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Free Vibration Analysis of Cracked Beam

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ABSTRACT
The presence of cracks causes changes in the physical properties of a structure which introduces flexibility, and thus reducing the stiffness of the structure with an inherent reduction in modal natural frequencies. Consequently it leads to the change in the dynamic response of the beam. In this paper, A model for free vibration analysis of a beam with an open edge crack has been presented. Variations of natural frequencies due to crack at various locations and with varying crack depths have been studied. A parametric study has been carried out. The cracked beams with different boundary conditions have been analyzed. The results obtained by experiments performed by previous studies are compared with those obtained by finite element analysis. The analysis was performed using ABAQUS software.
Keywords - Free vibration; Crack; Natural frequencies; ABAQUS

I. INTRODUCTION

Most of the members of engineering structures operate under loading conditions, which may cause damages or cracks in overstressed zones. The presence of cracks in a structural member, such as a beam, causes local variations in stiffness, the magnitude of which mainly depends on the location and depth of the cracks. The presence of cracks causes changes in the physical properties of a structure which in turn alter its dynamic response characteristics. The monitoring of the changes in the response parameters of a structure has been widely used for the assessment of structural integrity, performance and safety. Irregular variations in the measured vibration response characteristics have been observed depending upon whether the crack is closed, open or breathing during vibration.

The vibration behavior of cracked structures has been investigated by many researchers. The majority of published studies assume that the crack in a structural member always remains open during vibration. However, this assumption may not be valid when dynamic loadings are dominant. In such case, the crack breathes (opens and closes) regularly during vibration, inducing variations in the structural stiffness. These variations cause the structure to exhibit non-linear dynamic behavior. A beam with a breathing crack shows natural frequencies between those of a non-cracked beam and those of a faulty beam with an open crack.

Christides and Barr [1] developed a one-dimensional cracked beam theory at same level of approximation as Bernoulli-Euler beam theory. Ostachowicz and Krawczuk [2] presented a method of analysis of the effect of two open cracks upon the frequencies of the natural flexural vibrations in a cantilever beam.They replaced the crack section with a spring and then carried out modal analysis for each part of the beam using appropriate matching conditions at the location of the spring. Liang, Choy and Jialou Hu [3] presented an improved method of utilizing the weightless torsional spring model to determine the crack location and magnitude in a beam structure. Dimaragonas [4] presented a review on the topic of vibration of cracked structures. His review contains vibration of cracked rotors, bars, beams, plates, pipes, blades and shells. Shen and Chu [5] and Chati, Rand and Mukherjee [6] extended the cracked beam theory to account for opening and closing of the crack, the so called "breathing crack" model. Kisa and Brandon [7] used a bilinear stiffness model for taking into account the stiffness changes of a cracked beam in the crack location. They have introduced a contact stiffness matrix in their finite element model for the simulation of the effect of the crack closure which was added to the initial stiffness matrix at the crack location in a half period of the beam vibration. Saavedra and Cuitino [8] and Chondros, Dimarogonas and Yao [9] evaluated the additional flexibility that the crack generates in its vicinity using fracture mechanics theory. Zheng et al [10] the natural frequencies and mode shapes of a cracked beam are obtained using the finite element method. An overall additional flexibility matrix, instead of the local additional flexibility matrix, is added to the flexibility matrix of the corresponding intact beam element to obtain the total flexibility matrix, and therefore the stiffness matrix. Zsolt huszar [11] presented the quasi periodic opening and closings of cracks were analyzed for vibrating reinforced concrete beams by laboratory experiments and by numeric simulation. The linear analysis supplied lower and upper bounds for the natural frequencies. Owolabi, Swamidas and Seshadri [12] carried out experiments to detect the presence of crack in beams, and determine its location and size. Yoon, In-Soo Son and Sung-Jin [13] investigated the influence of two open cracks on the dynamic behavior of a double cracked simply supported beam both analytically and experimentally. Behzad, Ebrahimi and Meghdari [14] developed a continuous model for flexural vibration of beams with an edge crack perpendicular to the neutral plane. The model assumes that the displacement field is a superposition of the classical Euler-Bernoulli beam's displacement and of a displacement due to the crack. Rezaee and Hassannejad [15] new analytical method is developed to investigate the free vibration behaviour of a simply supported beam affected by a breathing crack as an alternative to the numerical methods. A SDOF model for the cracked beam is developed and the time-varying stiffness is modelled using a periodic function. Shifrin [16] presented a new technique is proposed for calculating natural frequencies of a vibrating beam with an arbitrary finite number of transverse open cracks. Most of the researchers studied the effect of single crack on the dynamics of the structure. A local flexibility will reduce the stiffness of a structural member, thus reducing its natural frequency. Thus most popular parameter applied in identification methods is change in natural frequencies of structure caused by the crack.

In this paper, the natural frequencies of cracked and uncracked beams have been calculated using Finite element software ABAQUS and compared with experiments performed by previous researchers. Parametric study has been carried out on beams with crack at various crack depths and crack locations.

II. METHODOLGY & VALIDATION

1. Crack modelling

Cracks in the beam create changes in geometrical properties so it becomes complex to study the effect of cracks in the beam. The crack modeling has been very important aspect. The analysis has been done using finite element method. FEM software package ABAQUS 6.12 has been used. Cracked beam has been modeled and free vibration analysis has been performed considering geometric and material non linearity. The crack is considered to be an open edge notch. Crack with a 0.5 mm width on the top surface of the beam has been modeled. It is assumed that crack have uniform depth across the width of the beam.

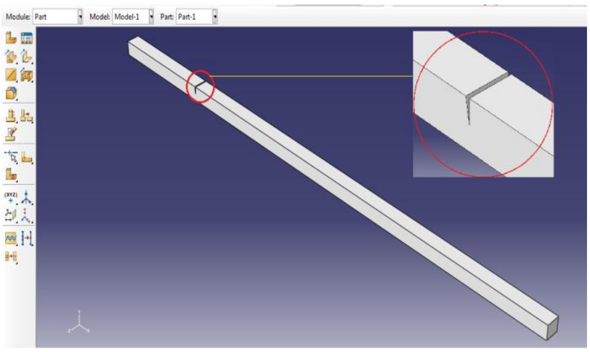

Figure 1 V-shaped edge crack with a 0.5 mm width on the top surface of the beam

2. Validation

The cracked concrete cantilever beam with a single crack has been considered for the free vibration analysis. The same beam has been solved using ANSYS and ABAQUS program and comparison of results have been made.

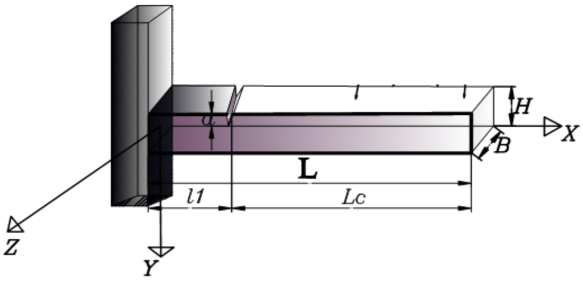

Figure 2 Cantilever beam with a crack.

Properties:
Width of the beam = 0.23 m
Depth of the beam = 0.5 m
Length of the beam = 3 m
Elastic modulus of the beam = 30 GPa
Poisson's Ratio = 0.3
Density = 25 KN/m^3
a = 5 mm
l_1= 1m

Table 1 Comparison of results obtained using ABAQUS and ANSYS software

MODE No.	Natural Frequencies obtained by	
	ABAQUS (cycles/time)	ANSYS (cycles/time)
1	4.51309	4.7407
2	9.65374	9.7412
3	27.6089	28.968
4	43.2749	46.18
5	54.6432	55.157

The frequencies calculated using ABAQUS and ANSYS were in good agreement with each other.

III. PARAMETRIC STUDIES

1. Simply supported beam

The effects of the crack on natural frequency of a simply supported steel beam were investigated for various crack depths and crack locations. The comparison has been made between the results obtained using ABAQUS 6.12 and experimental results taken from Yoon, In-Soo Son and Sung-Jin [13].

Properties:
Width of the beam = 0.01 m
Depth of the beam = 0.01 m
Length of the beam = 0.4 m
Elastic modulus of the beam = $2.16x10^{11}$ N/m^2
Poisson's Ratio = 0.3
Density = 7650 kg/m^3

Natural frequencies of a simply supported cracked beam for the first and second modes of vibration have been studied. For simplicity, the following dimensionless quantities are introduced:
Crack location ratio $\xi c = x_c/L$
Crack depth ratio H= a/h

Table 2 Natural frequency of a simply supported beam with a crack for the first mode

Crack poition ξc = xc/L	Crack depth H=(a/h)	Natural frequency ratio (ωc/ω)		Error%
		Experi-ment	Abaqus	
Un-cracked beam		1	1	0
0.2	0.1	0.996	0.997	0.15
	0.3	0.976	0.987	1.13
	0.5	0.944	0.956	1.23
0.3	0.1	1.000	1.000	0.00
	0.3	0.986	0.987	0.11
	0.5	0.937	0.945	0.84
0.5	0.1	0.996	0.997	0.15
	0.3	0.992	0.994	0.18
	0.5	0.984	0.988	0.38

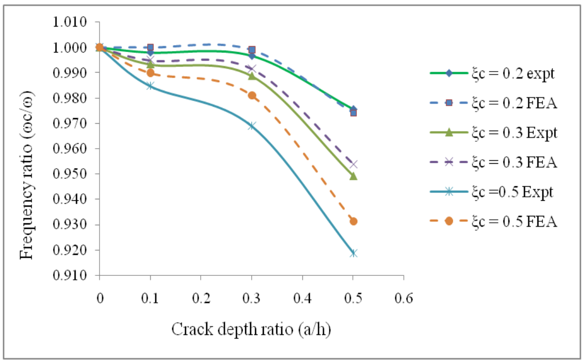

Figure 3 Natural frequency of a simply supported beam according to the crack depth for First mode

Table 3 Natural frequency of a simply supported beam with a crack for the second mode

Crack position ξc = xc/L	Crack depth H=(a/h)	Natural frequency ratio (ωc/ω)		Error%
		Experi-ment	Abaqus	
Un-cracked beam		1	1	0
0.2	0.1	0.996	1.000	0.45
	0.3	0.976	0.999	2.33
	0.5	0.944	0.974	3.24
0.3	0.1	1.000	0.995	0.50
	0.3	0.986	0.992	0.54
	0.5	0.937	0.954	1.81
0.5	0.1	0.996	0.990	0.55
	0.3	0.992	0.981	1.12
	0.5	0.984	0.931	5.37

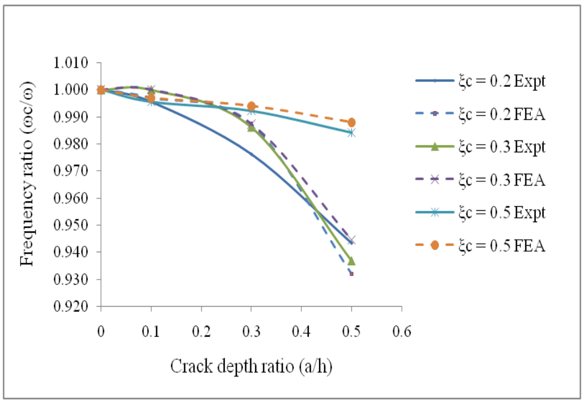

Figure 4 Natural frequency of a simply supported beam according to the crack depth for second mode.

2. Cantilever beam

The effects of the crack on natural frequency of a cantilever beam were investigated for various crack depths and crack locations. Natural frequencies of a cantilever cracked beam for the first second and third mode of vibration have been studied.
Properties:
Breath of the beam = 0.150 m
Depth of the beam = 0.250 m
Length of the beam = 2.2 m
Elastic modulus of the beam = 30×10^{9} N/m^{2}
Poisson's Ratio = 0.17
Density = 2500 kg/m^{3}
End condition = One end fixed and other end free

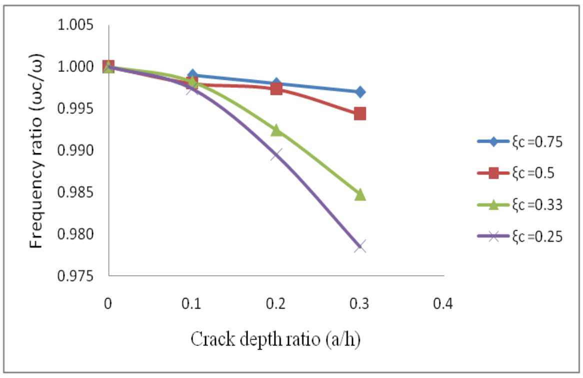

Figure 5 Natural frequency of a cantilever beam according to the crack depth for first mode.

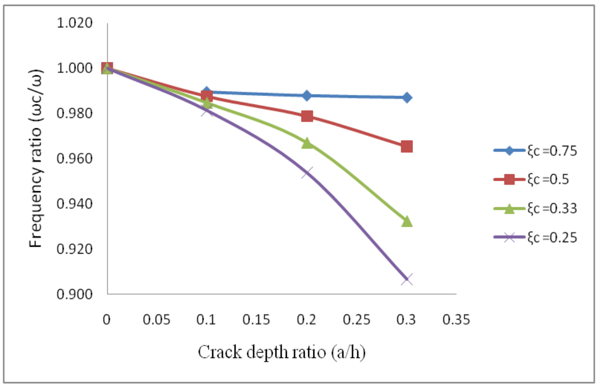

Figure 6 Natural frequency of a cantilever beam according to the crack depth for second mode.

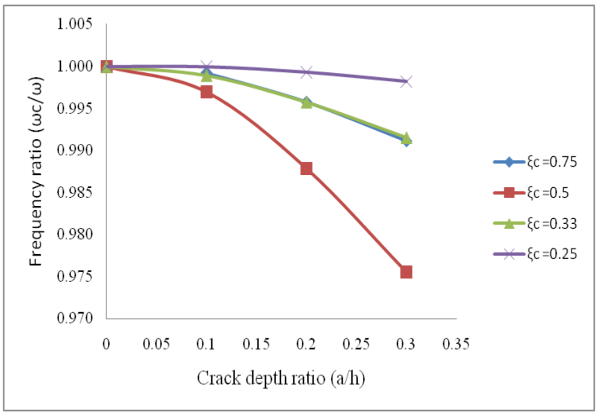

Figure 7 Natural frequency of a cantilever beam according to the crack depth for third mode.

IV. OBSERVATIONS

The natural frequency of a simply supported beam with a single crack for the first mode and second mode of vibration are shown in TABLE 2 and 3, respectively. In these tables comparison between experimental measurements and FE analysis using ABAQUS is also given. In TABLE 2, the maximum difference between the two results is less than 1.23%. It can be found that the FE analysis results are in a good agreement with the experimental measurements. In TABLE 3, natural frequency of the second mode of vibration, the two results are found to be almost identical. But, when the crack position is 0.5 and the crack depth is 0.5, the difference between the two results is 5.37 %. Fig. 3 and Fig. 4 shows when the crack position is constant, the natural frequency of a simply supported beam is inversely proportional to the crack depth. In Fig. 4, in the case of ξc=0.5 the natural frequency ratios of the second mode of vibration are unit due to the mode shape of the beam. In addition, the natural frequency of a simply supported beam is decreased rapidly when the crack depth ratio H is 0.5.

Fig. 5, Fig. 6 and Fig. 7 shows the ratio of natural frequencies of a cantilever beam with a single edge crack at various crack locations and crack depths for first, second and third modes of vibration respectively. Results show that there is an appreciable variation between natural frequency of cracked and uncracked cantilever beam. It is observed that largest effect has been felt at the fixed end. The first and second natural frequencies of the cantilever beam changed rapidly for a crack located at ξc=0.25. Whereas the third natural frequency was almost unaffected for a crack located at the ξc=0.25. It appears therefore that the change in frequencies is not only a function of crack depth, and crack location, but also of the mode number.

V. CONCLUSION

- It has been observed that the natural frequency changes substantially due to the presence of cracks depending upon location and size of cracks.
- When the crack positions are constant i.e. at particular crack location, the natural frequencies of a cracked beam are inversely proportional to the crack depth.
- It has been observed that the change in frequencies is not only a function of crack depth, and crack location, but also of the mode number.
- As largest effects are observed at the centre for simply supported beam and at the fixed end for the cantilever beam we can say, decrease in frequencies is more for a crack located where the bending moment is higher.
- In actual practice structural members such as beams are highly susceptible to transverse cross-sectional cracks due to fatigue. Therefore this study can be further extended to beams with multiple cracks

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