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Applications of Delaunay Triangulation (DT), Steiner Trees, Spanning Trees and DT-Minimum Spanning Trees in MANET Multicasting

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ABSTRACT
The issues of graph theory concepts always proved their mathematical vital roles in various fields such as data mining, cloud computing, computer networks. This work focuses on applications of graph theory structures such as Delaunay triangulations, Steiner trees and spanning trees in MANET multicasting. Because of its inherent simplicity and potential to be as natural models, graph theory has a very wide range of applications in engineering, physics, social, biological sciences, linguistics, and numerous other areas. A graph can be used to represent almost any physical situation involving discrete objects and a relationship among them.
In this paper, the graph theory structures namely minimum spanning tree is computed on the Delaunay triangulations constructed on the MANET. The analysis of minimum spanning tree and Delaunay triangulation is carried out and the simulation experiment is done using MATLAB.
Keywords: Steiner Tree, Delaunay Triangulations, Minimum Spanning trees, Multicasting in *MANET*.

I. INTRODUCTION

Graph theoretical ideas are highly utilized by computer science applications, principally in research areas of *computer* science such as data mining, image segmentation, clustering, image capturing, networking etc., For example a data structure tree finds its application in data mining, computer networks, artificial intelligence, etc can be designed in the form of tree which in turn utilized vertices and edges. Similarly modeling of network topologies can be done using graph concepts. Using graph mathematical models, a plenty of simulation research work is done in several areas including MANETs.

Several issues such as node density, mobility of the nodes, link formation between nodes, connectivity, clustering, network partition, flooding analysis, analysis of network energy, no. of packets transmitted, etc. are investigated employing the graph theory concepts as the models. In the simulation study of MANETs, concepts of graph theory such as random graph theory are utilized [1][2][3][4][5].

Graph theoretical concepts are widely used study to molecules, atoms,construction of bonds in chemistry and the study of atoms.Graph theory finds its application in sociology for example to measure actors prestige or to explore diffusion mechanisms.

In MANET, the network topology changes randomly and rapidly at unpredictable times due to node mobility. Consequently the network topology is susceptible to frequent link failure and network partitioning, which could add to extensive routing overhead, excessive transmission delay and packet loss among mobile nodes. It is highly necessary to come across a solution which is mobility resilient and still achieves the desired network performance. Therefore, investigation of properties of node mobility and its effect on topological dynamics of MANET gains importance. The performance of routing protocols depends on link status of any pair of nodes [21].

The study of link properties is based on mobility models, which specify node moving behaviours in a MANET [22]. The most broadly used mobility models in the present research of MANETs are Random mobility models [23], such as random waypoint (RWP) model [24]. It is worth mentioning that graph models make their relevance in wide variety of research areas including routing in MANET.
This paper is organized as follows: Section 2 presents relevant literature survey of applications of graph theory structures. Network model, simulation methodology and Simulation results are discussed in Section 3. Section 4 concludes the paper.

II. GRAPH THEORY APPLICATIONS IN MULTICASTING IN WIRELESS MOBILE AD HOC NETWOEKS

Multicasting is an effective way to communicate among multiple hosts in a network. It outperforms the basic broadcast strategy by sharing resources along general links, while sending information to a set of predefined multiple destinations concurrently. However, it is vulnerable to component failure in ad hoc network due to the lack of redundancy, multiple paths and multicast tree structure.

Graph theory finds its large applications in multicasting through spanning trees and Steiner trees. The objectives of using multicast spanning tree are as follows:

- Every member of the group should receive one, and only one, copy of the multicast packet. Receipt of multiple copies is not allowed.
- Nonmembers must not receive a copy.
- There must be no loops in routing; that is, a packet must not visit a router more than once.
- The path traveled from the source to each destination must be optimal (the shortest path).

Two types of trees are used for multicasting: source-based trees and group-shared trees. In the source-based tree method, a single tree is made for each combination of source and group. In other words, the formation of the tree is based on both the source and the group. If there are N different groups and M different sources in the system, there can be a maximum of N X M different trees, one for each source-group combination.

In the group-shared tree method, each group in the system shares the same tree. If there are N groups in the whole system, there is a maximum of N trees, one for each group.

Several research works in the literature have shown the extensive applications of minimum spanning trees, Steiner trees to address the problem of broadcasting and multicasting in large scale multi hop *ad hoc* wireless networks.

The work in [13] presents a minimum spanning tree based on energy aware multicast protocol (MSTEAM), which is a localized geographic multicast routing scheme designed for ad hoc and sensor networks. It uses locally-built minimum spanning trees (MST) as an efficient approximation of the optimal multicasting backbone. Using an MST is highly relevant in the context of dynamic wireless networks since its computation has a low time complexity (O (n log n)). The study in several research works including [16] proposes a *Minimum Spanning Tree* (MST) based topology control algorithm, called *Local Minimum Spanning Tree* (LMST), for multi-hop wireless networks with limited mobility.

Another important concept in graph theory is due to Jakob Steiner called Steiner tree problem (STP). In combinatorial optimization, the Steiner tree problem is to find the shortest interconnects for a given set of objects.

At the highest level, STP problem is superficially similar to the minimum spanning tree problem: given a set *V* of points (vertices), interconnect them by a network (graph) of shortest length, where the length is the sum of the lengths of all edges. The difference between the Steiner tree problem and the minimum spanning tree problem is that, in the Steiner tree problem, extra intermediate vertices and edges may be added to the graph in order to reduce the length of the spanning tree. These new vertices introduced to decrease the total length of connection are known as Steiner points or Steiner vertices. It has been proved that the resulting connection is a tree, known as the Steiner tree. There may be several Steiner trees for a given set of initial vertices [14] [17].

The Steiner tree problem has applications in circuit layout design and multicast applications including wireless networks. Further, Steiner trees can find its application in multicast communications for example as a suitable topology for multicast communications in point-to-point networks [14]. Most versions of the Steiner tree problem are NP-complete. However, some restricted cases can be solved in polynomial time [14].

In [4], the novel algorithm for computing Steiner K-Connected Minimum multicast Spanning Tree based on Connectivity Index is designed and demonstrated that restoring the connectivity when network gets disconnected using Steiner edges.

2.1 Delaunay Triangulations

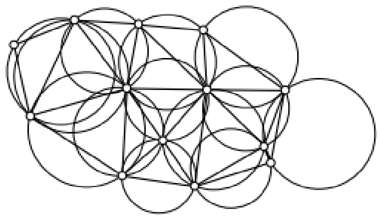

Figure 1 : Delaunay Traingulations

A Delaunay triangulation is set of points making various triangles where in there is no points inside the boundary of the triangles. Delaunay triangulation is a proximal method that satisfies the requirement that a circle drawn through the three nodes of a triangle will contain no other node as shown in the above Figure 1.

The Delaunay triangulation of a point set S, introduced by Boris Nikolaevich Delaunay in 1934, is characterized by the empty circumdisk property: no point in S lies in the interior of any triangle's circumscribing disk. Delaunay triangulations play an important role in computational geometry.

Let N be a set of points in the plane (in general position), and let T be a triangulation of N. Then T is a Delaunay Triangulation of N if and only if the circumcircle of any triangle of T does not contain any other point of N in its interior (i.e. T fulfills the circle criterion).

For N consisting of n points, all triangulations contain 2n-2-k triangles, 3n-3-k edges, where n = number of points in N and k = number of points on convex hull of N.

One of the advantages of Delaunay triangulations, as opposed to triangulations constructed heuristically is that they automatically avoid forming triangles with small included angles whenever this is possible.

The Delaunay triangulation of a set of points in the plane, and its dual the Voronoi diagram – are

probably one of the most basic spatial structures in computational geometry. Their underlying theory has been extensively developed, and a vast number of practical applications are based on them [9].
In [10], the Delaunay triangulation has been shown to possess a number of optimality properties.

2.2 Delaunay Triangulation Applications in MANET Routing

The research works in Ad-Hoc Wireless routing has attracted many researchers and a large depository of the research literature is available. It is worth to mention that research work is still in learning stage. Since MANET has enormous applications, substantial varieties of wireless devices are ever increasing, the expectations of different quality of services (QoS) are to be meet in the perspective of end user and the network are also increasing. Therefore, further work in this direction is still necessary, relevant and present works may also need updating.

In the literature survey, several routing algorithms which employs DT concept are found. Luan Lan and Hsu Wen-Jing [5] have presented a new topology construction scheme based on the idea of Local Delaunay Triangulation for efficient routing. The time complexity for the construction and maintenance of the graph is also well within acceptable limits.
The work in [10] has shown that Computational Geometry is relevant in attempts to build stable, low power routing schemes in MANET.
As discussed in [6], Unmanned Air Vehicles (UAVs) can provide important communication advantages to ground-based wireless ad hoc networks. The location and movement of UAVs are optimized to improve the connectivity of a wireless network. The work proposes simple algorithm that improves connectivity using Delaunay triangulation.

Several localized routing protocols have shown consistent performance in terms of packet delivery guarantee when the underlying network topology is a planar graph [7][8][9]. Relative neighborhood graph (RNG) or Gabriel graph (GG) is used typically as planar structure. However, it is well-known that the spanning ratios of these two graphs are not bounded by any constant (even for uniform randomly distributed points). Bose *et al.* [10][16] developed a localized routing protocol, called FACE, that guarantees that the distance travelled by the packets is within a constant factor of the minimum if Delaunay triangulation of all wireless nodes are used, in addition to guarantee the delivery of the packets. However, it is expensive to construct the Delaunay triangulation in a distributed manner.

In [8], for a set of wireless nodes, the network is modeled as a unit-disk graph (UDG) in which a link *uv* exists only if the distance ‖uv‖ is at most the maximum transmission range. Further [8], designs a novel localized networking protocol that constructs a planar 2.5-spanner of UDG, called the localized Delaunay triangulation LDEL, as network topology. It contains all edges that are both in the unit-disk graph and the Delaunay triangulation of all nodes. It is shown in [8] that the entire communication cost of networking protocol is $O(n \log n)$ bits, which is within a constant factor of the optimum to construct any structure in a distributed manner. The experiments in [8] demonstrate that the delivery rates of some of the existing localized routing protocols are improved when localized Delaunay triangulation is used instead of several earlier proposed topologies. In addition to this simulations experiments show that the travelled distance of the packets is significantly less when the FACE routing algorithm is applied on LDEL than applied on GG.

Luan Lan, Hsu Wen-Jing and Zhang Rui designed a location-aware routing protocol named MGPSR (Modified Greedy Perimeter Stateless Routing) for Mobile Ad Hoc Networks. MGPSR offers the crucial correctness guarantee of the well known Greedy Perimeter Stateless Routing (GPSR) protocol.

The recent research development of Bose et al [10], has a localized routing protocol that guarantees that the distance travelled by the packets is within a constant factor of the minimum if Delaunay triangulation of all wireless nodes is used, in addition, to guarantee the delivery of the packets. Conversely, the construction of the Delaunay triangulation in a distributed manner is costly.

III. NETWORK MODEL, SIMULATION METHODOLOGY & RESULTS

3.1 Network Model

In this work, a mobile ad hoc network where nodes communicate with their neighbors using wireless links with the following assumptions is used for computation of minimum spanning tree on the Delaunay Triangulations constructed on MANET with n nodes.

Assumptions:- The neighborhood of a node is the set of nodes which can receive a packet transmitted by the nodes. Any packet transmitted by a node is received by all its neighbors. The source node of a multicasting is believed to know the entire information essential to construct the multicast tree. We use the term edge and link interchangeably. The cost of an edge from u to v is same as v to u where (u, v) = E.

A network is modeled as an undirected, connected and weighted graph G = (V, E) with node set V and edge (link or arc) set E. An edge e Є E from u Є V to v Є V is represented by e = (u, v). Each link e = (u, v) Є E is associated with the cost of an edge Ψ(e) . The total cost of the spanning tree computed on DTs formed on the MANETs,
ΨC (MST-DT) = ∑ Ψ (e (i)) where e (i) Є (MST-DT).

During the course of literature study several researchers have iterated the importance of graph models. The random graph of Paul Erdos and Alfred

Renyi is one of the oldest and best wilful models of a network, and possesses the considerable advantage of being exactly solvable for many of its regular properties [20].

The MANET so constructed in this experiment is studied by adopting the Random Geometric Graph (RGG) Model. In this model the nodes at close range have a higher probability of being connected than nodes at farther distances.

3.2 Methodology

In order to study problem of constructing multiple minimum spanning tree on the DTs constructed, the relevant aspects of connected ad-hoc network are simulated. The simulation process starts with a minimal 1-connected network graph with n (which is fixed for each simulation) number of nodes. Then progressively, more links are added to build the network $\leq n(n-1)/2$ edges.

The methodology adopted is as follows:

1. Choose two nodes randomly and add the links between them.
2. Label the link with its weight.
3. Build the entire network with the maximum of n(n-1)/2 edges.
4. Compute the cost of the entire network (sum of the weight link of all the edges in a network).
5. Construct the Delaunay Triangulations on the entire network.
6. Compute the spanning tree having minimum cost, MST_DT (i.e. by selecting the link of the lowest weight)

The algorithm to compute MST on DTs constructed on MANET with n nodes is designed in the MATLAB environment.

3.3 Simulation Results

By adopting the methodology explained in Section 3 experiments are carried out (see Figure 2-Figure 6).

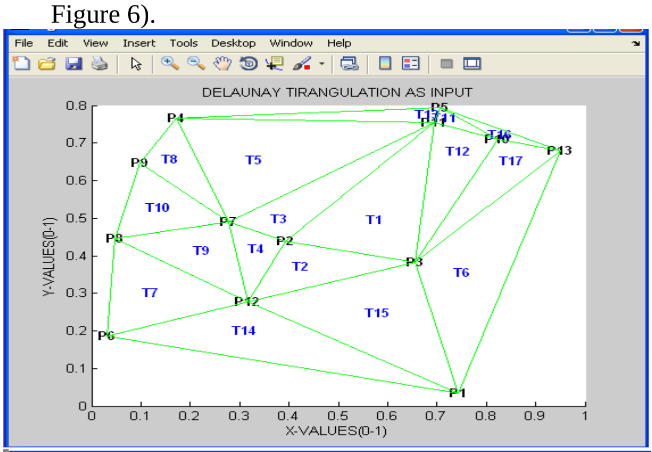

Figure 2: MANET Delaunay Triangulations [13 Nodes]

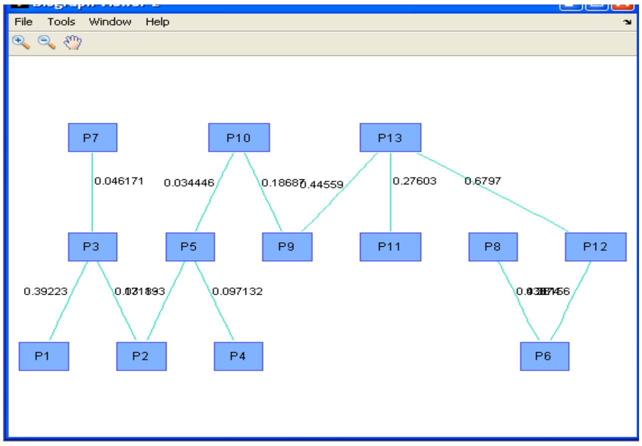

Figure 3 : Minimum Spanning Tree on DTs of Figure 2

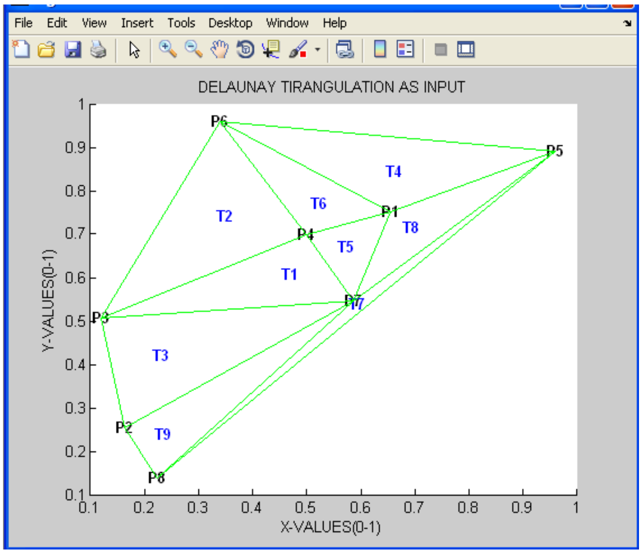

Figure 4: MANET Delaunay Triangulations constructed[8 Nodes]

Figure 2 shows the Delaunay Triangulations constructed in a MANET of 13 nodes represented as P1, P2…P13. Minimum spanning tree of 13 nodes represented with weights displayed on each of the edge is computed (shown in Figure 3) which is a multicast path corresponding to the MANET shown in Figure 2. Figure 4 shows the Delaunay Triangulation for MANET of 8 nodes represented as P1, P2,…,P8 along with the Triangles formed by joining these nodes and its corresponding minimum spanning tree is as shown in the Figure 5 below.

The simulation experiments are carried out on the MANET with 3 nodes to 25 nodes and their respective minimum spanning trees are computed. An instance of the DTs constructed on the MANET with 25 nodes is shown in the Figure 6.

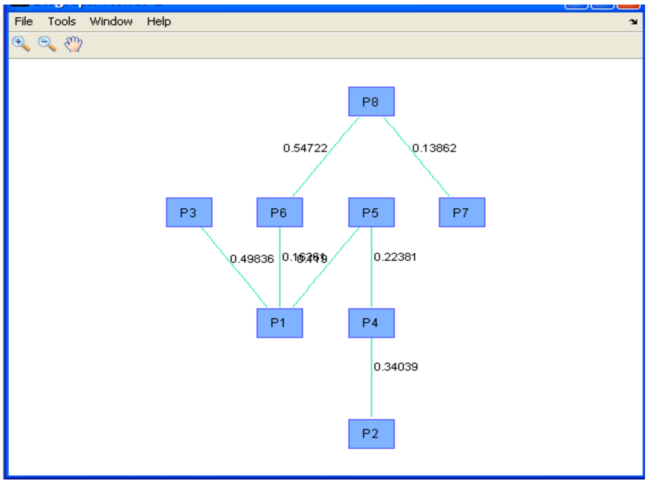

Figure 5 : Minimum Spanning Tree on DTs of Figure 4

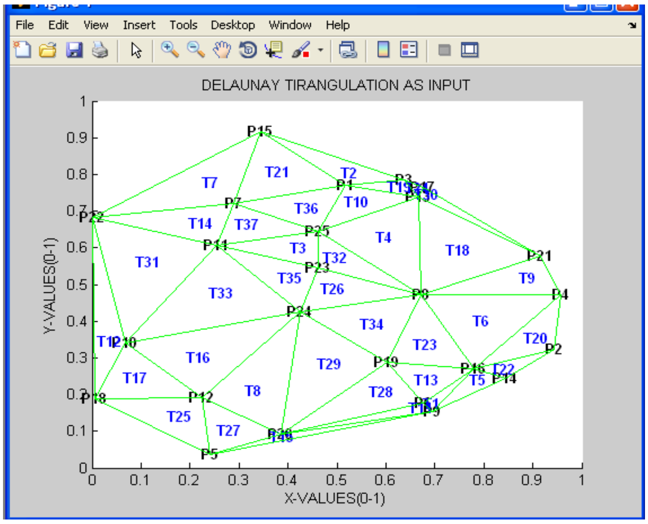

Figure 6: MANET Delaunay Triangulations constructed

IV. CONCLUSION

In this research attempt, the work highlights the applications of graph theory structures. In MATLAB environment, random network is modeled, Delaunay Triangulations are constructed, and corresponding spanning trees are computed. The minimum spanning tree computed are useful as multicast paths including MANETs. The study summarizes and point outs the applications of graph theory structures and models.

V. ACKNOWLEDGMENTS

Dr. ARUNKUMAR B.R., Professor and Head of the department, Master of Computer Applications, BMS Institute of Technology, Bangalore, Karnataka, India, Member BOS, VTU MCA , 2013-2016 has published nearly 21 research papers in National/International Journals and 11 papers in National/International Conferences including IEEE international conferences, wishes to place on record his sincere gratitude to BMS Education Trust, The principal, Dr.S.Venkateswaran, BMSIT, The Chief Mentor, Dr.S.C.Sharma, BMS Institute of Technology, Bangalore and expresses thanks to all those who helped in bringing out this paper.

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