

On Z_{\square} -Open Sets and Decompositions of Continuity

A. M. Mubarki¹, M. A. Al-Juhani², M. M. Al-Rshudi³, M. O. AL-Jabri⁴

¹Department of Mathematics, Faculty of Science, Taif University,

^{2,3,4} Department of Mathematics, Faculty of Science, Taibah University Saudi Arabia

ABSTRACT

In this paper, we introduce and study the notion of Z_{α} -open sets and some properties of this class of sets are investigated. Also, we introduce the class of A^*L -sets via Z_{α} -open sets. Further, by using these sets, a new decompositions of continuous functions are presented.

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I. INTRODUCTION

J. Tong [21] introduced the notion of B-set and B-continuity in topological spaces. The concept of A^* -sets, DS -set, A^* -continuity, DS-continuity introduced by E. Ekici [4, 8] and used them to obtain a new decomposition of continuity. Noiri et.al [17] introduced the notion of η -set and η -continuity in topological spaces. The main purpose of this paper is to obtain a new decompositions of continuous functions. We introduce and study the notion of Z_{α} -open sets and A^*L -sets. The relationships among of Z_{α} -open sets, A^*L -sets and the related sets are investigated. By using these notions, we obtain a new decompositions of continuous functions. Also, some characterizations of these notions are presented.

II. PRELIMINARIES

A subset A of a topological space (X, τ) is called regular open (resp. regular closed) [20] if $A = \text{int}(\text{cl}(A))$ (resp. $A = \text{cl}(\text{int}(A))$). The δ -interior [22] of a subset A of X is the union of all regular open sets of X contained in A and is denoted by $\delta\text{-int}(A)$. A subset A of a space X is called δ -open [22] if it is the union of regular open sets. The complement of a δ -open set is called δ -closed. Alternatively, a set A of (X, τ) is called δ -closed [22] if $A = \delta\text{-cl}(A)$, where $\delta\text{-cl}(A) = \{x \in X: A \cap \text{int}(\text{cl}(U)) \neq \emptyset, U \in \tau \text{ and } x \in U\}$. Throughout this paper (X, τ) and (Y, σ) (simply, X and Y) represent non-empty topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space (X, τ) , $\text{cl}(A)$, $\text{int}(A)$ and $X \setminus A$ denote the closure of A , the interior of A and the complement of A respectively. A space X is called submaximal [3] if every dense subset of X is open. A space (X, τ) is called extremally disconnected (briefly, E. D.) [19] if the closure of every open set of X is open. A subset A of a space X is called δ -dense [6] if and only if $\delta\text{-cl}(A) = X$. A subset A of a space X is called a -open [4] (resp. α -open [16], preopen

[13], δ -semiopen [18], semiopen [12], Z -open [11], b -open [1] or γ -open [10] or sp -open [3], e -open [5]) if $A \subseteq \text{int}(\text{cl}(\delta\text{-int}(A)))$ (resp. $A \subseteq \text{int}(\text{cl}(\text{int}(A)))$, $A \subseteq \text{int}(\text{cl}(A))$, $A \subseteq \text{cl}(\delta\text{-int}(A))$, $A \subseteq \text{cl}(\text{int}(A))$, $A \subseteq \text{cl}(\delta\text{-int}(A)) \cup \text{int}(\text{cl}(A))$, $A \subseteq \text{int}(\text{cl}(A)) \cup \text{cl}(\text{int}(A))$, $A \subseteq \text{cl}(\delta\text{-int}(A)) \cup \text{int}(\delta\text{-cl}(A))$). The complement of a -open (resp. α -open, preopen, δ -semiopen, semiopen) sets is called a -closed [4] (resp. α -closed [16], pre-closed [13], δ -semi-closed [18], semi-closed [2]). The intersection of all a -closed (resp. α -closed, pre-closed, δ -semi-closed, semi-closed) sets containing A is called the a -closure (resp. α -closure, pre-closure, δ -semi-closure, semi-closure) of A and is denoted by $a\text{-cl}(A)$ (resp. $\alpha\text{-cl}(A)$, $\text{pcl}(A)$, $\delta\text{-scl}(A)$, $\text{scl}(A)$). The union of all a -open (resp. α -open, preopen, δ -semiopen, semiopen) sets contained in A is called the a -interior (resp. α -interior, pre-interior, δ -semi-interior, semi-interior) of A and is denoted by $a\text{-int}(A)$ (resp. $\alpha\text{-int}(A)$, $\text{pint}(A)$, $\delta\text{-sint}(A)$, $\text{sint}(A)$). The family of all δ -open (resp. a -open, α -open, preopen, δ -semiopen, semiopen) is denoted by $\delta O(X)$ (resp. $aO(X)$, $\alpha O(X)$, $PO(X)$, $\delta SO(X)$, $SO(X)$).

Lemma 2.1. Let A, B be two subset of (X, τ) . Then the following are hold:

- (1) $\alpha\text{-cl}(A) = A \cup \text{cl}(\text{int}(\text{cl}(A)))$ and $\alpha\text{-int}(A) = A \cap \text{int}(\text{cl}(\text{int}(A)))$ [1],
- (2) $\delta\text{-scl}(A) = A \cup \text{int}(\delta\text{-cl}(A))$ and $\delta\text{-sint}(A) = A \cap \text{cl}(\delta\text{-int}(A))$ [17],
- (3) $\text{pcl}(A) = A \cup \text{cl}(\text{int}(A))$ and $\text{pint}(A) = A \cap \text{int}(\text{cl}(A))$ [1].

Definition 2.1. A subset A of a space (X, τ) is called:

- (1) a A^* -set [4] if $A = U \cap V$, where U is open and V is a -closed,
- (2) a DS-set [8] if $A = U \cap V$, where U is open and V is δ -semi-closed,
- (3) a B-set [21] if $A = U \cap V$, where U is open and V is semi-closed,
- (4) a η -set [17] if $A = U \cap V$, where U is open and

V is α -closed,
 (5) a δ^* -set [7] if δ -int(A) is δ -closed.

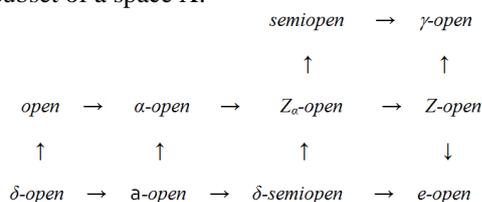
III. Z_α -OPEN SETS

Definition 3.1. A subset A of a topological space (X, τ) is called

- (1) Z_α -open if $A \subseteq \text{int}(\text{cl}(\text{int}(A))) \cup \text{cl}(\delta\text{-int}(A))$,
- (2) Z_α -closed if $\text{cl}(\text{int}(\text{cl}(A))) \cap \text{int}(\delta\text{-cl}(A)) \subseteq A$.

The family of all Z_α -open (resp. Z_α -closed) subsets of a space (X, τ) will be as always denoted by $Z_\alpha O(X)$ (resp. $Z_\alpha C(X)$).

Remark 2.1. The following diagram holds for a subset of a space X:



The converse of the above implications need not necessary be true as shown by [1, 3, 4, 5, 10, 11, 16, 18] and the following examples.

Example 3.1. Let $X = \{a, b, c, d, e\}$ with topology $\tau = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}, \{a, b, c, d\}, X\}$. Then:

- (1) A subset $\{a, b, e\}$ of X is Z_α -open but it is not δ -semiopen and it is not α -open,
- (2) A subset $\{b, e\}$ of X is semiopen but it is not Z_α -open.

Example 3.2. Let $X = \{a, b, c, d, e\}$ with topology $\tau = \{\emptyset, \{a, b\}, \{c, d\}, \{a, b, c, d\}, X\}$. Then the subset $\{a, b, c\}$ is a Z-open set but it is not Z_α -open.

Theorem 3.1. Let (X, τ) be a topological space. Then a Z_α -open set A of X is α -open if one of the following conditions are hold:

- (1) (X, τ) is E.D.,
- (2) A is δ^* -set of X,
- (3) $X \setminus A$ is δ -dense of X.

Proof. (1) Since, $A \in Z_\alpha O(X)$ and X is E.D., then $A \subseteq \text{int}(\text{cl}(\text{int}(A))) \cup \text{cl}(\delta\text{-int}(A)) \subseteq \text{int}(\text{cl}(\text{int}(A))) \cup \text{int}(\text{cl}(\delta\text{-int}(A))) = \text{int}(\text{cl}(\text{int}(A)))$ and therefore $A \in \alpha O(X, \tau)$,

(2) Let A be a δ^* -set and Z_α -open. Then $A \subseteq \text{int}(\text{cl}(\text{int}(A))) \cup \text{cl}(\delta\text{-int}(A)) = \text{int}(\text{cl}(\text{int}(A))) \cup \delta\text{-int}(A) = \text{int}(\text{cl}(\text{int}(A)))$. Therefore A is α -open,

(3) Let $A \in Z_\alpha O(X)$ and $X \setminus A$ be a δ -dense set of X. Then $\delta\text{-int}(A) = \emptyset$ and hence $A \subseteq \text{int}(\text{cl}(\text{int}(A)))$. Therefore A is α -open.

Lemma 3.1. Let (X, τ) be a topological space. Then the following statements are hold.

- (1) The union of arbitrary Z_α -open sets is Z_α -open,
- (2) The intersection of arbitrary Z_α -closed sets is Z_α -closed.

Remark 3.2. By the following example we show that

the intersection of any two Z_α -open sets is not Z_α -open.

Example 3.3. Let $X = \{a, b, c\}$ with topology $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$. Then $A = \{a, c\}$ and $B = \{b, c\}$ are Z_α -open sets. But $A \cap B = \{c\}$ is not Z_α -open.

Definition 3.2. Let (X, τ) be a topological space. Then :

(1) The union of all Z_α -open sets of X contained in A is called the Z_α -interior of A and is denoted by $Z_\alpha\text{-int}(A)$,

(2) The intersection of all Z_α -closed sets of X containing A is called the Z_α -closure of A and is denoted by $Z_\alpha\text{-cl}(A)$.

Theorem 3.2. Let A be subset of a topological space (X, τ). Then the following are statements are equivalent:

- (1) A is Z_α -open set,
- (2) $A = Z_\alpha\text{-int}(A)$,
- (3) $A = \alpha\text{-int}(A) \cup \delta\text{-sint}(A)$.

Proof. (1) \leftrightarrow (2). Obvious,

(1) \rightarrow (2). Let A be a Z_α -open set. Then $A \subseteq \text{int}(\text{cl}(\text{int}(A))) \cup \text{cl}(\delta\text{-int}(A))$. BY Lemma 2.1, $\alpha\text{-int}(A) \cup \delta\text{-sint}(A) = (A \cap \text{int}(\text{cl}(\text{int}(A)))) \cup (A \cap \text{cl}(\delta\text{-int}(A))) = A \cap (\text{int}(\text{cl}(\text{int}(A))) \cup \text{cl}(\delta\text{-int}(A))) = A$,

(2) \rightarrow (1). Let $A = \alpha\text{-int}(A) \cup \delta\text{-sint}(A)$. Then by Lemma 2.1, we have

$A = (A \cap \text{int}(\text{cl}(\text{int}(A)))) \cup (A \cap \text{cl}(\delta\text{-int}(A))) \subseteq \text{int}(\text{cl}(\text{int}(A))) \cup \text{cl}(\delta\text{-int}(A))$. Therefore A is Z_α -open set.

Theorem 3.2. Let A be subset of a topological space (X, τ). Then the following are statements are equivalent:

- (1) A is a Z_α -closed,
- (2) $A = Z_\alpha\text{-cl}(A)$,
- (3) $A = \alpha\text{-cl}(A) \cap \delta\text{-scl}(A)$.

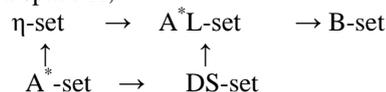
Proof. It is clear.

IV. A^*L -SETS

Definition 4.1. A subset A of a space (X, τ) is said to be an A^*L -set if there exist an open set U and an Z_α -closed set V such that $A = U \cap V$.

The family of A^*L -sets of X is denoted by $A^*L(X)$.

Remark 4.1. (1) The following diagram holds for a subset A of a space X,



(2) Every open set and every Z_α -closed set is A^*L -set,
 (3) None of the above implications is reversible as shown by [4, 7, 16] and the following examples.

Example 4.1. Let $X = \{a, b, c, d, e\}$ with topology $\tau = \{\emptyset, \{a\}, \{c\}, \{a, b\}, \{a, c\}, \{a, b, c\}, X\}$. Then the set $\{b, c, e\}$ is an B-set but it is not an A^*L -set. Also, the set $\{b, e\}$ it is an A^*L -set but it is not DS-set and it is not open. Further, the set $\{a\}$ is A^*L -set but not Z_α -closed.

Example 4.2. Let $X = \{a, b, c, d\}$ with topology $\tau =$

$\{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$. Then the set $\{b, c\}$ is an A^*L -set but not an η -set.

Theorem 4.1. Let A be a subset of a space (X, τ) . Then $A \in A^*L(X)$ if and only if $A = U \cap Z_\alpha\text{-cl}(A)$, for some open set U .

Proof. Let $A \in A^*L(X)$. Then $A = U \cap V$, where U is open and V is Z_α -closed. Since $A \subseteq V$, then $Z_\alpha\text{-cl}(A) \subseteq Z_\alpha\text{-cl}(V) = V$. Thus $U \cap Z_\alpha\text{-cl}(A) \subseteq U \cap V = A \subseteq U \cap Z_\alpha\text{-cl}(A)$. Therefore, $A = U \cap Z_\alpha\text{-cl}(A)$.

Conversely. Since $A = U \cap Z_\alpha\text{-cl}(A)$, for some open set U and $Z_\alpha\text{-cl}(A)$ is Z_α -closed, then by Definition 4.1, A is A^*L -set.

Lemma 4.1 [12]. Let A be a subset of a space (X, τ) . Then, A is semi-closed if and only if $\text{int}(A) = \text{int}(\text{cl}(A))$.

Theorem 4.2. Let X be a topological space and $A \subseteq X$. If $A \in A^*L(X)$, then $\text{pint}(A) = \text{int}(A)$.

Proof. Let $A \in A^*L(X)$. Then, $A = U \cap V$, where U is open and V is Z_α -closed. Since V is Z_α -closed, then V is semi-closed. Hence by Lemmas 2.1, 4.1, we have $\text{pint}(A) = A \cap \text{int}(\text{cl}(A)) \subseteq U \cap \text{int}(\text{cl}(V)) = U \cap \text{int}(V) = \text{int}(A)$. Thus, $\text{pint}(A) = \text{int}(A)$.

Theorem 4.3. Let A be a subset of a space (X, τ) . Then the following are equivalent:

- (1) A is open,
- (2) A is α -open and A^*L -set,
- (3) A is preopen and A^*L -set.

Proof. (1) \rightarrow (2) and (2) \rightarrow (3) Obvious,

(3) \rightarrow (1). Let A be a preopen set and A^*L -set. Then by Theorem 4.2, we have $\text{pint}(A) = \text{int}(A)$. But, A is preopen, then $A = \text{pint}(A) = \text{int}(A)$. Thus A is open.

Theorem 4.4. For an extremally disconnected space X . The following are equivalent:

- (1) A is open,
- (2) A is Z_α -open and A^*L -set,
- (3) A is preopen and A^*L -set.

Proof. It follows directly from Theorems 3.1, 4.3.

Theorem 4.5. Let (X, τ) be a topological space. Then the following are equivalent:

- (1) X is submaximal,
- (2) Every dense subset of X is an A^*L -set.

Proof. (1) \rightarrow (2). Let X be a submaximal space. Then every dense subset of X is an open sets, so is an A^*L -set.

(2) \rightarrow (1). It is known that every dense set is preopen. Also, by hypothesis, every dense is A^*L -set. So, by Theorem 4. 3, it is open. Therefore, X is submaximal.

Theorem 4.6. Let X be a topological space. Then the following are equivalent:

- (1) X is indiscrete,
- (2) The A^*L -set of X are only trivial ones.

Proof. (1) \rightarrow (2). Let A be an A^*L -set of X . Then there exists an open set U and an Z_α -closed set V such that $A = U \cap V$. If $A \neq \emptyset$, then $U \neq \emptyset$. We obtain $U = X$ and $A = V$. Hence $X = Z_\alpha\text{-cl}(A) \subseteq A$ and $A = X$,

(2) \rightarrow (1). Every open set is an A^*L -set. So, open sets in X are only the trivial ones. Hence, X is indiscrete.

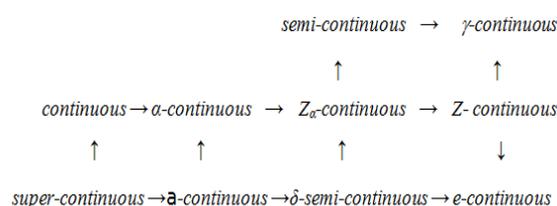
V. DECOMPOSITIONS OF CONTINUOUS FUNCTIONS

Definition 5.1. A function $f:(X, \tau) \rightarrow (Y, \sigma)$ is said to be Z_α -continuous if

$f^{-1}(V)$ is Z_α -open in X , for every $V \in \sigma$.

Definition 4.2. A function $f:(X, \tau) \rightarrow (Y, \sigma)$ is called super-continuous [15] (resp. α -continuous [4], α -continuous [14], pre-continuous [13], δ -semi-continuous [9], semi-continuous [12], γ -continuous [10], e -continuous [5], Z -continuous [11]) if $f^{-1}(V)$ is δ -open (resp. α -open, α -open, preopen, δ -semiopen, semiopen, γ -open, e -open, Z -open) of X , for each $V \in \sigma$.

Remark 5.1. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a function. Then The following diagram is hold:



The implications of the above diagram are not reversible as shown by [4, 9, 10, 11, 15] and the following examples.

Example 5.1. Let $X = \{a, b, c, d\}$ with topology $\tau = \{\emptyset, \{a\}, \{c\}, \{a, c\}, \{a, b\}, \{a, b, c\}, X\}$. Then:

(1) the function $f:(X, \tau) \rightarrow (X, \tau)$ which defined by $f(a) = a, f(b) = d$ and $f(c) = c, f(d) = b$ is semi-continuous but it is not Z_α -continuous,

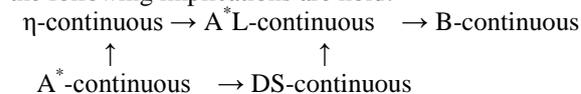
(2) the function $f:(X, \tau) \rightarrow (X, \tau)$ which defined by, $f(a) = a, f(b) = b$, and $f(c) = f(d) = c$ is Z_α -continuous but it is not δ -semi-continuous.

Example 5.2. In Example 3.2, the function $f:(X, \tau) \rightarrow (X, \tau)$ which defined by, $f(a) = c, f(b) = f(c) = d$ and $f(d) = f(e) = e$ is Z -continuous but it is not Z_α -continuous. Also, the function $f:(X, \tau) \rightarrow (X, \tau)$ which defined by, $f(a) = a, f(b) = b, f(c) = c$ and $f(d) = f(e) = d$, is Z_α -continuous but it is not α -continuous.

Definition 5.3. A function $f:(X, \tau) \rightarrow (Y, \sigma)$ is said to be A^*L -continuous if $f^{-1}(V)$ is an A^*L -set of X , for every $V \in \sigma$.

Definition 5.4. A function $f:(X, \tau) \rightarrow (Y, \sigma)$ is called B-continuous [21] (resp. η -continuous [17], DS-continuous [8]) if $f^{-1}(V)$ is a B-set (resp. η -set, DS-set) in X , for each $V \in \sigma$.

Remark 5.2. (1) Let $f: X \rightarrow Y$ be a function. Then the following implications are hold:



(2) Every continuous is A^*L -continuous.

(3) These implications are not reversible as shown by [4, 8] and the following examples.

Example 5.3. Let $X = \{a, b, c, d, e\} = Y$ with topology $\tau = \{\emptyset, \{a\}, \{c\}, \{a, c\}, \{a, b\}, \{a, b, c\}, X\}$

and $\sigma = \{\varphi, \{c\}, \{d, e\}, \{c, d, e\}, Y\}$. Then:
 (1) the function $f : (X, \tau) \rightarrow (Y, \sigma)$ which defined by, $f(a) = a, f(b) = d, f(c) = e, f(d) = b$ and $f(e) = c$ is B-continuous but it is not A^*L -continuous,
 (2) the function $f : (X, \tau) \rightarrow (Y, \sigma)$ which defined by, $f(a) = f(c) = a, f(b) = d, f(d) = b$ and $f(e) = e$ is A^*L -continuous but it is not DS-continuous and it is not continuous.

Example 5.4. In Example 4.2, the function $f : (X, \tau) \rightarrow (X, \tau)$ which defined by $f(a) = f(d) = d$ and $f(b) = f(c) = b$ is A^*L -continuous but it is not η -continuous.

Theorem 5.1. The following are equivalent for a function $f: X \rightarrow Y$:

- (1) f is continuous,
- (2) f is α -continuous and A^*L -continuous,
- (3) f is pre-continuous and A^*L -continuous.

Proof. It is an immediate consequence of Theorem 4.3.

Theorem 5.2. Let X be an extremely disconnected space and $f: X \rightarrow Y$ be a function. Then following are equivalent:

- (1) f is continuous,
- (2) f is Z_α -continuous and A^*L -continuous,
- (3) f is α -continuous and A^*L -continuous,
- (4) f is pre-continuous and A^*L -continuous.

Proof. It is an immediate consequence of Theorem 4.4.

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