Investigation Into Cable-Structure Interaction For Extradosed Bridge

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ABSTRACT

Cable supported Bridge structures have distinctive dynamic behavior compared to any other type of bridge, especially the dynamic behavior. Support excitations sets structure to vibrate; cable excitations can be caused by rain, wind or stochastical vibration due to plying vehicles or due to vibration of deck. In modern cable-stayed / Extradosed bridges, the stay cables are often closely spaced, with the cable lengths and tensions gradually varying from position to position. The natural frequencies of their self-vibrations are therefore likely to be rather closely placed as well. Such boundary-induced vibrations of the stay cables are likely to complicate the overall dynamic behavior of the bridge. The paper focuses on dynamic behavior of Extradosed Bridges, dynamic behavior of Extradosed cables and possible interaction resulting into coupled mode of vibrations.

Keywords **-** Extradosed cable stayed bridge, Earthquake, Dynamic response, cable-structure interaction

I. INTRODUCTION

A) Extradosed Bridge

The recent research has shown that a Extradosed bridge, variant of cable stayed bride where cables add substantial prestress to the deck because of the shallow pylon, are found to be economical for spans upto 250m. Figure 1 shows the arrangement of cable for Girder, Extradosed and cable-stayed bridge.

The intrados is defined as the interior curve of an arch, or in the case of cantilever-constructed girder bridge, the soffit of the girder. Similarly, the extrados is defined as the uppermost surface of the arch. The term 'Extradosed' was coined by Jacques Mathivat (1988) to appropriately describe an innovative cabling concept he developed for the Arrêt-Darré Viaduct, in which external tendons were placed above the deck instead of within the cross-section as would be the case in a girder bridge. To differentiate these shallow external tendons, which define the uppermost surface of the Bridge, from the stay cables found in a cable-stayed bridge, Mathivat called them 'Extradosed' prestressing.

Some features of Extradosed Bridge as given below;

- External appearance resembles cable-stayed bridge – but structural characteristics are comparable to those of conventional girder bridge
- The Girder Depth are lesser than that of conventional girder bridges
- The stay cables (prestressing tendons outside the girder) need no tension adjustment necessary for cable-stayed bridges, and can be treated as usual tendons as in girder bridges
- The height of pylon is half as that of cable-stayed bridge and hence easier to construct
- With small stress fluctuation under live load the anchorage method for stay cables can be same as that of tendons inside girder and thereby achieve economy

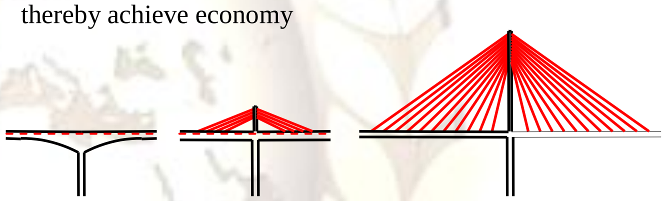


Fig. 1 Cable arrangements in Girder Bridge, Extradosed Bridge and Cable stayed bridge

B) Dynamic response

Vibrations caused of support excitations has been matter research with some failures (For example Aratsu Bridge, Tenzopan bridge, Erasmus Bridge etc) underlining the importance and need of study of dynamic behavior bridge vibrations as well as cables vibrations. With the rapid increase in span length, combined trend and also trend of using high strength materials have resulted in slender structures and a concern is being raised over dynamic behavior of such structures, in case of cable supported structures it is more pronounced as this further includes vibrations of structure and cable elements also. An accurate analysis of natural frequencies is fundamental to the solution of its dynamic responses. In modern cable-stayed / Extradosed bridges, the stay cables are often closely spaced, with the cable lengths and tensions gradually varying from position to position. The natural frequencies of their self-vibrations are therefore likely to be rather closely placed as well. Such boundary-induced vibrations of the stay cables are likely to complicate the overall dynamic behavior of the bridge. Although there is no universal agreement about the causes of vibration, possible explanations essentially follow two different ways of thinking. According to the first rationale, vibrations are due to external environmental actions acting directly on the stays. In particular, the wind-tunnel experiments by Hikami and Shiraishi (1988) have shown that it is the combination of rain and wind, rather than their separate action that provokes aerodynamic instability. The excitation is due to the change in shape of the stay-sheath profile, produced by the wind-induced formation of a water rill at the extrados of the cable. A completely different explanation points instead to an interaction between the vibrations of the stays and the oscillations of their extremities anchored to girder and pylons. If some resonance conditions get satisfied, energy can flow to the stays and provoke their large-amplitude oscillations.

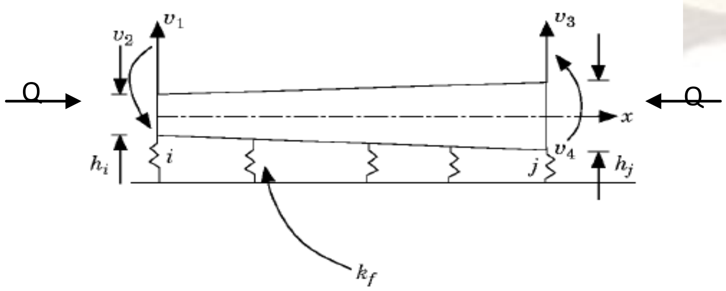
II. GOVERNING EQUATIONS

A) Vibration of structure

When finite element is used, each stay cable is modeled as either a single truss element with an equivalent modulus or number of cable elements with the original modulus. The deck and tower are modeled as Bernoulli-Euler beam elements with axial forces due to prestress imparted by horizontal component of cable force due to its shallow cables.

I. STIFFNESS AND MASS MATRIX FORMULATION FOR EXTRADOSED BRIDGE

Consider a typical Extradosed bridge as shown in figure 1; let us take a small section as shown in the figure 2 below. The boundary conditions for this element can be considered as that of beam on elastic the provided by cable. Further this beam will be subjected to prestressing force due to horizontal component of cable forces, as shown in figure 2 foundation to relate effect of elastic support



below;

Fig.2 Beam on elastic foundation

Now, consider an element *i-j* of length *L* of a beam on an elastic foundation as shown in Figure.7 having a uniform width *b* and a linearly varying thickness *h(x)* It will be a simple matter to consider an element having a linearly varying width if the need arises. Neglecting axial deformations this beam on an elastic foundation element has two degrees of freedom per node a lateral translation and a rotation about an axis normal to the plane of the paper and thus possesses a total of four degrees of freedom. The (4x4) stiffness matrix *k* of the element is obtained by adding the (4x4) stiffness matrices k_B, k_F and k_Q pertaining to the usual beam bending stiffness and foundation stiffness and stiffness due to prestressing force (Q) respectively Since, there are four end displacements or degrees of freedom a cubic variation in displacement is assumed in the form

$$v = Aa \qquad \text{Eq. (1)}$$

Where, $A= (1\ x\ x^2\ x^3)$ and $a^T= (a_1\ a_2\ a_3\ a_4)$ (Displacement variation within element)

The four degrees of freedom corresponding to the displacements v_1, v_3 and the rotations v_2 v_4 at the longitudinal nodes are given by

$$q=Ca \quad \text{(Nodal displacements)} \qquad \text{Eq. (2)}$$

Where $q^T= (v_0\ v_1\ v_2\ v_3)$ and *C* is the connectivity matrix for an element *ij* between *x*=0 and *x*=*L as given in* Figure 2

From equations (Eq.1) and (Eq.2)

$$V=AC^{-1}q \qquad \text{Eq. (3)}$$

If *E* is the Young|s modulus and $I=bh(x)^3/12$ is the second moment of area of the beam Cross-section about an axis normal to the plane of the paper the bending moment *M* in the element is given by

$$M = D\frac{\partial^2 v}{\partial x^2} = DBC^{-1}q \qquad \text{Eq. (4)}$$

Where *D*=*EI(x)* and $B=d^2A/dx^2=(0, 0, 2, 6x)$

a. Stiffness due to bending

The potential energy U_B due to bending is

$$U_B = \frac{1}{2}\int_0^l \frac{d^2v}{dx^2} M dx \qquad \text{Eq. (5)}$$

And the stiffness is given by

$$kb = \frac{\partial^2 U_B}{\partial d^2} \qquad \text{Eq. (6)}$$

From equations (Eq.5) and (Eq.6) we get,

$$k\bar{b} = \int_0^l B^T DBdx \ \text{(Elemental)} \qquad \text{Eq. (7)}$$

$$Kb = (C^{-1})^T k\bar{b} C^{-1} \ \text{(Assmebled)} \qquad \text{Eq. (8)}$$

b. Stiffness due to elastic foundation

The potential energy of foundation stiffness is given by,

$$U_f = \frac{1}{2}\int_0^l V^T kfV dx \qquad \text{Eq. (9)}$$

and then the stiffness is given by,

$$kf = \frac{\partial^2 U_f}{\partial d^2} \quad \text{Eq. (10)}$$

From equations (Eq.3) in (Eq.10) we get,

$$\bar{kf} = \int_0^l A^T kf A dx \text{ (Elemental)} \quad \text{Eq. (11)}$$

$$Kf = (C^{-1})^T \bar{kf} C^{-1} \text{ (Assembled)} \quad \text{Eq. (12)}$$

c. Stiffness due to prestressing force

The potential energy of prestressing force is given by,

$$U_Q = \frac{1}{2}\int_0^l Q\left(\frac{\partial v}{\partial x}\right)^2 dx \quad \text{Eq. (13)}$$

Then the stiffness is given by,

$$k_Q = \frac{\partial^2 U_Q}{\partial d^2} \quad \text{Eq. (14)}$$

Substituting equation (3) in (14) we get,

$$\bar{k}_Q = \int_0^l A^T k_Q A dx \quad \text{Eq. (15)}$$

$$K_Q = (C^{-1})^T k_Q C^{-1} \quad \text{Eq. (16)}$$

Finally complete stiffness is given by,

$$K = K_B + K_f + K_Q \quad \text{Eq. (17)}$$

Element mass matrix is the equivalent nodal mass that dynamically represents the actual distributed mass of the element. This is kinetic energy of the element.

$$T = \frac{1}{2}\int_0^l (\dot{v})^T \rho dV \dot{v} \quad \text{Eq. (18)}$$

Where, $\dot{v}$ = Lateral velocity and ρ = mass density

$$T = \frac{\rho}{2}(\dot{q})^T (C^{-1})^T \left\{\int_0^l A^T hxA dx\right\} (C)^{-1} q$$

Eq. (19)

Then, the mass matrix is given by,

$$m = (C^{-1})^T \overline{m} C^{-1} \quad \text{Eq. (20)}$$

$$\text{and } \overline{m} = \rho \int_0^l A^T hxA dx \quad \text{Eq. (21)}$$

for free vibration of this beam,

$$[M]\{\ddot{q}\} + [C]\{\dot{q}\} + [K]\{q\} = 0 \quad \text{Eq. (22)}$$

and for forced vibration,

$$[M]\{\ddot{q}\} + [C]\{\dot{q}\} + [K]\{q\} = \{f\} = [N]^{-1} f_0$$

Eq. (23)

For Extradosed bridge, since the cable are shallower and the effect of prestressing force is more the effecting of prestress shall be taken in to account as shown in the equation above.

B) Vibration of Cables

i) With equivalent modulus

In global analysis of cable stayed / Extradosed bridges, one common practice is to model each cable as a single truss element with an equivalent modulus to allow for sag. The element stiffness matrix in local coordinates for such a cable element can be written as,

$$k_c = \frac{A_c E_{eq}}{l_c}\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{Eq. (24)}$$

The equivalent modulus of elasticity is given be

$$E_{eq} = \frac{E_c}{1 + (wH_c)^2 A_c E_c / 12T^3} \quad \text{Eq. (25)}$$

Where, l_c is chord length, H_c is the horizontal projection length, A_c is the cross-sectional area, E_c is the effective material modulus of elasticity, w is the weight per unit length and T is the updated cable tension of the cable. A certain cable profile has been assumed to account for the effect of cable sag. However, once the equivalent modulus has been obtained, the profile will not have a role to play in the final analysis, and hence the method cannot model transverse vibrations of the cable.

ii) With original modulus

Another approach for accounting for the transverse vibrations of cables is to model each cable by number of cables elements with the original modulus. Following the sign conventions adopted by Broughton and Ndumbara (1994), the element incremental stiffness matrix in local coordinates can be written as

$$K_c = \frac{E_c A_c}{L_0(L_0+e)^2}\begin{bmatrix} (L_0+u_c)^2 & v(L_0+u_c) & -(L_0+u_c)^2 & -v(L_0+u_c) \\ v(L_0+u_c) & v_c^2 & -v(L_0+u_c) & -v_c^2 \\ -(L_0+u_c)^2 & -v(L_0+u_c) & (L_0+u_c)^2 & v(L_0+u_c) \\ -v(L_0+u_c) & -v_c^2 & v(L_0+u_c) & v_c^2 \end{bmatrix}$$
$$+ \frac{T}{(L_0+e)^3}\begin{bmatrix} v_c^2 & -v(L_0+u_c) & -v_c^2 & v(L_0+u_c) \\ -v(L_0+u_c) & (L_0+u_c)^2 & v(L_0+u_c) & -(L_0+u_c)^2 \\ -v_c^2 & v(L_0+u_c) & v_c^2 & -v(L_0+u_c) \\ v(L_0+u_c) & -(L_0+u_c)^2 & -v(L_0+u_c) & (L_0+u_c)^2 \end{bmatrix}$$

Eq. (26)

Where the updated element basic tension T and the element extension e along the deformed element longitudinal axis are given, respectively, by

$$T = T_0 + E_c A_c / L_0 e \quad \text{Eq. (27)}$$

$$e = \sqrt{(L_0+u_c)^2 + v_c^2} - L_0 \quad \text{Eq. (28)}$$

T_0 is the original cable element pre-tension, L_0 is the original cable element length, and u_c and v_c are, respectively, the relative displacements of one node acting along and perpendicular to the cable chord with respect to the other node.

iii) Mass matrix for cable

The cable element mass matrix is the same for both the single-element and multiple-element modeling methods. The mass matrix is given as follows,

$$m_c = \frac{m_{cc}}{6}\begin{bmatrix} 2 & 0 & 1 & 0 \\ 0 & 2 & 0 & 1 \\ 1 & 0 & 2 & 0 \\ 0 & 1 & 0 & 2 \end{bmatrix} \quad \text{Eq. (29)}$$

In which m_{cc} is the total mass of the cable element

C) Vibration of stay cables

To demonstrate the abilities of various methods in predicting local cable vibrations, each stay cable was analyzed as an inclined stay cable fixed/pinned at both ends to evaluate the natural frequencies of local vibrations. It is noted however that the real situation is slightly different, as the end anchorages themselves are movable. The first symmetric and anti-symmetric in-plane transverse vibration frequencies ω in radians per second can be computed, respectively, as

$$\omega = \frac{\omega^*}{l}\sqrt{\frac{T_\theta}{m}}$$ For symmetric in-plane vibration

and,

$$\omega = \frac{2\pi}{l}\sqrt{\frac{T_\theta}{m}}$$ For anti-symmetric in-plane vibration,

Where, $$\tan\left(\frac{\omega^*}{2}\right) = \left(\frac{\omega^*}{2}\right) - \frac{4}{\lambda^2}\left(\frac{\omega^*}{2}\right)^3$$

Where l denotes the chord length, T_θ is static cable tension, m is the cable mass per unit length

Resonance instability occurs when one of the parameters that influence the systems natural vibrations varies with time due to the action of external causes. This phenomenon has been suspected to cause large amplitude oscillations of the stays, in which it is the variation of their axial stress due to girder movements that provokes the instability i.e. the frequency of pulsating load and natural frequency of cable coincides. In general, environmental or traffic loads may provoke large-amplitude oscillations only when in resonance with the cable vibration modes.

III. STRUCTURAL DESCRIPTION AND IDEALIZATION

A) Dynamic behaviour of Extradosed Bridge

Since, Extradosed bridges take part in an intermediate zone between prestressed bridges and cable-stayed bridges, their structural behavior may be similar to these kinds of typologies, depending on design criteria adopted during the project stage. Generally a rigid deck Extradosed bridge shall have a similar behavior to the prestressed bridges, thus avoiding high stress oscillations of stay cables and, consequently, avoiding fatigue conditions associated with anchorages and tendons present in a slender deck Extradosed bridge, which behavior is quite close to the cable-stayed bridge. In order to analytically study dynamic structural behavior with respect to ground acceleration numerical studies were conducted for free vibration on typical Extradosed Bridge.

Nonlinear static analysis of a Extradosed Bridge is first performed to get the internal forces in the bridge deck, towers and stay cables. Non-linearities such as those arising from the sag of stay cables, beam-column behaviour of the bridge deck and towers, and geometrical large displacements can be taken into account in the static analysis. The global stiffness matrix and global mass matrix are then available following the nonlinear static analysis. The subspace iteration algorithm or similar may therefore be employed to determine the natural frequencies and their corresponding mode shapes.

Two arrangement of Extradosed Bridge structures are considered for numerical studies, the details of which are as given in table-1. Software SAP-2000 has been used for dynamic analysis of Bridge as well as stay cables. The mode shapes and natural frequency are found out. The Type-1 model represents typical Extradosed Bridge with span to pylon height ratio of 10 and the deck thickness is considered to be L/35 at support and L/55 at midspan as given by H. Otsuka et al. (2002), the angle of inclination of cables is between 16 to 27 degrees , the type-2 model represent a hybrid type Extradosed bridge with some of the cables having angle of inclination more than 27 degrees and with flexible steel deck expected to behave like those of cable stayed bridge.

Table-1:- Details of models used for numerical study

Span arrangement	**Pylon height above deck**	**Span to pylon height ratio**	**Cable arrangement type**
Type-1 48+120+48m Concrete	12m Up to CG of cable	10	Radiating (Figure-3)
Type-2 110+260+110m Steel	20m Up to CG of cable	8	Harp (Figure-4)

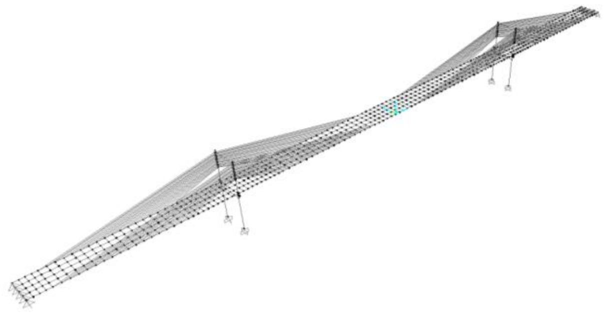


Fig-3 3-D Model 48+120+48 Extradosed Bridge

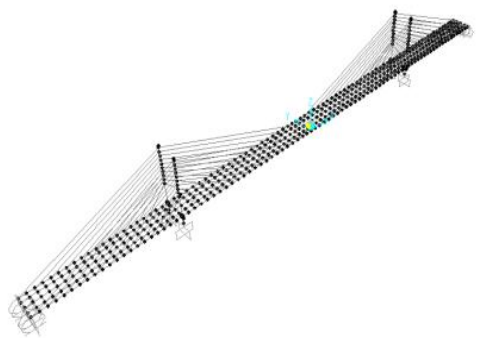
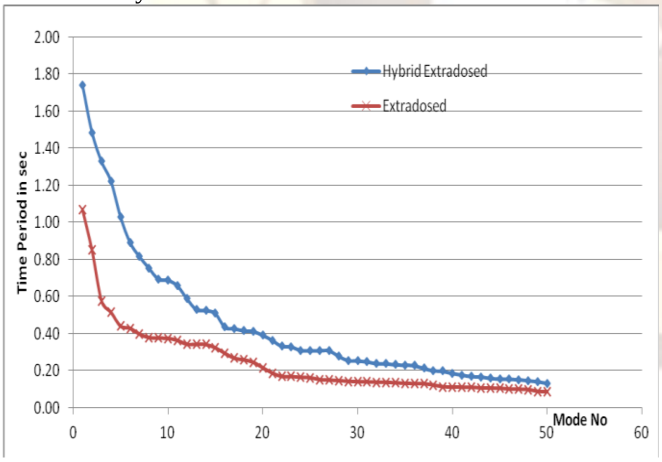


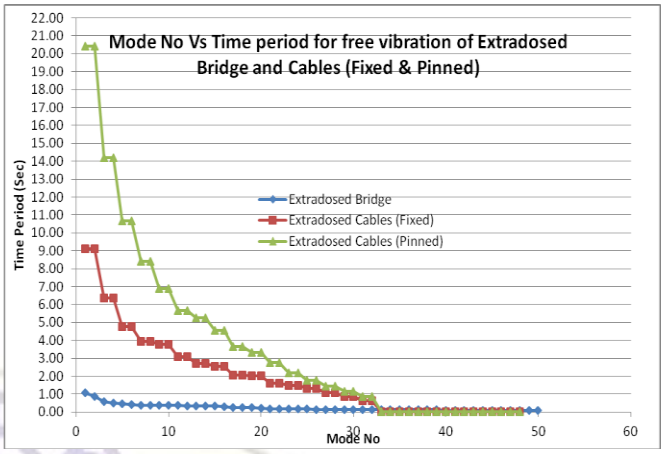
Fig.4. 3-D Model 110+260+110 Extradosed Bridge

Each stay cable is modelled as single truss element with equivalent modulus of elasticity. Two type of boundary conditions are considered i.e. both ends pinned and both ends fixed the actual boundary condition is likely to vary in between these two. No effect of damper is considered in this numerical study.

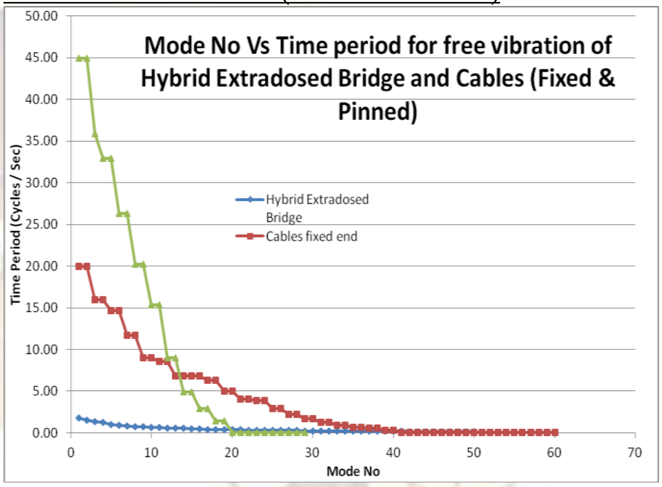


Graph-1- Mode no Vs Time period for type-1 and type-2 structure

Similarly the results are plotted for free vibrations of cables also and then in order to investigate the possible resonance the graphs are superimposed to find intersections.



Graph-2- Mode shapes vs time period for 48+120+48m Extradosed (structure and cable)



Graph-3- Mode shapes vs time period for 110+260+110m Extradosed (structure and cable)

IV. DISCUSSIONS & CONCLUSION

An accurate analysis of natural frequencies and mode shapes of cable supported structures such as Extradosed Bridge is fundamental to the solution of its dynamic responses due to seismic, wind and traffic loads. Now days, from economic considerations, the stay cables are often closely spaced, with the cable lengths and tensions gradually varying from position to position. The natural frequencies of their self-vibrations are therefore rather closely spaced. This may cause boundary-induced vibrations of the stay cables. This complicates the overall dynamic behavior of cable stayed structures. In addition to pure local vibrations of stay cables, some new frequencies are also present indicating strongly the existence of coupled vibration modes, these coupled vibration modes cannot be predicted by equations. The frequencies of cables with actual boundary conditions are expected to lie in-between those of with fixed and pinned ends. The effect damper is not considered in this study and may be considered separately when desired. For investigating the possibility of coupled mode of vibration the time periods for various modes of vibration are superimposed for structure and cables. The intersection zone (intersection of stay cable vibrations and bridge vibrations) suggests the possibility of coupled vibrations.

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