REVIEW ARTICLE OPEN ACCESS

Torsional-Distortional Performance of Multi-Cell Trapezoidal Box Girder with All Inclined Web Members

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Abstract
In this study, the torsional and distortional performance of a three triangular cell trapezoidal box girder section was studied using Vlasov's theory of thin walled structures. The potential energy of a system under equilibrium was used to obtain the governing differential equations of equilibrium for torsionl-distortional analysis of the box structure. The strain modes diagrams representing torsional and distortional interactions of the box girder structure were obtained as well as the distortional bending moment diagram for the box girder structure. These were used to compute Vlasov's coefficients contained in the differential equations of equilibrium. The fourth order differential equations obtained were solved using method of trigonometric series with accelerated convergence to obtain the distortional and torsional deformations which were compared with torsional and distortional deformations of a single cell mono symmetric box girder section of similar overall dimensions and plates thicknesses. The maximum distortional deformation was found to be 168% lower than that of a single cell mono-symmetric box girder section of the same size and dimensions. The inclined internal web members, which also act as diaphragms, brought about increase in the pure torsional and distortional components of the applied torsional load, resulting to marginal increase in the torsional deformation but major decrease in the distortional deformation.
Keywords: deformation, distortion, torsion, trapezoidal box girder, triangular cells, Vlasov's theory.

I. Introduction

The cross section of box girder bridges may take the form of single cell, multi cell (cellular) or multi spine (multiple box) profiles. Multi cell box girder type provides greater torsional stiffness than multi spine type due to the high efficiency of the contiguous cells in resisting eccentric loadings. Trapezoidal box girder bridges are often used in curved bridges due to the large torsional stiffness that result from the closed cross section.

In general, a reinforced concrete bridge structure may consist of deck slabs, T beams (deck girder), through and box girder, rigid frames and flat slab types. Combinations of these with precasting or prestressing produce additional structural forms and enhance bridge versatility, [1].

In the selection of the proper type of bridge, cost is usually the determining criterion. Box girder decks are cast-in-place units that can be constructed to follow any desired alignment in plan, so that straight, skew, and curved bridges of various shapes are common in the highway system.

Research work [2], [3], [4], [5] and [6] on thin-walled box girder structures covered essentially three types of cross section. (a) Single cell steel box girder structure, straight and curved. (b) Twin cell reinforced concrete box girder, straight and curved. (c) Two-box and three-box multiple spine box girder, steel and reinforced concrete. Literature on cellular box girders with three or more cells appears to be scarce. Those available for two-celled box girder are in reinforced concrete area. Thus, there appears to be a dearth of information on the torsional-distortional behaviour of thin-walled box girder bridge structure with three or more cells in cross section.

Specific areas covered by the available literature on torsional and or tortional performance of mono symmetric box girgers include:(a) torsional stiffness and shear flow, (b) distortional design of multi-box and multi-cellular sections, (c) distortional behaviour and brace forces, (d) effect of warping on longitudinal and transverse normal stresses, (e) design coefficients for loads and aids to practical design, (f) analysis of deformable sections / design curves, (g) torsional load distribution, (h) approximate method to determine torsional moments in non deformable sections, (i) bending moment expressions, shear and torsional moments for live loads (j) bracing requirements for open sections.

With these trends, the authors felt that available literature on torsional-distortional elastic behaviour of thin-walled deformable box girder structure is surprisingly few. This could be attributed to the fact that most authors assume that the use of intermediate stiffeners and diaphragms on thin-walled closed and quasi-closed structures is an effective way of handling non-uniform torsion and its attendant problems of warping and distortion. The authors also believe that the use of triangular celled trapezoidal box girder will improve the torsional-distortional qualities of mono-symmetric box girders and at the same time make the use of diaphragms and intermediate stiffeners irrelevant. Hence, this research seeks to evaluate the torsional and distortional deformations of a simply supported multi- triangular-cell trapezoidal box girder bridge.

By comparing its torsional and distortional deformations with those of an equivalent single cell trapezoidal box girder section having the same overall dimensions and plate thicknesses, an insight is obtained concerning the efficiency of such box girder section in resisting torsional and distortional loads. Vlasov's [7] theory as modified by Verbernov [8] was adopted in the analysis.

II. Formulation of Equilibrium Equations

The potential energy of a box girder structure under the action of a distortional load of intensity q is given by;

$$\pi = U + W_E \tag{1}$$

Where,

π = the total potential energy of the box girder structure,

U = Strain energy,

W_E = External potential or work done by the external loads.

From strength of material study, the strain energy of a structure is given by:

$$U = \frac{1}{2}\int_L\int_S\left[\left(\frac{\sigma^2(x,s)}{E} + \frac{\tau^2(x,s)}{G}\right)t(s) + \frac{M^2(x,s)}{EI_{(s)}}\right]dxds \tag{2}$$

Work done by external load is given by;

$$W_E = qv(x,s)dxds$$

$$= \int_s\int_x q\Sigma V_h(x)\varphi_h(s)dsdx = \int_x \Sigma q_h V_h dx \tag{3}$$

Substituting (2) and (3) into (1) we obtain that:

$$\pi = \frac{1}{2}\left[\int_L\int_S\left[\frac{\sigma^2(x,s)}{E} + \frac{\tau^2(x,s)}{G}\right]t(s) + \frac{M^2(x,s)}{EI(s)} - qv(x,s)\right]dxds \tag{4}$$

where,

$\sigma(x,s)$ = Normal stress

$\tau(x,s)$ = Shear stress

$M(x,s)$ = Transverse distortional bending moment

q = Line load per unit area applied in the plane of the plate

Moment of inertia of plates forming the box girder is given by: $I_{(s)} = \frac{t^3(s)}{12(1-\nu^2)}$ (5)

E = Modulus of elasticity

G = Shear modulus

ν = poisson ratio

t = thickness of plate

Using Vlasov's displacement fields [7] and the basic stress-strain relations of theory of elasticity, the expressions for normal and shear stresses become:

$$\sigma(x,s) = E\sum_{i=1}^{m}\varphi_i(s)U_i'(x) \tag{6a}$$

$$\tau(x,s) = G\left[\sum_{i=1}^{m}\varphi_i'(s)U_i(x) + \sum_{k=1}^{n}\psi_k(s)V_k'(x)\right] \tag{6b}$$

where U(x) and V(x) are unknown functions governing the displacements in the longitudinal and transverse directions respectively, and φ and ψ are generalized warping and distortional strain modes respectively. These strain modes are known functions of the profile coordinates and are chosen in advance for any type of cross section

The potential energy of the box girder structure is given by [7]:

$$\pi = \frac{1}{2}E\Sigma a_{ij}U_i'(x)U_j'(x)dx +$$
$$+\frac{1}{2}G\left[\Sigma b_{ij}U_i(x)U_j(x) + \Sigma c_{kj}U_k(x)V_j'(x)\right]dx$$
$$+ \quad \frac{1}{2}G\left[\Sigma c_{ih}U_i(x)V_h'(x) + \Sigma r_{kh}V_k'(x)V_h'(x)\right]dx$$
$$+$$
$$+\frac{1}{2}E\Sigma s_{hk}V_k(x)V_h(x)dx \ -$$
$$-\Sigma q_h V_h dx \tag{8}$$

Where, $a_{ij} = a_{ji} = \int\varphi_i(s)\varphi_j(s)dA$ (a)

$b_{ij} = b_{ji} = \int\varphi_i^I(s)\varphi_j^I(s)dA$ (b)

$c_{kj} = c_{jk} = \int\varphi_k^I(s)\psi_j(s)dA$ (c)

$c_{ih} = c_{hi} = \int\varphi_i^I(s)\psi_k(s)dA$ (d)

$r_{kh} = r_{hk} = \int\psi_k(s)\psi_h(s)dA;$ (e)

$$s_{kh} = s_{hk} = \frac{1}{E}\int \frac{M_k(s)M_h(s)}{EI_{(s)}}ds \qquad \text{(f)}$$

$$q_h = \int q\psi_h ds \qquad \text{(h)}$$

(9)

The transverse bending moment generated in the box structure due to distortion is given by [7]:

$$M(x,s) = \sum_{k=1}^{n} M_k(s)V_k(x) \qquad (10)$$

Where $M_k(s)$ is the bending moment generated in the cross sectional frame of unit width due to a unit distortion, $V(x) = 1$.

By minimizing the potential energy of the box girder structure, (8), with respect to the functional variables U(x) and V(x) using Euler Lagrange technique [9], we obtain the torsional-distortional equilibrium equations for displacement analysis of the box girder. Thus:

$$\kappa \sum_{i=1}^{m} a_{ij}U_i''(x) - \sum_{i=1}^{m} b_{ij}U_i(x) - \sum_{k=1}^{n} c_{kj}V_k'(x) = 0$$

$$\sum c_{ih}U_i^{I}(x) + \sum r_{kh}V_k^{II}(x) - \kappa\sum s_{hk}V_k(x) + \frac{1}{G}\sum q_h = 0$$

(11) where $\kappa = \frac{E}{G} = 2(1+\nu)$

Equation (11) is Vlasovs generalized differential equations of distortional equilibrium for the box girder structure. Its matrix form is as follows:

$$\kappa\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}\begin{Bmatrix} U_1'' \\ U_2'' \\ U_3'' \end{Bmatrix} - \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}\begin{Bmatrix} U_1 \\ U_2 \\ U_3 \end{Bmatrix}$$

$$-\begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} \\ c_{21} & c_{22} & c_{23} & c_{24} \\ c_{31} & c_{32} & c_{33} & c_{34} \end{bmatrix}\begin{Bmatrix} V_1' \\ V_2' \\ V_3' \\ V_4' \end{Bmatrix} = 0 \qquad \text{(12a)}$$

$$\begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{bmatrix}\begin{Bmatrix} U_1' \\ U_2' \\ U_3' \end{Bmatrix} - \kappa\begin{bmatrix} s_{11} & s_{12} & s_{13} & s_{14} \\ s_{21} & s_{22} & s_{23} & s_{24} \\ s_{31} & s_{32} & s_{33} & s_{34} \\ s_{41} & s_{42} & s_{43} & s_{44} \end{bmatrix}\begin{Bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{Bmatrix}$$

$$+\begin{bmatrix} r_{11} & r_{12} & r_{13} & r_{14} \\ r_{21} & r_{22} & r_{23} & r_{24} \\ r_{31} & r_{32} & r_{33} & r_{34} \\ r_{41} & r_{42} & r_{43} & r_{44} \end{bmatrix}\begin{Bmatrix} V_1'' \\ V_2'' \\ V_3'' \\ V_4'' \end{Bmatrix} = \frac{1}{G}\begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} \qquad \text{(12b)}$$

The elements, a_{ij}, b_{ij}, c_{ij}, and r_{ij}, in the matrix equations are the Vlasov's coefficients and are determined from the strain modes diagrams and by using Morh's integral for diagram multiplication. The s_{hk} coefficients depend on the bending deformation of the plates forming the box girder section. They are obtained first, by drawing the distortional bending moment diagram of the box girder section and then by applying Morh's integral for diagram multiplication.

While drawing the strain modes diagrams we note that φ_1 diagram is a property of the cross section obtained by plotting the displacement of the members of the cross section when the vertical axis is rotated through a unit radian. Similarly, values of φ_2 are obtained for the members of the cross section by plotting the displacements of the cross section when the horizontal axis is rotated through a unit radian. φ_3 is the warping function of the cross section. It is the out of plane displacement of the cross section when the box girder is twisted about its axis through the pole, one radian per unit length, without bending in either x or y directions and without longitudinal extension. Other strain modes, $\varphi_1' = \psi_1$, $\varphi_2' = \psi_2$, $\varphi_3' = \psi_3$ are obtained by numerical differentiation of the respective strain mode diagrams, φ_1, φ_2 and φ_3. Strain mode ψ_4 is the displacement diagram of the box cross section when the section is rotated one radian in say, a clockwise direction, about its centroidal axis. The procedure for evaluation of strain mode diagrams is fully explained in [10]

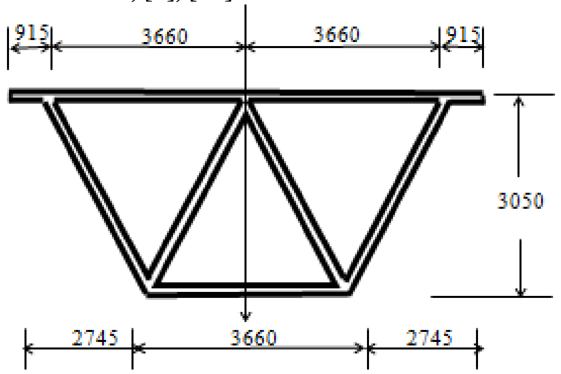
III. Evaluation of Vlasov's Coefficients

A study [12] of the interaction of the strain fields with each other reveal that the relevant strain field interactions for torsional-distortional analysis of mono symmetric box girder sections are those involving strain field 3 (distortion) and strain field 4 (rotation). Thus, the coefficients involved are those dependent on the interaction of φ_3, $\varphi_3' = \psi_3$, and ψ_4 diagrams only, which are shown in Fig. 1. Using Morh's integral for displacement computations, the following coefficients were obtained for the box girder section shown in Fig. 1(a).

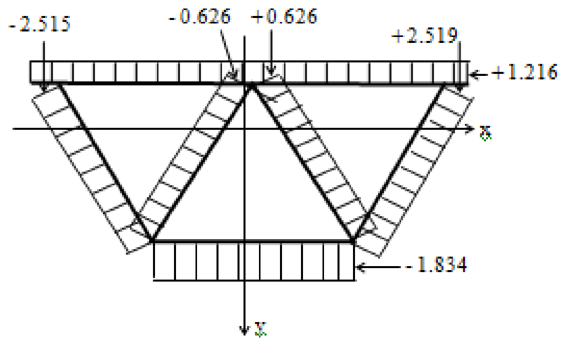
$$a_{33} = \varphi_3 * \varphi_3 = 2.103,$$
$$b_{33} = c_{33} = r_{33} = \varphi_3^{'} * \varphi_3^{'} = 1.943,$$
$$c_{34} = c_{43} = r_{34} = r_{43} = \varphi_3^{'} * \psi_4 = 1.775,$$
$$r_{44} = \psi_4 * \psi_4 = 14.754,$$
$$s_{33} = (M_3 * M_3) I_S = 1.205 I_S$$

3.1 Distortional bending moment diagram

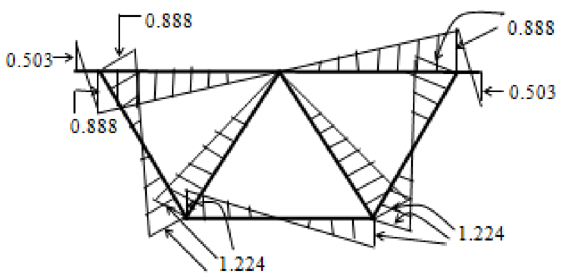
The procedure for evaluation of distortional bending moment diagram and hence, the s_{hk} coefficients of any box girder section are available in literatures, [7], [12].



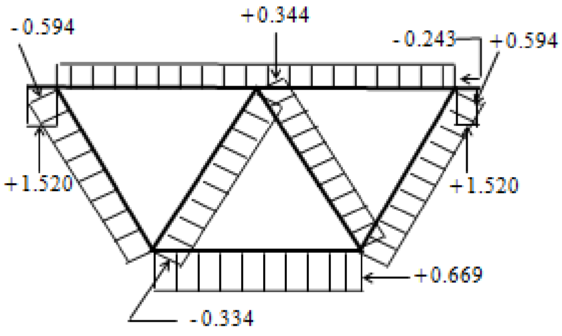
(a) Multi cell trapezoidal box girder



(b) Rotation diagram, ψ_4



(c) Warping function, $\omega_3(s)$ diagram



(d) distortion diagram, ψ_3

Fig. 1: Generalized strain modes for the multi-cell trapezoidal box girder section

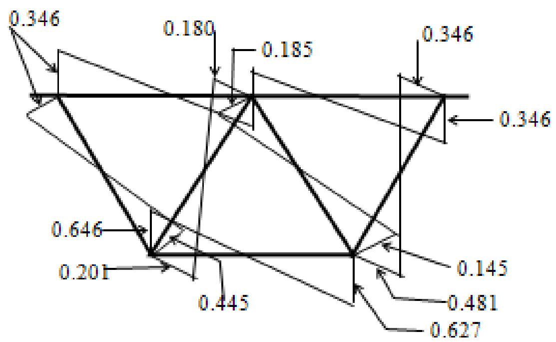


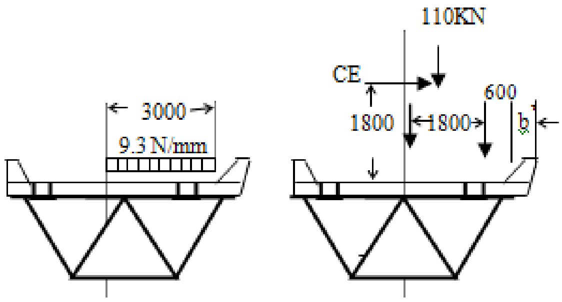
Fig.2: Distortional bending moment diagram for the multi-cell box girder

IV. Evaluation of Distortional and Pure Torsional Components of the Torsional Load

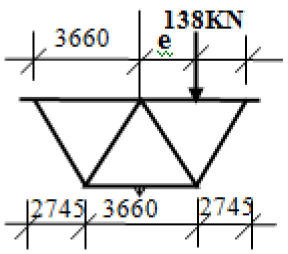
Live loads were considered according to AASHTO [11], following the HL-93 loading. Uniform lane load of 9.3N/mm distributed over a 3000mm width plus tandem load of two 110 KN axles were applied as shown in Figs 4 and 5. The loads were positioned at the outermost possible location to generate the maximum torsional effects.

The torsional load on the bridge girder was 138 kN, Fig.3(c), acting at eccentricity e = $(b/2 - 1.46)$ m.

The torsional moment, $M_T = 138\,e$, Fig.4(a), is decomposed into bending component and torsional component, Figs. 4(b) and 4(c) respectively. The torsional component comprises of distortional component and pure rotational component, Fig.5. From study [12] it was found that the bending component of the torsional moment does not affect or influence the pure torsional and the distortional components of the applied torsional load. Hence, the bending component of the applied torsional moment is irrelevant in this study, so the dead load of the entire box structure is ignored. Thus, the pure torsional component and the distortional component of the applied torsional moment are investigated and evaluated using Table 1.

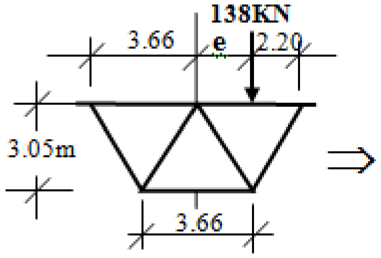


(a) Lane load Load (b) Tandem load

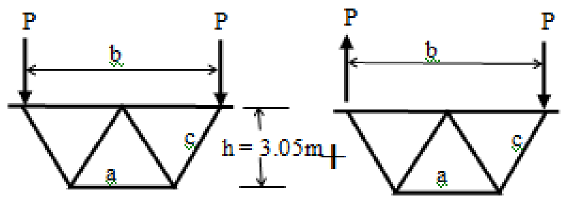


(c) Combined Lane load and tandem load

Fig. 3 Positioning of Live Loads for Single Lane Bridge

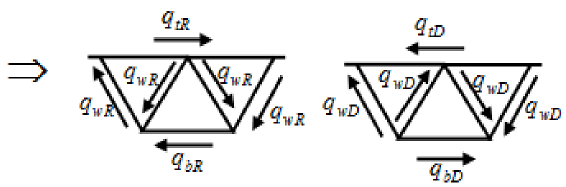


(a) Bridge eccentric Load



(b) Bending component (c) Torsional component

Fig. 4 Components of Bridge Eccentric Load



(a) Pure torsional component (rotation) (b) Distortional component (warping)

Fig. 5 Components of the Torsional Load

V. Torsional-Distortional Analysis

Substituting the values of Vlasov's coefficients into eqns. (12a) and (12b) and multiplying out we obtain, after simplification, the relevant torsional-distorsional differential equations of equilibrium for the analysis of the box girder section as follows:

$$\alpha_1 V_3^{iv} + \alpha_2 V_4^{iv} - \beta_1 V_4^{''} = K_1$$
$$\beta_2 V_4^{''} - \gamma V_3 = K_2 \qquad (13)$$

Table 1: Pure torsional and distortional components of applied torsional load [3]

Type / Magnitude of load	Components in plate members	Pure torsional component	Distortional component
Torsional moment by vertical forces (Pb)	Top plate: P_t =2Ptan	$Pb^2/ 2A_o$	$Pa^2 /2 A_o$
	Bottom plate: P_b = 0	Pab/ $2A_o$	Pab / $2A_o$
	Web plate: P_w=P /cos	Pbc/ $2A_o$	Pac / $2A_o$
Torsional moment by horizontal forces (Ph)	Top plate: P_t = P	Pb/(a+b)	Pa / (a+b)
	Bottom plate: P_b = P	Pa/(a+ b)	Pb / (a+b)
	Web plate P_w = 0	Pc / (a+b)	Pc / (a+b)

($2A_o$ = h(a+b), where a and b are the widths of the parallel sides of the trapezium, h is the distance between them)

Where,

$\alpha_2 = ka_{33}c_{43} = 9.332$;

$\alpha_2 = ka_{33}r_{44} = 77.567$;

$\beta_1 = b_{33}r_{44} - c_{34}c_{43} = 25.56$;

$\beta_2 = r_{34}c_{43} - c_{33}r_{44} = -25.516$;

$\gamma = c_{43}ks_{33} = 3.565*10^{-3}$;

$K_1 = b_{33}q_4 / G = 3.331*10^{-4}$

$K_2 = (c_{33}q_4 - c_{43}q_3)/G = 2.842*10^{-4}$

Substituting the values of these parameters into eqn. (13) we obtain:

$$9.332V_3^{iv} + 77.569V_4^{iv} - 25.516V_4^{''} = 3.331*10^{-4}$$
$$-25.516V_4^{''} - 3.565*10^{-3}V_3 = 2.842*10^{-4} \qquad (14)$$

Integrating by method of trigonometric series with accelerated convergence we obtain:

$$V_3(x) = 1.218*10^{-2}\sin(\pi x/L)$$
$$V_4(x) = 3.252*10^{-3}\sin(\pi x/L) \qquad (15)$$

where L = span of the bridge = 50m

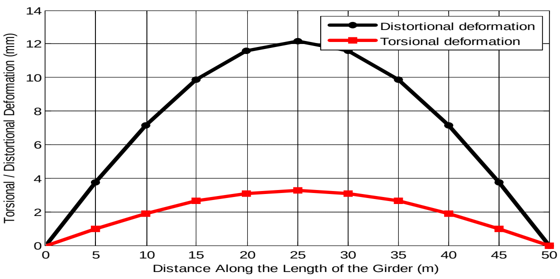


Fig.6: Torsional and distortional deformations along the length of the girder

Table 2. Comparison of torsional and distortional deformations of single and multi-cell trapezoidal box girder sections

Single cell trapezoidal box girder [10]				Triple triangular cell trapezoidal box girder			
Distortional load and deformation		Pure tortional load and deformation		Distortional load and deformation		Pure tortional load and deformation	
q_3 (KN)	V_3(max) (mm)	q_4 (KN)	V_4(max) (mm)	q_3 (kN)	V_3(max) (mm)	q_4 (kN)	V_4(max) (mm)
196	32.68	1446	2.80	265	12.18	1646	3.25

VI. Discussion of Results

Figure 2 shows the distortional bending moment diagram for the multi cell trapezoidal box girder section. The solution to the general fourth order differential equations of torsional- distirtional analysis of the box girder section (eqn.13) is given by eqn. (15), plotted as Fig.6 with the title 'variation of torsional and distortional deformation of the box girder structure through out its length'. The maximum torsional deformation was 3.25mm over a 50m span bridge, while the maximum distortion was 12.18mm, all occurring at the mid span.

In table 2, the performance of the chosen triangular celled box girder structure is compared with those of a single cell mono-symmetric box girder structure with similar over all dimensions and plate thicknesses [10]. We observed that the distortional and pure torsional components of the applied torsional load for the study profile were higher than those of single cell mono symmetric profile [10], yet, def its distortional deformation was much lower than those of mono symmetric section. Arguably, the two extra inclined internal web members in the study profile brought about increase in the torsional rigidity of the box girder which is expected to reduce torsional and distortional deformations. While this was true for distortional deformation it was not so for torsional deformation.

The increase in the distortional and pure tortional components of the applied torsional load can be attributed to the fact that torsional components of the applied torsional load increase with increase in the profile warping function which the two internal webs in the study profile tend to promote. Specifically, the distortional load component for the study profile increased by 35% over that of the single cell mono-symmetric profile, while the maximum deformation reduced by 168%. Also, pure torsional component of the torsional load increased by 13.8% while the torsional deformation also increased by 16%.

VII. Conclusion

The maximum distortional deformation of a multi- triangular- cell trapezoidal box Girder Bridge was found to be 168% lower than that of a single cell mono-symmetric box girder section of the same overall size and plate thicknesses.

The effect of the inclined internal web members (which also act as diaphragms) was increase in the pure torsional and distortional components of the applied torsional load, resulting to minor increase in the torsional deformation and major decrease in the distortional deformation. The ratio of distortional deformation to torsional deformation (which is an indicator of structural efficiency) was drastically reduced from 11.67 to 4.67 by the introduction of the two internal webs forming the triangular cells in the study profile. The lower this indicator value the more efficient the structure behaves.

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