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Heat Transfer Effects Due To Viscous Dissipation On Unsteady Boundary Layer Fluid Flow Past A Stretching Sheet Subject To Transverse Magnetic Field

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ABSTRACT
In the present investigation heat transfer character due to viscous dissipation and magnetic field applied to unsteady boundary layer fluid flow past a stretching sheet is investigated. The equations governing the unsteady boundary layer fluid flow are constructed and non-dimensionalized. The non-dimensional equations are discretized using implicit finite difference method of Crank-Nicolson type. The results obtained are interpreted and discussed using graphs. The study is carried out for different Hartmann and Eckert number (Pr = 0.71 and 7.0). Heat transfer effects are studied by plotting the temperature profiles and local Nusselt number. It is observed that increase in Eckert number enables increase in temperature profile and subdues the local Nusselt number.

Keywords **-** MHD flow, stretching sheet, unsteady boundary layer flow, viscous dissipation

I. INTRODUCTION

Heat transfer along with mass transfer over a stretching sheet has received significant attention among researchers and applied scientists due to its various applications in industries. Applications include engineering design such as design of building components for energy consideration, control of pollutant spread in ground water, heat exchangers, solar power collectors. Along with other physical effects, Magnetic and viscous dissipation effect also play significant role in the behavior of the boundary layer flow[3,4].

Effect of transverse magnetic field applied to fluid flow past a moving surface has been widely studied by various researchers. Pop and Na[2] have studied the effect of magnetic field over a permeable stretching sheet. Anderrson and Bech[3] have included the magnetic effect over non-Newtonian fluid past a stretching sheet. Chiam[4] studied the heat transfer and magnetic effect over the boundary layer fluid flow past a stretching sheet. Heat transfer effects were also studied by Datti et al.[8], Siddheshwar et al.[6], Mukhopadhyay et al.[7].

The contribution of viscous dissipation being lower compared to other physical parameter namely magnetic field, it is often neglected as part of study. It must be noted that viscous dissipation cannot be always be neglected as can observed from the investigation carried out by Salama et al.[11], Jha et al.[12], Ragueb et al.[13], Pantokratoras et al.[14], Bareletta and Magyari [15], Chen and Tso [16], and Zhang and Ouyang [17]. The importance of viscous dissipation is also highlighted in the research articles by Celata et al.[9] and Kumaran et al.[10].

In the present investigation, heat and mass transfer effects are studied over unsteady boundary layer fluid flow in the presence of transverse magnetic field and viscous dissipation. As far as the authors knowledge is concerned the present investigation has not been done earlier and hence carried out here.

II. GOVERNING EQUATIONS

Consider a stationary viscous incompressible fluid maintained at a temperature T'_∞ over a stretching sheet which is initially held stationary. x', y' are the horizontal and vertical directions. At $t' > 0$, the sheet is stretched horizontally with a linear velocity proportional to $\beta' x'$, where β' is the stretching parameter. Also, a transverse magnetic field of strength B_0 is applied in the vertical direction to the fluid flow. Using the assumptions for boundary layer approximations [1], the governing equations describing the conservation of mass, momentum and energy along with viscous dissipation are given by,

$$\frac{\partial u'}{\partial x'} + \frac{\partial v'}{\partial y'} = 0,$$

$$\frac{\partial u'}{\partial t'} + u'\frac{\partial u'}{\partial x'} + v'\frac{\partial u'}{\partial y'} = \frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma B_0^2}{\rho}u',$$

$$\frac{\partial T'}{\partial t'} + u'\frac{\partial T'}{\partial x'} + v'\frac{\partial T'}{\partial y'} = \frac{\alpha}{\nu}\frac{\partial^2 T'}{\partial y'^2} + \left(\frac{\mu}{\rho c_p}\right)\left(\frac{\partial u'}{\partial y'}\right)^2,$$

$t' \le 0:$
$x', y' \ge 0:$

$u' = 0, \quad v' = 0, \quad T' = T'_\infty$,
$t' > 0$:
$y' = 0, \forall x' \geq 0$:
$u' = \beta' x', \quad v' = 0, \quad T' = T'_w$,
$y' \to \infty, x' = 0$:
$u' = 0, \quad T' \to T'_\infty$,
$y' \to \infty, x' > 0$:
$u' \to 0, \quad T' \to T'_\infty$,

where $\sigma, B_0, \rho, \alpha, \nu, \mu, c_p, T'_\infty, T'_w$ are electrical conductivity of the fluid, imposed magnetic strength, density of the fluid, thermal diffusivity, kinematic viscosity, coefficient of viscosity, specific heat at constant pressure, temperature of the fluid at infinity, and sheet temperature adjacent to the fluid.

Introducing the following non-dimensional variables

$$t = \beta' x', x = x'\sqrt{\frac{\beta'}{\nu}}, \quad y = y'\sqrt{\frac{\beta'}{\nu}}, u = \frac{u'}{\sqrt{\nu\beta'}},$$

$$v = \frac{v'}{\sqrt{\nu\beta'}}, T = \frac{T' - T'_\infty}{T'_w - T'_\infty}, M = Ha^2 = \frac{\sigma B_0{}^2}{\beta'\rho},$$

$$\Pr = \frac{\kappa}{\nu}, Ec = \frac{\mu\beta'}{\rho c_p\left(T_w{}' - T_\infty{}'\right)}$$

the original governing equations reduces to

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} - Mu$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{1}{\Pr}\frac{\partial^2 T}{\partial y^2} + Ec\left(\frac{\partial u}{\partial y}\right)^2$$

along with the initial and boundary conditions given by

$t \leq 0$:
$x, y \geq 0$:
$u = 0, \quad v = 0, \quad T = 0$,
$t > 0$:
$y = 0, \forall x \geq 0$:
$u = x \quad v = 0 \quad T = 1$,
$y \to \infty, \quad x = 0$
$u = 0, \quad T \to 0$,
$y \to \infty, \quad x > 0$,
$u = 0, \quad T \to 0$,

where the variables are the corresponding dimensionless quantities and M, $\Pr$, Ec are the magnetic parameter, Prandtl number and Eckert number respectively.

Further, after obtaining the solutions of the dependent variables it is useful to calculate the values local Nusselt number which is given (in dimensionless form) by the formula,

$$Nu_x = -x\left(\frac{\partial T}{\partial y}\right)_{y=0}.$$

III. NUMERICAL SOLUTIONS

In order to obtain the numerical solutions of the above equation, the equations are discretized using implicit finite difference method of Crank-Nicolson type. The scheme is unconditionally stable and is convergent. The order of convergence is $\Delta t^2 + \Delta x + \Delta y^2$ i.e., it is second order in time, first order along horizontal direction and second order along vertical direction.

The domain of computation was fixed as x = 0 to 1, and y as 0 to 25. The time interval was taken as 0.008, interval along x-direction as 0.0667(= 1/150), interval along y-direction as 0.1. The converges criteria was set as 0.00005.

The local Nusselt number are obtained using Newton-Cotes closed formula for numerical differentiation at x = 1.

IV. RESULT AND DISCUSSION

In this section the nature of the fluid flow is illustrated. It must be noted that the nature of velocity with respect to the magnetic field is already studied in the literature and neglected here for brevity. However, the effect of magnetic field on the temperature profile and local Nusselt number is discussed and illustrated with various plots (Figs. (1-21)).

In order to prove the authenticity of the results the computed values of the velocity profiles and local skin friction are compared with the exact solution of the velocity and local skin friction at steady time with magnetic effect. The Figs.(1-4) shows the difference between the computed values and exact solutions of dimensionless velocity and local skin friction for various values of magnetic parameter. The plots show that the obtained results are good for small values of magnetic parameter M.

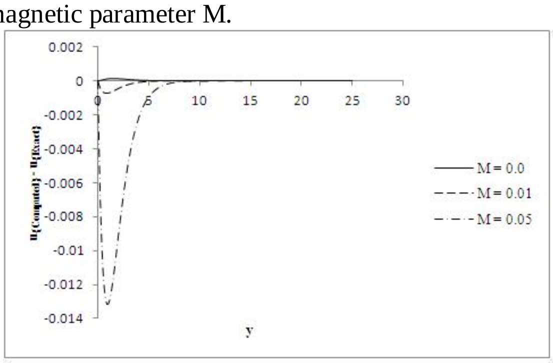

Fig. (1) Error for horizontal velocity

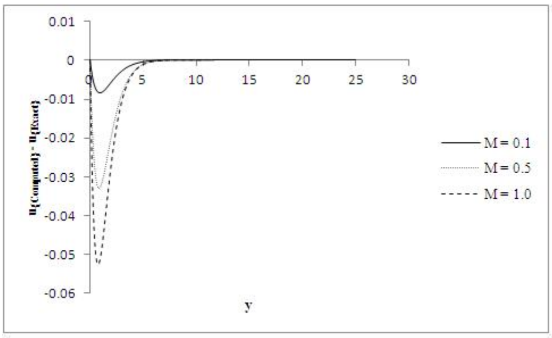

Fig. (2) Error for horizontal velocity

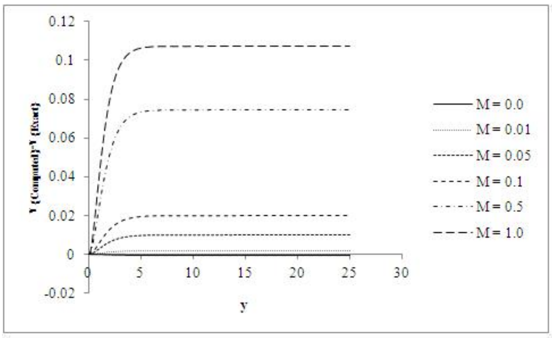

Fig. (3) Error for vertical velocity

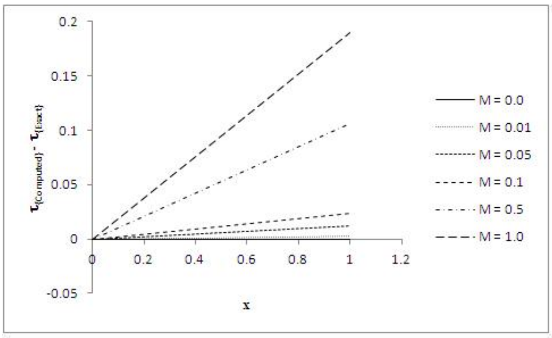

Fig. (4) Error for local skin friction

The Figs. (5-7) depict the difference of temperature profiles at Pr = 0.71 and M = 0.0, 0.05, 1.0 for Ec = 0.0, 0.2, 0.4, 0.8,1.0. It is observed that the effect of viscous dissipation has mild effect on the heat transfer effect that is as the viscous dissipation increases the heat transfer effect increases. Similar observation is made with respect to M also.

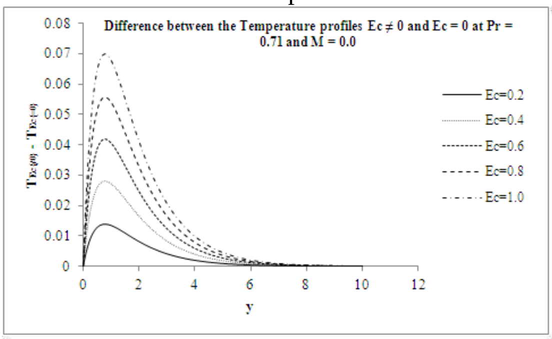

Fig. (5) Temperature difference at Pr = 0.71, M = 0.0

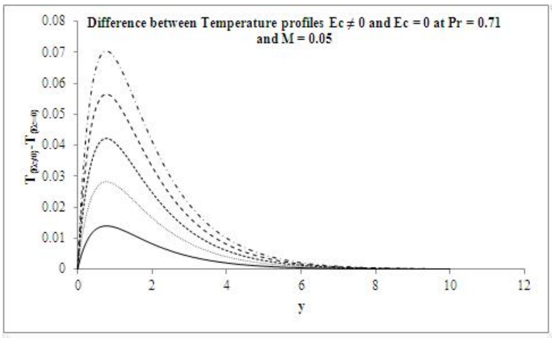

Fig. (6) Temperature difference at M = 0.05

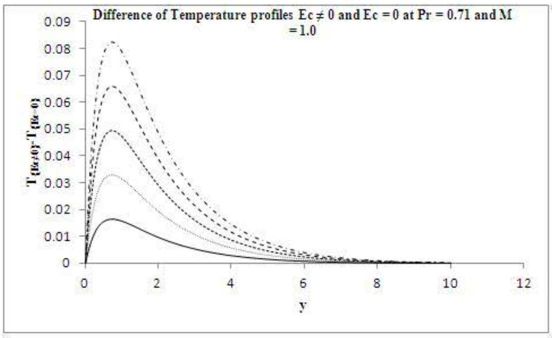

Fig. (7) Temperature difference at M = 1.0

The physical nature of the fluid does not change in the case of Pr = 7.0 except for the magnitude. This can be observed from the subdued temperature profiles (compared to Pr = 0.71) shown in Figs. (8-10).

The Figs. (11-13) describe the transient nature of the fluid flow at Pr = 0.71, M = 0.0, 0.05, 1.0, and Ec = 1.0. The plot describes the difference in magnitude of the profile between the 50^{th} iteration and 100^{th}, 150^{th} , 200^{th},….(i.e., iteration Mod 50), steady state. It can be observed from the plots that as the steady state is approached the change in difference of transient temperature diminishes.

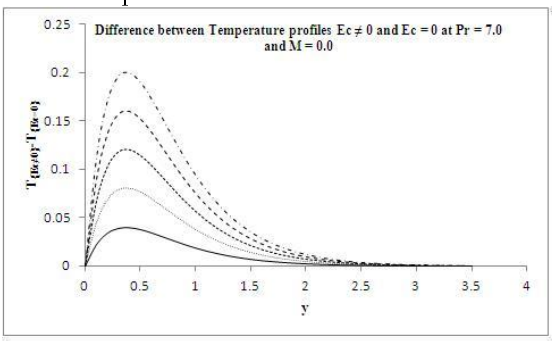

Fig. (8) Temperature difference at Pr = 7.0, M = 0.0

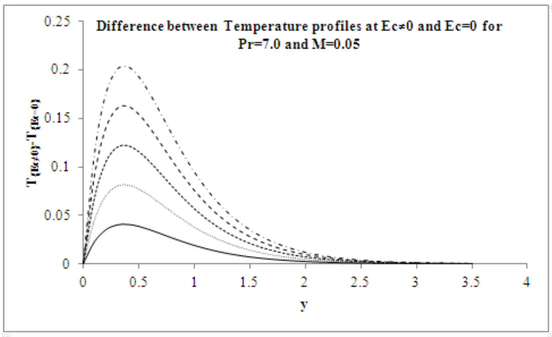

Fig. (9) Temperature difference at M = 0.05

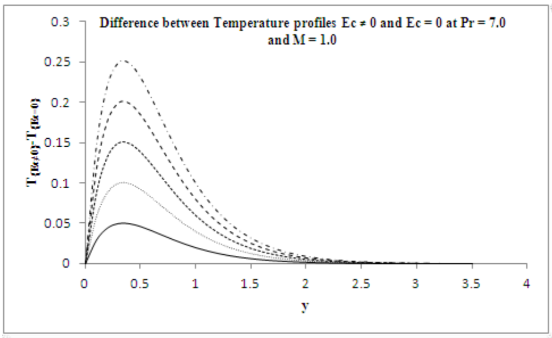

Fig. (10) Temperature difference at M = 1.0

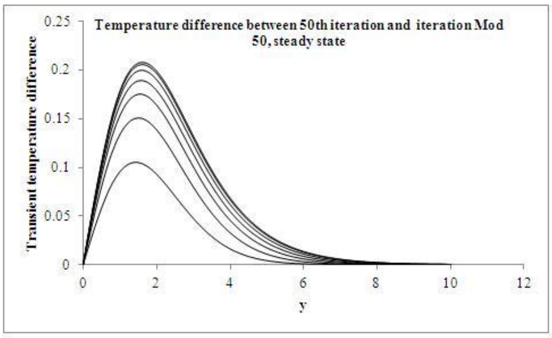

Fig. (11) Transient temperature difference at Pr = 0.71, M = 0.0, and Ec = 1.0

Similar observations can be made from the Figs. (14-16). It must be noted that as the magnetic parameter, M and Eckert number, Ec increases the steadiness gets delayed.

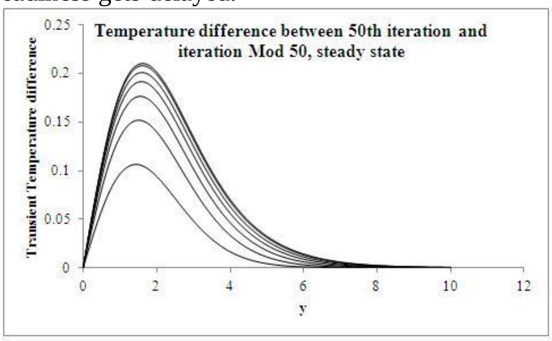

Fig. (12) Transient temperature difference at Pr = 0.71, M = 0.05, and Ec = 1.0

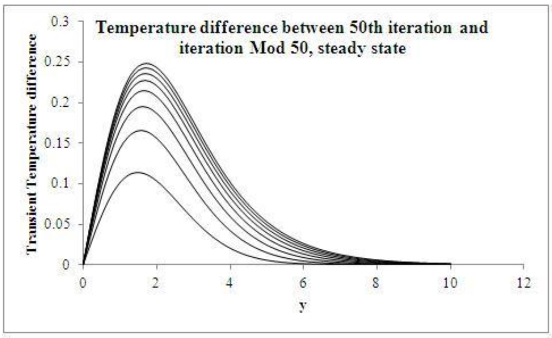

Fig. (13) Transient temperature difference at Pr = 0.71, M = 1.0, and Ec = 1.0

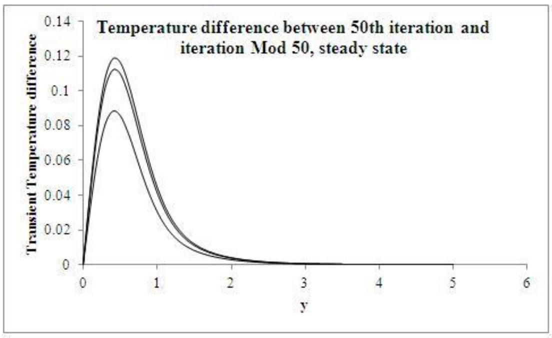

Fig. (14) Transient temperature difference at Pr = 7.0, M = 0.0, and Ec = 1.0

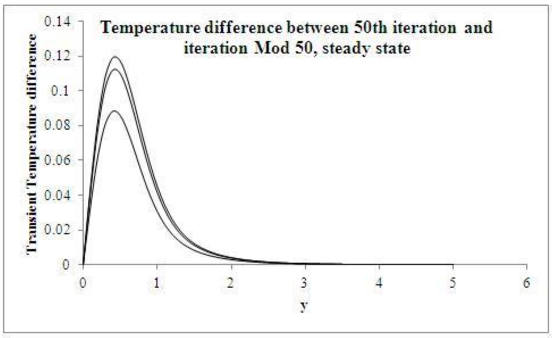

Fig. (15) Transient temperature difference at Pr = 7.0, M = 0.05, and Ec = 1.0

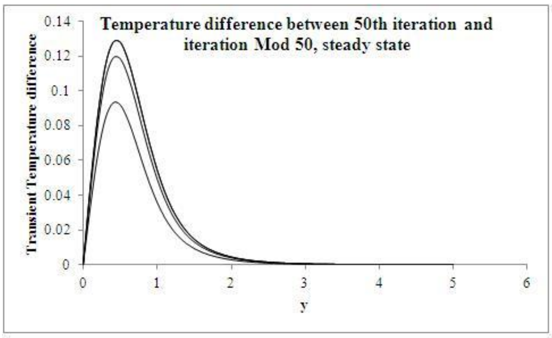

Fig. (16) Transient temperature difference at Pr = 7.0, M = 1.0, and Ec = 1.0

The profiles in Figs. (17-22) describe the local skin friction profiles for Pr = 0.71 and Pr = 7.0 for M = 0.0, 0.05, 1.0 and Ec = 0.0, 0.2, 0.4, 0.6, 0.8, 1.0. It is clear from the observation of the plots that as magnetic parameter and viscous dissipation increases the Nusselt number decreases.

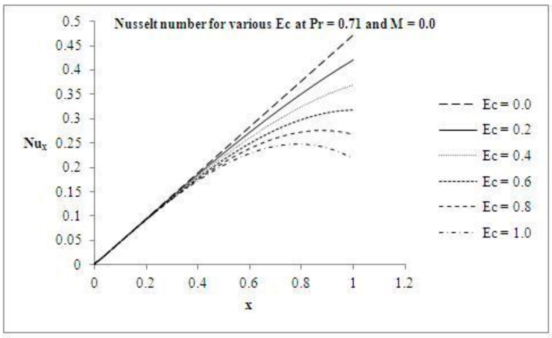

Fig. (17) Steady local Nusselt number at Pr = 0.71, M = 0.0, and Ec = 0.0, 0.2, 0.4, 0.6, 0.8, 1.0

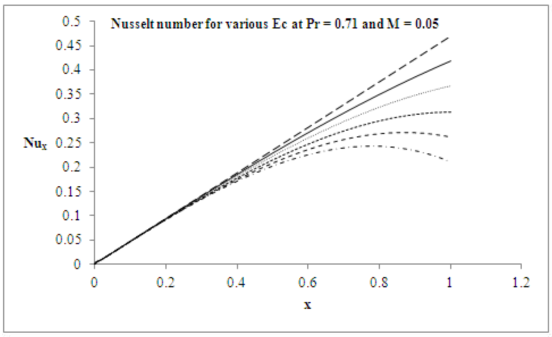

Fig. (18) Steady local Nusselt number at Pr = 0.71, M = 0.05, and Ec = 0.0, 0.2, 0.4, 0.6, 0.8, 1.0

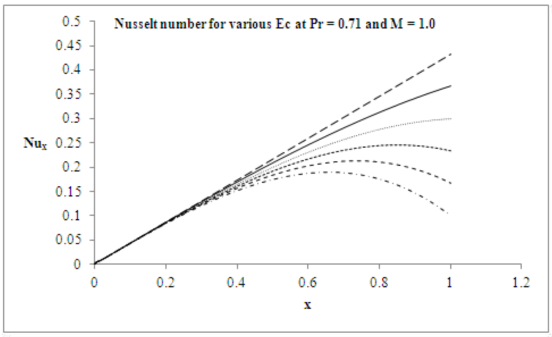

Fig. (19) Steady local Nusselt number at Pr = 0.71, M = 1.0, and Ec = 0.0, 0.2, 0.4, 0.6, 0.8, 1.0.

It can be observed (Figs. (20-22)) that as Pr changes from 0.71 to 7.0 the local Nusselt number increases significantly.

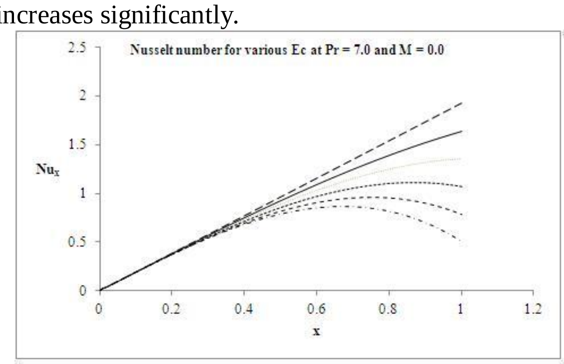

Fig. (20) Steady local Nusselt number at Pr = 7.0, M = 0.0, and Ec = 0.0, 0.2, 0.4, 0.6, 0.8, 1.0

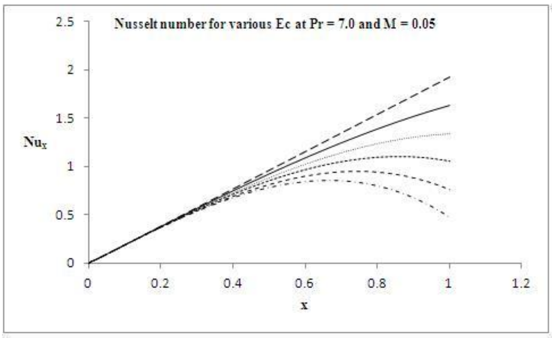

Fig.(21) Steady local Nusselt number at Pr = 7.0, M = 0.05, and Ec = 0.0, 0.2, 0.4, 0.6, 0.8, 1.0

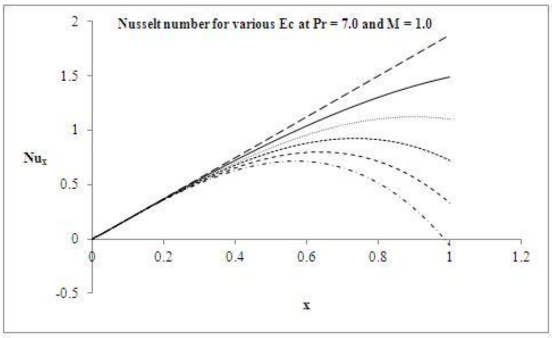

Fig. (21) Steady local Nusselt number at Pr = 7.0, M = 1.0, and Ec = 0.0, 0.2, 0.4, 0.6, 0.8, 1.0

V. CONCLUSION

In this investigation, the effect of Eckert number and Magnetic field on unsteady boundary layer fluid (Air and Water) are studied. It is observed that the effect of Prandtl number increase leads to much better profile as Eckert number increases. Eckert number tends to decrease the local Nusselt number which is again enhanced by increase in strength of magnetic field.

REFERENCES

Books:

[1] H. Schlichting, K. Gersten, *Boundary layer theory*, *8th Edition*, 2000, Springer.

Journals:

[2] I. Pop, T-. Y. Na. A note on MHD flow over a stretching permeable surface. *Mech Res Commun* 25(3):1998;263–269.

[3] H. I. Andersson, K. H. Bech, MHD flow of a power-law fluid over a stretching sheet. *Int. J. Nonlin. Mech*, 1992;27(6):929–936.

[4] T. C. Chiam, Magnetohydrodynamic heat transfer over a non-isothermal stretching sheet. *Acta Mech.*, 1997;122:169–79.

[5] R. Cortell, A note on magnetohydrodynamic flow of a power-law fluid over a stretching sheet. Appl Math. Comput. 2005;168:557–66.

[6] P. G. Siddheshwar, U. S. Mahabaleswar, Effects of radiation and heat source on MHD flow of viscoelastic liquid and heat transfer

over a stretching sheet. Int. J. Nonlin. Mech., 2005;40:807–20.

[7] S. Mukhopadhyay, G. C. Layek, Sk. A. Samad, Study of MHD boundary layer flow over a heated stretching sheet with variable viscosity. Int. J. Heat Mass Transf. 2005;48:4460–6.

[8] P. S. Datti, K. V. Prasad, M. S. Abel, J. Ambuja, MHD visco-elastic fluid flow over a non-isothermal stretching sheet. Int. J. Engg. Sci. 2004;42935–46

[9] G. P. Celata, G.L. Morini, V. Marconi, S. J. Mc Phail, G. Zummo, Using viscous heating to determine the friction factor in micro-channels experimental validation, Exp. Therm. Fluid Sci. 30 (2006) 725–731.

[10] V. Kumaran, A. K. Banerjee, A. Vanav Kumar and I. Pop, Unsteady MHD flow and heat transfer with viscous dissipation past a stretching sheet, *Int. Commun. Heat Mass Transf.*, 38, 2011, 335-339.

[11] A. Salama, I. A. Abbas, M. F. El-Amin, S. Sun, Comparison study between the effects of different terms contributing to viscous dissipation in saturated porous media, *Int. J. Therm. Sci.*, 64, 2013, 195-203

[12] B. K. Jha, A. O. Ajibade, Effect of viscous dissipation on natural convection flow between vertical parallel plates with time-periodic boundary conditions, *Commun. in Nonlin. Sci. and Num. Sim.*, 17(4), 2012, 1576-1587

[13] H. Ragueb, K. Mansouri, A numerical study of viscous dissipation effect on non-Newtonian fluid flow inside elliptical duct, *Energy Conv. Mgmt*, 68, 2013, 124-132.

[14] A. Pantokratoras, Effect of viscous dissipation and pressure stress work in natural convection along a vertical isothermal plate. New results *Int. J. Heat Mass Transf.*, 46(25), 2003, 4979-4983

[15] A. Barletta, E. Magyari, Forced convection with viscous dissipation in the thermal entrance region of a circular duct with prescribed wall heat flux, *Int. J. Heat Mass Transf.*, 50(1–2), 2007, 26-35.

[16] G. M. Chen, C.P. Tso, Effects of viscous dissipation on forced convective heat transfer in a channel embedded in a power-law fluid saturated porous medium, *Int. Commun. Heat Mass Transf.*, 38(1),2011,57-62.

[17] X-. H. Zhang, J. Ouyang, Meshless analysis of heat transfer due to viscous dissipation in polymer flow, *Engg. Anal. With Boundary Elements*, 32(1), 2008, 41-51.