RESEARCH ARTICLE OPEN ACCESS

Analysis of Stress and Deflection of Cantilever Beam and its Validation Using ANSYS

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Abstract
This study investigates the deflection and stress distribution in a long, slender cantilever beam of uniform rectangular cross section made of linear elastic material properties that are homogeneous and isotropic. The deflection of a cantilever beam is essentially a three dimensional problem. An elastic stretching in one direction is accompanied by a compression in perpendicular directions. The beam is modeled under the action of three different loading conditions: vertical concentrated
load applied at the free end, uniformly distributed load and uniformly varying load which runs over the whole span. The weight of the beam is assumed to be negligible. It is also assumed that the beam is inextensible and so the strains are also negligible. Considering this assumptions at first using the Bernoulli-Euler's bending-moment curvature relationship, the approximate solutions of the cantilever beam was obtained from the general set of equations. Then assuming a particular set of dimensions, the deflection and stress values of the beam are calculated analytically. Finite element analysis of the beam was done considering various types of elements under different loading conditions in ANSYS 14.5. The various numerical results were generated at different nodal points by taking the origin of the Cartesian coordinate system at the fixed end of the beam. The nodal solutions were analyzed and compared. On comparing the computational and analytical solutions it was found that for stresses the 8 node brick element gives the most consistent results and the variation with the analytical results is minimum.
Keywords: Cantilever, loading, ANSYS, element, Cartesian Coordinate System.

I. Introduction

In this paper cantilever beam [1] has been analyzed. All the following cases represents statically determinate beam since the reactions at the support can be determined from the equation of statics. The measure to which a structural member gets deviated from the initial position is called deflection. The deflected distance of a member under a load is directly related to the slope of the deflected shape of the member under that load. While the beam gets deflected under the loads, bending occurs in the same plane due to which stresses are developed. Here the deflection of the beam element is calculated by using the Euler-Bernoulli's beam equation [2] and the bending stresses using the general standard bending equation analytically. ANSYS [3] has been used to do the computational analysis. It is general purpose finite element analysis [4] software which enables the product development process at less computational and financial expenditure. Researchers [5-9] have used Ansys for the calculation and validation of experimental results.

II. Theoretical Calculations

First a uniform rectangular cross-sectional beam of linear elastic isotropic homogeneous material has been considered. The beam is taken mass less and inextensible hence have developed no strains. It is subjected to a vertical point load at the tip of its free end and the differential equation is developed mathematically. Similarly it is done with the same value of uniformly distributed load and uniformly varying load over the whole span. Using the Bernoulli-Euler's elastic curve equation [1] the following relationship is obtained:

$EI\ (d^2y/dx_2) = M$ (1)

Where E is modulus of elasticity which is of constant value.
I is moment of inertia$=bh^3/12$, b=width of beam, h=height of beam.
M=moment developed.

Case 1: Cantilever beam of length L subjected to a vertical point load 'F' at its free end .

Fig. 1: Cantilever with vertical load at free end

The B.M equation at section X-X at a distance x from fixed end is given by:

$Eid^2y/dx_2 = -F(L-x)$ (2)

On integrating and solving the above eq. with required boundary conditions we get the downward deflection of beam as: $FL^3/3\ EI$.

Assuming L=100m, b=10m,h=10m,ν=0.3,E=2×10^5N/m, F=500N.
Analytic Deflection, s_B= 1.000004 m.
Using the equation: (M/I) = (E/R) = (s/Y), Analytic Stress developed s_B=300N/m^2

Case 2: Cantilever beam subjected to a uniformly distributed load 'q' per unit run over the whole length.

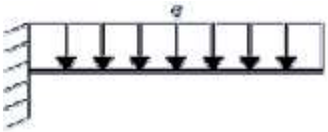

Fig. 2: Cantilever with Uniformly distributed load.
The B.M equation at section X-X at a distance x from fixed end is given by
$EId^2y/dx^2= -W/2(L-x)^2$ (3)

On integrating and solving the above eq. with required boundary conditions we get
the downward deflection of beam as : $qL^4/8EI$.
Assuming
L=100m,b=10m,h=10m,ν=0.3,E=2×10,F=500N

Deflection, d_B= 37.5 m.
Using the equation: (M/I)=(E/R)=(s/Y), Stress developed s5N/m2,s_B=150 N/m^2

Case 3: Cantilever beam subjected to a uniformly varying load 'q_0 'per unit run over the whole length.

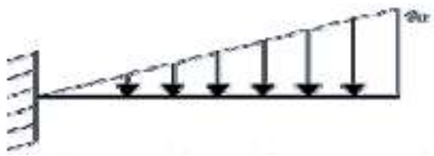

Fig. 3: Cantilever with Uniformly varying load.

The B.M equation at section X-X at a distance x from fixed end is given by:
$EId^2y/dx_2= -q_o(L-x)^3/6L$ (4)

On integrating and solving the above eq. with required boundary conditions we get the downward deflection of beam as: $11q_oL_4/120EI$.
Assuming
L=100m,b=10m,h=10m,ν=0.3,E=2×10N/m,F=500N.
Deflection, dB= 27.5 m.

Case 2: Cantilever beam subjected to a uniformly distributed load 'q' per unit run over the whole length.

III. Computational Results

Case 1: Cantilever beam of length L subjected to a vertical point load 'F' at its free end
(a)Element 1:-brick 8node 185

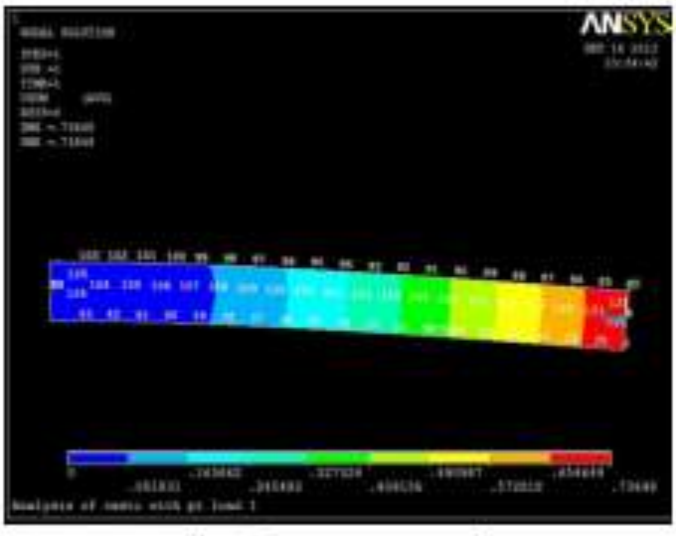

Fig. 4: Displacement values

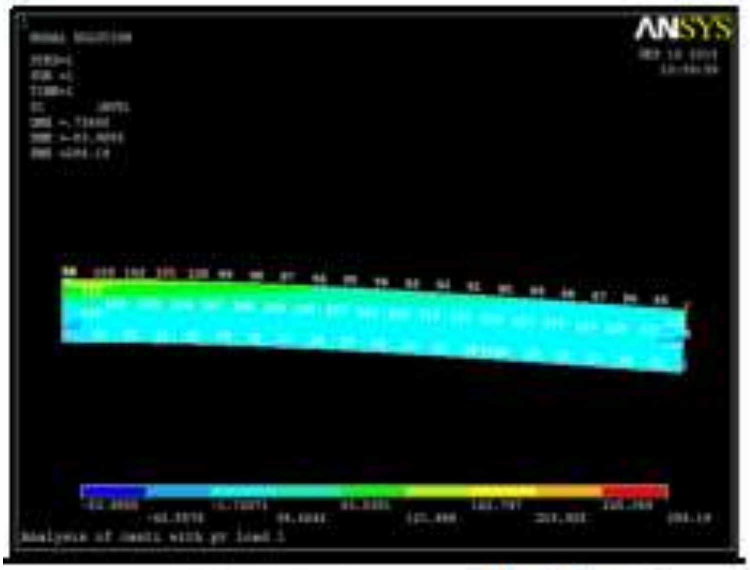

Fig. 5: Stress distribution

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Hence, Max deflection obtained= 0.73648 m,
Von-mises stress obtained= 286.19 N/m.
(b) Element 2:-Tet 10node 187

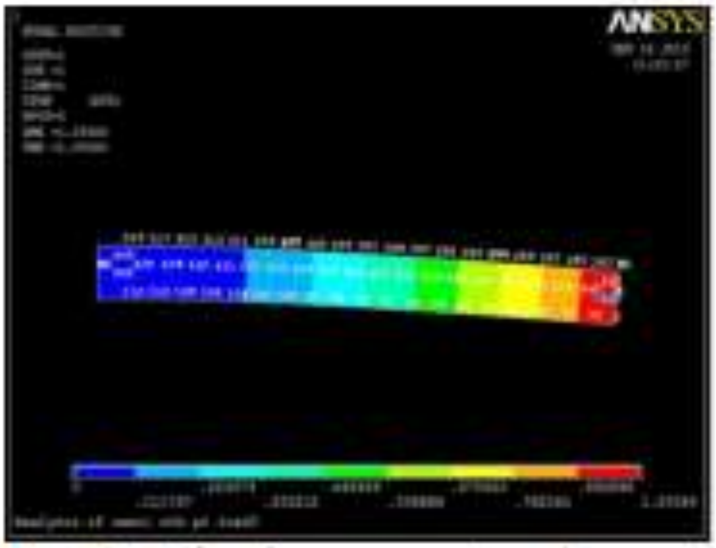
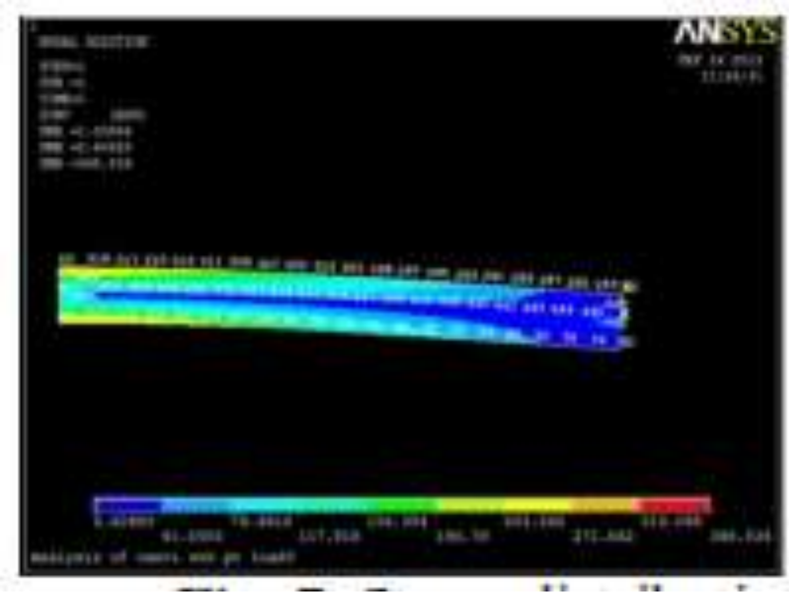


Fig. 6: Displacement values



. **Fig. 7**: Stress distribution

Hence, Max deflection obtained= 1.00564 m, Von-mises stress obtained= 348.534 N/m^2
.
Case 2: Cantilever beam subjected to a uniformly distributed load ‘q’ per unit run over the whole length.
(a)Element 1:-brick 8node 185.

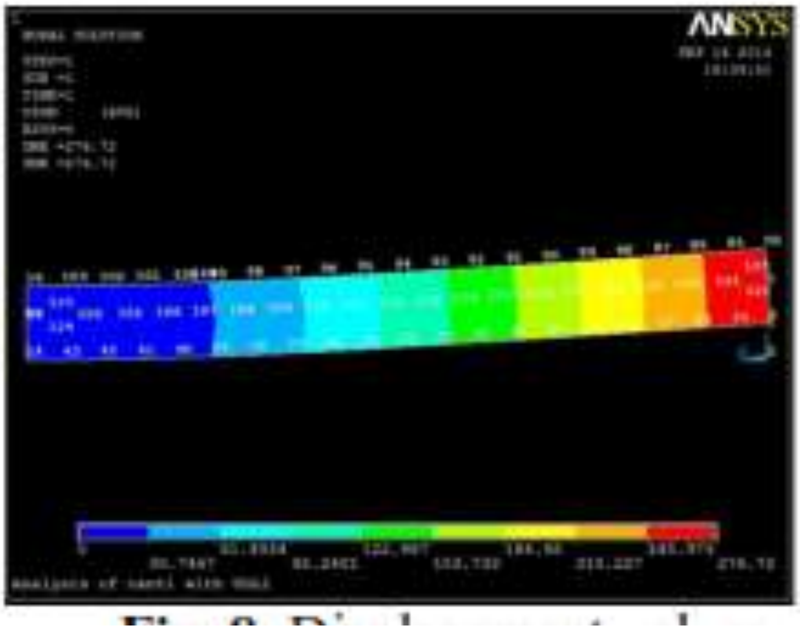


Fig. 8: Displacement values

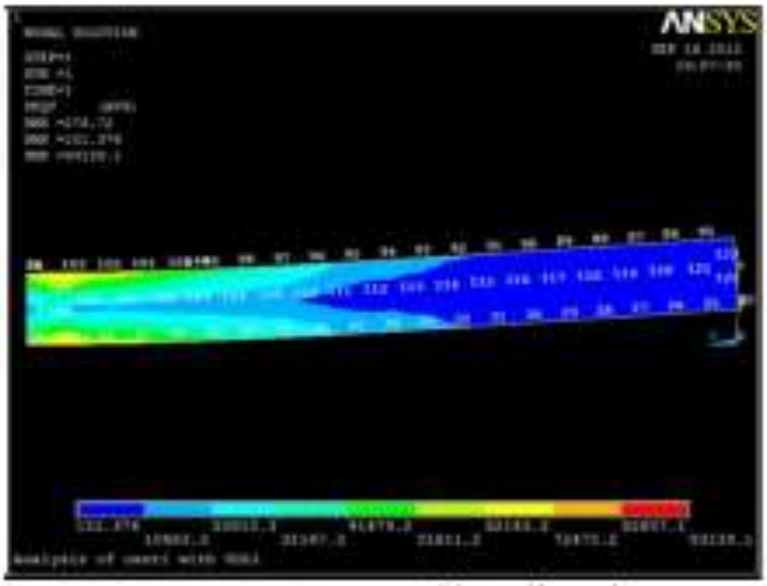


Fig. 9: Stress distribution

Hence, Max deflection obtained= 18.277 m, Von-mises stress obtained= 151.376N/m^2

(b) Element 2:-Tet 10node 187

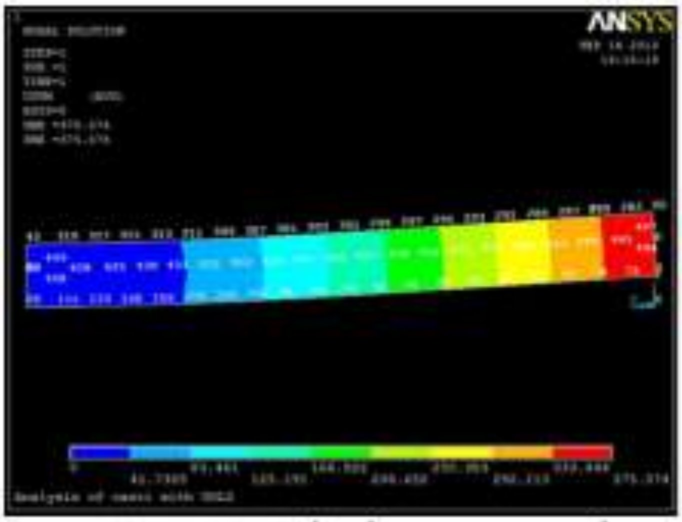


Fig. 10: Displacement values.

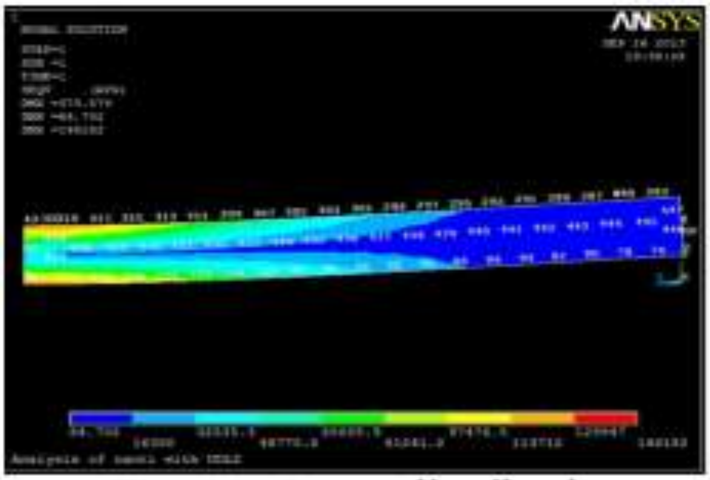


Fig. 11: Stress distribution

Hence, Max deflection obtained= 24.858 m, Von-mises stress obtained= 112.77 N/m.
Case 3: Cantilever beam subjected to a uniformly varying load ‘q_0 ’per unit run over the whole length.
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(a) Element 1:-brick 8node 185

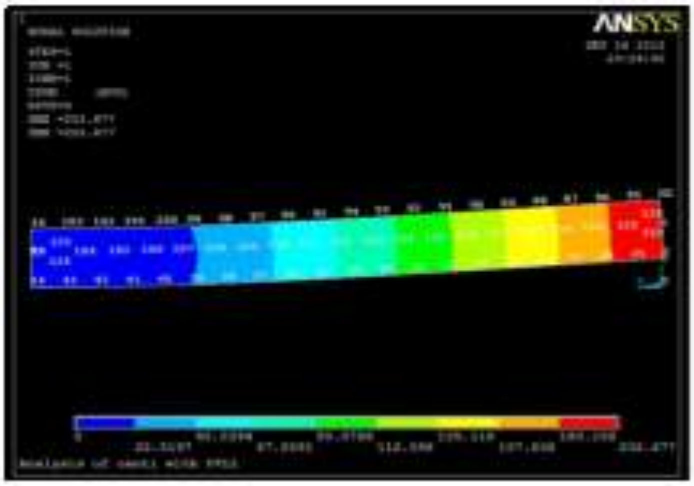

Fig. 12: Displacement values.

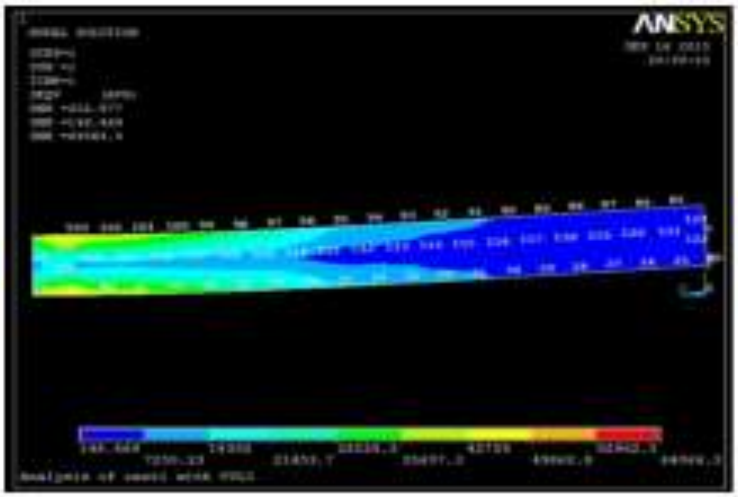

Fig. 13: Stress distribution.

Hence, Max deflection obtained= 13.685 m, Von-mises stress obtained= 150.76N/m^2. (b) Element 2:-Tet 10node 187

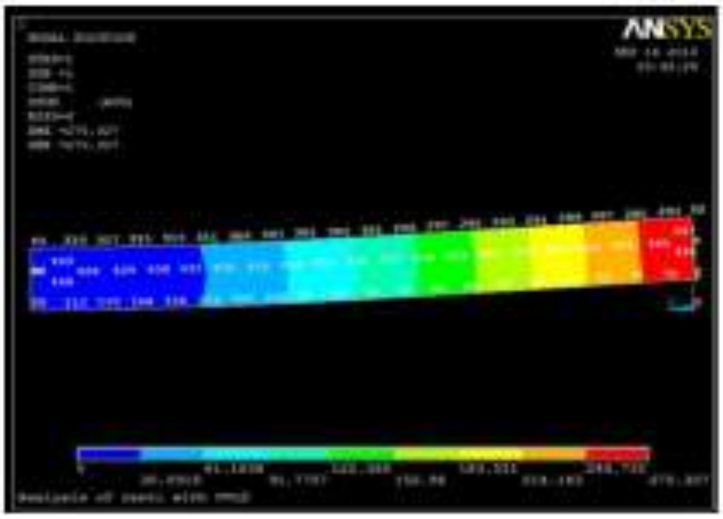

Fig. 14: Displacement values.

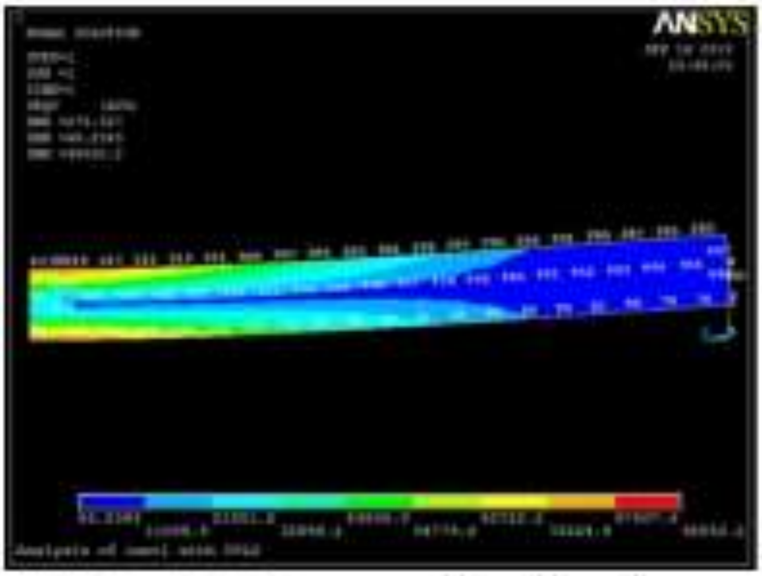

Fig. 15: Stress distribution.

IV. Results

Table 1 gives the comparison of analytical results with the computational results. The analytical results for all loading conditions have been compared with computational results considering the two most universally used elements i.e. 8node brick element and 10node Tetrahedral element.

Table 1: Comparison of results

	Element1 (E1)	Element1 (E2)	Analytic Calculation	Error with E1 (%)	Error with E1 (%)	Element1 (E1)	Element1 (E2)	Analytic Calculation	Error with E1 (%)	Error with E1 (%)
Point Load	0.736	1.005	1.000	26.4	0.5	286.19	348.534	300	4.603	16.178
UDL	18.277	24.858	37.5	51.261	33.72	151.376	112.77	150	-0.97	-24.82
UVL	13.685	18.643	27.5	50.236	-32.27	150.76	72.84	100	-50.7	-27.16

V. Conclusions:

From the above table it can be concluded that using Ansys the deflection is more accurate when element 2 i.e. 10node Tetrahedral element is used but for stresses 8node brick element gives a better results. Hence for determination of deflection 10node Tetrahedral element should be used whereas for stresses 8node brick element is more appropriate.

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