RESEARCH ARTICLE OPEN

Effect of first order chemical reaction in a vertical double-passage channel

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ABSTRACT:
Fully developed laminar mixed convection flow in a vertical channel in the presence of first order chemical reactions has been investigated. The channel is divided into two passages by means of a thin, perfectly plane conducting baffle and hence the velocity, temperature and concentration will be individual in each stream. The coupled, nonlinear ordinary differential equations are solved analytically using regular perturbation method valid for small values of Brinkman number. The effects of thermal Grashoff number, mass Grashoff number, Brinkman number and chemical reaction parameter on the velocity, temperature and concentration fields at different positions of the baffle are presented and discussed in detail. The increase in thermal Grashof number, mass Grashoff number and Brinkman number enhances the flow, whereas the chemical reaction parameter suppress the flow at all baffle positions.

I. INTRODUCTION:

Convective heat transfer in a vertical channel has practical importance in many engineering systems. Examples are solar energy collection, the design of heat exchangers and the cooling of electronic systems. A convection situation, in which the effects of both forced and free convection are significant is commonly referred to as mixed convection or combined convection. The effect is especially pronounced in situations where the forced fluid flow velocity is moderate and/or the temperature difference is very large. In mixed convection flows, the forced convection effects and the free convection effects are of comparable magnitude. Thus, in this situation, both the forced and free convection occurs simultaneously, that is mixed convection occurs in which the effect of buoyancy forces on a forced flow or the effect of forced flow on buoyant flow is significant. Merkin [1, 2] has studied dual solutions occurring in the problem of the mixed convection flow over a vertical surface through a porous medium with constant temperature for the case of opposing flow. Aly et al. [3] and Nazar and Pop [4] have investigated the existence of dual solutions for mixed convection in porous medium.

The techniques of mixed convection to enhance heat transfer is an important main objective in numerous engineering applications of the micro and nano fluids including pumping systems, chemical catalytic reactors, electronic system cooling, plate-fin heat exchangers, etc. Moreover, the arrangement of baffles may be used to cool down the temperature in the passage for the thermally developed flow. When the channel is divided into several passages by means of plane baffles, as usually occurs in heat exchangers or electronic equipment, it is quite possible to enhance the heat transfer performance between the walls and fluid by the adjustment of each baffle position and strengths of the separate flow streams. In such configurations, perfectly conductive and thin baffles may be used to avoid significant increase of the transverse thermal resistance. Stronger streams may be arranged to occur within the passages near the channel wall surfaces in order to cool or heat the walls more effectively. Even though the subject of channel flow have been investigated extensively, few studies have so far evaluated for these effects. Recently, Prathap Kumar et al. [5, 6] and Umavathi [7] analyzed the free convection in a vertical double passage wavy channel for both Newtonian and non-Newtonian fluids.

The growing need for chemical reactions in chemical and hydro metallurgical industries requires the study of heat and mass transfer in the presence of chemical reactions. There are many transport processes that are governed by the simultaneous action of buoyancy forces due to both thermal and mass diffusion in the presence of chemical reaction effect. These processes are observed in nuclear reactor safety and combustion systems, solar collectors as well as chemical and metallurgical engineering. Das et al. [8] have studied the effects of mass transfer on the flow started impulsively past an infinite vertical plate in the presence of wall heat flux and chemical reaction. Muthucumaraswamy and Ganeshan [9, 10] have studied the impulsive motion of a vertical plate with heat flux/mass and diffusion of chemically reactive species. Seddeek [11] has studied the finite element method for the effects of chemical reaction, variable viscosity, thermophoresis, and heat generation/absorption on a boundary layer hydro magnetic flow with heat and mass transfer over a heat surface.

When the previous studies are reviewed, one can see there was no much study performed on heat and mass transfer on a vertical enclosure with baffle. Therefore, an analysis is made to understand heat and mass transfer in a vertical channel by introducing a perfectly thin baffle for viscous fluid in the presence of first order chemical reaction.

II. MATHEMATICAL FORMULATION:

Consider a steady, two-dimensional laminar fully developed mixed convection flow in an open ended vertical channel filled with pure viscous fluid and is concentrated. The *X*-axis is taken vertically upward, and parallel to the direction of buoyancy, and the *Y*-axis is normal to it (see Fig. 1). The walls are maintained at a constant temperature. The fluid properties are assumed to be constant. The channel is divided into two passages by means of thin, perfectly conducting plane baffle and each stream will have its own pressure gradient and hence the velocity will be individual in each stream.

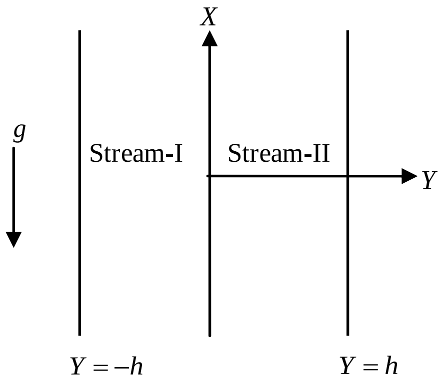

Figure 1. Physical configuration.

The governing equations for velocity, temperature and concentrations are

Stream-I

$$\rho g \beta_T \left(T_1 - T_{W_2}\right) + \rho g \beta_{C1}(C_1 - C_{01}) - \frac{\partial P}{\partial X} + \mu \frac{d^2 U_1}{dY^2} = 0 \tag{1}$$

$$K \frac{d^2 T_1}{dY^2} + \mu \left(\frac{dU_1}{dY}\right)^2 = 0 \tag{2}$$

$$D \frac{d^2 C_1}{dY^2} - K_1 C_1 = 0 \tag{3}$$

Stream-II

$$\rho g \beta_T \left(T_2 - T_{W_2}\right) + \rho g \beta_{C2}(C_2 - C_{02}) - \frac{\partial P}{\partial X} + \mu \frac{d^2 U_2}{dY^2} = 0 \tag{4}$$

$$K \frac{d^2 T_2}{dY^2} + \mu \left(\frac{dU_2}{dY}\right)^2 = 0 \tag{5}$$

$$D \frac{d^2 C_2}{dy^2} - K_2 C_2 = 0 \tag{6}$$

subject to the boundary conditions on velocity, temperature and concentration as

$U_1 = 0$, $T_1 = T_{W_1}$, $C_1 = C_1'$ at $Y = -h$

$U_2 = 0$, $T_2 = T_{W_2}$, $C_2 = \bar{C}_2$ at $Y = h$

$U_1 = 0$, $U_2 = 0$, $T_1 = T_2$, $\frac{dT_1}{dY} = \frac{dT_2}{dY}$, $C_1 = \bar{C}_1$,

$C_2 = C_2'$ at $Y = h^*$ (7)

Introducing the following non-dimensional variables

$$u_i = \frac{U_i}{\bar{U}_1}, \quad \theta_i = \frac{T_i - T_{W_2}}{T_{W_1} - T_{W_2}}, \quad Gr = \frac{g\beta_T \Delta T h^3}{\upsilon^2},$$

$$Gc_1 = \frac{g\beta_{C1}\Delta C_1 h^3}{\upsilon^2}, \quad Gc_2 = \frac{g\beta_{C2}\Delta C_2 h^3}{\upsilon^2}, \quad \mathrm{Re} = \frac{\bar{U}_1 h}{\upsilon},$$

$$Br = \frac{\bar{U}_1^2 \mu}{K\Delta T}, \quad p = \frac{h^2}{\mu \bar{U}_1}\frac{\partial P}{\partial X}, \quad i = 1,2, \quad \Delta T = T_{W_2} - T_{W_1},$$

$$\phi_1 = \frac{C - C_{01}}{C_1' - C_{01}}, \quad \phi_2 = \frac{C - C_{02}}{C_1' - C_{02}}, \quad \Delta C_1 = C_1' - C_{01},$$

$$\Delta C_2 = C_2' - C_{02}, \quad Y = \frac{y}{h}, \quad y^* = \frac{h^*}{h}, \quad n_1 = \frac{\overline{C_1} - C_{01}}{C_1^1 - C_{01}},$$

$$n_2 = \frac{\overline{C_2} - C_{02}}{C_2^1 - C_{02}}. \tag{8}$$

One obtains the momentum, energy and concentration equations in the non-dimensional form in stream-I and stream-II as

Stream-I

$$\frac{d^2 u_1}{dy^2} + GR_T \theta_1 + GR_{C1}\phi_1 - p = 0 \tag{9}$$

$$\frac{d^2 \theta_1}{dy^2} + Br\left(\frac{du_1}{dy}\right)^2 = 0 \tag{10}$$

$$\frac{d^2 \phi_1}{dy^2} - \alpha_1^2 \phi_1 = 0 \tag{11}$$

Stream-II

$$\frac{d^2 u_2}{dy^2} + GR_T \theta_2 + GR_{C2}\phi_2 - p = 0 \tag{12}$$

$$\frac{d^2 \theta_2}{dy^2} + Br\left(\frac{du_2}{dy}\right)^2 = 0 \tag{13}$$

$$\frac{d^2 \phi_2}{dy^2} - \alpha_2^2 \phi_2 = 0 \tag{14}$$

subject to the boundary conditions,

$u_1 = 0$, $\theta_1 = 1$, $\phi_1 = 1$ at $y = -1$

$u_2 = 0$, $\theta_2 = 0$, $\phi_2 = n_2$, $y = 1$

$u_1 = 0$, $u_2 = 0$, $\theta_1 = \theta_2$, $\frac{d\theta_1}{dy} = \frac{d\theta_2}{dy}$, $\phi_1 = n_1$,

$\phi_2 = 1$ at $y = y^*$ (15)

where $\alpha_1 = h\sqrt{\frac{K_1}{D}}$, $\alpha_2 = h\sqrt{\frac{K_2}{D}}$, $GR_T = \frac{Gr}{\text{Re}}$ and $GR_{C1} = \frac{Gc_1}{\text{Re}}$, $GR_{C2} = \frac{Gc_2}{\text{Re}}$.

II. SOLUTIONS:

Equations (9), (10), (12) and (13) are coupled non-linear ordinary differential equations. Approximate solutions can be found by using the regular perturbation method. The Brinkman number is chosen as the perturbation parameter. Adopting this technique, solutions for velocity and temperature are assumed in the form

$$u_i(y) = u_{i0}(y) + Bru_{i1}(y) + Br^2u_{i2}(y) + ... \quad (16)$$

$$\theta_i(y) = \theta_{i0}(y) + Br\theta_{i1}(y) + Br^2\theta_{i2}(y) + ... \quad (17)$$

Substituting Equations (16), (17) in Equations (9), (10), (12) and (13) and equating the coefficients of like power of Br to zero and one, we obtain the zeroth and first order equations as

Stream-I

Zeroth order equations

$$\frac{d^2\theta_{10}}{dy^2} = 0 \quad (18)$$

$$\frac{d^2u_{10}}{dy^2} + GR_T\theta_{10} + GR_{C1}\phi_1 - p = 0 \quad (19)$$

First order equations

$$\frac{d^2\theta_{11}}{dy^2} + \left(\frac{du_{10}}{dy}\right)^2 = 0 \quad (20)$$

$$\frac{d^2u_{11}}{dy^2} + GR_T\theta_{11} = 0 \quad (21)$$

Stream-II

Zeroth order equations

$$\frac{d^2\theta_{20}}{dy^2} = 0 \quad (22)$$

$$\frac{d^2u_{20}}{dy^2} + GR_T\theta_{20} + GR_{C2}\phi_2 - p = 0 \quad (23)$$

First order equations

$$\frac{d^2\theta_{21}}{dy^2} + \left(\frac{du_{20}}{dy}\right)^2 = 0 \quad (24)$$

$$\frac{d^2u_{21}}{dy^2} + GR_T\theta_{21} = 0 \quad (25)$$

The corresponding zeroth order and first order boundary conditions are

$$u_{10} = 0,\ \theta_{10} = 1 \text{ at } y = -1$$
$$u_{20} = 0,\ \theta_{20} = 0 \text{ at } y = 1 \quad (26)$$
$$u_{10} = 0,\ u_{20} = 0,\ \theta_{10} = \theta_{20},\ \frac{d\theta_{10}}{dy} = \frac{d\theta_{20}}{dy} \text{ at } y = y*$$

$$u_{11} = 0,\ \theta_{11} = 0 \text{ at } y = -1$$
$$u_{21} = 0,\ \theta_{21} = 0 \text{ at } y = 1$$
$$u_{11} = 0,\ u_{21} = 0,\ \theta_{11} = \theta_{21},\ \frac{d\theta_{11}}{dy} = \frac{d\theta_{21}}{dy}, \text{ at } y = y* \quad (27)$$

The solutions of zeroth and first order equations (18) to (25) using the boundary conditions as given in equations (26) and (27) are given as follows

Stream-I

$$\theta_{10} = c_1y + c_2 \quad (28)$$

$$u_{10} = A_1 + A_2y + r_1y^2 + r_2y^3 + r_3Cosh(\alpha_1y) + r_4Sinh(\alpha_1y) \quad (29)$$

Stream-II

$$\theta_{20} = c_3y + c_4 \quad (30)$$

$$u_{20} = A_4 + A_3y + r_5y^2 + r_6y^3 + r_7Cosh(\alpha_2y) + r_8Sinh(\alpha_2y) \quad (31)$$

First order equations

Stream-I

$$\theta_{11} = E_1y + E_2 + p_1y^2 + p_2y^3 + p_3y^4 + p_4y^5 + p_5y^6 + p_6Cosh(2\alpha_1y) + p_7Sinh(2\alpha_1y) + p_8Cosh(\alpha_1y) + p_9Sinh(\alpha_1y) + p_{10}yCosh(\alpha_1y) + p_{11}ySinh(\alpha_1y) + p_{12}y^2Cosh(\alpha_1y) + p_{13}y^2Sinh(\alpha_1y) \quad (32)$$

$$u_{11} = E_5y + E_6 + R_1y^2 + R_2y^3 + R_3y^4 + R_4y^5 + R_5y^6 + R_6y^7 + R_7y^8 + R_8Cosh(2\alpha_1y) + R_9Sinh(2\alpha_1y) + R_{10}Cosh(\alpha_1y) + R_{11}Sinh(\alpha_1y) + R_{12}yCosh(\alpha_1y) + R_{13}ySinh(\alpha_1y) + R_{14}y^2Cosh(\alpha_1y) + R_{15}y^2Sinh(\alpha_1y) \quad (33)$$

Stream-II

$$\theta_{21} = E_4 + E_3y + s_1y^2 + s_2y^3 + s_3y^4 + s_4y^5 + s_5y^6 + s_6Cosh(2\alpha_2y) + s_7Sinh(2\alpha_2y) + s_8Cosh(\alpha_2y) + s_9Sinh(\alpha_2y) + s_{10}yCosh(\alpha_2y) + s_{11}ySinh(\alpha_2y) + s_{12}y^2Cosh(\alpha_2y) + s_{13}y^2Sinh(\alpha_2y) \quad (34)$$

$$u_{21} = E_8 + E_7y + w_1y^2 + w_2y^3 + w_3y^4 + w_4y^5 + R_5y^6 + w_6y^7 + w_7y^8 + w_8Cosh(2\alpha_2y) + w_9Sinh(2\alpha_2y) + w_{10}Cosh(\alpha_2y) + w_{11}Sinh(\alpha_2y) + w_{12}yCosh(\alpha_2y) + R_{13}ySinh(\alpha_2y) + w_{14}y^2Cosh(\alpha_2y) + w_{15}y^2Sinh(\alpha_2y) \quad (35)$$

Solutions of equations (11) and (14) can be obtained directly and are given as follows

$$\phi_1 = B_1Cosh(\alpha_1y) + B_2Sinh(\alpha_1y) \quad (36)$$

$$\phi_2 = B_3Cosh(\alpha_2y) + B_4Sinh(\alpha_2y) \quad (37)$$

III. RESULTS AND DISCUSSIONS:

The velocity, temperature, and concentration fields for a viscous fluid in a vertical channel in the presence of first order chemical reaction containing a thin conducting baffle is studied analytically. The non-linear coupled ordinary differential equations governing the flow have been solved by regular perturbation method. The thermal Grashoff number GR_T, mass Grashoff numbers $GR_C\left(=GR_{C1}=GR_{C2}\right)$, pressure gradient p, chemical reaction parameter α are fixed as 5, 5,-5, and 1 respectively, for all the graphs except the varying one.

The effect of thermal Grashoff number GR_T on the velocity and temperature is shown in Figures 2a,b,c and 3a,b,c respectively at three different baffle positions. As the thermal Grashoff number increases, the velocity increases near the left (hot) wall, and also at the right (cold) wall at all the baffle positions in both the streams. The maximum point of velocity is in stream-II for the baffle position at $y^*=-0.8$, where as it is in stream-I for the baffle position at $y^*=0.0$ and 0.8. From Figure 3a,b,c it is seen that when the baffle position is near the hot wall, the temperature increases in stream-I where as in stream-II it increases up to $y=0.5$ and decreases from $y=0.5$ as seen in Figure 3a. The temperature increases continuously in both the streams when the baffle is at the center of the channel and at the right wall as seen in Figure 3b and 3c respectively. The increase in temperature with increase in thermal Grashoff number is obvious. As the increase in thermal Grashof number increases the buoyancy force and hence increases the velocity and temperature fields.

The effect of mass Grashoff number $GR_C\left(=GR_{C1}=GR_{C2}\right)$ on the flow is shown in Figure 4a,b,c and Figure 5a,b,c at all the baffle positions. The effect of GR_C is to increases the velocity and temperature in both the streams at all baffle positions. When the baffle position is near the left and right walls, there is no much effect of mass Grashoff number on the flow field. There is a distinct increase in velocity and temperature field in stream-II when the baffle position is near the left wall, in stream-I and in stream-II when the baffle position is in the middle of the channel and in stream-I when the baffle position is near the right wall. From Figure 5a,b,c it is seen that there is reversal of temperature field in stream-II when the baffle position is near the left and middle of channel, where as there is no such reversal of the temperature field when the baffle position is at the right wall. This is due to the fact that the left wall is at higher temperature when compared to right wall. Further increase in mass Grashoff number implies increase in concentration buoyancy force and hence increases the flow field. The similar result was also observed by Muturaj and Srinivas [12] without inserting the baffle in a vertical wavy channel.

The effect of Brinkman number Br on the flow field is shown in Figures 6a,b,c and 7a,b,c. As the Brinkman number increases both the velocity and temperature increases in both the streams at all the baffle positions. However the increase in velocity and temperature is highly significant in stream-I when the baffle position is near the left wall, in both the streams when the baffle position is in the middle e of the channel and in stream-I when the baffle position is at the right wall. This is due to the fact that as the Brinkman number increases the viscous dissipation also increases which results in the increase of the flow field.

The effect of first order chemical reaction parameter $\alpha\left(=\alpha_1=\alpha_2\right)$ on the velocity, temperature and concentration is shown in Figures 8a,b,c, 10a,b,c, and 11a,b,c respectively. As α increases the velocity, temperature and concentration decreases in both the streams at all the baffle positions. This is due to the facts that increase in α increase the concentration of the fluid which results in the reduction of heat and mass transfer. The increase of chemical reaction parameter was to reduce the velocity, concentration field as observed by Shivarih and Anand Rao [13] for unsteady MHD free convection flow past a vertical porous plate.

IV. CONCLUSIONS:

The effect of first order chemical reaction on heat and mass transfer of viscous fluid in a vertical double-passage channel by inserting perfectly thin conducting baffle was analysed. According to the results following conclusions can be drawn.

[1] Increasing the values of thermal Grashoff number increases the velocity and temperature in both the streams at all the baffle positions.

[2] Increasing the value of mass Grashoff number increases the velocity in both the streams at all the baffle positions. The temperature increases in stream-I where as it reveres its direction near the right wall when the baffle position is at the left wall. The temperature field increase in both the streams as mass Grashoff number increases, when the baffle position is in the middle of the channel and at the right wall.

[3] With the increase in Brinkman number, increases the flow field in both the streams at all the baffle positions.

[4] The increase in the chemical reaction parameter decreases the velocity, temperature and concentration in both the streams at all the baffle positions.

[5] By inserting the baffle the heat transfer can be enhanced depending on the position of the baffle.

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REFERENCES:

[1] Merkin JH (1980) Mixed convection boundary layer flow on a vertical surface in a saturated porous medium. J Eng Math 14:301–303.

[2] Merkin JH (1985) On dual solutions occurring in mixed convection in a porous medium. J Eng Math 20:171–179.

[3] Aly EH, Elliot L, Ingham DB (2003) Mixed convection boundary layer flow over a vertical surface embedded in a porous medium. Eur J Mech B, Fluids 22:529–543.

[4] Nazar R, Pop I (2004) Mixed convection boundary layer flow over a vertical surface in a porous medium with variable surface heat flux. In: Recent advances and applications in heat and mass transfer. IMECE, Kuwait, pp 87–97.

[5] J. Prathap Kumar, J.C. Umavathi, Ali J. Chamkha and H. Prema (2011) Free convection in a vertical double passage wavy channel filled with a Walters fluid (model B’). Int. J. Energy & Technology, 3: 1–13.

[6] J. Prathap Kumar, J.C. Umavathi, and H. Prema (2011) Free convection of Walter’s fluid flow in a vertical double-passage wavy channel with heat source. Int. J. Eng. Sci. Techno., 3: 136-165.

[7] J. C. Umavathi (2011) Mixed convection of micropolar fluid in a vertical double-passage channel. Int. J. Eng. Sci. Techno., 3: 197-209.

[8] Das UN, Deka RK, Soundalgekar VM (1994) Effects of mass transfer on the flow past an impulsively started infinite vertical plate with constant heat flux and chemical reaction. Forsch Ingenieurwes 60:284–287.

[9] Muthucumaraswamy R, Ganesan P (2000) On impulsive motion of a vertical plate with heat flux and diffusion of chemically reactive species. Forsch Ingenieurwes 66:17 23.

[10] Muthucumaraswamy R, Ganesan P (2001) First order chemical reaction on flow past an impulsively started vertical plate with uniform heat and mass flux. Acta Mech 147:45 57.

[11] Seddeek MA (2005) Finite element method for the effects of chemical reaction, variable viscosity, thermophoresis and heat generation/absorption on a boundary layer hydromagnetic flow with heat and mass transfer over a heat surface. Acta Mech 177:1–18.

[12] R.Muthuraj, S. Srinivas heat and mass transfer in a vertical wavy channel with traveling thermal waves and porous medium. Computers and mathematicas with Applications 59 (2010)3516-3528.

[13] S. Shivaiah, J. Anand Rao .Chemical reaction effect of an unsteady MHD free convection flow past a vertical porous plate in the presence of suction or injection. Appl.Mech., Vol.39, No.2, pp. 185-208, Belgrade 2012.

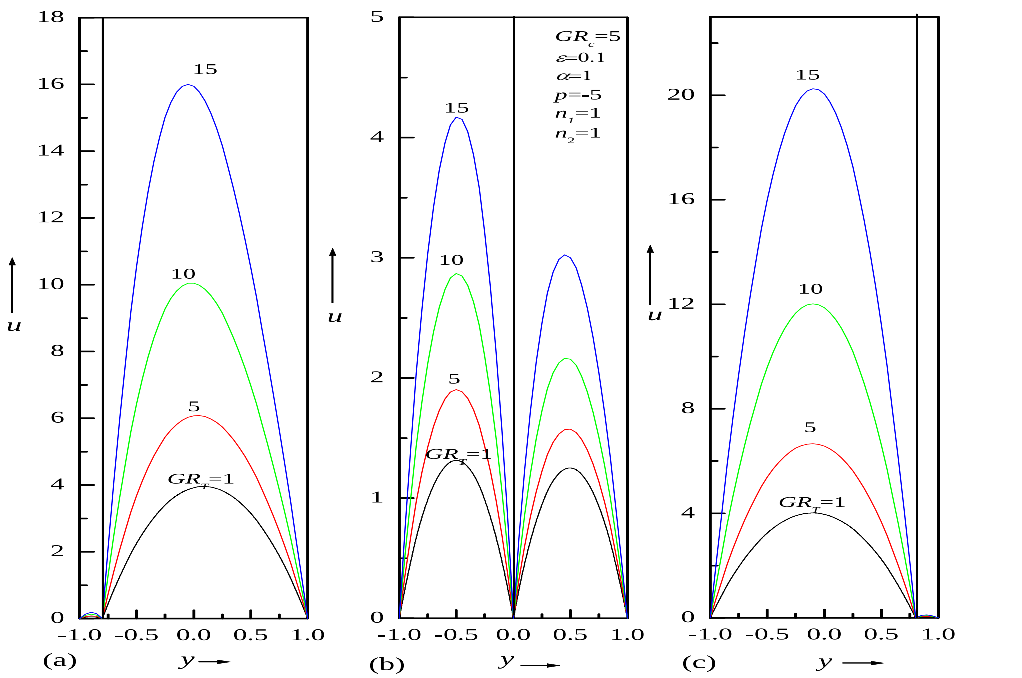


Figure2 . velocity distribution for different values of thermal Grashoff number GR_T at (a) $y^*=-0.8$ (b) $y^*=0.0$ (c) $y^*=0.8$

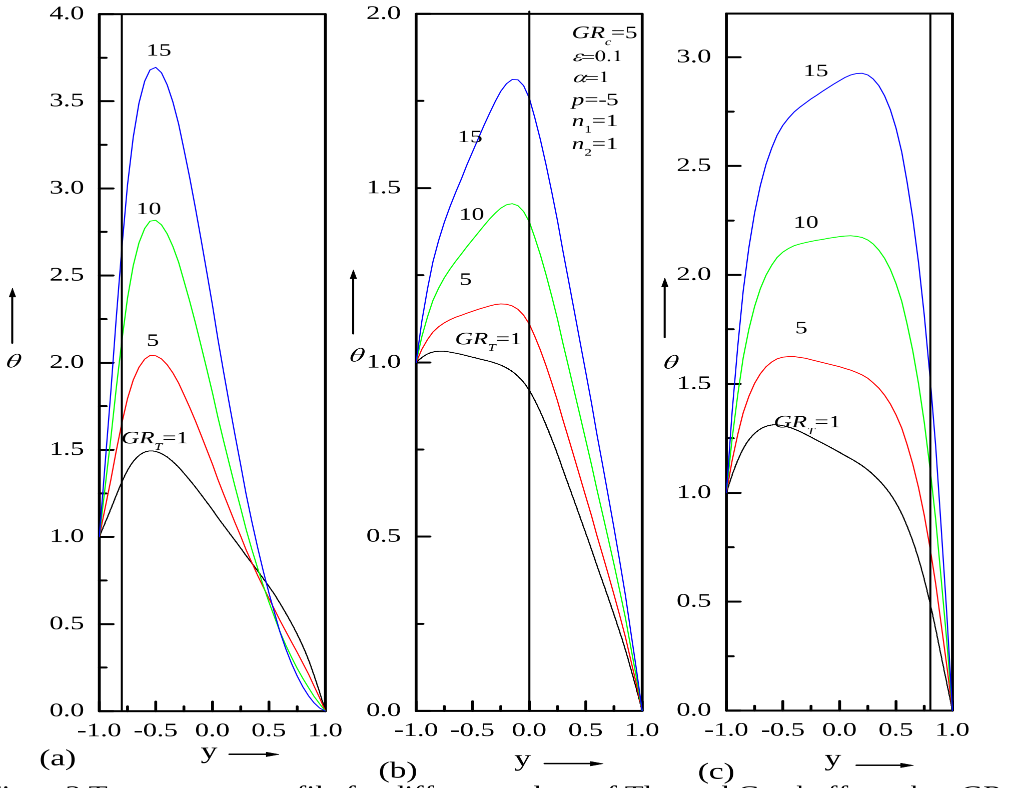


Figure3.Temperature profile for different values of Thermal Grashoff number GR_T at (a) y*=0.8 (b) y*=0 (c) y*=0.8

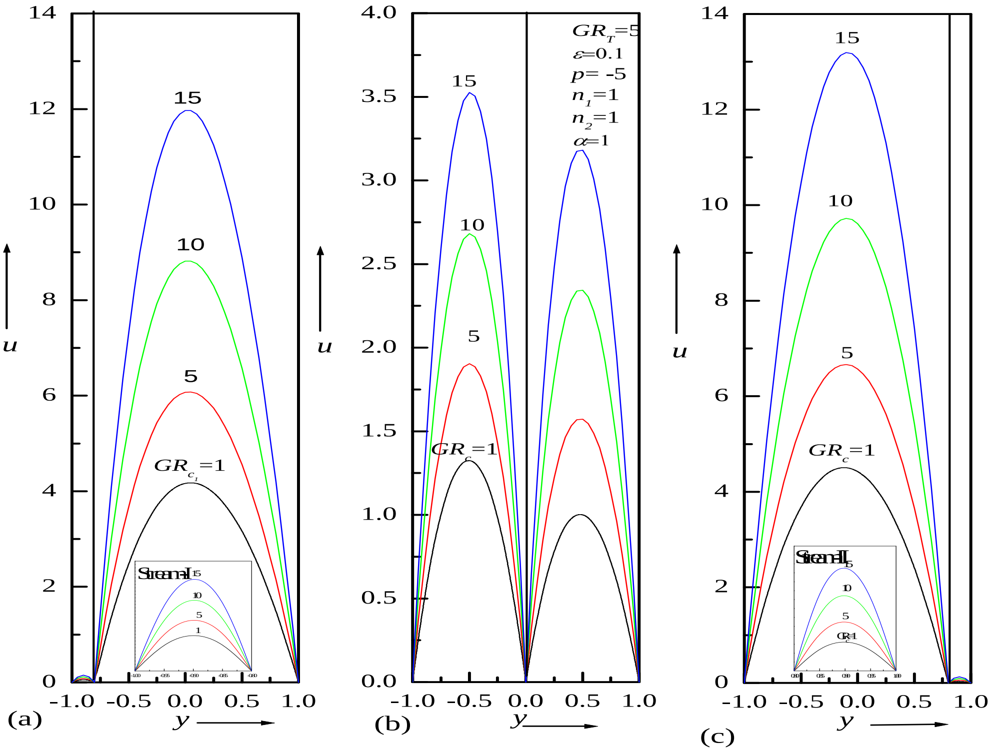


Figure4.Velocity profile for different values of mass Grashoff number GR_C at (a) y*=-0.8 (b) y*=0 (c) y*=0.8

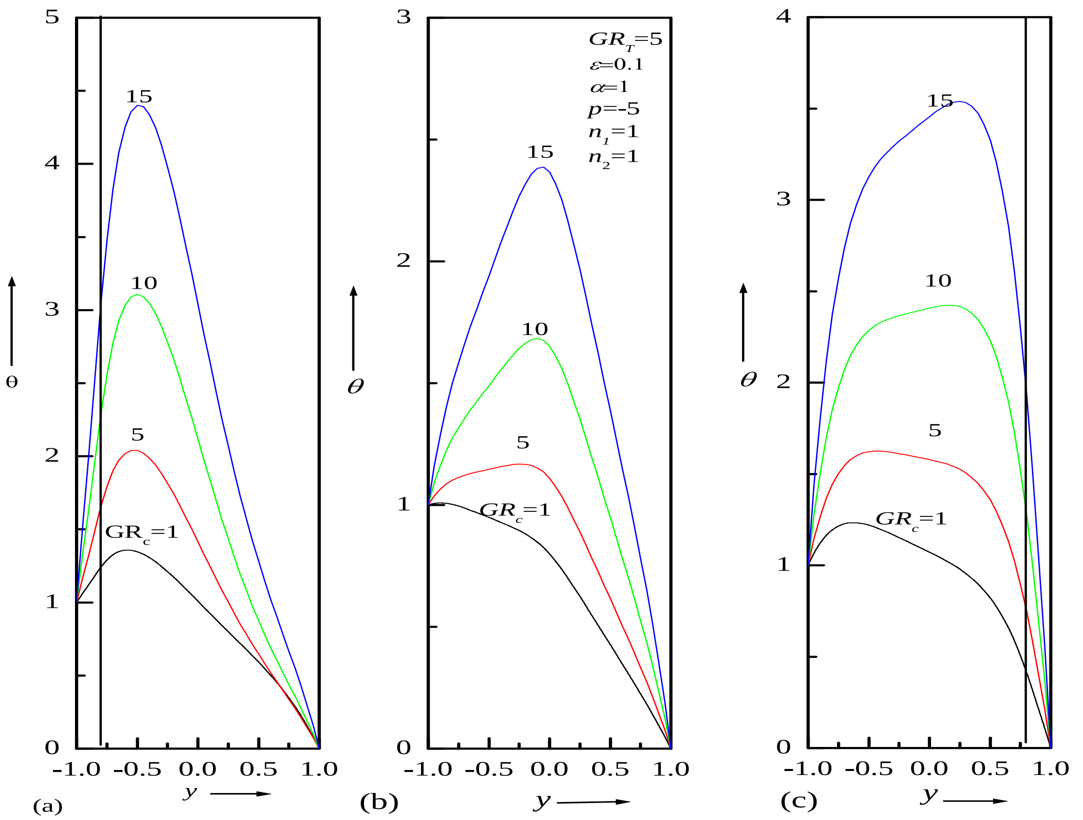


Figure5.Temperature profile for different values of mass Grashoff number GR_c at (a) $y^*=-0.8$ (b) $y^*=0$ (c) $y^*=0.8$

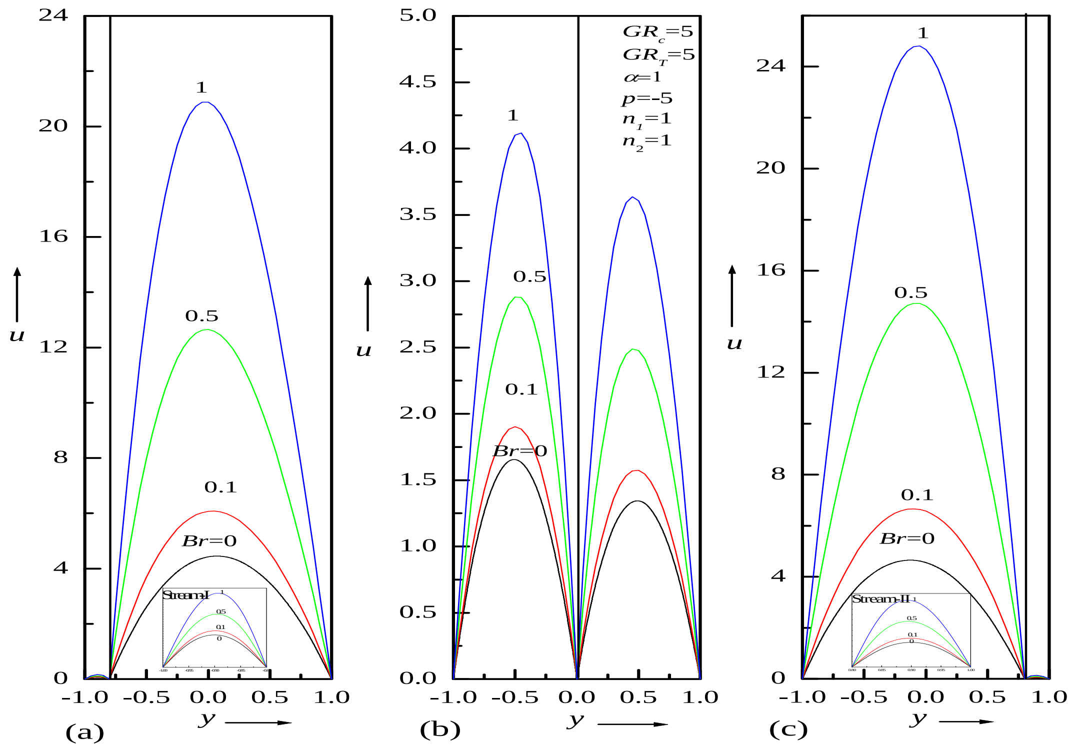


Figure6.Velocity for different values of Brinkman number Br (a)$y^*=-0.8$ (b)$y^*=0$ (c)$y^*=0.8$

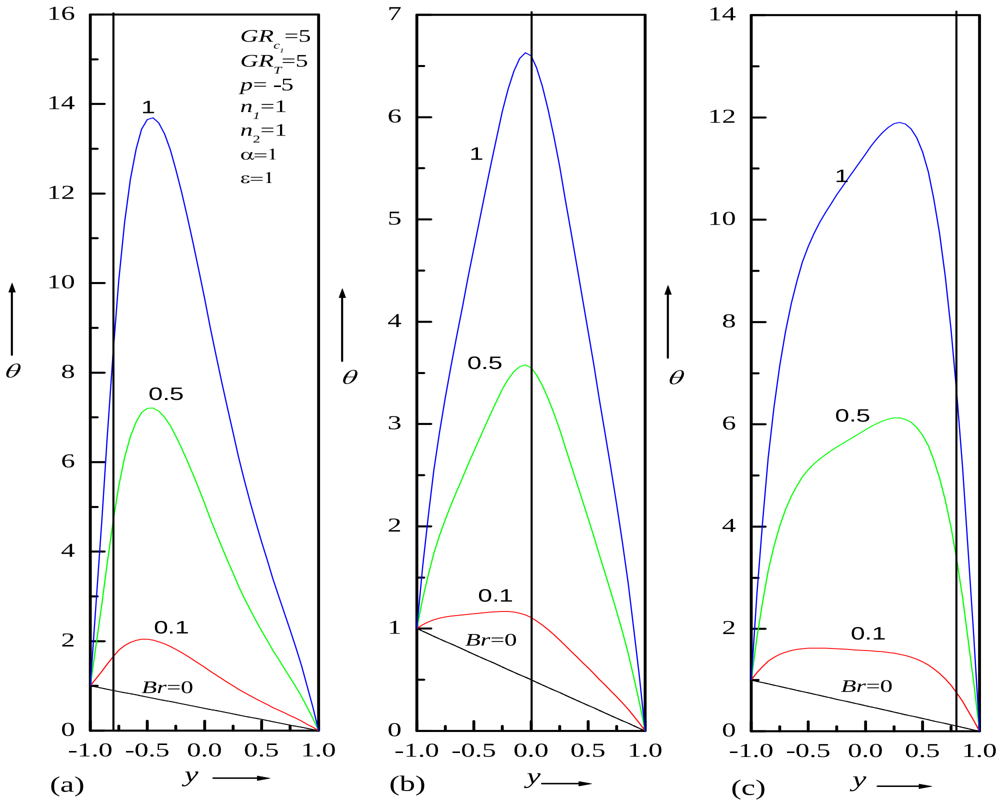


Figure7.Temperature profile for different values of Brinkman number Br at (a) y*=-0.8 (b) y*=0 (c) y*=0.8

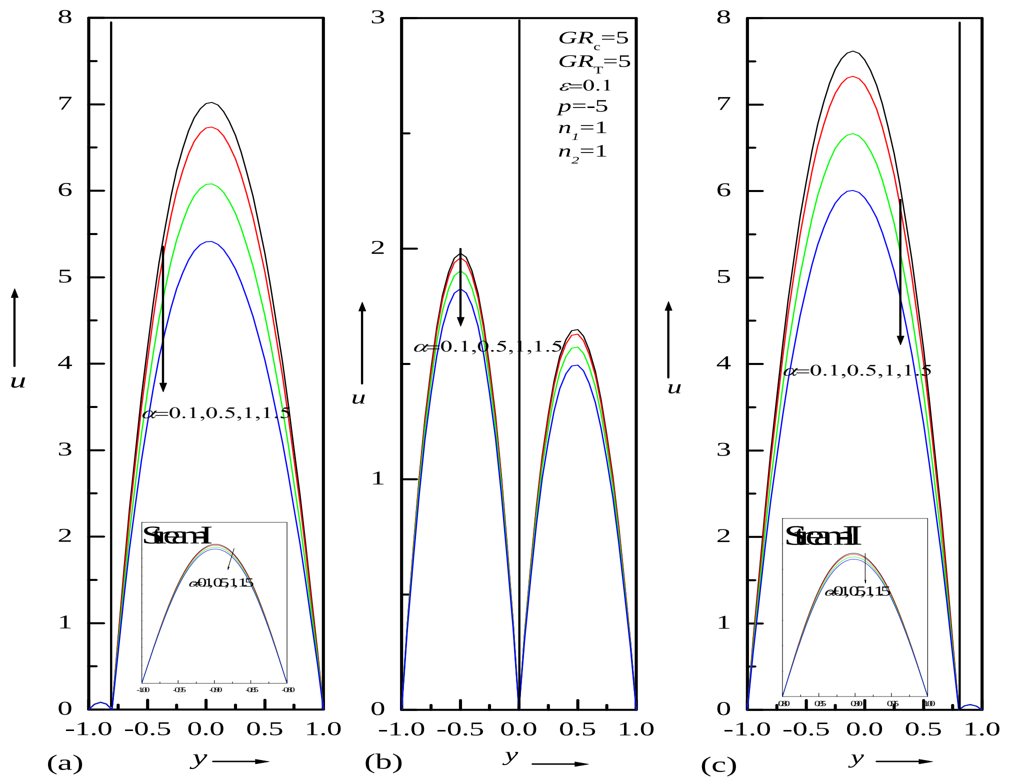


Figure8.Velocity profile for different values of chemical reaction parameter α at (a) y*=-0.8 (b) y*=0 (c) y*=0.8

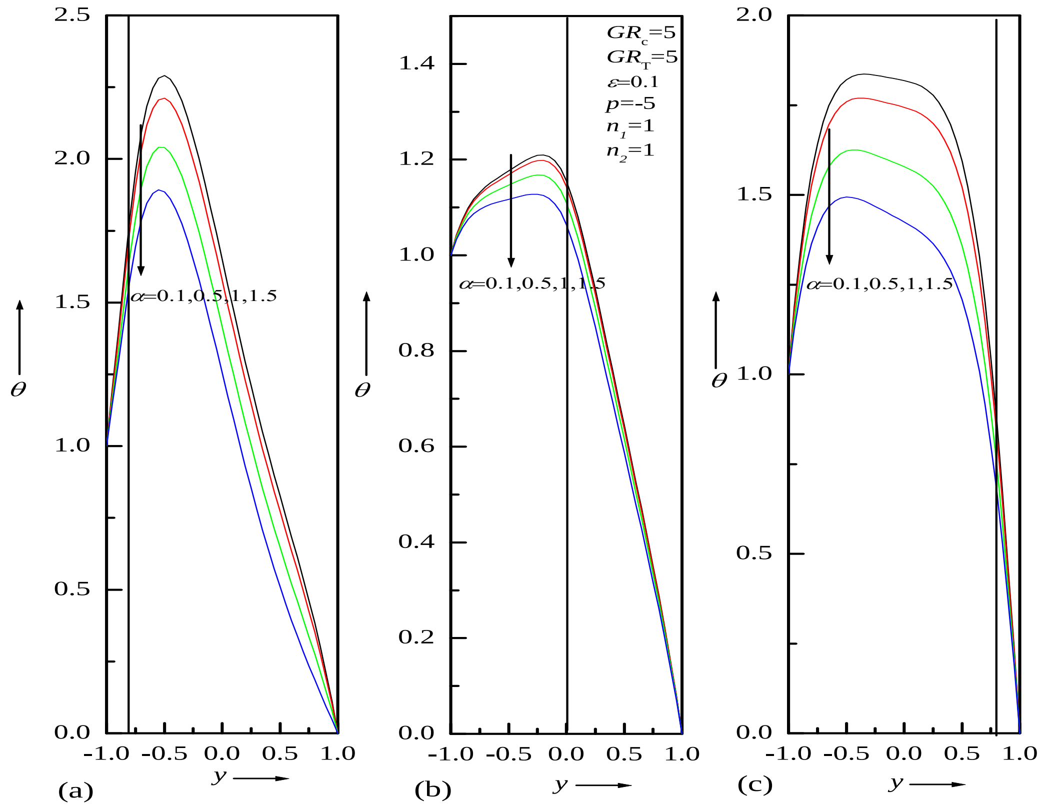
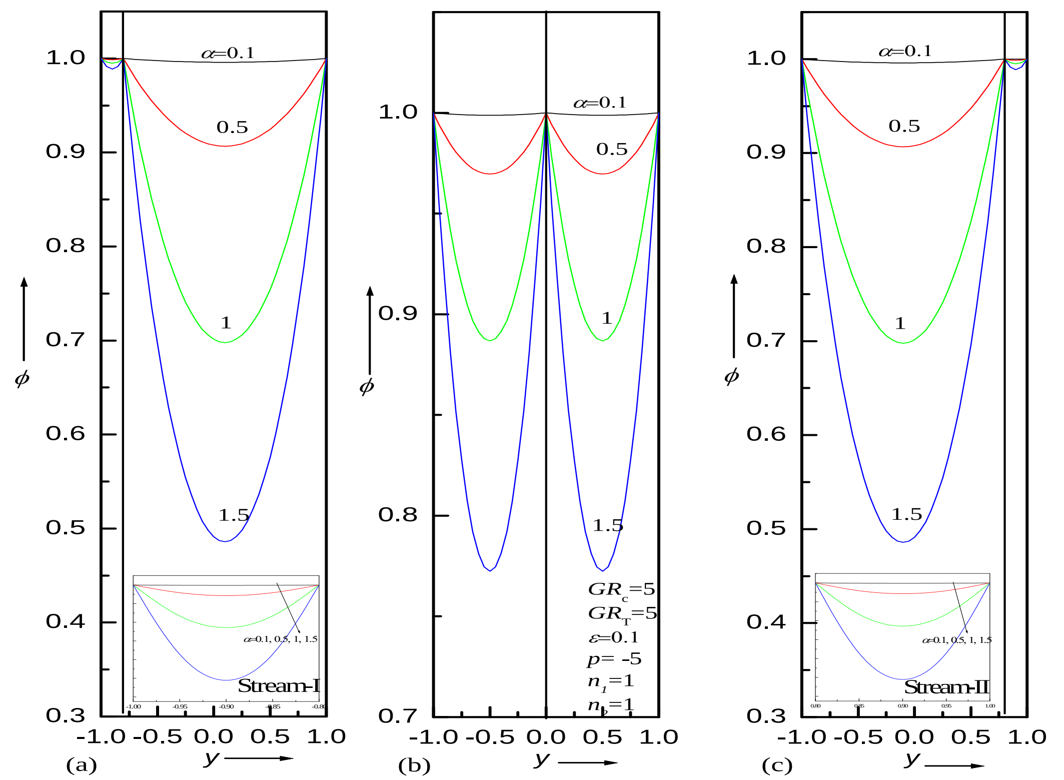


Figure9.Temperature profile for different values of chemical reaction parameter α at (a) y^*=-0.8 (b) y^*=0 (c) y^*=0.8



gure10.Concentration profile for different values of chemical reaction parameter α

NOMENCLATURE:

Br Brinkman number
C_p specific heat at constant pressure
C_1, C_2 the concentration in stream-I and stream-II
C_{01}, C_{02} reference concentrations
D diffusion coefficients
g acceleration due to gravity
Gr Grashoff number $\left(\frac{h^3 g\beta\Delta T}{\nu^2}\right)$
Gr_{C1} Grashoff number $\left(\frac{g\,\beta_{c1}\,\Delta C_1\,h^3}{\gamma^2}\right)$
Gr_{C1}, Gr_{C2} modified Grashoff number
h channel width
h^* width of passage
k thermal conductivity of fluid
p non-dimensional pressure gradient
Re Reynolds number $\left(\frac{\overline{U_1}h}{\gamma}\right)$
$\overline{U_1}$ reference velocity
U_1, U_2 dimensional velocity distributions
u_1, u_2 nondimensional velocities in stream-I, stream-II
T_1, T_2 dimensional temperature distributions
T_{w_1}, T_{w_2} temperatures of the boundaries
y^* baffle position

GREEK SYMBOLS

α_1, α_2 chemical reaction parameters
β_T coefficients of thermal expansion
β_C coefficients of concentration expansion
$\Delta T, \Delta C$ difference in temperatures and concentration
θ_i non-dimensional temperature
γ kinematics viscosity
ϕ_1, ϕ_2 non-dimensional concentrations
ρ density
μ viscosity

SUBSCRIPTS

i refer quantities for the fluids in stream-I and stream-II, respectively.

APPENDIX:

$$B_1 = \frac{Sinh(\alpha_1 y^*) + n\,Sinh(\alpha_1)}{Sinh(\alpha_1 y^*)Cosh(\alpha_1) + Sinh(\alpha_1)Cosh(\alpha_1 y^*)} \qquad B_2 = \frac{n\,Cosh(\alpha_1) - Cosh(\alpha_1 y^*)}{Sinh(\alpha_1 y^*)Cosh(\alpha_1) + Sinh(\alpha_1)Cosh(\alpha_1 y^*)}$$

$$B_3 = \frac{Sinh(\alpha_2) - n_2 Sinh(\alpha_2 y^*)}{Sinh(\alpha_2)Cosh(\alpha_2 y^*) - Sinh(\alpha_2 y^*)Cosh(\alpha_2)}, \quad B_4 = \frac{1 - B_3 Cosh(\alpha_2 y^*)}{Sinh(\alpha_2 y^*)}\; c_1 = -\frac{1}{2}, \quad c_2 = \frac{1}{2}, \quad c_3 = -\frac{1}{2},$$

$$c_4 = \frac{1}{2}, \quad r_1 = \frac{(p - GR_T\,c_2)}{2}, \quad r_2 = -\frac{GR_T c_1}{6}, \quad r_3 = -\frac{GR_{C1}\,B_1}{\alpha_1^2} \quad r_4 = -\frac{GR_{C1}\,B_2}{\alpha_1^2}, \quad r_5 = \frac{-GR_T c_4 + p}{2}, r_6 = \frac{GR_T c_3}{6},$$

$$r_7 = -\frac{GR_{C2}\,B_3}{\alpha_2^2}, \; r_8 = -\frac{GR_{C2}\,B_4}{\alpha_2^2}, \; A_2 = A_1 - r_1 + r_2 - r_3\,Cosh(\alpha_1) + r_4\,Sinh(\alpha_1),$$

$$A_1 = -\frac{\left(r_1\left(y^{*2} - 1\right) + r_2\left(y^{*3} + 1\right) + r_3\left(Cosh(\alpha_1 y^*) - Cosh(\alpha_1)\right) + r_4\left(Sinh(\alpha_1 y^*) + Sinh(\alpha_1)\right)\right)}{1 + y^*}$$

$$A_3 = \frac{\left(r_5\left(-y^{*2} + 1\right) + r_6\left(-y^{*3} + 1\right) + r_7\left(Cosh(\alpha_2 y^*) - Cosh(\alpha_2)\right) + r_8\left(Sinh(\alpha_2 y^*) - Sinh(\alpha_2)\right)\right)}{y^* - 1}$$

$$A_4 = -A_3 - r_5 - r_6 - r_7\,Cosh(\alpha_2) - r_8\,Sinh(\alpha_2), \qquad p_1 = -\frac{\left(2A_1^2 + r_4^2\alpha_1^2 - r_3^2\alpha_1^2\right)}{4}, \qquad p_2 = -\frac{2A_1 r_1}{3},$$

$$p_3 = -\frac{\left(4r_1^2 + 6A_1 r_2\right)}{12}, \qquad p_4 = -\frac{3r_1 r_2}{5}, \qquad p_5 = -\frac{3r_2^2}{10}, \qquad p_6 = -\frac{\left(r_3^2 + r_4^2\right)}{8}, \qquad p_7 = -\frac{r_3 r_4}{4},$$

$$p_8 = -\frac{\left(2A_1 r_4 \alpha_1^2 - 8r_1 r_3 \alpha_1 + 36r_2 r_4\right)}{\alpha_1^3}, \qquad p_9 = -\frac{\left(2A_1 r_3 \alpha_1^2 - 8r_1 r_4 \alpha_1 + 36r_2 r_3\right)}{\alpha_1^3}, p_{10} = -\frac{\left(4r_1 r_4 \alpha_1 - 24r_2 r_3\right)}{\alpha_1^2},$$

$$p_{11} = -\frac{\left(4r_1 r_3 \alpha_1 - 24r_2 r_4\right)}{\alpha_1^2}, p_{12} = -\frac{6r_2 r_4}{\alpha_1}, p_{13} = -\frac{6r_2 r_3}{\alpha_1}, \; s_1 = -\frac{(2A_3^2 + r_8^2\alpha_2^2 - r_7^2\alpha_2^2)}{4}, \; s_2 = \frac{-2A_3 r_5}{3},$$

$$s3 = \frac{-(4r_5^2 + 6A_3 r_6)}{12}, \; s_4 = -\frac{3r_6 r_5}{5}, s_5 = \frac{-3r_6^2}{10} \; s_6 = -\frac{r_7^2 + r_8^2}{8}, \; s_7 = \frac{-r_7 r_8}{4}, \; s_8 = -\frac{(2A_3 r_8 \alpha_2^2 - 8r_5 r_7 \alpha_2 + 36r_6 r_8)}{\alpha_2^3},$$

$$s_9 = -\frac{(2A_3 r_7 \alpha_2{}^2 - 8 r_5 r_8 \alpha_2 + 36 r_6 r_7)}{\alpha_2{}^3}\; s_{10} = \frac{-(4 r_5 r_8 \alpha_2 - 24 r_6 r_7)}{\alpha_2{}^2}, s_{11} = \frac{-(4 r_5 r_7 \alpha_2 - 24 r_6 r_8)}{\alpha_2{}^2}, s_{12} = -\frac{6 r_6 r_8}{\alpha_2}, s_{13} = -\frac{6 r_6 r_7}{\alpha_2},$$

$$G_9 = -\left(p_1 - p_2 + p_3 - p_4 + p_5 + p_6\, Cosh\left(2\alpha_1\right) - p_7\, Sinh\left(2\alpha_1\right) + p_8\, Cosh\left(\alpha_1\right)\right.$$
$$\left. - p_9\, Sinh\left(\alpha_1\right) - p_{10}\, Cosh\left(\alpha_1\right) + p_{11}\, Sinh\left(\alpha_1\right) + p_{12}\, Cosh\left(\alpha_1\right) - p_{13}\, Sinh\left(\alpha_1\right)\right)$$

$$G_{10} = -\left(s_1 + s_2 + s_3 + s_4 + s_5 + s_6\, Cosh\left(2\alpha_2\right) + s_7\, Sinh\left(2\alpha_2\right) + s_8\, Cosh\left(\alpha_2\right)\right.$$
$$\left. + s_9\, Sinh\left(\alpha_2\right) + s_{10}\, Cosh\left(\alpha_2\right) + s_{11}\, Sinh\left(\alpha_2\right) + s_{12}\, Cosh\left(\alpha_2\right) + s_{13}\, Sinh\left(\alpha_2\right)\right),$$

$$G_{11} = G_4 + G_3 y + s_1 y^{*2} + s_2 y^{*3} + s_3 y^{*4} + s_4 y^{*5} + s_5 y^6 + s_6 Cosh\left(2\alpha_1 y^*\right) + s_7 Sinh\left(2\alpha_1 y^*\right) + s_8 Cosh\left(\alpha_1 y^*\right)$$
$$+ s_9 Sinh\left(\alpha_1 y^*\right) + s_{10} y^* Cosh\left(\alpha_1 y^*\right) + s_{11} y Sinh\left(\alpha_1 y^*\right) + s_{12} y^{*2}\, Cosh\left(\alpha_1 y^*\right) + s_{13} y^{*2}\, Sinh\left(\alpha_1 y^*\right)$$
$$- p_1 y^{*2} - p_2 y^{*3} - p_3 y^{*4} - p_4 y^{*5} - p_5 y^{*6} - p_6\, Cosh\left(2\alpha_1 y^*\right) - p_7\, Sinh\left(2\alpha_1 y^*\right) - p_8\, Cosh\left(\alpha_1 y^*\right),$$
$$- p_9\, Sinh\left(\alpha_1 y^*\right) - p_{10}\, y^* Cosh\left(\alpha_1 y^*\right) - p_{11} y^* Sinh\left(\alpha_1 y^*\right) - p_{12} y^{*2}\, Cosh\left(\alpha_1 y^*\right) - p_{13} y^{*2}\, Sinh\left(\alpha_1 y^*\right)$$

$$G_{12} = 2 s_1 y^* + 3 s_2 y^{*2} + 4 s_3 y^{*3} + 5 s_4 y^{*4} + 6 s_5 y^{*5} + 2\alpha_2 s_6\, Sinh\left(2\alpha_2 y^*\right) + 2\alpha_2 s_7\, Cosh\left(2\alpha_2 y^*\right) + s_8 \alpha_2\, Sinh\left(\alpha_2 y^*\right)$$
$$+ \alpha_2 s_9\, Cosh\left(\alpha_2 y^*\right) + s_{10}\left(y^* \alpha_2 Sinh\left(\alpha_2 y^*\right) + Cosh\left(\alpha_2 y^*\right)\right) + s_{11}\left(y^* \alpha_2\, Cosh\left(\alpha_2 y^*\right) + Sinh\left(\alpha_2 y^*\right)\right)$$
$$+ s_{12}\left(2 y^* Cosh\left(\alpha_2 y^*\right) + \alpha_2 y^{*2}\, Sinh\left(\alpha_2 y^*\right)\right) + s_{13}\left(2 y^* Sinh\left(\alpha_2 y^*\right) + \alpha_2 y^{*2}\, Cosh\left(\alpha_2 y^*\right)\right) - 2 p_1 y^*$$
$$- 3 p_2 y^{*2} - 4 p_3 y^{*3} - 5 p_4 y^{*4} - 6 p_5 y^{*5} - 2\alpha_1 p_6\, Sinh\left(2\alpha_1 y^*\right) - 2\alpha_1 p_7\, Cosh\left(2\alpha_1 y^*\right) - p_8 \alpha_1\, Sinh\left(\alpha_1 y^*\right)$$
$$- \alpha p_9\, Cosh\left(\alpha_1 y^*\right) - p_{10}\left(y^* \alpha_1 Sinh\left(\alpha_1 y^*\right) + Cosh\left(\alpha_1 y^*\right)\right) - p_{11}\left(y^* \alpha_1\, Cosh\left(\alpha_1 y^*\right) + Sinh\left(\alpha_1 y^*\right)\right)$$
$$- p_{12}\left(2 y^* Cosh\left(\alpha_1 y^*\right) + \alpha y^{*2}\, Sinh\left(\alpha_1 y^*\right)\right) - p_{13}\left(2 y^* Sinh\left(\alpha_1 y^*\right) + \alpha y^{*2}\, Cosh\left(\alpha_1 y^*\right)\right)$$

$$E_1 = -\frac{\left(y^* G_{12} + G_9 - G_{10} - G_{11} - G_{12}\right)}{2}, \quad E_2 = \frac{\left(G_9 + G_{10} + G_{11} + G_{12}\left(1 - y^*\right)\right)}{2}, \quad E_4 = G_{10} - E_3, \quad R_1 = -\frac{GR_T\, E_2}{2},$$

$$E_3 = \frac{\left(-G_9 + G_{10} + G_{11} - G_{12}\left(1 + y^*\right)\right)}{2}, \quad R_2 = -\frac{GR_T\, E_1}{6} \quad R_3 = -\frac{GR_T\, p_1}{12}, \quad R_4 = -\frac{GR_T\, p_2}{20}, \quad R_5 = -\frac{GR_T\, p_3}{30},$$

$$R_6 = -\frac{GR_T\, p_4}{42}, \quad R_7 = -\frac{GR_T\, p_5}{56}, \quad R_8 = -\frac{GR_T\, p_6}{4\alpha_1^2}, \quad R_9 = -\frac{GR_T\, p_7}{4\alpha_1^2}, \quad R_{10} = -\frac{\left(p_8 \alpha_1^2 - 2 p_{11} \alpha_1 + 6 p_{12}\right) GR_T}{\alpha_1^4}$$

$$R_{11} = -\frac{\left(p_9 \alpha_1^2 - 2 p_{10} \alpha_1 + 6 p_{13}\right) GR_T}{\alpha_1^4}, \quad R_{12} = -\frac{\left(p_{10} \alpha_1 - 4 p_{13}\right) GR_T}{\alpha_1^3}, \quad R_{13} = -\frac{\left(p_{11} \alpha_1 - 4 p_{12}\right) GR_T}{\alpha_1^3}, \quad R_{14} = -\frac{GR_T\, p_{12}}{\alpha_1^2},$$

$$R_{15} = -\frac{GR_T\, p_{13}}{\alpha_1^2}, \quad w_1 = -\frac{GR_T\, E_4}{2}, \quad w_2 = -\frac{GR_T\, E_3}{6}, w_3 = -\frac{GR_T\, s_1}{12}, w_4 = -\frac{GR_T\, s_2}{20}, w_5 = -\frac{GR_T\, s_3}{30}\;\; w_6 = -\frac{GR_T\, s_4}{42},$$

$$w_7 = -\frac{GR_T\, s_5}{56}, \quad w_8 = -\frac{GR_T\, s_6}{4\alpha_2^2}, \quad w_9 = -\frac{GR_T\, s_7}{4\alpha_2^2}, \quad w_{11} = -\frac{\left(s_9 \alpha_2^2 - 2 s_{10} \alpha_2 + 6 s_{13}\right) GR_T}{\alpha_2^4}, \quad w_{12} = -\frac{\left(s_{10} \alpha_2 - 4 s_{13}\right) GR_T}{\alpha_2^3},$$

$$w_{13} = -\frac{\left(s_{11} \alpha_2 - 4 s_{12}\right) GR_T}{\alpha_2^3}, \quad w_{14} = -\frac{GR_T\, s_{12}}{\alpha_2^2}, \quad w_{15} = -\frac{GR_T\, s_{13}}{\alpha_2^2},$$

$$G_{13} = -\left(R_1 - R_2 + R_3 - R_4 + R_5 - R_6 + R_7 + R_8 Cosh\left(2\alpha_1\right) - R_9 Sinh\left(2\alpha_1\right) + R_{10} Cosh\left(\alpha_1\right)\right.$$
$$\left. - R_{11}\, Sinh\left(\alpha_1\right) - R_{12}\, Cosh\left(\alpha_1\right) + R_{13}\, Sinh\left(\alpha_1\right) + R_{14} Cosh\left(\alpha_1\right) - R_{15} Sinh\left(\alpha_1\right)\right),$$

$$G_{15} = R_1 y^{*2} + R_2 y^{*3} + R_3 y^{*4} + R_4 y^{*5} + R_5 y^{*6} + R_6 y^{*7} + R_7 y^{*8} + R_8\, Cosh\left(2\alpha_1 y^*\right) + R_9\, Sinh\left(2\alpha_1 y^*\right)$$
$$+ R_{10}\, Cosh\left(\alpha_1 y^*\right) + R_{11}\, Sinh\left(\alpha_1 y^*\right) + R_{12}\, y^* Cosh\left(\alpha_1 y^*\right) + R_{13} y^* Sinh\left(\alpha_1 y^*\right) + R_{14} y^{*2}\, Cosh\left(\alpha_1 y^*\right),$$
$$+ R_{15} y^{*2}\, Sinh\left(\alpha_1 y^*\right)$$

$$G_{14} = -\left(w_1 + w_2 + w_3 + w_4 + w_5 + w_6 + w_7 + w_8\, Cosh\left(2\alpha_2\right) + w_9\, Sinh\left(2\alpha_2\right) + w_{10}\, Cosh\left(\alpha_2\right)\right.$$
$$\left. + w_{11}\, Sinh\left(\alpha_2\right) + w_{12}\, Cosh\left(\alpha_2\right) + R_{13}\, Sinh\left(\alpha_2\right) + w_{14}\, Cosh\left(\alpha_2\right) + w_{15}\, Sinh\left(\alpha_2\right)\right),$$

$$G_{16} = w_1 y^{*2} + w_2 y^{*3} + w_3 y^{*4} + w_4 y^{*5} + w_5 y^{*6} + w_6 y^{*7} + R_7 y^{*8} + w_8\, Cosh\left(2\alpha_2 y^*\right) + w_9\, Sinh\left(2\alpha_2 y^*\right)$$
$$+ w_{10}\, Cosh\left(\alpha_2 y^*\right) + w_{11}\, Sinh\left(\alpha_2 y^*\right) + w_{12}\, y^* Cosh\left(\alpha_2 y^*\right) + w_{13} y^* Sinh\left(\alpha_2 y^*\right) + w_{14} y^{*2}\, Cosh\left(\alpha_2 y^*\right),$$
$$+ w_{15} y^{*2}\, Sinh\left(\alpha_2 y^*\right)$$

$$E_5 = \left(G_{15} - G_{13}\right) / \left(1 + y^*\right), \; E_7 = \left(G_{14} - G_{16}\right) / \left(1 - y^*\right), \; E_6 = G_{13} + E_5, \; E_8 = G_{14} - E_7.$$