RESEARCH ARTICLE OPEN ACCESS

Energy-Efficient Power Allocation And Performance Estimation For MIMO-MRC Systems

Ananda Reddy Mekala[1], Penchalaiahyadla[2], Irala Suneetha[3], Anitha Mekala[4]
[1,2,3,4](Department of Electronics and CommunicatoinAnnamacharya Institute of Technology and Sciences, Tirupati)

ABSTRACT
Adaptive power allocation problem where it minimizes the energy-per-good bit (EPG) of a system employing a multiple-input multiple-output maximum ratio combining (MIMO-MRC) scheme is formulated. Closed-form results are obtained for the optimum transmit power and minimum EPG as a function of the number of antennas employed and the quality of the channel. The cumulative distribution function (CDF) of the minimum EPG in closed form is obtained to assess the performance of the solution in a statistically varying channel. The energy-efficiency trade-off between enhanced diversity and the increased circuit power consumption of multiple antennas is explored. In particular, EPG CDF in a numerical example is used to find the most energy-efficient number of antennas for a given probability of outage. Both Rayleigh and Rician MIMO fading channels are considered.
Keywords-Energy minimization, MIMO, beamforming, MIMO MRC, fading channels, adaptive power allocation. Introduction.

I. INTRODUCTION

Adaptive power allocation or more generally adaptive modulation, where the transmitter adapts its transmit power, coding, modulation or any combination thereof in response to the channel fading, is a technique that can provide substantial gains in spectral efficiency and reliability. Recently adaptive modulation has been used to provide substantial gains in energy efficiency especially in cases where the energy consumed by the communication circuitry is not negligible. It is well known that multiple antennas at the transmitter and receiver or multiple-input multiple output (MIMO) can provide dramatic improvements in spectral efficiency and reliability without requiring an increase in bandwidth or power. However, using multiple radio chains incurs a higher circuit power consumption. In this paper the trade-off between the diversity advantage of multiple antennas and the higher circuit power consumption by focusing on energy efficiency of the wireless link is studied.

Most works in the area of energy-efficient communications either involve solving a convex optimization problem or evaluate the solution by an exhaustive search. Some of this work has also been extended to the MIMO scenario, where

Alamouti space-time block code (STBC)is used. In contrast to all of these works, closed form solutions are provided. Also unlike, perfect channel

state information (CSI) at the transmitter and hence focus on beamforming is assumed . In contrast to which relied on simulation to analyze performance, a complete statistical characterization of the solution (closed-form CDF of minimum EPG) is made possible in this work by simplifications in the EPG model (described in section II-C). The contributions of this letter are:

1) Problem formulation: the objective function, energy-per good bit(EPG), is extended to a MIMO-MRC system
2) Closed-form solutions for energy-efficient transmit power, spectral efficiency and minimum EPG
3) Statistical performance analysis: CDF of the minimum EPG is characterized for any fading distribution
4) Numerical example: demonstrates application of closed form results by characterizing the best MIMO configuration as a function of the ratio of circuit power to transmit power costs.

The paper is organized as follows. In section II, the EPG objective function for the MIMO-MRC system is Introduced and the energy minimization problem is formulated. In section III, we derive closed-form results for the optimum transmit power and minimum EPG. In section IV, we characterize the statistical performance of the minimum EPG by deriving its CDF. In section V, we present a numerical example where we use the results derived to evaluate the most energy-efficient MIMO-MRC configuration (in terms of the number of antennas) under both Rayleigh and Rician fading conditions. In this paper, the terms rate, bits-per-symbol and spectral efficiency are considered equivalent and are used interchangeably.

A. Notation

Vectors and matrices are denoted with boldface in lower andupper case, respectively. The operations $()^T$ and $()^\dagger$ representtranspose and conjugate transpose operations, respectively.The symbol ∇ represents the gradient operation.

II. PROBLEM FORMULATION

A. System preliminaries

Bits from a packet of length L bits are coded and modulated, where coded bits are mapped to a constellation symbol, x, at an average power GP, where P is the component of transmit power that will be optimized and G is defined as the constant transmit power that is needed to overcome deleterious effects such as path loss, implementation loss and thermal noise, but also takes into account the benefits of coding gain (or equivalently SNR gap) and antenna gain. The constant transmit power G is defined as

$$G = M_L \Gamma(\mathrm{BER}) \left(\frac{\lambda}{4\pi}\right)^{-2} d^k N_0 N_f B \qquad (1)$$

Where M_L is the link margin to account for implementation imperfections in the system, λ is the wavelength at the carrier frequency, d is the distance between the transmitter and receiver and κ is the path-loss exponent based on a reference distance of 1 meter. We also scale G by the variance of the additive thermal noise observed at the receiver. This noise is modeled as additive white Gaussian noise (AWGN) with variance $\sigma^2 = N_0 N_f B$, where $\frac{N_0}{2}$ is the two-sided noise power spectral density (PSD), N_f is the noise figure of the receiver and B is the bandwidth of the system. Assuming a fixed operating bit error rate (BER), BER, we can compute the SNR gap approximation Γ(BER) for a given coded modulation scheme. As a simple example, an approximate SNR gap (from Shannon limit) for uncoded QAM is $\Gamma(\mathrm{BER}) = \frac{\log(\mathrm{BER}^{-1})}{1.5}$. It is assumed a packet based system with ARQ in a long-term static fading channel. We compute packet error rate (PER) using the formula given by PER = 1 −(1 −BER)^L, which is an approximation. We assume that the receiver and transmitter have perfect channel state information (CSI).

B. Channel model

In a MIMO system with $n_R$receive antennas and n_T transmit antennas, the received signal y$\epsilon\mathbb{C}^{n_R}$, after some simplifications, is a linear transformation of transmit signal s$\epsilon\mathbb{C}^{n_T}$ plus an additive noise w$\epsilon\mathbb{C}^{n_R}$

$$y = \sqrt{P}Hs + w \qquad (2)$$

where H$\epsilon\mathbb{C}^{n_R X\, n_T}$ is the MIMO channel matrix and whas i.i.d. complex Gaussian elements with zero-mean and unitvariance. We consider scalar beamforming schemes in thiswork, i.e. s is a function of only one information symbol. Thetransmit signal depends on unit-norm transmit beam forming vector v and scalar x ∈ A (information symbol from unitenergyconstellation A) as s = vx. The received signal y islinearly combined by receive beamforming vector u† to yield the scalar decision statistic$\tilde{y} = u^\dagger y = \sqrt{P}u \dagger Hvx + u^\dagger w$. Assuming coherent detection at the receiver, the resulting channel model is given by

$$\tilde{y} = \sqrt{P\gamma}x + \tilde{w} \qquad (2)$$

Where $\tilde{w}$ is zero-mean unit-variance AWGN and $\gamma = \left|u^\dagger Hv\right|^2$, effectively is the channel gain-to-noise ratio (CNR). By applying the Cauchy-Schwartz inequality, we have

$$\left|u^\dagger Hv\right|^2 \leq \|u\|^2 \|Hv\|^2 = \|Hv\|^2 = v^\dagger H^\dagger Hv$$
$$\leq \lambda_{max}(H^\dagger H)$$

Thus, the maximum CNR is achieved by setting v to the eigen vector corresponding to the maximum eigenvalue of H†H, λ_{max}(H†H), and u = Hv. For a Single Input Multiple Output (SIMO) configuration (where $n_T = 1$and $n_R > 1$),this scheme is called Maximum Ratio Combining (MRC), fora Multiple Input Single Output (MISO) configuration (where$n_T > 1$and $n_R = 1$), this scheme is called Maximum RatioTransmission (MRT) and for a general MIMO configuration,this scheme is called transmit-beamforming (TB) or MIMO-MRC, which is illustrated in Fig. 1.

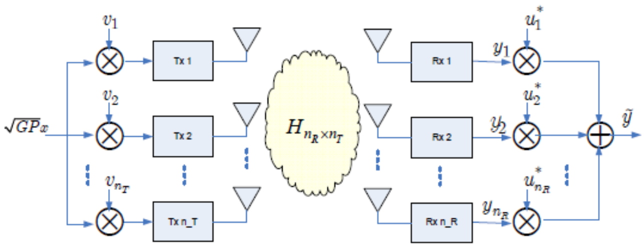

Fig. 1. The architechture of $n_R \times n_T$MIMO-MRC configuration.

C. Objective function: energy-per-goodbit (EPG)

In this paper, we formulate EPG as a function of the variable P, which corresponds to transmit power without the constant scale factor G. Due to normalizations in G(see (1)), effectively the signal to noise ratio (SNR) at the receiver is γP. The bits-per-symbol or spectral efficiency isgiven by $b = \log_2(1 + \gamma P)$. The EPG for a single carrier MIMO-MRC system is defined for γ >0 as

$$E_a(P) = \frac{k_t P + N_A k_c}{\log_2(1 + \gamma P)} \qquad (4)$$

Where k_t and k_c are non-negative constants which are interpreted as transmit and circuit EPG respectively for unit transmit power and unit spectral efficiency, and are given by$k_t = \frac{\alpha G}{(1-PER)B}$and $k_c = \frac{P_c}{(1-PER)B}$, respectively. Parametersin the model include, $\alpha = \frac{OBO}{\eta_{max}}$, where OBO is the outputbackoff from saturation power level of the power amplifier(PA) and η_{max} is the maximum PA efficiency (or drainefficiency), P_c is the average power consumption in a singletransmit or receiver chain (the rest of the transmit and receiveelectronics excluding the PA). For a MIMO system, P_c is computed as

$$P_c = \frac{n_T P_{c,tx} + n_R P_{c,rx} + P_{co}}{N_A}$$

Where $P_{c,tx}$ and $P_{c,rx}$ are the power consumed in a singletransmit chain and receive chain, Pco is the (common) power consumed independent of the number of antennas and the total number of antennas in the system, $N_A = n_T + n_R$.In contrast to the EPG model , the above model assumes a constant PA back-off. This assumption can be a practical one where the back-off is a function of the maximum supported constellation. Orthogonal frequency division multiplexing (OFDM) is another example where a constant back-off may be employed in practice. It is to be noted that the results presented in this paper are directly applicable for OFDM systems under a frequency-flat fading channel model. A constant back-off also may be prescribed for coded modulation systems, where the code rate is adapted and the constellation size is fixed. The constant PA back-off assumption facilitates the derivation of a closed-form solution for the distribution function of the optimum EPG.

D. Optimization problem

For a given MIMO configuration and channel realization, H, the MIMO-MRC scheme results in a CNR γ. The goal is then to find the power allocation, $P = P^*$, that results in the minimum EPG, $E_a(P^*)$, subject to certain quality of service (QoS) constraints, that is,

$$E_a(P^*) = min\{ E_a(P) | P \in X \} = \frac{k_t P^* + N_A k_c}{\log_2(1+\gamma P^*)} \quad (5)$$

where P^* is the optimum feasible solution over the convex set X defining the set of feasible power allocations.

III. ENERGY–EFFICIENCT POWER ALLOCATION

A. Pseudoconvexity of objective function

A definition is provided for a pseudo-convex function and then present some theorems which will be useful for our purposes.

Definition 1: Let X be a nonempty open convex set in R_n and let h : X → R be differentiable on X. Then h is pseudoconvex if for each $x_1, x_2 \in X$ we have $(x_1 - x_2)^T \nabla h(x_2) \geq 0 \Longrightarrow h(x_1) \geq h(x_2)$.

Theorem 1: Let h be a differentiable pseudo convex function over X ⊂ R_n which is an open convex set and suppose that ∇h(x∗) = 0 for some x∗∈ X. Then x∗ is a global minimum of h over X.

Proof: From the definition it is clear that $h(x) \geq h(x^*)$ for all $x \in X$. Although pseudoconvex functions do not have to be convex, they possess an important attribute of convex functions,

Theorem 2: If f(x) is a real-valued, non-negative, differentiable, convex function and, g(x) is a real-valued, positive, differentiable, concave function both defined on an open convex set $X \subset R_n$, then $h(x) = \frac{f(x)}{g(x)}$ on X is pseudoconvex.

Proposition1: $E_a(P)$ is pseudo convex on $X = \{P \in R \,|P > 0\}$.

Proof: $E_a(P)$ is of the form f(P)g(P), where $f(P) = k_t P + N_A k_c$ is affine and non-negative since k_t and k_c are non-negative, and $g(P) = log_2(1 + \gamma P)$ is concave in P and $g(P) > 0$ over X. Note that $E_a(P)$ is only defined for γ > 0. By theorem 2, $E_a(P)$ is pseudoconvex on X.

B. Optimum power allocation and minimum EPG

Using proposition 1 and theorem 1, the optimum power allocation, P^* is the solution to $\frac{\partial E_a(P)}{\partial x} = 0$. Assuming that γ > 0 (since $E_a(P)$ is not defined otherwise) and after performing some simple manipulations P^* is the solution to

$$(1+\gamma P^*)(\log(1+\gamma P^*) - 1) = N_A \mu_d \gamma - 1$$

whereμ_d is defined as the ratio of circuit to transmit EPG for unit rate and power, i.e. $\mu_d = \frac{k_c}{k_t}$. Using the Lambert W function and (11) from the appendix, the optimum power thatminimizes the EPG as a function of γ is

$$P^*(\gamma) = \frac{exp\left(1 + W\left(\frac{N_A \mu_d \gamma - 1}{\exp(1)}\right)\right)}{\gamma} \quad (6)$$

In practical systems, the spectral efficiency may be restricted to a finite set of values as dictated by allowed constellation sizes and code rates. In these cases, a nearest neighbor rounding of the solution can be employed. The expression for the minimum EPG is obtained by plugging the solutions in (4) and applying (12) from the appendix

$$E_a{}^*(\gamma) = k_t log(2) \frac{exp\left(1 + W\left(\frac{N_A \mu_d \gamma - 1}{\exp(1)}\right)\right)}{\gamma} \quad (7)$$

From the expression above, we can immediately conclude that the minimum EPG is positive for all γ > 0. Also for any power allocation P, it is clear that the EPG in (4) is strictly decreasing in γ. Thus, intuitively, we expect the MIMO-MRC scheme (which maximizes γ over all possible beam-forming schemes) to be the EPG optimal beam-forming scheme. In fact, it can be shown that the minimum EPG $E_a{}^*(\gamma)$ is a decreasing function of γ.

IV. PERFORMANCE ESTIMATION

In the previous section it was shown that the optimum power allocation and consequently the minimum EPG that results, is a function of the channel fade power γ. If we treat γ as a random variable with CDF $F_\gamma(x) = \Pr(\{\gamma \leq x\})$, then E∗ a(γ) is a function of this random variable γ and as a result is a random variable itself.

A. CDF of minimum EPG

In order to derive the CDF of the minimum EPG, $F_{E_a^*}(q_0) = \Pr(\{E_a^* \le q_0\})$, we make a change of variable in (7) by introducing $t = W\left(\frac{N_A \mu_d \gamma - 1}{\exp(1)}\right)$, then the inequality $E_a^* \le q_0$ is equivalent to

$$\frac{\exp(t+1)}{t\exp(t+1)+1} \le \frac{q_0}{N_A k_c log(2)}$$

where we have used (9) in the appendix to express γ as a function of t. Due to the positivity of EPG (for nonzero γ), we can assume $F_{E_a^*}(0) = 0$ ($\{\gamma = 0\}$) is an event of zero probability) and restrict our attention to $q_0 > 0$. The equivalent inequality

$$t = W\left(\frac{N_A \mu_d \gamma - 1}{\exp(1)}\right) \ge \mathrm{W}\left(-\exp\left(-1 - \frac{1}{c_0}\right) + \frac{1}{c_0}\right) \triangleq c_1$$

By applying the inverse of the Lambert W function on bothsides yields,

$$\{E_a^*(\gamma) \le q_0\} = \left\{\gamma \ge \frac{1 + c_1 \exp(c_1+1)}{N_A \mu_d}\right\}$$

Thus the CDF of the minimum EPG can be expressed as afunction of the fading distribution.

$$F_{E_a^*}(q_0) = 1 - F_\gamma\left(\frac{1 + c_1\exp(c_1+1)}{N_A \mu_d}\right)$$

B. Outage EPG as a performance measure

When the fading power follows a probability distribution, there may be a range of channels that are very poor in quality or cost too much EPG. In these cases, it may make sense to turn off the radio completely (outage event) until the channel is of a better quality. Service requirements specify the maximum tolerated outage probability p_o. A percentile is the value of a variable below which a certain percent of observations fall. Thus a $(1 - p_o) \times 100\%$ percentile EPG or equivalently $p_o \times 100\%$ outage EPG is the highest cost (in EPG) that a user will incur in order to communicate a bit. In a similar manner, outage spectral efficiency gives the slowest rate at which a user will communicate. Outage transmit power gives the maximum power that will be used. In the next section, various antenna configurations will be compared in terms of the outage EPG achieved. The most energy-efficient configuration is defined as that configuration which yields the least outage EPG, given a certain specified p_o.

V. NUMERICAL EXAMPLE

The parameters (unless specified otherwise) used in the numerical example are given in table I. EPG is measured in dBmJ which is the energy in dB relative to 1 mJ. We assume that wavelength is calculated as $\lambda = 3\times108f_c$. A realization of the channel gain γ is generated by finding the maximum eigenvalue of the Wishart matrix H†H. Each (i, j)-th element in a MIMO channel matrix realization is an i.i.d. random variable with the following distribution

$$[H]_{i,j} \sim \sqrt{\frac{K}{K+1}} + \sqrt{\frac{1}{K+1}}\mathcal{CN}(0,1)$$

where K is called the Rician K-factor and is the power (strength) of the constant line-of-sight (LOS) component relative to the random non-LOS component

TABLE I
PARAMETER VALUES USED IN NUMERICAL EXAMPLE

Parameter Name	Symbol	Value
Packet length	L	2000 bits
System bandwidth	B	100 kHz
Path-loss exponent	κ	3.5
Carrier frequency	f_c	2.4 GHz
Noise PSD	$N_0/2$	−204 dBW/Hz
Noise figure	N_f	10 dB
Circuit power	P_c	200 mW
Output backoff	OBO	3 dB
Max. PA efficiency	$\eta_{\max}$	0.35
Link margin	M_l	10 dB
Operating BER	BER	10^{-5}

A. CDF of EPG for various configurations

In order to validate the theoretical expression for the CDF of the minimum EPG, we calculated the CDF using simulation by generating 80, 000 channel realizations for each configuration.

Figure 2 shows a zoomed-in view of the EPG CDF corresponding to a distance of 25 meters. In Fig. 2, we consider MIMO configurations of 1 × 1, 1 × 2 and 2 × 2 and both Rayleigh fading and Rician fading with K = 10. It can be seen that the theoretical CDF values (solid and dashed lines) match the simulation values (symbols) for all configurations and channel fading conditions considered. In order to evaluate the EPG CDF, we have utilized recent results on the distribution of maximum eigenvalue for non central Wishart matrices. For the Rayleigh fading case, it can be observed that the 99th percentile EPG for the 2×2 case is around −28.2 dBmJ, where as it is around −23.7 dBmJ for the 1×1 configuration. Thus for the Rayleigh fading case, the enhanced reliability (diversity) provided by multiple antennas is vital to providing energy-efficient communication. For the Rician case, we observe that the CDFs, in general, show less spread due to the constant line-of-sight component and thus lower values of EPG than the Rayleigh fading case. For the Rician case, we also observe that the smaller configurations possess a lower lower 99th percentile EPG than the 2 × 2 configuration. This is because the larger configuration has a higher circuit power consumption.

B. Most energy-efficient MIMO configuration

The most energy-efficient MIMO configuration or optimum MIMO configuration,

$n_{T,EE} \times n_{R,EE} \in N_{MIMO}$, is defined to be the MIMO configuration that yields the smallest outage EPG (at a specific po). The optimum configuration is chosen from a discrete and (in practice) finite set of allowed MIMO configurations NMIMO. Since we use a closed-form formula to evaluate the outage EPG, evaluating the optimum MIMO configuration does not require time consuming simulations. We consider balanced MIMO configurations limited to a maximum of four antennas at either

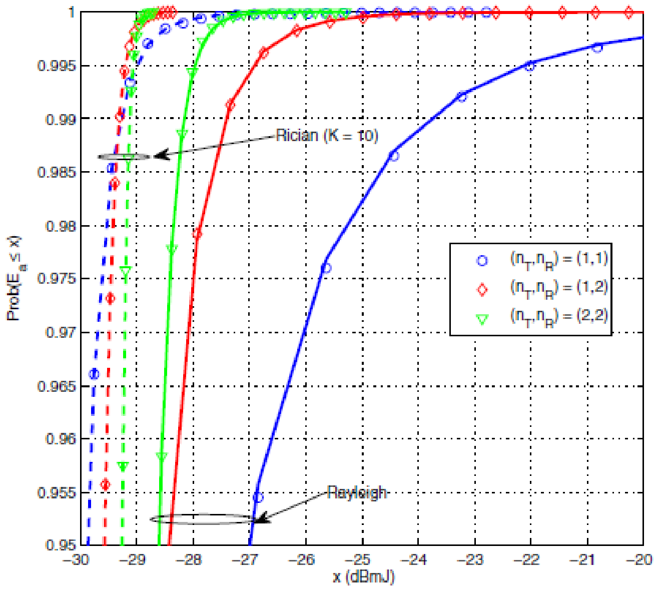

Fig. 2. Cumulative Distribution Function of Energy PerGoodbit at a distance of 40 m. Solid lines correspond to Rayleigh fading. Dashed lines correspond to Rician fading (K = 10).

end. For a given total number of antennas, balanced MIMO configurations provide superior diversity performance.We consider balanced MIMO configurations limited to a maximum of four antennas at either end. For a given total number of antennas, balanced MIMO configurations provide superior diversity performance. Since we have assumed symmetric circuit energy costs at both transmitter and receiver, an X × Y configuration is equivalent to a Y × X configuration. It should be noted that the equivalence will generally not hold when there is spatial correlation. In our case, the search space can be reduced by removing equivalent configurations. Thus the set of allowed MIMO configurations are $N_{MIMO} = \{1 \times 1, 1 \times 2, 2 \times 2, 2 \times 3, 3 \times 3, 3 \times 4, 4 \times 4\}$. Notice that the number of antennas, $N_A = n_T + n_R$ uniquely determines the configuration. If $N_A{}^*$ denotes the optimum or most energy-efficient number of antennas,then $n_{T,EE}\, x\, n_{R,EE} = \left\lceil \frac{N_A{}^*}{2} \right\rceil x \left\lfloor \frac{N_A{}^*}{2} \right\rfloor$.

Figure 3 shows the optimum MIMO configuration at a 1% outage level (po = 0.01) over a range of µd, over Rayleigh (K = 0) and Rician (K = 10) fading. The largest MIMO configuration is preferred (for both fading scenarios) when transmit power cost dominates circuit power cost, i.e., when µd<< 1. In order to meet the outage requirement, diversity helps to reduce the probability that bad channels need to be used for transmission. As μ_d gets smaller than the range shown, we have observed that largerMIMO configurations can provide improvements under suitable conditions. On the other hand, when circuit power cost is very large (μ_d around 105 for this example), SISO (1×1) is the preferred configuration. For Rician fading, SISO is optimal for a larger range of µd which indicates that the LOS component provides some amount of reliability. In general, there is an optimum MIMO configuration depending on the value of μ_d.

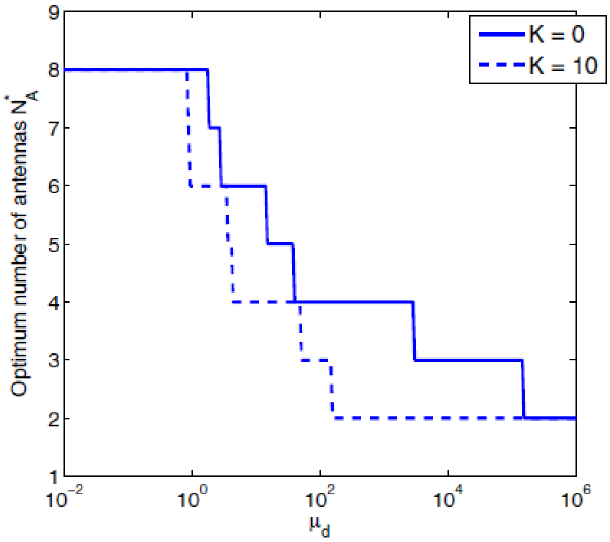

Fig. 3.Energy-efficient MIMO configuration N^*_A= $n_{T,EE}$ +$n_{R,EE}$for both Rayleigh (K = 0, solid) and Rician (K = 10, dashed) fading.

Figure 4 shows the corresponding 1%-outage EPG (equivalently 99th percentile EPG) for the optimum MIMO configuration, the largest MIMO configuration (4 × 4) and the SISO configuration under Rayleigh fading. The Rician fading curves are similar but exhibit lower values of outage EPG and is not shown here. It can be seen that the outage EPGfor the SISO configuration rapidly increases as the transmit power cost increases. Although not as dramatic, the largest MIMO configuration is seen to be suboptimal when circuit power costs dominate.

Figure 5 shows the 1%-outage spectral efficiency achievedby the the optimum MIMO configuration, the largest MIMO configuration (4×4) and the SISO configuration (1×1) under Rayleigh fading. Also shown is the average spectral efficiency for the optimum MIMO configuration.We can observe that larger MIMO

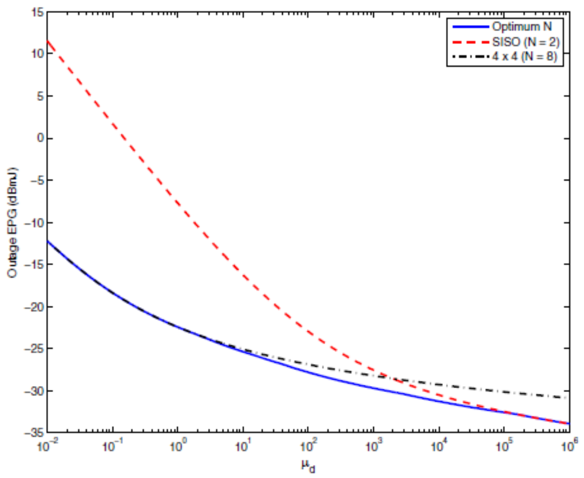

Fig. 4. Outage EPG (Rayleigh fading) for the Optimum, 4 × 4 and 1 × 1 configurations as a function of the ratio of circuit and transmit EPG for a unit total power and rate, $\mu_d = k_c/k_t$.

configurations provide larger outage spectral efficiencies, even though these larger configurations are not utilizing multiplexing gain. The jagged nature of the spectral efficiency for the optimum MIMO configuration is due to the optimum number of antennas changing as a function of µd. Ignoring the jaggedness, we can observe that the spectral efficiency increases as the circuit power cost proportion increases and this makes sense because circuit energy isminimized by minimizing transmission time or maximizing spectral efficiency.

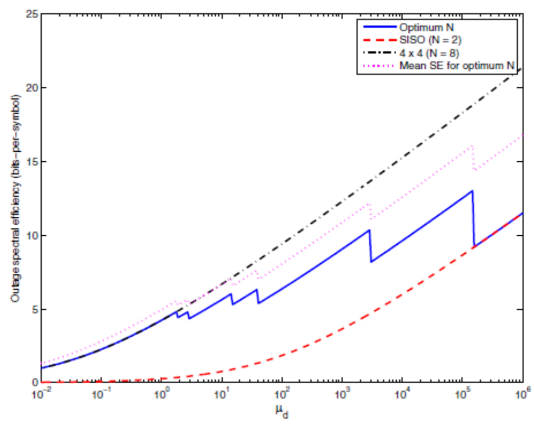

Fig.5.Optimum, 4×4 and 1×1 configurations,Outage spectral efficiency (Rayleigh fading)

VI. CONCLUSION

An energy minimization problem was formulated by considering an objective function that corresponded to the energyper-goodbit (EPG) of a wireless system employing a MIMO-MRC scheme.A closed-form solution of the energy-efficient power allocation was derived. A closed-form expression for theminimum EPG was also obtained and was used to derivethe CDF of the minimum EPG as a function of the CDF of the fading distribution. A numerical example applied recent statistical results on the maximum eigenvalue of Wishart matrices to evaluate the most energy-efficient MIMO-MRC configuration. Depending on the ratio of transmit energy to circuit energy, it was observed that there is an optimum number of antennas (or diversity order) that provides the most energy-efficient operation. Although the presence of a LOS component lessens the need for diversity (more antennas) compared to NLOS operation, it was observed that as transmit energy becomes more dominant compared to circuit energy, larger MIMO configurations provided better energy-efficient operation in both Rician and Rayleigh fading scenarios. Larger MIMO configurations also provided higher and steadier spectral efficiencies compared to SISO configuration.

REFERENCES

[1]. G. Caire, G. Taricco, and E. Biglieri, "Optimum power control over fading channels," *IEEE Trans. Inf. Theory, vol. 45, no. 5, pp. 1468–1489, July 1999.*

[2]. C. Schurgers, O. Aberthorne, and M. Srivastava, "Modulation scaling for energy aware communication systems," in *Proc. 2001 International Symposium on Low Power Electronics and Design, pp. 96–99.*

[3]. R. M. Corless, G. H. Gonnet, D. E. G. Hare, D. J. Jeffrey, and D. E.Knuth, "*On the Lambert W function*," Advances in ComputationalMathematics, vol. 5, pp. 329–359, 1996.

[4]. S. Cui, A. Goldsmith, and A. Bahai, "Energy-constrained modulation optimization," *IEEE Trans. Wireless Commun., vol. 4, no. 5, pp. 2349–2360, Sep. 2005.*

[5]. A. Goldsmith and S.-G. Chua, "Variable-rate variable-power MQAM for fading channels," *IEEE Trans. Commun., vol. 45, no. 10, pp. 1218–1230, Oct. 1997.*

[6]. E. Telatar, "Capacity of multi-antenna Gaussian channels," *European Trans. Telecommun., pp. 585–595, 1999.*

[7]. P. Dighe, R. Mallik, and S. Jamuar, "Analysis of transmit-receive diversity in Rayleigh fading," *IEEE Trans. Commun., vol. 51, no. 4, pp. 694–703, Apr. 2003.*

[8]. S. Cui, A. Goldsmith, and A. Bahai, "Energy-efficiency of MIMO and cooperative MIMO techniques in sensor networks," *IEEE J. Sel. Areas Commun., vol. 22, no. 6, pp.*

[9]. R. Prabhu, B. Daneshrad, and L. Vandenberghe, "Energy minimization of a QAM system," *in Proc. 2007 IEEE Wireless Communications and Networking Conference, pp. 729–734.*

[10]. V. Tarokh, N. Seshadri, and A. R. Calderbank, "Space-time codes for high data

rate wireless communications: performance criterion and code construction," *IEEE Trans. Inf. Theory, vol. 44, no. 2, pp. 744–765, Mar.1998*.

[11]. C. Lin, V. Raghavan, and V. Veeravalli, "To code or not to code acrosstime: space-time coding with feedback," *IEEE J. Sel. Areas Commun.,vol. 26, no. 8, pp. 1588–1598, Oct. 2008*.

[12]. A. Maaref and S. Aissa, "Optimized rate-adaptive PSAM for MIMOMRC systems with transmit and receive CSI imperfections," *IEEETrans. Commun., vol. 57, no. 3, pp. 821–830, Mar. 2009*.

[13]. R. Prabhu and B. Daneshrad, "Energy minimization of a QAM systemwith fading," *IEEE Trans. Wireless Commun., vol. 7, no. 12, pp. 4837–4842, Dec. 2008*.