[\sum_{k=0}^{N/4-1} S(k)(-1)^{0} + \sum_{k=N/4}^{N/2-1} S(k)(-1)^{1} + \sum_{k=N/2}^{3N/4-1} S(k)(-1)^{0} + \sum_{k=3N/4}^{N-1} S(k)(-1)^{1} \right];$$
;

Or

$$S(Q) = \frac{1}{32} = \left[\sum_{k=0}^{N/4-1} S(k) - \sum_{k=N/4}^{N/2-1} + \sum_{k=N/2}^{3N/4-1} - \sum_{k=3N/4}^{N-1} S(k) \right];$$

Since $N = 32$

$$S(Q) = \frac{1}{32} = \left[\sum_{k=0}^{7} S(k) - \sum_{k=8}^{15} + \sum_{k=16}^{23} S(k) - \sum_{k=24}^{31} S(k) \right]$$

(7)

To simplify the analysis, we assume in the first approximation that the *S(k)* is a periodic function that varies with a period of *T* and the discretization (sampling) steps are coherent to period *T*. In practice, the last idea is adopted in the majority of cases in this way, i.e. the moment of the first intra period reference (when *k=0*) coincides with the beginning of the period of the signal *S(t)*. Then, taking into account the fact that under the given conditions among the discrete values the integer interval Δt is defined as $\Delta t = T / N$, we write the dependency between the components in the expression (7) as follows:

K=0 when *S(0)=0* and *K=16 when S(16)=0*

As a result, we get

$$\sum_{k=0}^{7} S(k) - \sum_{k=8}^{15} + S(k) \neq 0$$

$$\sum_{k=16}^{23} S(k) - \sum_{k=24}^{31} + S(k) \neq 0 \qquad (8)$$

$$S(Q) = \frac{1}{32} = \sum_{k=0}^{31} S(k) - \sum_{k=24}^{31} S(k) \neq 0 \qquad (9)$$

To determine the cause of the last inequality, let us analyze the difference (8). Then we write this expression as follows:

$$\sum_{k=0}^{7} S(k) - S(8) \sum_{k=9}^{15} + S(k) \neq 0$$

$$\sum_{k=10}^{23} S(k) - S(24) - \sum_{k=23}^{31} S(k) \neq 0$$

For polyharmonic signals encountered in radio engineering and power engineering
except for components with a repetition period of
$T^{t} = \pi / \omega$ should be

$$S(Q) = \frac{1}{32} = \sum_{k=0}^{31} S(k) = -[S(8) + S(24)],$$

And in general form we get

$$S(Q) = \frac{1}{N} = \sum_{k=0}^{N-1} S(k) = -[S(N/4) + S(3/4)]$$

We come to the conclusion that, among the periodic functions with period *T*, the *N/4* and *3N/4* samples for functions that are skew-symmetric relatively time *T/2* or those corresponding to the middle of the interval must be discarded. To prove this rule, we assume that *S(t)* is a signal that changes according to the monoharmonic law. Then we write

$$S(k) = A\sin\left(\frac{2\pi}{N}k\right)$$

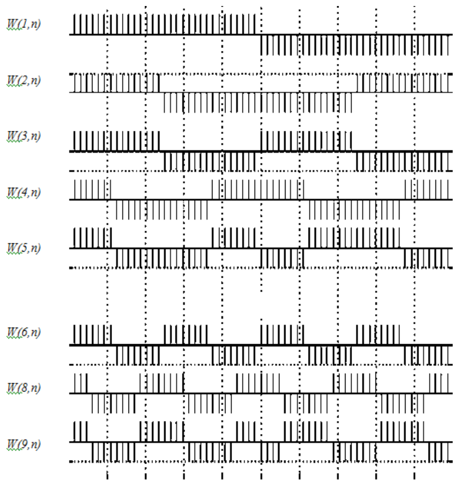

Figure 1. System of discret Walş functions.

This function is simultaneously skew-symmetric relatively to center of the repetition period 2π, and on the intervals $[0,\pi)$ and $[\pi,2\pi]$ symmetric relatively to these intervals. The points of the centers correspond to values $\frac{\pi}{2}$ and $\frac{3\pi}{2}$.

A specific error occurs when the third Walsh function *W(3,n)* changes its value at these points. As shown in Figure 2, b that the initial (first) discrete value of the third Walsh function is zero, so the final value of the multiplication of the function *S(t)* and the third Walsh function (Figure 2, a) is equals to zero. Thus, the values of the multiplication of *S(t)*W(3,n)* are different in the range *[0, N/4-1]* and

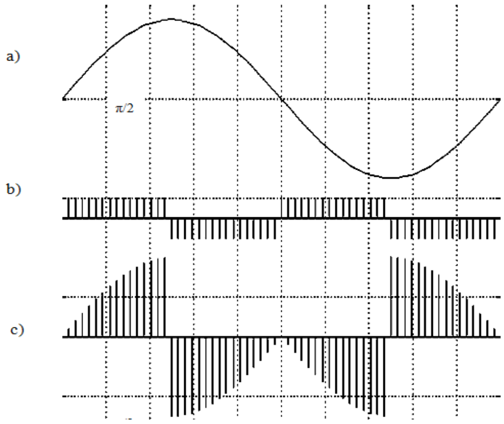

Figure 2. Grafical interpretation of violation effect of skew-symmetric the multiplication *S(t)*W(3,n)* relatively to point $\pi / 2$.

[N/2-1]. As can be seen from Fig.2c, the cause of the violation of oblique symmetry of the resultant function relatively to the points $k = \frac{N}{4}$ and $k = \frac{3N}{4}$ are the values of the function *S(t)* at these points. Indeed, the number of readings (samplings), which is not zero at one quarter of the period *(2*π*)* for the *N=64* and $\omega = 314$ rad./s ratios (Fig. 2, c), the number of selective values in the second quarter is 16, and its equilibrium, the skew symmetry relatively to the point $\pi / 2$ is violated.

It should be noted that the described effect belongs to the entire system of functions that are skew-symmetric relatively to center of the repetition period. To analysis and correction of the error, arising from the effect of sign change, an imitation model has been developed in the MATLAB (Simulink) program environment (Fig. 3).

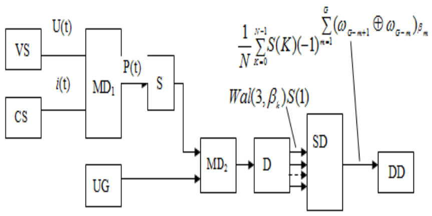

Figure 3. Structure model for the analysis of errors of the sign changing effect.

This model includes the following components: VS - voltage sensor; CS - current sensor; MD1 and MD2 multiplication devices; UG - Walsh generator; S - sampler (discretizer); D - delay circuit; SD - summing device; DD - digital display. The VS is a source of sinusoidal voltage, and CS generates a current flowing in the measured circuit. Both sensors use the Sine Wave block from the Simulink library.

The signals VS and CS vary according to the following law (Fig. 4):

$$U(t) = U_m \sin \omega t$$

$$i(t) = I_m \sin(\omega t - \varphi)$$

Where U_m, I_m - amplitude values of voltage and current signals, respectively; ω - frequency of voltage and current variation; φ is the phase shift between the voltage and current signals.

MD1 is used for obtain multiplication of the input voltage and current signals. Its output signal can be described as follow:

$$P(t) = U_m I_m \sin \omega t \sin(\omega t - \varphi)$$

The signal obtained at the output of the MD1 and proportional to the instantaneous value of the electrical power is applied to the discretizer S (fig.2). The discretizer serves as a quantizer of level and time. The output signal S at each discrete time *k* is the instantaneous value of the electrical power.

The output signal S at each discrete time *k* is mathematically described by the following expression:

$$S(k) = U_m I_m \sin(\frac{2\pi}{T} \Delta t \cdot k) \sin(\frac{2\pi}{T} \Delta t - \varphi) \qquad (10)$$

T - period of repetition of voltage and current signals: $\Delta t = T / N$; *N* - number of discrete values for the full period of time; *k* = 0,1,2..,*N-1*.

The output signal of the UG changes according to the following law:

$$U_{ug}(t) = (-1)^{\sum_{m=1}^{n} (\omega_{n-m+1} \oplus \omega_{n-m}) \beta_m}$$

And it performs the function of changing the sign. Thus, in the interval $[0, T/4)$ and $[T/2, 3T/4)$ or in discrete version in the interval $[0, N/4)$ and $[N/2, 3N/4)$ the value of the function is plus one, and in the interval $[T/4, T/2)$ və $[3T/4, T)$ or in the discrete version $[N/4, N/2)$ and $[3N/4, N-1]$ is minus one. As a result, each discrete value obtained at the output S, remains unchanged, or multiplied by minus one.

In the simulation, the values that correspond to the actual values are taken as the initial data: U_m=4 V; I_m =1,5A; T=0,02s; φ =0,571r; N=32.

Proceeding from these data, n is determined from the expression for $N=2^n$. In this case, for the output signal $U_{ug\,(t)}$ we get:

$$U_{ug(t)} = (-1)^{(\omega_5 \oplus \omega_4)\beta_1 + (\omega_4 \oplus \omega_3)\beta_2 + (\omega_3 \oplus \omega_2)\beta_3 + (\omega_2 \oplus \omega_1)\beta_4 + (\omega_1 \oplus \omega_0)\beta_5}$$

During the simulation process, the reactive component of the electrical power is determined and, due to the use of the Walsh function of the third order ω=3. In this case, in the binary description of the ω, the bits of the largest unit (LU) and LU-1 are equal to one, i.e. $\omega_5 = \omega_4 = 1$ and in the remaining bits, the ω value is zero, i.e. $\omega_1 = \omega_2 = \omega_3 = 0$.

Taking this into account, we get

$$\omega_5 \oplus \omega_4 = 0$$
$$\omega_4 \oplus \omega_3 = 1$$
$$\omega_3 \oplus \omega_2 = 0$$
$$\omega_2 \oplus \omega_1 = 0$$

The expression for $U_{ug}(t)$

$$U_{ug}(t) = (-1)^{\beta_2} \qquad (11)$$

The parameter β_2 corresponds to the second order Rademacher function. As a result, at the output of the MD2 we obtain discrete values that vary according to a certain law. An error occur during the sign change is clearly demonstrated in Fig.4, f. Let us clear the mathematical basis of this error. The concept of coherent measurements is widely used in digital signal processing. In this case, i.e. during measuring of electric power, is used the given concept. The fulfillment of the coherence condition assumes of the existence of proportionality between the interval and the amount of measurements in a given interval. This proportionality factor must enter to the group of integer, in the opposite case, an error occurs, called the spectral leakage. Thus, the ratio *T/N* must belong to a set of integers. It should be noted that given coherence condition is violated when the period of the analyzed input signal changes.

Even when the coherence condition is met, the error from the sign change exists. Assume that during the period *T* we have got *N* discrete samples (intra period samples). The time step Δt between samples is a constant value and is defined as:

$$\Delta t = \frac{T}{N}$$

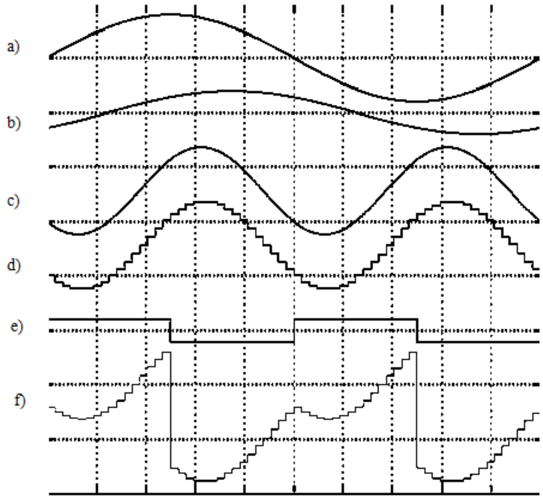

Figure 4. The time diagrams of output's signals of modules of simulation model.

And the moments of the time counts in the interval *[0,T]* vary as $k\Delta t$, where *k=0,1,2..,N-1*. In this case, although the number of samples in the first and second half of the signal period is the same, the number of samples in the first and second quarter of the time interval will be different. This is explained by the fact that the value of the initial reading will be zero for *k=0*. The reason is that at the beginning of the measurement process the signal value is zero (Fig. 4f).

The first measurement (sample), i.e. the value of the first sample at *k=1* corresponds to the time $t_1 = \Delta t$, i.e. end of the first step, and the second sample will correspond to the time $t_2 = 2\Delta t$, etc. To determine the number of samplings corresponding to the first half-period from the equality

$$\frac{2\pi}{N} k = \frac{\pi}{2}$$

we get *k=N/4*.

Indeed, if we take into account this value of *k*, we get:

$$t_{n/4} = \frac{N}{4}\Delta t = \frac{T}{4} = \frac{T}{4}$$

The samples that are symmetrical to the value of *(T/4)N/4* during the first half-period can be designated as follows

$k_{left} = \frac{N}{4} - i$ for samples from the left side and

$k_{right} = \frac{N}{4} + i$ for samples from the right side.

Here $i=1,2,... \frac{N}{4}-1, \frac{N}{4}$. If to take $i\frac{N}{4}$, we get for reading from the initial moment $k_{left} = \frac{N}{4} - \frac{N}{4} = 0$, and $k_{right} = \frac{N}{4} + \frac{N}{4} = \frac{N}{2}$. The values of the initial and *N/2* samples are symmetric relatively to *N/4* are equal to zero. The number of samples not equal to zero and located to the right and left are equal. The number m of samples on each side is determined as follows

$$m = \frac{N}{4} - 1.$$

Thus, if the samples at the point *k=N/4* is not taken into account, *(N/4-1)* of samples that are symmetric to a given point are equal to each other. Since the function itself is symmetric relatively to the perpendicular line passing through the point *k*=*N*/4, the integrals of the left and right parts are equal. That is why when multiplying this sign change function by the discretized values of the input signals (Fig. 4, a), the integral values of the signal at the intervals $[0, T/4)$ and $\left[\frac{T}{q}, \frac{T}{2}\right)$ are different. This difference is exactly equal to the value of the function at the point *k=N/4*. Thus, the error from the sign change effect is formulated and is justified.

Thus, if the signal subjected to digital processing satisfies the conditions of periodicity or quasi-periodicity and at the same time - to condition of the skew-symmetry relatively to the center (middle) of the repetition period, in signal processing on the half-period by dividing it by half of the sampling signal a specific error occurs. The absolute value of this error is equal to the discrete value of the signal on the middle of half-period.

And if the extreme values of the signal during a full period of given signal are discrete values with different signs, then the error due to the effect of changing the sign during the entire period is zero.

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