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Fuzzy Reliability Analysis for the Effect of TRH Based On Gamma Distribution

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ABSTRACT
The theoretical study of serum prolactin response to TRH during antithyroid treatment in hyperthyroid patients was determined. Formulae of fuzzy gamma distributions, fuzzy reliability functions and its α-cut sets are presented. Using a Fuzzy reliability analysis based on gamma distribution, we showed that changes from normal serum levels of T_3 and T_4 are associated with changes in prolactin responses to TRH.
***Keywords* -** Fuzzy Gamma distribution, Fuzzy reliability function, Thyrotropin Releasing Hormone (TRH)
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I. INTRODUCTION

The most frequently used functions in lifetime data analysis are the reliability or survival function. This function gives the probability of an item operating for a certain amount of time without failure. The reliability of a system can be determined on the basis of tests or the acquisition of operational data. However due to the uncertainty and inaccuracy of this data the estimation of precise values of probabilities is very difficult in many systems. For this reason the fuzzy reliability concept has been introduced and formulated in the context of fuzzy measure.

Many methods and models in classical reliability theory assume that all parameters of lifetime density function are precise. But in the real world randomness and fuzziness are mixed up in the lifetime of the systems. In 1965, Zadeh [1] introduced fuzzy set theory. Subsequently, the theory and the mathematics of fuzzy sets were fleshed –out and applied in many research fields [2]. The theory of fuzzy reliability was proposed and developed by several authors [3], [4], [5], [6].

In this paper we propose a fuzzy reliability analysis for the effect of serum prolactin response to TRH during antithyroid treatment in hyperthyroid patients based on the gamma distribution. We determined the α - cuts of a fuzzy reliability function using incomplete gamma functions [7].

II. FUZZY GAMMA DISTRIBUTION

If λ and r are unknown we must estimate them from a random sample and we obtain a fuzzy estimator $\overline{\lambda}$ for λ and $\overline{r}$ for r. Now consider the probability density function of fuzzy gamma distribution for the fuzzy numbers $\overline{\lambda}$ and $\overline{r}$,

$$\overline{f}(t,\lambda,r)=\frac{\lambda^{r}t^{(r-1)}e^{-\lambda t}}{\Gamma(r)},\ t\geq 0,\quad \lambda\in\overline{\lambda}[\alpha],\ r\in\overline{r}[\alpha]$$

The fuzzy probability of obtaining a value in the interval [c, d], c > 0 is $p(c\leq X\leq d)$ and it's α -cut is defined as

$$\overline{P}[c,d][\alpha]=\int_{c}^{d}\overline{f}(t,\lambda,r)dt \,/\, \lambda\in\overline{\lambda}[\alpha], r\in\overline{r}[\alpha]$$

$$\overline{p}[c,d][\alpha]=[p^{L}[\alpha], p^{U}[\alpha]]$$

Where $p^{L}[\alpha] = \text{Min}\{\int_{c}^{d}\overline{f}(t,\lambda,r)dt\}$

$$p^{U}[\alpha] = \text{Max}\{\int_{c}^{d}\overline{f}(t,\lambda,r)dt\}$$

III. FUZZY RELIABILITY FUNCTION

Assume that X and U are two crisp sets. Let failure rate function be fuzzy and represented by a fuzzy set $\overline{H}(t)$, $\overline{H}(t)=\{h,\mu_{\overline{H}(t)} h\in X\}$. The α – cut fuzzy set of $\overline{H}(t)$ is $\overline{H}(t)=\{\alpha\in X/\mu_{\overline{H}(t)}\geq\alpha\}$.

Note that $\overline{H_{\alpha}}(t)$ is a crisp set. Suppose that $\overline{H}(t)$ is a fuzzy number. Then for each choice of α – cut, we have an interval $\overline{H}_{\alpha}(t)=\{h_1(t), h_2(t)\}$. By the convexity of the fuzzy number, the bounds of the interval are function of α and can be obtained as

$\overline{h}_{1\alpha} = \min \mu_{\overline{H}(t)}(\alpha)$ and

$\overline{h}_{2\alpha} = \max \mu_{\overline{H}(t)}(\alpha)$ respectively.

Let $\phi : X \to U$ be a bounded continuous differentiable function from X to U. We wish to calculate the fuzzy set (fuzzy reliability functions) induced on U by applying ϕ for the set $\overline{H}(t)$. If we write u= ϕ (h), where $h \in X$ and

$$\overline{R}(t) = \{u, \mu_{\overline{R}(t)}(u) / u = \phi(h),\ u \in U\}$$

then the membership function of $\overline{R}(t)$ is defined by the extension principle $\mu_{\overline{R}(t)}(u) = \sup_{h \in X}\{\mu_{\overline{H}(t)}(h) / u = \phi(h)$.

We know that if $\overline{H}(t)$ is normal and convex and ϕ is bounded, then $\overline{R}(t)$ is also normal and convex. Therefore we can calculate the corresponding interval $[r_1(t)\ , r_2(t)] = \phi\left(\overline{H}_\alpha(t)\right)$ Where $r_1(t)$ and $r_2(t)$ correspond, respectively, to the global minimum and maximum of ϕ over the space $\overline{H}_\alpha(t)$ at the α level.

$r_1(t)$ = min $\phi(h)$, such that $h_1(t) \le h \le h_2(t)$
$r_2(t)$ = max $\phi(h)$, such that $h_1(t) \le h \le h_2(t)$

The crisp reliability function of an object is R(t) = P(T $\geq$ t) = 1 – F(t). Now we define the fuzzy reliability by means of the fuzzy distribution function

$$\overline{R}(t) = \overline{P}(\overline{T} > t) = 1 - \overline{F}(t)\ \forall\ t \in [0, \infty)$$

where $\overline{T}$ is a fuzzy random variable which describes the vagueness of the time "t" and the uncertainty of the probability distribution whose distribution functions is $\overline{F}(x) = \overline{P}(X < x)$ and X is the random variable with gamma parameters.

The reliability function for gamma distribution is defined by

$$R(t) = \frac{1}{\Gamma(r)} \int_t^\infty \lambda^r u^{r-1} e^{-\lambda u} du \,/\, \lambda \in \overline{\lambda}[\alpha], r \in \overline{r}[\alpha]$$

$$= \frac{1}{\Gamma(r)} \Gamma(r, \lambda t) \,/\, \lambda \in \overline{\lambda}[\alpha], r \in \overline{r}[\alpha]$$

The α - cut of fuzzy reliability function for gamma distribution is

$$\overline{R}(t)[\alpha] = [R_1(\alpha), R_2(\alpha)]$$

Where

$$R_1[\alpha] = \text{Max}\ \frac{1}{\Gamma(r)} \Gamma(r, \lambda t) \,/\, \lambda \in \overline{\lambda}[\alpha], r \in \overline{r}[\alpha]$$

$$R_2[\alpha] = \text{Min}\ \frac{1}{\Gamma(r)} \Gamma(r, \lambda t) \,/\, \lambda \in \overline{\lambda}[\alpha], r \in \overline{r}[\alpha]$$

IV. APPLICATION

Thyrotropin-releasing hormone (TRH) has been shown to be at least as potent and rapid a stimulus of prolactin release as of thyrotropin (TSN) release in normal man. The TSH release stimulated by the administration of synthetic TRH is exquisitely sensitive to inhibition by thyroid hormones [8].

10 Patients (9 females, 1 male; age 16-56) with diffuse thyroid enlargement were diagnosed as having hyperthyroidism on the basis of elevated serum T_3 and T_4 levels, low-normal serum TSH levels, and T_3 resin uptake ratio that demonstrated no excessive serum protein binding of thyroid hormones. Each hyperthyroid patient's prolactin response to TRH was determined before any antithyroid treatment and again after the serum T_3 and T_4 levels had been lowered by antithyroid treatment.

Eight patients were treated with varying doses of methimazole and two patients were treated with radioactive iodide. The second TRH test was performed as soon as the patient was clinically and chemically euthyroid, which was from 1 to 8 months after the initiation of antithyroid treatment. The Table.1 shows the mean serum prolactin response of 10 patients with hyperthyroidism to 400 μg TRH during antithroid treatment.

Table: 1 Serum Prolactin response to TRH during antithyroid treatment in 10 patients with hyperthyroidism

Time(min)	Serum Prolactin (μg/ml)
0	0
10	52
15	48
20	49
30	49
45	35
60	33
90	22
120	18
180	15

Before antithyroid treatment, the mean serum prolactin level was 17.0 ± 2.0 ng/ml before TRH and reached a peak level of 29.5 ± 5.1ng/ml 10 min after TRH. After euthyroidism had been produced with anti-thyroid medication the mean serum prolactin level before TRH was 11.5 ± 2.3 ng/ml and reached a peak level of 51.54 ± 8.2 ng/ml 10 min after TRH.

The fall in the mean pre-TRH prolactin level was not statistically significant. The increase in the prolactin response to TRH after antithyroid treatment was examined in greater detail by comparing the maximum prolactin of each hyperthyroid subject before and during antithyroid treatment, the maximum prolactin was higher during antithyroid treatment in 9 of the 10 subjects.

During antithyroid treatment, the scale parameter and shape parameter of gamma distribution are respectively $\lambda = 0.268$ and r = 4.874. Here we assume that t = 60

Let the corresponding Triangular fuzzy numbers are

$$\bar{\lambda} = [0.263,\ 0.268,\ 0.275]$$

$$\bar{r} = [4.75,\ 4.874,\ 4.920]$$

and the corresponding α cuts are

$$\bar{\lambda}[\alpha] = [0.263 + 0.005\alpha,\ 0.275 - 0.007\alpha]$$

$$\bar{r}[\alpha] = [4.75 + 0.074\alpha,\ 4.920 - .0.046\alpha]$$

Table : 2 Alpha – cuts of the fuzzy reliability

Values of α	$\bar{r}[\alpha]$	$\bar{\lambda}[\alpha]$	$R_1[\alpha]$	$R_2[\alpha]$
0	[4.75, 4.92]	[0.2630, 0.2750]	0.0362	0.0465
0.1	[4.7574,4.9154]	[0.2635, 0.2743]	0.0368	0.0460
0.2	[4.7648,4.9108]	[0.2640, 0.2736]	0.0371	0.0456
0.3	[4.7722,4.9062]	[0.2645, 0.2729]	0.0373	0.0451
0.4	[4.7796,4.9016]	[0.2650, 0.2722]	0.0379	0.0441
0.5	[4.7870,4.8970]	[0.2655, 0.2715]	0.0382	0.0437
0.6	[4.7944,4.8924]	[0.2660, 0.2708]	0.0384	0.0432
0.7	[4.8018,4.8878]	[0.2665, 0.2701]	0.0387	0.0428
0.8	[4.8092,4.8832]	[0.2670, 0.2694]	0.0390	0.0423
0.9	[4.8166,4.8786]	[0.2675, 0.2687]	0.0392	0.0419
1	[4.8240,4.8740]	[0.2680, 0.2680]	0.0398	0.0414

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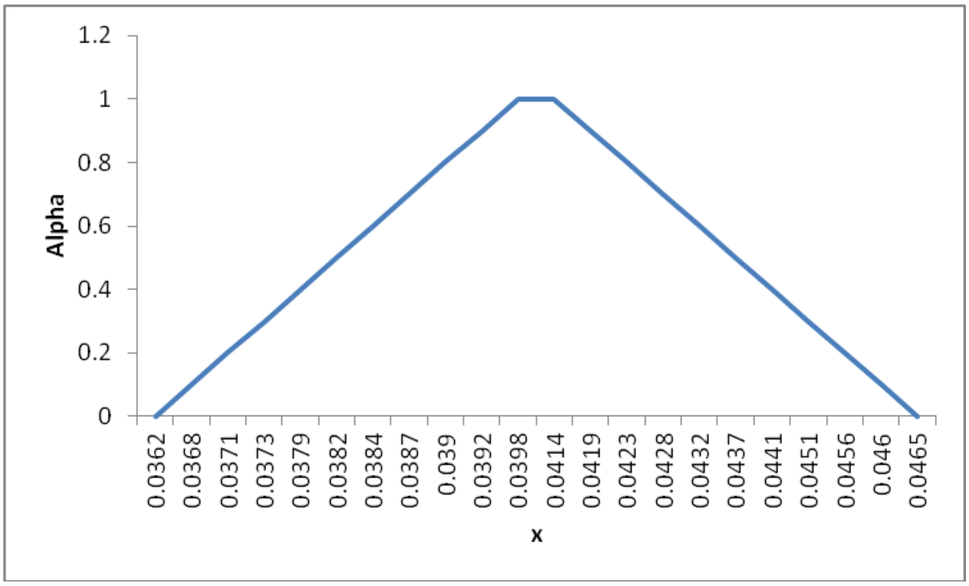

Fig:1 Fuzzy Probability

V. CONCLUSION

The α - cut of fuzzy reliability function based on the gamma distribution of serum prolactin response to TRH during antithyroid treatment was determined. The fuzzy probability curve corresponding to the α - cut showed that changes from normal serum levels of T_3 and T_4 are associated with changes in prolactin responses to TRH.

REFERENCES

[1]. Zadeh, L. A., Fuzzy sets, *Information and control*, 8, 1965, 338-352.

[2]. Singer, D., A fuzzy set approach to fault tree and reliability analysis, *Fuzzy sets and systems*, 34(2), 1990, 145-155.

[3]. Chen, C.H., Mon, L., Fuzzy System reliability analysis by interval of confidence, *Fuzzy sets and systems*, 56, 1993, 29-36.

[4]. Cai, K.Y., Wen, C.Y., Zhang, M. L., Fuzzy Variables as a basic for a theory of fuzzy reliability in the possibility context, *Fuzzy sets and systems*, 42, 1991, 145-172.

[5]. Utkin. L.V. and Gurov, S.V., A general formal approach, for fuzzy reliability analysis in the possibility context, *Fuzzy sets and systems*, 83, 1995, 203-213,.

[6]. Hammer, M., Application of fuzzy theory to electrical machine reliability, *Zeszyty Naukowe Politechniki ´Sl¸askiej, seria: Elektryka* 177, 2001, 161-166.

[7]. Karl Pearson, F. R. S., *Tables of the Incomplete Γ - Functions* (Department of Scientific and Industrial Research, London, 1922).

[8]. Snyder, P. J. and R. D. Utiger, Inhibition of thyrotropin response to thyrotropin-releasing hormone by small quantities of thyroid hormones, *J. Clin. Invest*., 51:2077,1972.