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Design of Linear and Planar Fractal Arrays

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Abstract
This paper describes the theories and techniques for designing linear and planar fractal arrays and compare their radiation pattern with conventional arrays. Fractals are recursively generated object having fractional dimension. We have compared radiation pattern of cantor linear arrays with conventional linear arrays using MATLAB program. Another program was developed to characterize the radiation pattern of periodic, random and Sierpinski fractal arrays.
Index Terms – *Uniform, linear, planar, cantor, sierpinski fractal*

I. INTRODUCTION

There has been an ever growing demand, in both the military as well as the commercial sectors, for antenna design that possesses the following highly desirable attributes:
i) Compact size
ii) Low profile
iii) Conformal
iv) Multi- band or broadband

There are a variety of approaches that have been developed over that year, which can be utilized to achieve one or more of these design objectives. The term fractal, which means broken or irregular fragments, was originally coined by Mandelbrot to describe a family of complex shapes that possess an inherent self similarity in their geometrical structure. The original inspiration for the development of fractal geometry came largely from in depth study of the patterns of nature. This geometry, which has been used to model complex objects found in nature such as clouds and coastlines, has space-filling properties that can be utilized to miniaturize antennas. One such area is fractal electromagnetic theory for the purpose of investigating a new class of radiation, propagation, and scattering problems. Modern telecommunication systems require antennas with wider bandwidths and Smaller Dimensions than conventionally possible. This has initiated antenna research in various directions, one of which is by using fractal shaped antenna elements. In recent years several fractal geometries have been introduced for antenna applications with varying degrees of success in improving antenna characteristics. Some of these geometries have been particularly useful in reducing the size of the antenna, while other designs aim at incorporating multi-band.

Fractal geometry was first discovered by Benoit Mandelbrot as a way to mathematically define structures whose Dimension cannot be limited to whole numbers. It is that branch of mathematics which studies properties and behaviour of Fractals [1]. These geometries have been used to characterize objects in nature that are difficult to define with Euclidean geometries including length of coastlines, branches of trees etc. These geometries have been used to characterize structures in nature that were difficult to define with Euclidean geometries According to Webster's Dictionary a fractal is defined as being "derived from the Latin fractus, meaning broken, uneven: any of various extremely irregular curves or shape that repeat themselves at any scale on which they are examined."

The primary motivation of fractal antenna engineering is to extend antenna design and synthesis concepts beyond Euclidean geometry. In this context, the use of fractals in antenna array synthesis and fractal shaped antenna elements have been studied. Obtaining special antenna characteristics by using a fractal distribution of elements is the main objective of the study on fractal antenna arrays. It is widely known that properties of antenna arrays are determined by their distribution rather than the properties of individual elements. Since the array spacing (distance between elements) depends on the frequency of operation, most of the conventional antenna array designs are band-limited. Self-similar arrays have frequency independent multiband characteristics. Fractal and random fractal arrays have been found to have several novel features. Variation in fractal dimension of the array distribution has been found to have effects on radiation characteristics of such antenna arrays. The use of random fractals reduces the fractal dimension, which leads to a better control of side lobes. Synthesizing fractal radiation patterns has also been explored. It has been found that the current distribution on the array affects the fractal dimension of the radiation pattern. It may be concluded that fractal properties such as self-similarity and dimension play a key role in the design of such arrays. [2]

II. LITERATURE SURVEY ON FRACTAL GEOMETRIES AND ARRAYS

The first fractals that will he considered is the popular Sierpinski gasket. The first few stages in the construction of the Sierpinski gasket are shown in Figure 1 Another popular fractal is known as the Koch snowflake. This fractal also starts out as a solid equilateral triangle in the plane, as illustrated in of Figure 2. A number of structures based on purely deterministic or random fractal trees have also proven to be extremely useful in developing new design methodologies for antennas and frequency selective surfaces. An example of a deterministic ternary (three branch) fractal tree is shown in Figure 4.The space-filling properties of the Hilbert curve and related curves make them attractive candidates for use in the design of fractal antennas. The first four steps in the construction of the Hilbert curve are shown in figure 3. The Koch snowflakes and islands have been primarily used to develop new designs for miniaturized-loop as well as micro strip patch antennas. New designs for miniaturized dipole antennas have also been developed based on a variety of Koch curves and fractal trees. Finally, the self-similar structure of Sierpinski gaskets and carpets has been exploited to develop multi-band antenna elements.

Fig. 1. Several stages of Sierpinski Geometry

Fig. 2. The first few stages of KOCH Snowflake

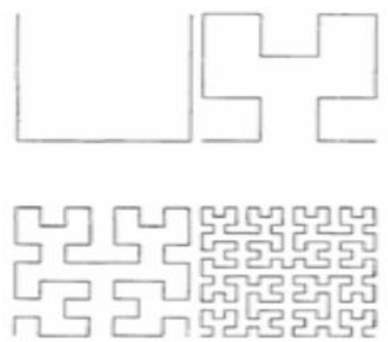


Fig. 3. The first few stages of Hlibert Curve

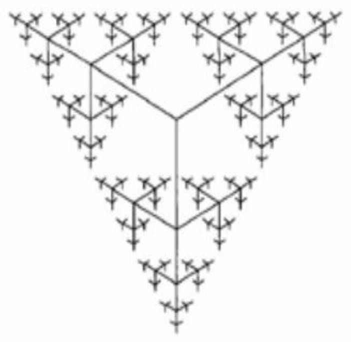


Fig.4. A stage ternary fractal tree

Apparently, the earliest published reference to use the terms fractal radiators and fractal antennas to denote fractal-shaped antenna elements appeared in May, 1994 prior to this, the terminology had been introduced publicly during an invited IEEE seminar held at Bucknell University in November, 1993. The application of fractal geometry to the design of wire antenna elements was first reported in a series of articles by Cohen. These articles introduce the notion of fractalizing the geometry of a standard dipole or loop antenna. This is accomplished by systematically bending the wire in a fractal way, so that the overall arc length remains the same, but the size is correspondingly reduced with the addition of each successive iteration. It has been demonstrated that this approach, if implemented properly, can lead to efficient miniaturized antenna designs. For instance, the radiation characteristics of Minkowski dipoles and Minkowski loops were originally investigated. Properties of the Koch fractal monopole were later considered. It was shown that the electrical performance of Koch fractal monopoles is superior to that of conventional straight-wire monopoles, especially when operated in the small-antenna frequency regime. A fast approximation technique for evaluating the radiation characteristics of the Koch fractal dipole was presented in Cohen et.al. Monopole configurations with fractal top loads have also been considered as an alternative technique for achieving size miniaturization. Finally, the effects of various types of symmetries on the performance of Koch dipole antennas were studied by Cohen.

III. DESIGN OF FRACTAL ARRAYS

All classic array design methods are limited by the fact that any array design is inherently frequency dependant. Intuitively, the self similarity of fractals at multiple scales seems to propose some kind of solution. While the fractal methods investigated here cannot yield frequency independent designs, similar behavior in log-periodic bands can be achieved.

To facilitate practical investigation, a number of Matlab scripts were written. The functionality of these scripts includes:

- The geometrical expansion of fractal arrays.
- 2D and 3D pattern calculations using either arbitrary array geometry, or a generator for fractal expansion.
- Directivity calculation of arbitrary arrays or fractal arrays.

One method of obtaining multi-band behavior from an array, is to synthesize an array factor that is a fractal curve. The fractal curve would show self-similarity at multiple scales. The visible region of the array factor would of course change with frequency. In certain bands the visible region could coincide with part of the curve which is self-similar. If this has been realized, both the directivity and side lobe levels should be constant between bands

An array factor can be defined as a fractal curve, and then be synthesized using classic array synthesis methods, such as described in texts. Array factors based on the Koch curve, and synthesized using the inverse Fourier transform are investigated in the earlier literature. Other methods might yield designs which are easier to thin than others, resulting in less complicated arrays. While this method does keep directivity and side lobes unchanged through the bands, its radiation intensity changes with frequency. This implies that the radiation resistance is not held constant through the bands. Furthermore, the finite extent of any real array means that at lower frequencies the array factor will be a less refined representation of the fractal curve it imitates. This might entail a difference in directivity and side lobe levels as compared to higher frequencies, and, with further reduction in operating frequency, a complete loss of self-similarity between bands.

Some array geometries easily lend themselves to the generation of fractal-like array factors. These geometries generally have array factors which resemble a fractal Weierstrass function. Fractal array factors which approach a Weierstrass function can be synthesized by using concentric-ring arrays. For a ring with a large number (M) of elements, and main beam normal to the ring, its array factor can be written as

$$F(u) \approx I_t J_o(ka_n u) \qquad -(1)$$

where I_t is the total current on the ring, J_0 is the Bessel function of the first kind, order zero, k is the wave number, u is sin θ, and a is the radius of the ring. If we now take N rings, with M elements each, and total current I_n on the N^{th} ring, the array factor becomes

$$F(u) = \sum_{n=1}^{N} I_n J_o(ka_n u) \qquad -(2)$$

If we now compare (1) and (2)

$$g(x) = J_o(x)$$

$$I_n = \eta$$

$$Ka_n = \eta_n \qquad -(3)$$

Where $1 < D < 2$ and $\eta > 1$.

The truncation (in other words, finite N) would cause the graph of F to appear smooth under magnification, and not completely undifferentiable like a real Weierstrass function. While g(x) is by definition periodic, the Bessel is quasi-periodic. F (u) therefore still possess the fractal property to some extent.

It is possible to generate an array using recursive fractal expansion of a generating sub array. The concept is similar to the one described in previous section. The main difference here is, that instead of shrinking the result with each iteration, it is expanded by a factor δ.

Here we use a generating array with N elements, relative current excitation I_n, and excitation phase α_n for the nth element.

A stepwise explanation of the iterative process is:

1. Geometrically scale the result of the previous iteration by a factor δ.
2. Scale each element's excitation phase by δ.
3. Replace each element of the scaled array, with a copy of the generating sub array.
4. Multiply the relative current excitation of each element of each generating array element, by the relative excitation of the element it replaces.
5. Add the excitation phase of the element replaced, to the excitation phase of the generating sub array's elements phase.
6. If elements overlap, add their excitation currents together. At this step, excitation currents and phase need to be combined, and then added using complex addition. The new relative excitation and phase will then be had.

For an array of expansion order P, this process is completed P − 1 times. During the first iteration, the generating array is used in step 1. One can see this construction as an array of arrays. At each iterative step, you have an initiating array, with the generating array as individual elements. A very useful result of this construction method, is that the array factor of any array constructed thusly, can be expressed as

$$AF_p(\psi) = \prod_{P=1}^{P} GA(\delta^{p-1}\psi) \quad - (4)$$

where GA (ψ) represents the array factor of the generating sub-array.

Multi-band characteristics for a doubly infinite array can be demonstrated using previous equations

$$AF_p(\psi) = \prod_{p-\infty}^{\infty} GA(\delta^{p-1}\psi) \quad - (5)$$

Then

$$AF(\delta^{\pm q}\psi) = \prod_{p-\infty}^{\infty} GA(\delta^{\pm q-1}\psi)$$

$$= \prod_{p-\infty}^{\infty} GA(\delta^{n-1}\psi) = AF(\psi) \quad - (6)$$

with q a natural number.

Scaling an array factor is equivalent to scaling the frequency of operation by the same factor. Thus, the array factor being equal when scaled by δ±q, the array will operate the same at f±q= δ±qf0, where f0 is the design frequency, and q is a natural number. Any real array would have to be finite, thus this characteristic would not hold exactly. One might expect some self-similarity between bands yet.

Another useful characteristic of these fractal expanded geometries is that, once the main beam location of the generating sub-array has been fixed, it remains constant through fractal expansion.

It has been shown that a doubly-infinite fractal array will have perfect multiband characteristics. Approaching a doubly-infinite fractal array does not only require an infinite number of elements, it also requires infinitely small elements. We can see this from previous equation, since p < 1 implies a shrinkage, rather than expansion factor of δ.

The question then is, can acceptable multi-band performance be obtained from a fractal array? To answer this question, fractal arrays with the triadic Cantor generator was investigated. An expansion factor of δ = 3 was used to keep the elements from overlapping. Similar performance is thus expected at bands spaced a factor 3 apart. Arrays at the third and seventh stage of expansion were investigated. Performance characteristics of the fractal arrays were compared in frequency bands corresponding to f0, f0/3 and f0/9, where f0 is the design frequency of the generating sub array. These are also compared to the performance of the generating sub array at f0.

The multi-band behavior is somewhat disappointing. The only parameters which really correspond between bands are the locations of the nulls. While there are fewer nulls at the lower frequency bands, the ones which do exist coincide with those of higher bands.

This can easily be explained in light of (4). For P = 3 we have

$$AF_P(\psi) = GA(\psi) * GA(3\psi) * GA(9\psi) \quad - (7)$$

Each time the frequency is lowered, a smaller part of the first product terms in (4) becomes visible. We can think of each generating sub-array collapsing into a single element at each lower frequency band. This analogy is shown clearly by the three iteration array.

IV. PERFORMANCE EVALUATION

(i) Linear Arrays

In this report we will examine a particular class of fractals known as the Cantor set. Comparison of Cantor set with conventional linear arrays are made by accounting various factors like Array Factor, Directivity, Half power beam width and Side-lobe level.

A linear array generally has N equally spaced elements with uniform excitations. It has a constant inter-element phase shift of radiating electromagnetic waves from the plane of array. By choosing appropriate elements to be turned on or off we can achieve the fractal behavior which is called as Cantor fractal set.

n	Element pattern	Active elements	Total elements
1	101	2	3
2	101000101	4	9
3	101000101000000000101000101	8	27
4	101000101000000000101000101 000000000000000000000000000 101000101000000000101000101	16	81

Table 1 : The element distribution for fractal linear array

Now let us compare a 16-element conventional array with its corresponding fractal version. In fractal configuration, as seen in Table 1, in order to obtain 16 active elements a total of 81 elements are generated by fractal iterations in which a majority are switched off with only 16 elements are left in radiating mode. So by separating these active elements at multiples of quarter-wavelength separations between two such elements we can obtain an array factor which can be compared with conventional linear arrays. The remaining inactive elements which are not used for beam forming purposes can be utilized for other purposes. If we consider the design frequency f_o of such cantor set be 8.1 GHz, we can notice that such fractal array operates at another three frequencies apart from the design frequency. The array factor of conventional linear array and Cantor fractal set with the help of MATLAB program.

The conventional linear arrays form a beam pattern for its design frequency with lower side-lobe levels as shown in Fig.5. However, as the frequency changes from its design frequency these arrays loses its beam focusing ability and behaves more like a point source.

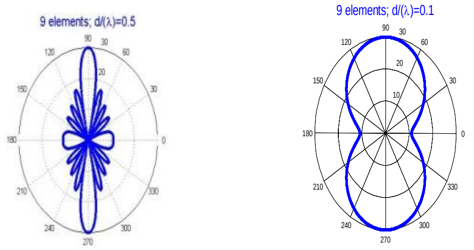


Fig.5: Radiation Pattern of conventional linear array

(ii) PLANAR ARRAYS

Another program is developed to analyze the radiation pattern of Planar, Random and Sierpinski Gasket. Several stages of Sierpinski geometry have been discussed earlier. Now let us consider N number of elements to be arranged in deterministic pattern, Random distribution and Sierpinski Gasket.

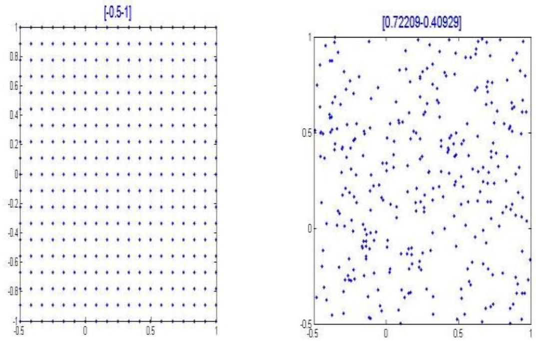


Fig.6: 324 periodically and randomly placed elements

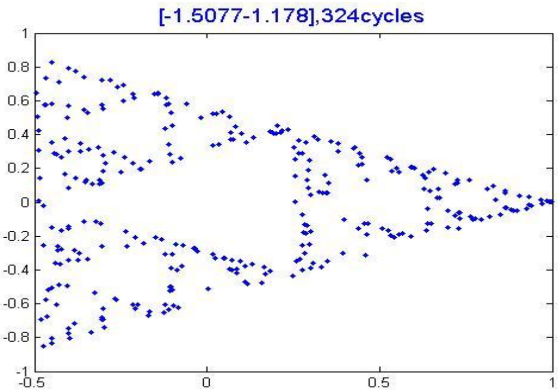
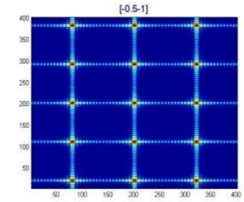


Fig.7: Random point generated Sierpinski Gasket

As shown in Fig.6, 324 elements have been placed in a rectangular grid of 1.5X2 square units first periodically and later in a random fashion. Fig. 8 & 9 shows the array factor and radiated field of orderly placed elements in which dark colored is the lowest point and white is the highest point. Fig. 10 shows the array factor of randomly placed elements. The elements are plotted randomly by using Mat lab's random number generator. It is clearly visible that side-lobes are generally lower than the ones seen in the first set of ordered points. There is also a 180-degree symmetry, which is more apparent about the main beam.



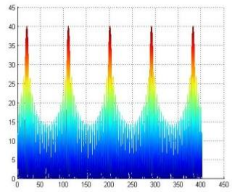


Fig.8: Radiated Field where white Fig.9:
Array Factor plot for highest point and dark is lowest
Periodic Planar Array

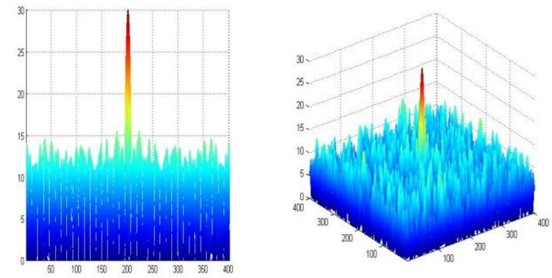


Fig.10: Array Factor plot for Random Array in 2D & 3D view

Periodic Array have side-lobes of equal amplitude as seen above. Random arrays have the advantage of reduced side-lobes. To characterize an optimised antenna array, the radiation properties of both the ordered fractals and the random fractals have redeeming qualities. Yet, there continues to be a gap betwewen the ordered and the disordered. Fractals attempt to bridge that gap where they are constructed in an organised pattern, yet they also attain the qualities of random points depending on their construction and orientation of points. By simulating Sierpinski model using Matlab script we organize 324 elements as shown in Fig.7 and we achieve the array factor and radiating pattern as shown in Fig.11.

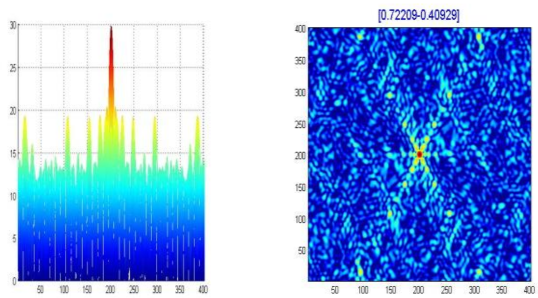


Fig 11:Array Factor plot and Radiated Field for sierpinski gasket

The qualities of an optimised antenna array deals mainly with the quality of the main beam, and the side lobe levels. As seen in the periodic array, the main beam was good in that there was no interference from smaller side-lobes coming from the side. This main beam degradation can be seen in the random array where the main beam no longer maintains its point-like appearance. The main beam characteristics of the periodic array were imitated by the fractal array.

Side-lobe levels for periodic elements are higher yet farther apart because of large amount of elements. The random array achieves lower side-lobes with significantly less elements. Like the random array, the fractal had lower side-lobes at lower stages of growth.

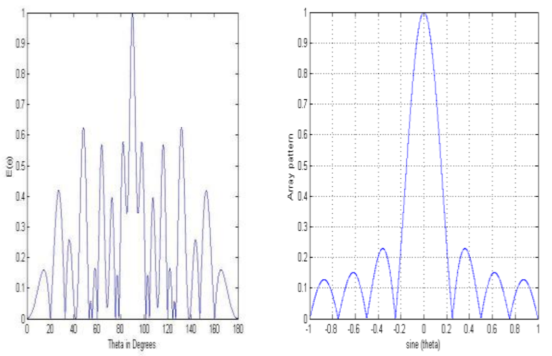
V. RESULTS

Simulated Results of Cantor Array

In this work, MATLAB programming language version 7.9 (R2009) used to simulate and design the conventional and fractal linear array antenna and their radiation pattern.

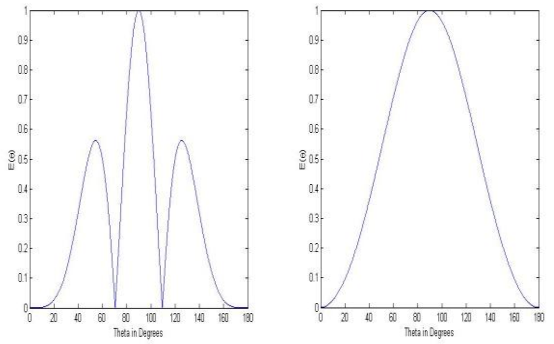
Let, a linear array will be design and simulate at a frequency F_0 equal to 8100MHz, (then the wavelength $\lambda 0$ = 0.037m), with quarter-wavelength ($d = \lambda 0/4$) spacing between array elements and 16 active elements in the array and progressive phase shift between elements (α) equal to zero. The level four of Cantor linear array (101) have the number of active elements of 16 and the total elements number of 81. This array will operate at four frequencies depending on the Eq. (5). These frequencies are 8100MHz, 2700MHz, 900MHz, and 300MHz.

Depending on the frequencies of the fractal Cantor linear array will be design and simulate of conventional linear array antenna then compare the radiation field pattern properties for them. The array factor for fractal and linear array antenna is plotted with uniformly amplitude distribution which they are feeding to active elements. The field patterns are illustrating as shown in the following figures.



(a) Fractal array (b) Conventional array

Fig.12: Plot showing the radiation characteristics for the resonating frequency of 8.1 GHz.



(a) Fractal array (b) Conventional array

Fig.12: Plot showing the radiation characteristics for the frequency of 0.9 GHz.

f(GHz)	D (dB)	Θ_H (degrees)	SLL (dB)
8.1	9.12	17.00	-13.14
2.7	4.63	53.50	-13.55
0.9	0.89	--	$-\infty$
0.3	0.10	--	$-\infty$

Table.2: Conventional Linear Phased Array

f(GHz)	D (dB)	Θ_H (degrees)	SLL (dB)
8.1	12.04	4.00	-5.45
2.7	9.03	12.95	-5.44
0.9	6.02	24.50	-5.44
0.3	3.01	80.45	$-\infty$

Table.3: Fractal Linear Phased Array

VI. CONCLUSION

At design frequency F_0= 8100MHz, the field pattern for conventional linear array antenna has low side lobes and narrow beam width, in other word, the system work as a normal array antenna. But at frequencies very low from the design frequency such as 300MHz, the array antenna operates as a point source. While fractal linear array antenna at all frequencies not operates as a point source so we conclude that the fractal Cantor linear array have capable to operating in multiband while, the conventional linear array have not capable to operate in multiband. Also the field pattern of the fractal linear array antenna has high side lobe level, lower half power beam width.

A novel approach to the design of frequency-independent radiating systems has been presented in this paper. Fractal structures are used in the design because of their self-similarity properties. The effort has been focussed in describing a technique to design low side-lobe and multiband arrays, which has always been difficult due to the sensitivity of most current design techniques to variations on the operating wavelength.

Two main approaches have been followed in this paper: The placement of the array elements on a fractal set of points (The cantor set) And the design of array factors with a fractal (sierpinski) shape.Although the cantor arrays have been shown to have similar pattern at several bands, some important properties such as mainlobe width and directivity are not held constant through the bands. On the otherhand, such structures have shown to be useful to synthesize low sidelobe patterns with uniform amplitude current distribution arrays.

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