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Harmonic Minimization in Multilevel Inverters Using Particle Swarm Optimization

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ABSTRACT—Harmonic minimization in multilevel inverters is a complex optimization problem that involves nonlinear transcen- dental equations having multiple local minima. In this paper, a solution to the harmonic minimization problem using a novel par- ticle swarm optimization (PSO) approach based on species-based PSO (SPSO) is presented. The original SPSO is modified, which increased the robustness of algorithm to find global optimum of the search space. The proposed method is able to find the optimum switching angles when their number is increased, while it is not possible to determine them using either conventional iterative tech- niques or resultant theory method. Theoretical results are verified by experiments and simulations for an 11-level H-bridge inverter. Results show that the proposed method effectively minimizes a large number of specific harmonics, and the output voltage results in very low total harmonic distortion and switching frequency.
Index Terms—Multilevel inverter, selective harmonic elimina- tion (SHE), species-based particle swarm optimization (SPSO).

I. Introduction

MULTILEVEL inverters have attracted a great deal of attention in medium-voltage and high-power applica- tions due to their lower switching losses, higher efficiency, and more electromagnetic compatibility than those of conven- tional two-level inverters [1]–[4]. The desired output voltage of these converters is synthesized from several levels of dc volt- ages. Among multilevel inverter structures, topologies based on series-connected H-bridge inverters are particularly attractive due to their modularity and simplicity of control [1], [2].

To control the output voltage and reduce the undesired har- monics, different sinusoidal pulsewidth modulation (PWM) and space-vector PWM schemes are suggested for multilevel invert- ers [5], [6]; however, PWM techniques are not able to eliminate low-order harmonics completely. Another approach is to choose switching angles so that specific lower order dominant harmon- ics are suppressed. This method is known as selective harmonic elimination (SHE) or programmed PWM technique in techni- cal literatures [7]–[10]. A fundamental issue associated with such method is to obtain the arithmetical solution of nonlinear transcendental equations that contain trigonometric terms and naturally present multiple solutions.

Numerical iterative techniques, such as Newton–Raphson method, are applied to solve the SHE problem [2], [7], [8]; how- ever, such techniques need a good initial guess that should be very close to the exact solution. Although the Newton–Raphson method works properly if a good initial guess is available, pro- viding a good guess is very difficult in most cases. This is because the search space of the SHE problem is unknown, and one does not know whether a solution exists or not, and if exists, what is the good initial guess.

A systematic approach to solve the SHE problem based on resultant theory method is proposed in [10]–[13], where tran- scendental equations that describe the SHE problem are con- verted into an equivalent set of polynomial equations, and then, resultant theory method is utilized to find all possible sets of solutions for this equivalent problem. However, as the number of harmonics to be eliminated increases (up to six harmonics for symmetrical equations set [11], [12]), the degrees of the poly- nomial in these equations become so large that solving them using contemporary computer algebra software tools such as Mathematica and Maple is not possible [11].

Another approach to deal with the SHE problem is based on modern stochastic search techniques such as genetic algorithm (GA) and particle swarm optimization (PSO) [13], [14]. However, by increasing the number of switching angles, the complexity of search space increases dramatically, and both the methods trap the local optima of search space. Of course, the exact limitation for the number of switching angles cannot be determined in evolutionary based algorithms; however, it can be said that as the number of switching angles increases, the probability of finding the optimum switching angles decreases.

Recently, a new active harmonic elimination method is also proposed to eliminate higher order harmonics in multilevel inverters. In this method, first, the switching angles to eliminate the lower order harmonics of staircase voltage waveform are calculated, which is called fundamental switching frequency method. Then, residual higher order harmonics are eliminated by additional PWM switching patterns [15]. Unfortunately, this method uses very high frequency switching to eliminate higher order harmonics; also, it needs a very complicated control procedure to generate the gate signals for power switches.

Harmonic elimination based on the modulation-based method by introducing a variable triangle carrier is presented in [16]. However, the technique has not been extended for multilevel converters.

More recently, real-time calculation of switching angles with analytical proof is presented to minimize the total harmonic dis- tortion (THD) of output voltage of multilevel converters [17]. However, the proof is given for staircase voltage waveform, and the equation derived to calculate the switching angles is valid only for fundamental switching frequency method. Also, the presented analytical proof is valid for minimizing all harmonics, including triples, and not only for minimizing nontriple harmonics, which are suitable for three-phase applications.

In order to increase the degrees of freedom (DOFs) and elim- ination of more harmonics without changing the physical hard- ware of inverter, SHE-PWM was proposed [18], [19]. In this method, each active device can be switched more than once per cycle, and more harmonics than in the case of fundamental fre- quency switching method can be eliminated. The main problem associated with the SHE-PWM method is that when the num- ber of switching angles is increased, none of aforementioned methods can be used to calculate the switching angles.

In [20], a general formula to the SHE-PWM problem was presented, and a hybrid GA was employed for harmonic elim- ination of PWM voltage waveform. However, the results are provided only for six and nine switching angles in a quarter of cycle, and the efficiency of the presented algorithm is not ex- amined for larger number of switching angles as this makes the algorithm more complicated.

Obviously, by effectively solving the harmonic elimination problem with large number of switching angles, the SHE-PWM method can generate high-quality voltage waveform as well as less switching frequency as compared to other modulation techniques.

In this paper, an algorithm based on species-based PSO (SPSO) is developed to deal with the problem where the number of switching angles is increased and their determination using conventional iterative methods in addition to GA and simple PSO techniques is not possible. Also, the original SPSO algorithm is modified, which increases the robustness of the algorithm to find the global optimum of search space. Sim- ulation and experimental results are provided for an 11-level cascaded multilevel inverter to show the validity of the proposed method.

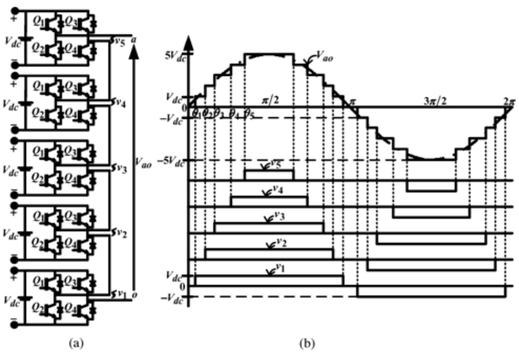

Fig. 1. Structure and the staircase output voltage waveform of a single-phase cascaded H-bridge inverter.

II. Cascaded H-Bridge Multilevel Inverters

Fig. 1(a) shows the structure of a single-phase 11-level cascaded inverter that consists of series connection of H-bridge inverter units. Each unit has its own separated dc source (SDCS), which may be obtained from an ultracapacitor, battery, fuel cell, solar cell, etc. In order to generate the multilevel voltage waveform, the ac terminal voltages of different H-bridge inverters are connected in series, and the overall output voltage v_{an} is given by the sum of H-bridge voltages, i.e., $v_{an} = v_1 + v_2 + v_3 + v_4 + v_5$. Each full bridge inverter can generate three different dc voltage levels of $+V_{dc}$, 0, and $-V_{dc}$ in its ac terminal, which depends on the state of four power switches: Q_1, Q_2, Q_3, and Q_4. The number of levels of output phase voltage m in a cascade multilevel inverter is $m = 2s + 1$, where s is the number of SDCSs. The synthesized single-phase staircase voltage of an 11-level cascade inverter is illustrated in Fig. 1(b). To obtain the three-phase configuration, the outputs of three single-phase cascaded inverters can be connected in Y or Δ shape.

III. Harmonic Minimization Problem in Multilevel Inverters

Commonly, the staircase voltage waveform [as shown in Fig. 1(b)] is chosen for the SHE modulation technique in multilevel converters [3], [4], [10]–[14]. In this method, the number of switching in a quarter of cycle is limited to s; therefore, the number of harmonics that can be eliminated from the output voltage of inverter is limited to $s - 1$ for this method. In order to increase the DOFs and elimination of more harmonics than in the case of fundamental frequency switching method without resorting to the increase of hardware, the SHE-PWM method is proposed in [18], which is also called virtual-stage PWM [19]. This technique for multilevel inverters is one of the powerful theories to generate high-quality voltage waveform with less switching frequency as compared to other PWM methods. The

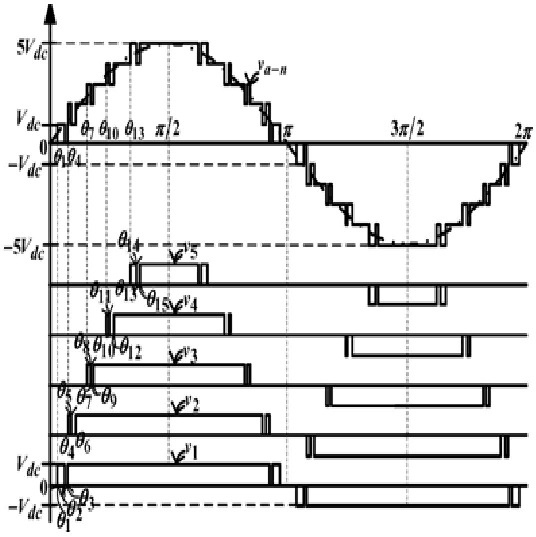

Fig. 2. Output phase voltage of inverter by using the SHE-PWM method.

general formula to eliminate lower order harmonics considering nonequal dc sources is also proposed in [20].

Generally, the number of switching in various levels can be different from each other. However, for simplicity, the number of switching is considered equal in this paper for different levels. As an example, Fig. 2 shows the output voltage of inverter for three times of switching at each level. If the number of switching at each level is denoted by k, switching frequency of the SHE-PWM method will be k times the fundamental frequency. Therefore, the number of harmonics that can be eliminated from the output voltage is equal to $k \times s - 1$. For the 11-level and $k = 3$ example shown in Fig. 2, there are 15 DOF, and 14 undesired harmonics can be eliminated from the output voltage of inverter. It is worth pointing out that the staircase output voltage can be obtained using the SHE-PWM method in the particular case of $k = 1$.

The Fourier expansion for generated voltage waveform using the SHE-PWM method is given by

$$V(\omega t) = \sum_{n=1,3,5,\ldots}^{\infty} \frac{4V_{dc}}{n\pi}\left(\cos(n\theta_1) \pm \cos(n\theta_2) \pm \cos(n\theta_3) \pm \cdots \pm \cos(n\theta_k)\right)\sin(n\omega t). \quad (1)$$

The switching angles θ_1–θ_{ks} must satisfy the following condition:

$$0 \leq \theta_1 \leq \theta_2 \leq \cdots \leq \theta_{ks} \leq \frac{\pi}{2}. \quad (2)$$

In (1), the positive signs indicate the rising edges of the voltage waveform and *vice versa*. The objective here is to choose

lower order harmonics are suppressed, and at the same time, the amplitude of fundamental becomes equal to the desired value. For the three-phase power system applications, elimination of triple harmonics is not necessary because these harmonics are eliminated from the line–line voltage, automatically. By solving the ks transcendental equations set (3) in the unknown θ_1, $\theta_2, \ldots, \theta_{ks}$, it is possible to eliminate the nontriple lower order harmonics up to $3ks$ 2 when ks is odd and up to $3ks$ 1 order harmonic when ks is even, where modulation index M is de-

fined as $M = V_1/sV_{dc}$ and V_1 is the magnitude of fundamental frequency of given voltage

$$\begin{cases} \cos(\theta_1) \pm \cos(\theta_2) \pm \cdots \pm \cos(\theta_{ks-1}) \pm \cos(\theta_{ks}) = (s\pi/4)M \\ \cos(5\theta_1) \pm \cos(5\theta_2) \pm \cdots \pm \cos(5\theta_{ks-1}) \pm \cos(5\theta_{ks}) = 0 \\ \cos(7\theta_1) \pm \cos(7\theta_2) \pm \cdots \pm \cos(7\theta_{ks-1}) \pm \cos(7\theta_{ks}) = 0 \\ \vdots \\ \cos((3ks-2)\theta_1) \pm \cos((3ks-2)\theta_2) \pm \cdots \\ \quad \pm \cos((3ks-2)\theta_{ks}) = 0. \end{cases} \quad (3)$$

IV. Particle Swarm Optimization

A. Basic PSO

Kennedy and Eberhart first introduced PSO in 1995 as a new heuristic method [21]. Basically, the PSO was inspired by the sociological behavior associated with swarms such as flock of birds and fish schooling. The individuals in the population are called particles. Each particle is a potential solution for the optimization problem and tries to search the best position through flying in a multidimensional space. The sociological behavior that is modeled in the PSO system is used to guide the swarm, thereby probing the most promising areas of search space. Each particle is determined by two vectors in D-dimensional search space; the position vector $X_i = [x_{i1}, x_{i2}, \ldots, x_{iD}]$ and the velocity vector $V_i = [v_{i1}, v_{i2}, \ldots, v_{iD}]$. Each particle in the swarm refines its search through its present velocity, previous experience, and the experience of the neighboring particles. The best position of particle i founded so far is called personal best and is denoted by $P_i = [p_{i1}, p_{i2}, \ldots, p_{iD}]$, and the best position in entire swarm is called global best and denoted by $P_g = [p_{g1}, p_{g2}, \ldots, p_{gD}]$. At first, the velocity of the ith particle on the dth dimension is updated by using (4), and then, (5) is used to modify the position of that particle

$$v_{id}(t+1) = \chi[v_{id}(t) + \phi_1 r_1 (p_{id} - x_{id}(t)) + \phi_2 r_2 (p_{gd} - x_{id}(t))] \quad (4)$$

$$x_{id}(t+1) = x_{id}(t) + v_{id}(t+1) \quad (5)$$

where ϕ_1 and ϕ_2 are the cogitative and social parameters, respectively. In these equations, r_1 and r_2 are random values uniformly distributed within [0, 1].

Moreover, in order to guarantee the convergence of the PSO algorithm and improve the convergence characteristic, the constriction factor χ is proposed and defined as [22], [23]

$$\chi = \frac{2k}{2 - \phi - \sqrt{\phi^2 - 4\phi}}, \quad \phi = \phi_1 + \quad 4 \quad (6)$$

Two general approaches to define the neighborhood and determine P_g are known as *gbest* and *lbest* versions of PSO. In the *gbest* version, the motion of each particle is influenced by the best-fit particle in the entire swarm population; while on the contrary, in the *lbest* version, each particle is attracted to the best-fit particle in its neighborhood. How the best-fit particle is determined as a leader of subpopulation has a significant effect on the performance of the *lbest* type of PSO algorithm. Recently, a novel method that uses the notion of species has been applied to determine the best-fit particles and their neighborhoods [24], which is called species-based PSO (SPSO), and has better performance and less calculation time than the well-known *k*-means-based clustering algorithm to determine the subpopulations, which was proposed by Kennedy [25].

B. Species-Based PSO

In the notion of species, the population is classified into groups according to their similarity measured by Euclidean distance. The dominating particle in each species is called species seed and attracts all particles that are located in the r_s distance of itself.

The algorithm to determine species seed from the population is performed at each iteration, and can be described as follows.

Step 1: Set the species seed set S to Ø.

Step 2: Generate a list L_{sorted} of all particles in decreasing fitness values (i.e., from the best-fit to worst-fit).

Step 3: Place the first particle of L_{sorted} into the set S as the newly identified seed s_i.

Step 4: Find all particles in the L_{sorted} that are located in the r_s Euclidean distance of s_i and save them as the members of *i*th species.

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Step 5: Remove all particles that belong to the *i*th species from L_{sorted}.

Step 6: Repeat step 2, till no particle is left in the L_{sorted}.

After this, once the species and species seeds have been determined from the aforementioned procedure, each species seed is set as the *lbest* of all those particles that belong to the same species at each iteration. Neutral particles in any species always follow a pure PSO position and velocity update rules. Moreover, any additional particles that have converged on the same local optima in one of the species are considered as redundant and are replaced with the new randomly generated particles in the search space. This increases the ability of the algorithm to better explore the other parts of search space [24]. Taking the earlier descriptions into consideration, the SPSO algorithm can be summarized as follows.

Step 1: Generate an initial population with randomly generated particles.

Step 2: Evaluate all particle individuals in the population.

Step 3: Sort all particles in descending order of their fitness values (i.e., from the best-fit to least-fit ones).

Step 4: Determine the species seeds for the current population (as described previously).

Step 5: Assign each species seed identified as the *lbest* to all individuals identified in the same species.

Step 6: Replace redundant particles in species.

Step 7: Adjust particle positions according to (4) and (5).

Step 8: Go back to step 2, until the termination condition is met.

C. Proposed Adaptive Adjustment of Niche Radius

The number of generated species at each iteration depends on the parameter r_s, which denotes the radius measured in Eu-

TABLE I

PSEUDOCODE OF THE ADAPTIVE ADJUSTMENT OF NICHE RADIUS r_s

Input: r_s, n_s, n_s^* and vr_s
If $n_s < n_s^*$ Then
$\quad r_s = r_s - vr_s$
Else If $n_s > n_s^*$ Then
$\quad r_s = r_s + vr_s$
End If

clidean distance from the center of a species (i.e., species seed) to its boundary. If r_s is small, the large number of isolated species with small species size could be formed, which enables the algorithm to find more local optima but with low-quality solutions because of low number of particles and *vice versa*. Because of very high complexity of the SHE-PWM problem, finding a global optima will be sufficient. The original SPSO is modified to increase the ability of the algorithm to find the global optima effectively. This is done by adaptive adjustment of niche radius r_s so that the desired number of generated species n_s^* decreases over the course of search, and finally, reaches to one species (entire swarm) at the end of the search procedure.

For this reason, it is proposed that the desired number of species n_s^* must decrease linearly according to (7). Then, r_s varies adaptively at each iteration step, so that the number of generated species n_s at the current iteration follows the desired number of species n_s^*

$$n_s^* = n_{s1}^* - (n_{s1}^* - n_{s2}^*) \frac{iter}{iter_{max}} \tag{7}$$

where n_{s1}^* and n_{s2}^* are the initial and final values of the desired number of species, respectively, $iter_{max}$ is the maximum number of iterations, and $iter$ is the current number of iterations. The values of n_{s1}^* and n_{s2}^* are initialized at the first step of the algorithm. By setting n_{s2}^* equal to zero, only one species at the last stage of the searching procedure will be formed, which includes the entire swarm. The pseudocode of Table I is incorporated in the original SPSO algorithm and executed after the new species is determined. The constant parameter vr_s indicates the variation of r_s at each iteration step. If the number of generated species at current iteration is less than n_s^*, the niche radius r_s decreases such that the number of generated species at the next generation follows the desired number of species n_s^* and *vice versa*. If the value of vr_s is set too large, the optimization process will become numerically unstable. Also, by setting vr_s to very small values, n_s will not be able to follow n_s^* properly. In this paper, vr_s is chosen to be as follows:

$$vr_s = \frac{x_{max}}{iter_{max}} \tag{8}$$

where x_{max} denotes the maximum domain of search space. This modification is applied to the original SPSO algorithm and is called modified SPSO (MSPSO) in this paper.

V. IMPLEMENTATION OF THE MSPSO TECHNIQUE TO HARMONIC MINIMIZATION PROBLEM

The harmonic minimization problem in multilevel inverters is determination of the switching angles of inverter so that the specified lower order harmonics are suppressed. Therefore, the cost function is related to the fundamental and undesired harmonics. This problem can be categorized into one of the following cases.

A. Elimination of Lower Order Harmonics

The switching angles θ_1, θ_2, …, θ_{ks} in a multilevel inverter for the output waveform can be calculated in such a way that the equation set (4) is satisfied, and under these circumstances, odd and nontriple low-order harmonics up to $3ks\_2$, while ks is odd, and up to $3ks\_1$ order, when ks is even, can be eliminated from the output phase voltage of inverter. In this case, the number of harmonics that are chosen to minimize is equal to the number of switching in a quarter of cycle. The cost function to approach this aim can be defined as (9). This equation can also be used for the fundamental frequency switching scheme by setting $k = 1$.

B. Minimization of THD

Another approach to reduce the harmonics content is to minimize the THD for the output phase voltage of inverter. In this paper, up to 50 harmonics are considered to calculate the THD. Therefore, the cost function to minimize the THD can be expressed as (10); obviously, this equation can also be used for the fundamental frequency switching scheme by substituting $k = 1$.

The objective is to minimize

$$f(\theta_1, \theta_2, \ldots, \theta_{ks})$$

subject to

$$0 \le \theta_1 \le \theta_2 \le \cdots \le \theta_{ks} \le \frac{\pi}{2}$$

where s is the number of dc sources and k is the ratio of switching frequency to fundamental frequency; therefore, the number of switching in a quarter of cycle is equal to $k \times s$. Let $\theta_i = [\theta_{i1}, \theta_{i2}, \ldots, \theta_{iks}]$ be a trial vector representing the ith particle of the swarm to be evolved. The elements of θ_i are the solutions of harmonic minimization problem and its dth element corresponds to the dth switching angle of inverter. The fitness value of each particle is evaluated by means of the aforementioned objective function $f(\theta_1, \theta_2, \ldots, \theta_{ks})$.

The step-by-step procedure of the MSPSO method for harmonic minimization problem is as follows.

1) *Get the data for the system:* At the first step, the parameters required for the algorithm such as population size M, maximum iteration number $iter_{max}$, initial value of niche radius n^*_{s1}, n^*_{s2}, etc., are determined, and iteration counter is set to $iter = 1$.
2) *Generation of initial conditions of each particle:* Each particle in the population is randomly initialized between 0 and $\pi/2$; similarly, velocity vector of each particle has to be generated randomly within $-V_{max}$ and V_{max}.
3) Evaluation of particles: Each particle is evaluated using the fitness function of the harmonic minimization problem to minimize the cost function given by (9) or (10).
4) Determination of species and species seeds: The cur- rent population is divided into species, as described in Section IV, and after this, the species seed of each species is assigned as lbest of all particles that belong to this species.
5) Update the personal best position of particles: If the cur- rent position of the ith particle is better than its previous personal position best Pi, replace the Pi with current posi- tion Xi. Note that the lbest of each particle is updated in the previous step.
6) Adjustment of niche radius rs: Desired number of species is updated by (7); afterward, rs is adaptively adjusted by using the procedure described in Table I so that the number of current species ns follows n∗s .
7) Update the velocity and position vectors: Particles in any species always follow a pure PSO position and velocity update rules.
8) Termination criteria: If the iteration counter iter reaches to itermax , stop; else increase the iteration counter iter = iter + 1 and go back to step 3:

$$f(\theta_1, \theta_2, \ldots, \theta_{ks}) = 100 \times M - \frac{|V_1|}{sV_{dc}} + \frac{|V_5| + |V_7| + \cdots + |V_{3ks-2 \text{ or } 3ks-1}|}{sV_{dc}} \quad (9)$$

$$f(\theta_1, \theta_2, \ldots, \theta_{ks}) = 100 \times M - \frac{|V_1|}{sV_{dc}} + \frac{|V_5| + |V_7| + |V_{11}| + \cdots + |V_{49}|}{sV_{dc}} \quad (10)$$

VI. Computational Results

In [14], the PSO algorithm is successfully applied to calculate the optimum switching angles in multilevel converters. However, by increasing the number of switching angles, the algorithm traps the local minima of search space.

In this section, the proposed approach is applied to calculate the optimum switching angles of the SHE-PWM technique, where other conventional methods such as the iterative techniques as well as the resultant theory approach are not able to determine these angles. Moreover, the results are compared with the new active harmonic elimination technique with respect to switching frequency and the quality of output voltage waveform. Calculation of the switching angles by the proposed method is accomplished in the MATLAB programming environment. The switching frequency is considered threefold of the fundamental ($k = 3$). The objective here is to minimize the THD of generated output voltage waveform by determining the switching angles $\theta_1, \theta_2, \ldots, \theta_{15}$. The output voltage waveform for the case of $k = 3$ and $s = 5$ is shown in Fig. 2. Therefore, the objective function f according to (10) is as follows:

$$f(\theta_1, \theta_2, \ldots, \theta_{15}) = 100 \times M - \frac{|V_1|}{5V_{dc}} + \frac{|V_5| + |V_7| + |V_{11}| + \cdots + |V_{49}|}{5V_{dc}} \tag{11}$$

By minimizing the earlier objective function for a given modulation index M, the magnitude of fundamental is maintained at the specified value, and the THD of output voltage, which is calculated up to 50 orders for odd and nontriple harmonics, is minimized. The residual higher order harmonics occur at much higher frequencies. Thus, filtering is much easier and less expensive. Also, the generated harmonics might be above the bandwidth of some actual systems, which means that there is no power dissipation due to these harmonics.

As an example, the switching angles for $M = 1$ by using the proposed method is calculated as follows:

$$\theta_1 = 6.19^\circ, \theta_2 = 11.56^\circ, \theta_3 = 13.89^\circ, \theta_4 = 14.06^\circ$$
$$\theta_5 = 16.48^\circ, \theta_6 = 19.77^\circ, \theta_7 = 28.40^\circ, \theta_8 = 31.07^\circ$$
$$\theta_9$$
$$\theta_{13} = 61.10^\circ, \theta_{14} = 65.49^\circ, \theta_{15} = 68.08^\circ. \tag{12}$$

Resulting THD for the generated phase voltage using (13) is 0.2%, while the number of switching in the active harmonic elimination method [15] to eliminate harmonics up to 31st order is 150, which is tenfold of the proposed method. Also, the resulting THD in [15] for this modulation index is 2.17%, which is more than tenfold of the proposed method

$$\mathrm{THD} = \frac{\sqrt{\sum_{n=5,7,11,\ldots}^{49} V_n^2}}{V_1}. \tag{13}$$

The switching angles versus the modulation index M are calculated with step size of 0.005 and is shown in Fig. 3. Also, comparison of the THD values determined from the fundamen-

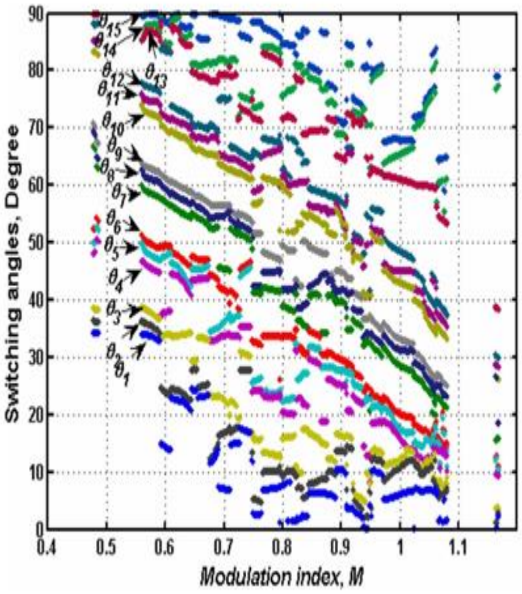


Fig. 3. Switching angles $\theta_1, \theta_2, \ldots, \theta_{15}$ versus modulation index M.

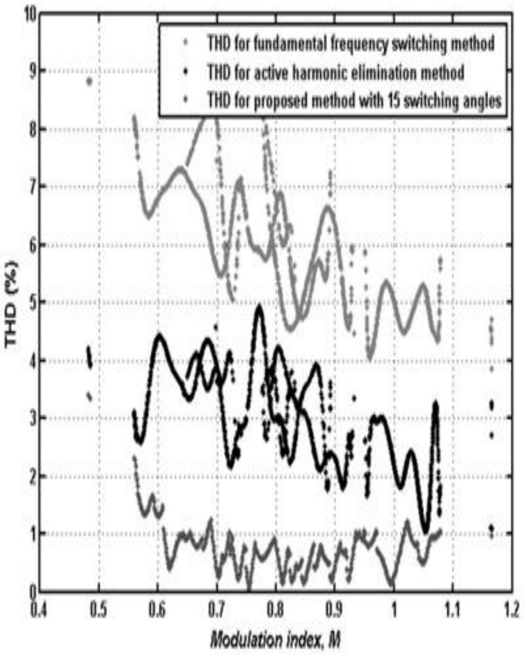


Fig. 4. Comparison of THD values for fundamental switching frequency method, active harmonic elimination method, and the proposed method with 15 switching angles versus M.

TABLE II
PARAMETER USED IN THE IMPLEMENTATION OF MSPSO

Parameter	value
n_{s1}^{*}	50
n_{s2}^{*}	0
$iter_{\max}$	1000
Population size	300
$V_{\max}$	.02
vr_s	0.02

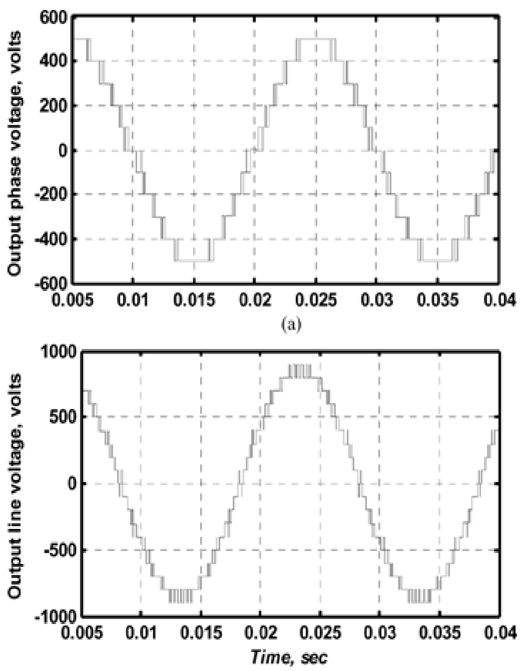
tal switching frequency scheme, active harmonic elimination method, and the proposed method with $k = 3$ is shown in Fig. 4. The THD values calculated by the proposed method are much less than those calculated by the fundamental frequency switching and active harmonic elimination methods in all ranges of the modulation indexes. Results show that the proposed algorithm effectively minimizes the undesired harmonics and the output voltage results in lower THD and lower switching frequency. The parameters of the MSPSO algorithm used in the implementation are listed in Table II.

VII. SIMULATION RESULTS

To validate the computational results for switching angles, a simulation is carried out in MATLAB/SIMULINK software tool for an 11-level cascaded H-bridge inverter.

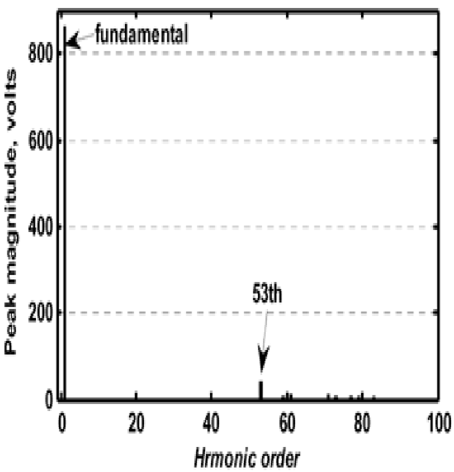
The voltage of dc link for each H-bridge units is considered to be 100 V, and simulation is done for $k = 3$ and $M = 1$, whose optimum switching angles is given in (12).

Output phase voltage, line–line voltage waveform, and frequency spectra of line–line voltage are shown in Fig. 5(a)–(c), respectively. From the frequency spectra of line–line voltage shown in Fig. 5(c), it can be seen that the magnitudes of lower order harmonics, up to 50th order, are negligible.



(a)

(b)



(c)

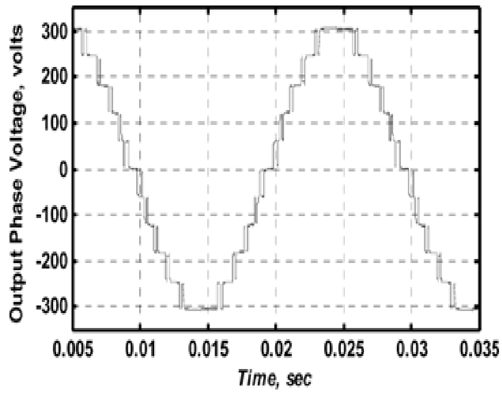
Fig. 5. Simulation results for 11-level inverter, $k = 3$, and $M = 1$. (a) Output phase voltage. (b) Output line–line voltage. (c) FFT analysis for line–line voltage.

Fig. 6. Prototype single-phase 11-level cascaded H-bridge inverter.

VIII. Experimental Results

To validate the simulation and theoretical results, a low-power, single-phase, 11-level cascaded inverter prototype is constructed. The circuit configuration in the experimental circuit is the same as shown in Fig. 1(a). The inverter uses 30 A, 200 V MOSFETs as the switching devices, and the dc-link voltage for each H-bridge is 62 V. Each H-bridge unit uses 4700 μF and 80 V capacitor in its dc bus. The gate control signals are generated by a dedicated unit, which is implemented on Spartan II series of Xilinx's field-programmable gate array (FPGA). Furthermore, an Atmel 8-bit AVR RISC microcontroller (ATmega16 L) is considered to interface with the operator and provide the switching times for FPGA. Fig. 6 shows the gate driver board and implemented prototype.

The first experiment was to minimize the THD of the output voltage waveform defined as (10), using the SHE-PWM



(a)

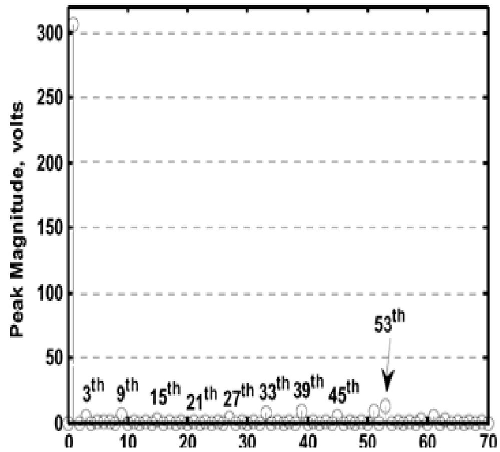


Fig. 7. Experimental results, $k = 3$ and $M = 1$. (a) Experimental output phase voltage. (b) Corresponding FFT analysis.

method and $k = 3$ for modulation index $M = 1$. Experimental output phase voltage waveform and the corresponding fast Fourier transform (FFT) analysis are shown in Fig. 7(a) and (b), respectively.

Another experiment was carried out for $k = 3$ and lower modulation index. Fig. 8 shows the results for $M = 0.634$. The output phase voltage and corresponding FFT analysis are shown in Fig. 8(a) and (b), respectively.

As seen from the FFT analysis for both experiments, all harmonics up to 50th order have been minimized, and the output voltage results in very low THD. Experimental results also validate the computational and simulation results.

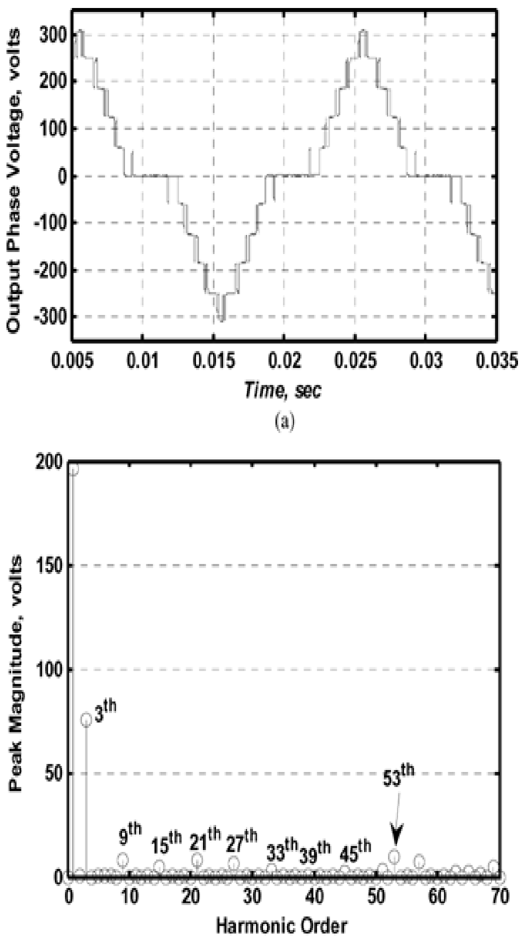


Fig. 8. Experimental results, k = 3 and M = 0.634. (a) Experimental output phase voltage. (b) Corresponding FFT analysis

IX. CONCLUSION

An MSPSO algorithm with adaptive adjustment of niche ra- dius has been proposed to determine the optimum switching angles of multilevel inverters. This algorithm has been suc- cessfully applied to the SHE-PWM problem that involves large number of switching angles, where other conventional methods are not able to solve it. Simulation and experimental results are provided for an 11-level cascaded H-bridge inverter to val- idate the accuracy of computational results. Results show that all undesired harmonics up to 50th order have been effectively minimized at the output voltage waveform of inverter. Compari- son of results with active harmonic elimination technique shows that the THD and the switching frequency of output voltage de- creased dramatically.

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