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H Infinity Based Iterative Learning Control Of Systems With Disturbances

Olfa Kouki[*, **], Chaouki Mnasri[**], Nizar Toujeni[**], Moncef Gasmi[**]
*(National Engineering School of Tunis, El Manar University, Tunisia)
** (Computer Laboratory of Industrial Systems, Carthage University, Tunisia)

ABSTRACT
The iterative learning control and the robust control are presented in this paper for the trajectory tracking control. An H infinity norm is used to find the initial robust control law applied to the plant then an iterative learning control is deduced which guaranteed the monotonic convergence of the system. A LMI (Linear Matrix Inequalities) technique is also used to analyses and synthesizes this control system.
Keywords- iterative learning control ILC; robust control; H_∞ norm; robust monotonic convergence and LMI technique.

I. INTRODUCTION

The iterative learning control “ILC” is a concept that has been introduced in the literature by several research works. It is an important approach which is categorized in the methodology of intelligent control. This approach is a simple and effective method for controlling systems that execute the same task repetitively, such as a motion stage for wire bonding [1], a robotic manipulator in a manufacturing environment [2], a chemical reactor in a batch processing application [3] and nanopositioning systems [4]. Then ILC concept means the capability to improving the control input using the data of previous iteration [5]. The goal of this approach is to generate a simple algorithm repetitively for an unknown process, until perfect tracking is realized [6].

To achieve this objective, different approach used ILC algorithms have been derived from the classical Arimoto-type ILC algorithm [7], such as the PID-type [8], the D-type [5] [6] [8] [9], the P-type [3], the PD-type and the PI-type [10] and the Newton-type [11]. These algorithms have been used to decreasing the system error from iteration to another until the convergence be realized but they cannot eliminate the uncertainty from the system. On the other hand, we need a robust control to guarantee the monotonic convergence.

To design a robust iterative learning control for uncertain system, several approaches have been used in discrete time and continuous time, such as the µ synthesis [2][12], the youla parameterization [2], the min-max method using the quadratic performance criterion [13] and the H_∞ approach [14][15]. This next one is an important approach used to improving the monotonic convergence of uncertain system with feed-back state.

A robust iterative learning control and a robust monotonic convergence are studied in this paper. We use the LMI technique to solve the robust monotonic convergence problem of uncertain linear continuous time system. The H_∞ norm is also used to ensure the system convergence.

We show in section 2 of this paper the problem formulation and the preliminary results. We present in this first, the system description and the control design objective. In section 3, the control design is studied, we divided this section into two parts, the first one represent the H infinity state feedback design and the next part illustrate the D type robust ILC design. A simulation example is showed in section 4 and we finished by a conclusion in section 5.

II. PROBLEM FORMULATION AND PRELIMINARY RESULT

First, our objective is to determine an initial robust control applied during the first iteration to eliminate the external uncertainty in the system. We use the H_∞ approach to calculate the control input with state feedback. Secondly, we want to determine the control input that cancels the error after a number of iterations.
We considered the following uncertain linear system:

$$\begin{cases} \dot{x}_k = Ax_k + Bu_k + Hw_k \\ y_k = Cx_k \end{cases} \quad (1)$$

The reference model is described as follows:

$$\begin{cases} \dot{x}_d = Ax_d + Bu_d \\ y_d = Cx_d \end{cases} \quad (2)$$

With x_k, w_k, u_k, y_k, x_d, u_d, and n represent respectively the system state, the disturbance input, the control input, the output, the desired state, the desired control and the system order, at iteration number k, with appropriate dimension.

We need in this work, an iterative learning control capable to controlling the system (1) to follow the model (2).
We defined the error model as follows:

$$\begin{cases} \dot{e}_k = \dot{x}_d - \dot{x}_k \\ ey_k = y_d - y_k \end{cases} \quad (3)$$

From (1) and (2), the error model becomes:

$$\begin{cases} \dot{e}_k = Ae_k + B\left(u_d - u_k\right) - Hw_k \\ ey_k = Ce_k \end{cases} \quad (4)$$

Our work is based on a set of lemmas:
Lemma. 1.
For a given $\gamma > 0$, there exists a control law $K \in \mathrm{R}^{p*n}$ such that:

- The closed loop system is asymptotically stable.
- The H_∞ norm of the transfer function in a closed loop between the input w and the output y is less than γ.

Lemma. 2. (bounded real lemma)
Let considers a stationary linear multivariable stable system written by the following minimal state representation:

$$\begin{cases} \dot{\mathrm{x}} = \mathrm{Ax} + \mathrm{Bw} \\ y = \mathrm{Cx} + \mathrm{Dw} \end{cases} \quad (5)$$

Where x is the system state, w is the input and y is the output. Then $\|\mathrm{H}\|_\infty < \gamma$ if and only if there exists a symmetric matrix P such that:

$$\begin{cases} P > 0 \\ \begin{bmatrix} A^T P + PA + C^T C & PB + C^T D \\ B^T P + D^T C & D^T D - \gamma^2 I \end{bmatrix} \le 0 \end{cases} \quad (6)$$

Lemma. 3. (Schur lemma)
Consider a block symmetric matrix [16]:

$$\begin{bmatrix} A & B^T \\ B & C \end{bmatrix} \quad (7)$$

Where A and C are square matrices, with C being negative definite. This matrix is negative definite if and only if:

$$A - B^T C^{-1} B \quad (8)$$

is negative semi-definite.
Property 1:
Let considers an invertible matrix T:

$$T \in R^{(n*n)}, S \le 0 \Leftrightarrow T^T S T \le 0 \quad (9)$$

III. CONTROL DESIGN

We would applying to the system a robust and iterative learning control at the same time to reduce the repetitive error in the plant that execute the same task repetitively. Then, our control is composed from two parts, the first is a robust control and the second is an iterative learning control witch guaranteed a robust monotonic convergence.
Let consider the following control expression:

$$\begin{cases} u_k = v_{1,k} + v_{2,k} \\ v_{1,k} = u_d + Ke_k \\ v_{2,k+1} = v_{2,k} + \Gamma \dot{e}y_k \end{cases} \quad (10)$$

Where $v_{2,0} = 0$ and K and Γ are the learning gains matrix.
The controls $v_{1,k}$ and $v_{2,k}$ represent respectively the robust control and the iterative learning control.
By integrating this control law in the error model, we obtain the following system:

$$\begin{cases} \dot{e}_k = (A - BK)e_k - Bv_{2,k} - Hw_k \\ ey_k = Ce_k \end{cases} \quad (11)$$

III. 1. H INFINITY STATE FEEDBACK DESIGN

We are looking for a control law with state feedback which stabilizes the system and minimizes the H_∞ norm. To achieve this objective, the learning gains matrix K and Γ must be determined.
To solve this problem, such an approach is based on solving a convex optimization problem under constraints of linear matrix inequalities (LMIs) is proposed.
At the first iteration the iterative learning control is zero, then:

$u_0 = v_{1,0}$

The error model becomes:

$$\begin{cases} \dot{e}_k = (A - BK)e_k - Hw_k \\ ey_k = Ce_k \end{cases} \quad (12)$$

III. 2. D TYPE ROBUST ILC DESIGN

The objective of the ILC is to refine the input signal from iteration to another until we obtain an output signal which approximates the desired signal.

Consider an input signal is applied to a system in an initial state. After applying the complete input the system returns to its initial state. The system output will be compared to a desired signal. The error is used to construct a new input signal (the same length) to be applied in the next iteration.
In the classical ILC the following standard assumptions are required [3]:

- Each test period ends in a fixed time.
- Repeat initial setting is satisfied. Then, the initial system state $x_k(0)$ can be set at the same point at the beginning of each iteration.
- The system dynamics invariance is ensured throughout the repetition (dynamic stability).

- The output is measured in a deterministic manner.
- The system dynamics are deterministic.

To give an idea of the results of the ILC, we consider the proposed ILC in 1984 by Arimoto [3]:

$$v_{2,k+1} = v_{2,k} + \Gamma \dot{e}y_k \tag{13}$$

Now we derive the learning rule. Consider:

$$\eta_{k+1} = \int_0^t x_{k+1}dt - \int_0^t x_k dt \tag{14}$$

Then:

$$\dot{\eta}_{k+1} = x_{k+1} - x_k$$

$$= \int_0^t (Ax_{k+1} + Bu_{k+1} + Hw_{k+1})dt - \int_0^t (Ax_k + Bu_k + Hw_k)dt$$

$$= \int_0^t [(A-BK)(x_{k+1} - x_k) + H\delta w_{k+1}]dt + B\int_0^t \Gamma \dot{e}y_k dt$$

$$= (A-BK)\eta_{k+1} + H\tilde{w}_{k+1} + B\Gamma ey_k$$

$$= A\eta_{k+1} + B(-K\eta_{k+1} + \Gamma ey_k) + H\tilde{w}_{k+1}$$

$$= A\eta_{k+1} + B\tilde{u}_{k+1} + H\tilde{w}_{k+1}$$

Where:

$$\tilde{w}_{k+1} = \int_0^t (w_{k+1} - w_k)dt \tag{15}$$

$$\tilde{u}_{k+1} = -K\eta_{k+1} + \Gamma ey_k \tag{16}$$

We will determine the deference between the error at iterations number k and k+1:

$$ey_{k+1} - ey_k = Ce_{k+1} - Ce_k$$
$$= -C\dot{\eta}_{k+1} \tag{17}$$

Then, the error at the last iteration becomes:

$$ey_{k+1} = ey_k - C\dot{\eta}_{k+1}$$
$$= -CA\eta_{k+1} - CB\tilde{u}_{k+1} + ey_k - CH\tilde{w}_{k+1} \tag{18}$$

Then, we consider the following system:

$$\begin{cases} \dot{\eta}_{k+1} = A\eta_{k+1} + B\tilde{u}_{k+1} + H\tilde{w}_{k+1} \\ ey_{k+1} = -CA\eta_{k+1} - CB\tilde{u}_{k+1} + ey_k - CH\tilde{w}_{k+1} \end{cases} \tag{19}$$

We can set the system (19) in the following form [17]**:**

$$\begin{cases} \dot{\eta}_{k+1} = \mathrm{A}\eta_{k+1} + \mathrm{B}u_{k+1} + \mathrm{B}_0 ey_k + \mathrm{B}_{11}\tilde{w}_{k+1} \\ ey_{k+1} = \mathrm{C}\eta_{k+1} + \mathrm{D}u_{k+1} + \mathrm{D}_0 ey_k + \mathrm{B}_{12}\tilde{w}_{k+1} \end{cases} \tag{20}$$

With:

$\mathrm{A} = A,\ \mathrm{B} = B,\ \mathrm{B}_0 = 0,\ \mathrm{B}_{11} = H,\ \mathrm{C} = -CA,$

$\mathrm{D} = -CB,\ \mathrm{D}_0 = I$ and $\mathrm{B}_{12} = -CH$

We define the following lyapunov function:

$$V = V_1 + V_2 \tag{21}$$

Where:

$$\begin{cases} V_1 = \eta_{k+1}{}^T P_1 \eta_{k+1} \\ V_2 = ey_{k+1}^T P_2 ey_{k+1} \end{cases} \tag{22}$$

We denote the following expression:

$$\Delta V = \dot{V}_1 + \Delta V_2 \tag{23}$$

We will determine:

$$\dot{V}_1 = \eta_{k+1}{}^T(\mathrm{A}^T P_1 + P_1\mathrm{A} - K^T\mathrm{B}^T P_1 - P_1\mathrm{B}K)\eta_{k+1} + \eta_{k+1}{}^T P_1 \mathrm{B}\Gamma ey_k$$
$$+ ey_k{}^T\Gamma^T\mathrm{B}^T P_1\eta_{k+1} + \eta_{k+1}{}^T P_1\mathrm{B}_{11}\tilde{w}_{k+1} + \tilde{w}_{k+1}{}^T\mathrm{B}_{11}{}^T P_1\eta_{k+1} \tag{24}$$

Now with the help of equality (20), we determine the following expression:

$$\Delta V_2 = ey_{k+1}^T P_2 ey_{k+1} - ey_k^T P_2 ey_k \tag{25}$$
$$= (\eta_{k+1}{}^T\mathrm{C}^T - \eta_{k+1}{}^T K^T\mathrm{D}^T + ey_k{}^T\Gamma^T\mathrm{D}^T + \tilde{w}_{k+1}{}^T\mathrm{B}_{12}{}^T$$
$$+ ey_k^T\mathrm{D}_0{}^T)P_2(\mathrm{C}\eta_{k+1} + \mathrm{D}u_{k+1} + \mathrm{D}_0 ey_k + \mathrm{B}_{12}\tilde{w}_{k+1}) - ey_k^T P_2 ey_k$$

$$= \begin{bmatrix} \eta_{k+1} \\ ey_k \\ \tilde{w}_{k+1} \end{bmatrix}^T \begin{bmatrix} \begin{pmatrix} \mathrm{C}^T P_2\mathrm{C} \\ -\mathrm{C}^T P_2\mathrm{DK} \\ -K^T\mathrm{D}^T P_2\mathrm{C} \\ +K^T\mathrm{D}^T P_2\mathrm{DK} \end{pmatrix} & (*) & (*) \\ \begin{pmatrix} \Gamma^T\mathrm{D}^T P_2\mathrm{C} \\ -\Gamma^T\mathrm{D}^T P_2\mathrm{DK} \\ +\mathrm{D}_0{}^T P_2\mathrm{C} \\ -\mathrm{D}_0{}^T P_2\mathrm{DK} \end{pmatrix} & \begin{pmatrix} \Gamma^T\mathrm{D}^T P_2\mathrm{D}\Gamma \\ +\Gamma^T\mathrm{D}^T P_2\mathrm{D}_0 \\ +\mathrm{D}_0{}^T P_2\mathrm{D}\Gamma \\ +\mathrm{D}_0{}^T P_2\mathrm{D}_0 \\ -P_2 \end{pmatrix} & (*) \\ \begin{pmatrix} \mathrm{B}_{12}{}^T P_2\mathrm{C} \\ -\mathrm{B}_{12}{}^T P_2\mathrm{DK} \end{pmatrix} & \begin{pmatrix} \mathrm{B}_{12}{}^T P_2\mathrm{D}\Gamma \\ +\mathrm{B}_{12}{}^T P_2\mathrm{D}_0 \end{pmatrix} & \begin{pmatrix} \mathrm{B}_{12}{}^T P_2\mathrm{B}_{12} \end{pmatrix} \end{bmatrix} \begin{bmatrix} \eta_{k+1} \\ ey_k \\ \tilde{w}_{k+1} \end{bmatrix} \tag{26}$$

The Hamilton Jacobi Bellman equality is described as follows:

$$\mathrm{H} = \Delta V + ey_k^T ey_k - \gamma^2\tilde{w}_{k+1}^T\tilde{w}_{k+1} \tag{27}$$

We can set the equality (27) in the following form:

$$\mathrm{H} = \begin{bmatrix} \Omega_{11} & (*) & (*) \\ \Omega_{21} & \Omega_{22} & (*) \\ \Omega_{31} & \Omega_{32} & \Omega_{33} \end{bmatrix} \tag{28}$$

The system is stable if and only if the Jacobi Bellman equality is definite negative.

To determine this matrix we will determine the components of the correspondent matrix.

$$\Omega_{11} = \mathrm{A}^T P_1 + P_1\mathrm{A} - K^T\mathrm{B}^T P_1 - P_1\mathrm{B}K$$
$$+ \mathrm{C}^T P_2\mathrm{C} - \mathrm{C}^T P_2\mathrm{DK} - K^T\mathrm{D}^T P_2\mathrm{C} \tag{29}$$
$$+ K^T\mathrm{D}^T P_2\mathrm{DK}$$

$$\Omega_{21} = \Gamma^T B^T P_1 + \Gamma^T D^T P_2 C - \Gamma^T D^T P_2 DK + D_0{}^T P_2 C - D_0{}^T P_2 DK \quad (30)$$

$$\Omega_{22} = \Gamma^T D^T P_2 D\Gamma + \Gamma^T D^T P_2 D_0 + D_0{}^T P_2 D\Gamma + D_0{}^T P_2 D_0 - P_2 + I \quad (31)$$

$$\Omega_{31} = B_{11}{}^T P_1 + B_{12}{}^T P_2 C - B_{12}{}^T P_2 DK \quad (32)$$

$$\Omega_{32} = B_{12}{}^T P_2 D\Gamma + B_{12}{}^T P_2 D_0 \quad (33)$$

$$\Omega_{33} = B_{12}{}^T P_2 B_{12} - \gamma^2 I \quad (34)$$

Now we will set the equality (28) in the following form:

$$\begin{bmatrix} \begin{bmatrix} \Omega_{11} & (*) \\ \Omega_{21} & \Omega_{22} \end{bmatrix} & \begin{bmatrix} (*) \\ (*) \end{bmatrix} \\ \begin{bmatrix} \Omega_{31} & \Omega_{32} \end{bmatrix} & \begin{bmatrix} \Omega_{33} \end{bmatrix} \end{bmatrix} \quad (35)$$

Next, we will identify each part of the (35):

$$\Omega_{33} = -\gamma^2 I + \begin{bmatrix} 0 \\ B_{12} \end{bmatrix}^T \begin{bmatrix} P_3 & 0 \\ 0 & P_2 \end{bmatrix} \begin{bmatrix} 0 \\ B_{12} \end{bmatrix} = -\gamma^2 I + \hat{D}_1^T \bar{S} \hat{D}_1 \quad (36)$$

$$\begin{bmatrix} \Omega_{31} & \Omega_{32} \end{bmatrix} = \begin{bmatrix} B_{11} \\ 0 \end{bmatrix}^T \begin{bmatrix} P_1 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 \\ B_{12} \end{bmatrix}^T \begin{bmatrix} P_3 & 0 \\ 0 & P_2 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ C - DK & D_0 + D\Gamma \end{bmatrix} = \hat{B}_1^T P + \hat{D}_1^T \bar{S} \hat{A}_2 \quad (37)$$

$$\begin{bmatrix} \Omega_{11} & (*) \\ \Omega_{21} & \Omega_{22} \end{bmatrix} = \begin{bmatrix} A - BK & B_0 + B\Gamma \\ 0 & 0 \end{bmatrix}^T \begin{bmatrix} P_1 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} P_1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} A + BK & B_0 + B\Gamma \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ C - DK & D_0 + D\Gamma \end{bmatrix}^T \begin{bmatrix} P_3 & 0 \\ 0 & P_2 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ C - DK & D_0 + D\Gamma \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & I \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ 0 & P_2 \end{bmatrix}$$

$$= \hat{A}_1^T P + P\hat{A}_1 + \hat{A}_2^T \bar{S} \hat{A}_2 + \bar{L}^T \bar{L} - R \quad (38)$$

With: $\bar{S} = \begin{bmatrix} P_3 & 0 \\ 0 & P_2 \end{bmatrix}$, $\hat{D}_1 = \begin{bmatrix} 0 \\ B_{12} \end{bmatrix}$, $\hat{B}_1 = \begin{bmatrix} B_{11} \\ 0 \end{bmatrix}$,

$P = \begin{bmatrix} P_1 & 0 \\ 0 & 0 \end{bmatrix}$, $\hat{A}_2 = \begin{bmatrix} 0 & 0 \\ C - DK & D_0 + D\Gamma \end{bmatrix}$,

$\hat{A}_1 = \begin{bmatrix} A - BK & B_0 + B\Gamma \\ 0 & 0 \end{bmatrix}$, $R = \begin{bmatrix} 0 & 0 \\ 0 & P_2 \end{bmatrix}$ and

$\bar{L}^T \bar{L} = \begin{bmatrix} 0 & 0 \\ 0 & I \end{bmatrix}$

Then, the Hamilton equality becomes:

$$\Theta = \begin{bmatrix} \begin{pmatrix} \hat{A}_1^T P + P\hat{A}_1 \\ + \hat{A}_2^T \bar{S} \hat{A}_2 \\ + \bar{L}^T \bar{L} - R \end{pmatrix} & \begin{pmatrix} P\hat{B}_1 \\ + \hat{A}_2^T \bar{S} \hat{D}_1 \end{pmatrix} \\ \begin{pmatrix} \hat{B}_1^T P \\ + \hat{D}_1^T \bar{S} \hat{A}_2 \end{pmatrix} & \begin{pmatrix} \hat{D}_1^T \bar{S} \hat{D}_1 \\ -\gamma^2 I \end{pmatrix} \end{bmatrix} \quad (39)$$

$$= \begin{bmatrix} \hat{A}_1^T P + P\hat{A}_1 + \bar{L}^T \bar{L} - R & P\hat{B}_1 \\ \hat{B}_1^T P & -\gamma^2 I \end{bmatrix} + \begin{bmatrix} \hat{A}_2^T \bar{S} \hat{A}_2 & \hat{A}_2^T \bar{S} \hat{D}_1 \\ \hat{D}_1^T \bar{S} \hat{A}_2 & \hat{D}_1^T \bar{S} \hat{D}_1 \end{bmatrix}$$

$$= \begin{bmatrix} \hat{A}_1^T P + P\hat{A}_1 + \bar{L}^T \bar{L} - R & P\hat{B}_1 \\ \hat{B}_1^T P & -\gamma^2 I \end{bmatrix} - \begin{bmatrix} \bar{S}\hat{A}_2 \\ \bar{S}\hat{D}_1 \end{bmatrix}^T (-\bar{S}^{-1}) \begin{bmatrix} \bar{S}\hat{A}_2 & \bar{S}\hat{D}_1 \end{bmatrix} \quad (40)$$

Appling the bounded real lemma, the equality (40) becomes as follow:

$$\Theta = \begin{bmatrix} -\bar{S} & \bar{S}\hat{A}_2 & \bar{S}\hat{D}_1 \\ \bar{S}\hat{A}_2^T & \hat{A}_1^T P + P\hat{A}_1 + \bar{L}^T \bar{L} - R & P\hat{B}_1 \\ \bar{S}\hat{D}_1^T & \hat{B}_1^T P & -\gamma^2 I \end{bmatrix} = \begin{bmatrix} -\bar{S} & \bar{S}\hat{A}_2 & \bar{S}\hat{D}_1 \\ \bar{S}\hat{A}_2^T & \hat{A}_1^T P + P\hat{A}_1 - R & P\hat{B}_1 \\ \bar{S}\hat{D}_1^T & \hat{B}_1^T P & -\gamma^2 I \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & \bar{L}^T \bar{L} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -\bar{S} & \bar{S}\hat{A}_2 & \bar{S}\hat{D}_1 \\ \hat{A}_2^T \bar{S} & \hat{A}_1^T P + P\hat{A}_1 - R & P\hat{B}_1 \\ \hat{D}_1^T \bar{S} & \hat{B}_1^T P & -\gamma^2 I \end{bmatrix} + \begin{bmatrix} 0 \\ \bar{L} \\ 0 \end{bmatrix}^T I \begin{bmatrix} 0 & \bar{L} & 0 \end{bmatrix}$$

With the help of the lemma (2), we obtain:

$$\Theta = \begin{bmatrix} -\bar{S} & \bar{S}\hat{A}_2 & \bar{S}\hat{D}_1 & 0 \\ \hat{A}_2^T \bar{S} & \begin{pmatrix} \hat{A}_1^T P \\ +P\hat{A}_1 - R \end{pmatrix} & P\hat{B}_1 & \bar{L}^T \\ \hat{D}_1^T \bar{S} & \hat{B}_1^T P & -\gamma^2 I & 0 \\ 0 & \bar{L} & 0 & -I \end{bmatrix} \quad (41)$$

Replace the variable with theirs appropriates expressions in the equality (41), it becomes as follow:

$$\Theta = \begin{bmatrix} -P_3 & (*) & (*) & (*) & (*) & (*) \\ 0 & -P_2 & (*) & (*) & (*) & (*) \\ 0 & \begin{pmatrix} C^T P_2 \\ +K^T D^T P_2 \end{pmatrix} & \begin{pmatrix} A^T P_1 \\ +P_1 A \\ -K^T B^T P_1 \\ -P_1 BK \end{pmatrix} & (*) & (*) & (*) \\ 0 & \begin{pmatrix} D_0^T P_2 \\ +\Gamma^T D^T P_2 \end{pmatrix} & \begin{pmatrix} B_0^T P_1 \\ +\Gamma^T B^T P_1 \end{pmatrix} & -P_2 & (*) & (*) \\ 0 & B_{12}^T P_2 & B_{11}^T P_1 & 0 & -\gamma^2 I & 0 \\ 0 & 0 & 0 & I & 0 & -I \end{bmatrix} \quad (42)$$

We define the following expression:

$$T = \begin{bmatrix} P_3^{-1} & 0 & 0 & 0 & 0 & 0 \\ 0 & P_2^{-1} & 0 & 0 & 0 & 0 \\ 0 & 0 & P_1^{-1} & 0 & 0 & 0 \\ 0 & 0 & 0 & P_2^{-1} & 0 & 0 \\ 0 & 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & 0 & I \end{bmatrix} \quad (43)$$

With the help of property 1, we will determine:

$$T^T \Theta T = \begin{bmatrix} -P_3^{-1} & (*) & (*) & (*) & (*) & (*) \\ 0 & -P_2^{-1} & (*) & (*) & (*) & (*) \\ 0 & P_1^{-1} C^T + P_1^{-1} K^T D^T & P_1^{-1} A^T + A P_1^{-1} - P_1^{-1} K^T B^T - BK P_1^{-1} & (*) & (*) & (*) \\ 0 & P_2^{-1} D_0^T + P_2^{-1} \Gamma^T D^T & P_2^{-1} B_0^T + P_2^{-1} \Gamma^T B^T & -P_2^{-1} & (*) & (*) \\ 0 & B_{12}^T & B_{11}^T & 0 & -\gamma^2 I & 0 \\ 0 & 0 & 0 & P_2^{-1} & 0 & -I \end{bmatrix}$$

Where $w_1 = P_1^{-1}$, $w_2 = P_2^{-1}$, $w_3 = P_3^{-1}$, $N_1 = KP_1^{-1}$ and $N_2 = \Gamma P_2^{-1}$

After applying the modification, we can set that:

$$T^T \Theta T = \begin{bmatrix} -w_3 & (*) & (*) & (*) & (*) & (*) \\ 0 & -w_2 & (*) & (*) & (*) & (*) \\ 0 & w_1 C^T + N_1^T D^T & w_1 A^T + A w_1 - N_1^T B^T - B N_1 & (*) & (*) & (*) \\ 0 & w_2 D_0^T + N_2^T D^T & w_2 B_0^T + N_2^T B^T & -w_2 & (*) & (*) \\ 0 & B_{12}^T & B_{11}^T & 0 & -\gamma^2 I & 0 \\ 0 & 0 & 0 & w_2 & 0 & -I \end{bmatrix} \quad (44)$$

The matrix (44) doesn't depend from w_3 then we can eliminate the first row and the first column from this matrix, we obtain the following expression:

$$\Pi = \begin{bmatrix} -w_2 & (*) & (*) & (*) & (*) \\ w_1 C^T + N_1^T D^T & w_1 A^T + A w_1 - N_1^T B^T - B N_1 & (*) & (*) & (*) \\ w_2 D_0^T + N_2^T D^T & w_2 B_0^T + N_2^T B^T & -w_2 & (*) & (*) \\ B_{12}^T & B_{11}^T & 0 & -\gamma^2 I & 0 \\ 0 & 0 & w_2 & 0 & -I \end{bmatrix} \quad (45)$$

Now, we replace the variables with theirs appropriate expression, we obtain:

$$\Pi = \begin{bmatrix} -w_2 & (*) & (*) & (*) & (*) \\ -w_1 A^T C^T - N_1^T B^T C^T & w_1 A^T + A w_1 - N_1^T B^T - B N_1 & (*) & (*) & (*) \\ w_2 - N_2^T B^T C^T & N_2^T B^T & -w_2 & (*) & (*) \\ -H^T C^T & H^T & 0 & -\gamma^2 I & 0 \\ 0 & 0 & w_2 & 0 & -I \end{bmatrix} \quad (46)$$

By the LMIs techniques, to determine the unknown variables P_1, P_2, K and Γ, we assume the sufficient conditions:

$w_1 \succ 0$, $w_2 \succ 0$, $K \succ 0$, $\Gamma \succ 0$ and $\Pi \prec 0$ (47)

IV. SIMULATION EXAMPLE

To illustrate the efficiency of the robust iterative learning control law presented in this paper, considers the system (1):

$$\begin{cases} \dot{x}_k = \begin{bmatrix} -0\cdot 8 & -0\cdot 22 \\ 1 & 0 \end{bmatrix} x_k + \begin{bmatrix} 0\cdot 5 \\ 1 \end{bmatrix} u_k + \begin{bmatrix} 1 \\ 1 \end{bmatrix} w_k \\ y_k = \begin{bmatrix} 1 & 0\cdot 5 \end{bmatrix} x_k \end{cases} \quad (48)$$

Our objective is to track the reference model given by:

$$\begin{cases} \dot{x}_d = \begin{bmatrix} -0\cdot 8 & -0\cdot 22 \\ 1 & 0 \end{bmatrix} x_d + \begin{bmatrix} 0\cdot 5 \\ 1 \end{bmatrix} u_d \\ y_d = \begin{bmatrix} 1 & 0\cdot 5 \end{bmatrix} x_d \end{cases} \quad (49)$$

The disturbance applied to the system is assumed as:

$w_k(t) = \sin(2 * \pi * t)$ (50)

The desired control is a sinusoidal signal of frequency 1 Hz:

$u_d(t) = 2 * \sin(2 * \pi * t)$ (60)

We assume zero initial conditions.

We present in the next section the simulation results during the iterations number 1, 10, 20 and 50. The fig.1, fig.2, fig.3 and fig.4 show the simulation result, for our proposed scheme, of the output and the desired trajectory. The fig.5, fig.6, fig.7 and fig.8 illustrate the error trajectory. The next figure shows the H infinity and gamma trajectory.

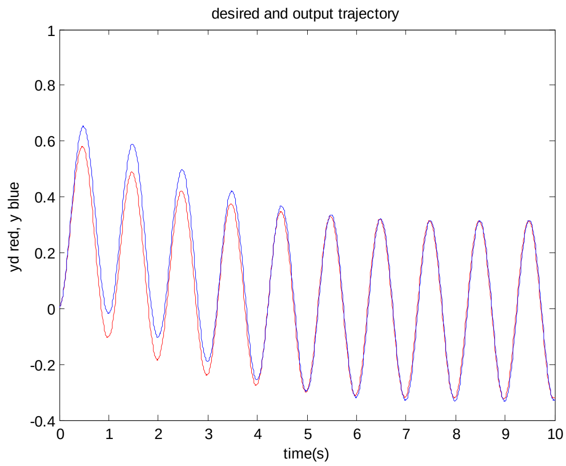

Fig. 1. The Output and the desired trajectory in the first iteration.

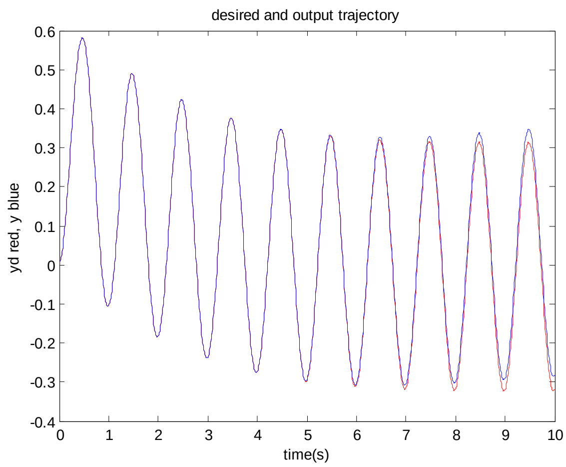

Fig. 2. The Output and the desired trajectory in the iteration number 10.

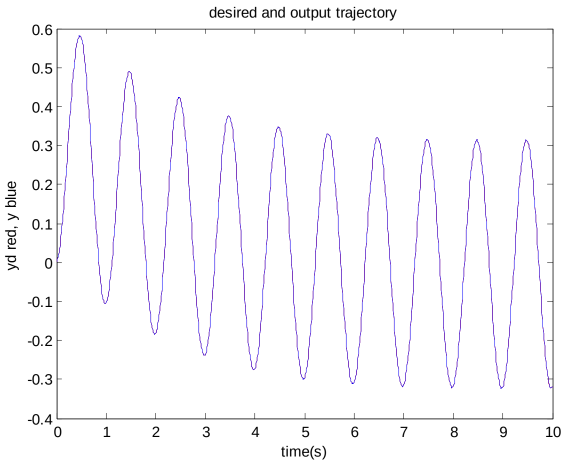

Fig. 3. The Output and the desired trajectory in the iteration number 20.

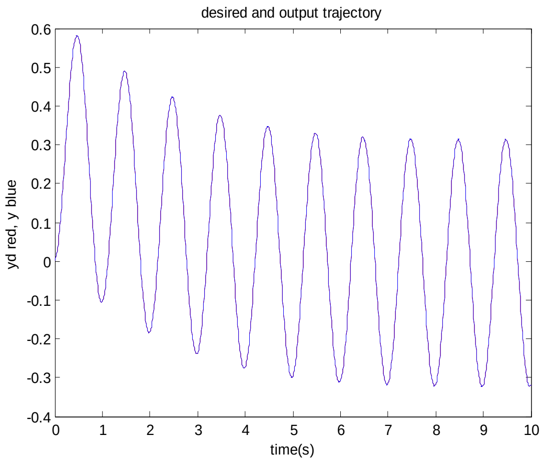

Fig. 4. The Output and the desired trajectory in the iteration number 50.

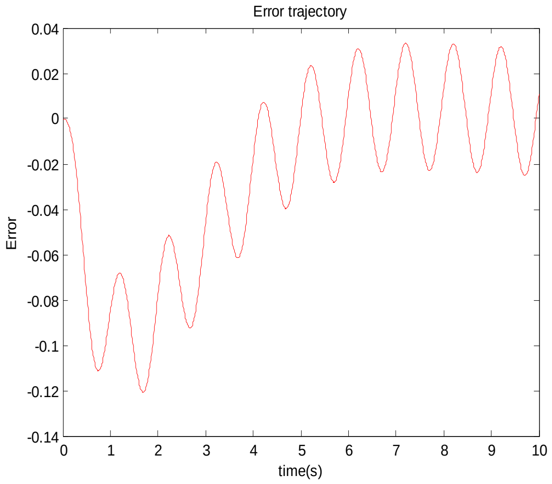

Fig. 5. The error trajectory in the first iteration.

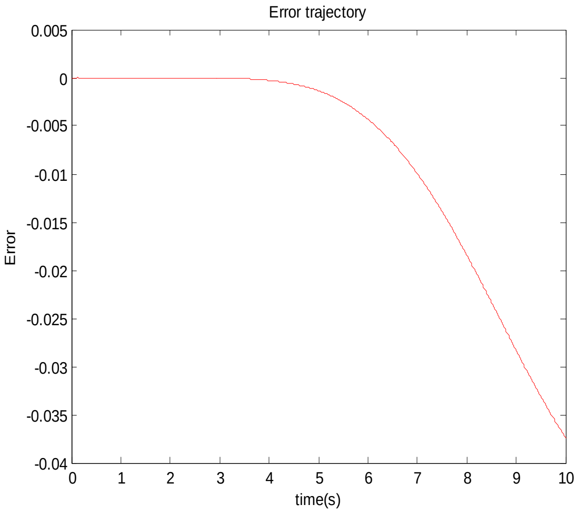

Fig. 6. The error trajectory in the iteration number 10.

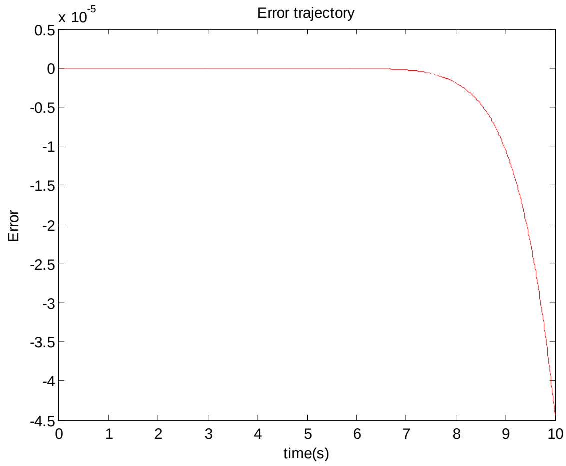

Fig 7. The error trajectory in the iteration number 20.

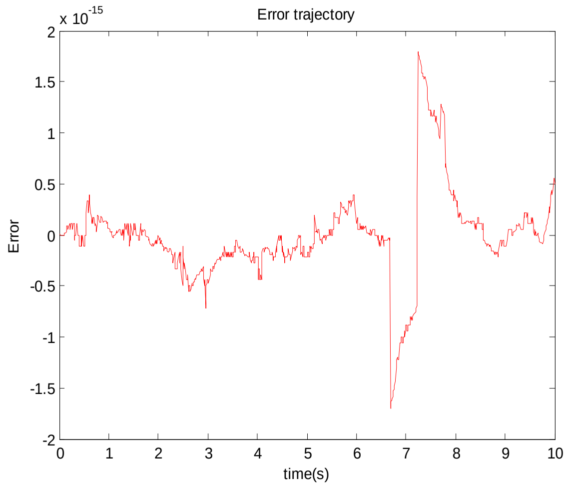

Fig. 8. The error trajectory in the iteration number 50.

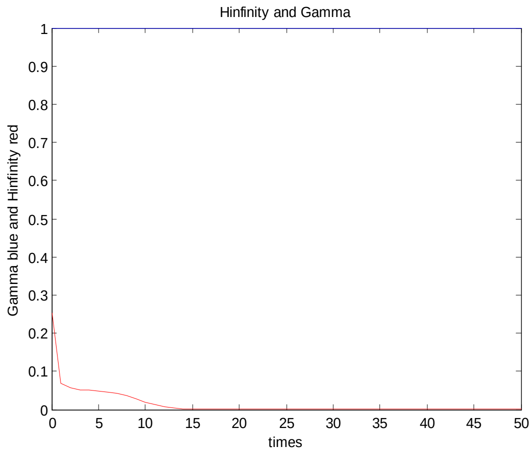

Fig. 9. H infinity and gamma trajectory.

We can conclude that our system is monotonically convergent. The error trajectory tends to zero. We respected the H infinity norm which is les then gamma during the iteration of the system.

V. Conclusion

A robust monotonic convergence for linear uncertain continuous time system is presented in this paper. A robust and an iterative learning control are studied to achieve this convergence. The updating law is physically based in an H infinity setting where the required conditions are LMI based witch can directly determine the learning gains.

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