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Performance Evaluation of various Control Techniques for Inverted Pendulum

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Abstract

The application of different types of FLC and conventional PID controllers to the Inverted pendulum problem is presented in this paper. The fuzzy logic controllers have been used to control many nonlinear systems. They are designed in various forms in the Matlab-Simulink environment with Mamdani type fuzzy inference system. The Inverted Pendulum system (also called "cart-pole system") is a challenging,nonlinear and unstable control system. By controlling the force applied to the cart in the horizontal direction, the inverted pendulum can be kept in various unstable equilibrium positions. Fuzzy control in association with PID control is found better amongst the fuzzy PD and fuzzy PD+I control.

Keywords-Fuzzy controller, Fuzzy inference system, Inverted pendulum, Mamdani, PID controller.

I. INTRODUCTION

THE inverted pendulum system is a multivariable,classic, nonlinear ,unstable and high order system. It is used by many researchers for designing and testing the control techniques. Some findings on the problem of Inverted pendulum-cart systems are presented in this paper. Inverted pendulums can be classified in two categories. One is the moving- cart (also referred to as moving-carriage or moving-wagon) type and the other is rotation type. The moving-cart inverted pendulum has a cart that can only move back and forth along the track. In this paper, control of the moving–cart type inverted pendulum problem has been considered. In conventional control theory, most of control problems are usually solved by mathematical tools based on system models. But in true sense, there are many complex systems whose accurate mathematical models are not available or difficult to formulate. As an alternative to conventional control approach, the fuzzy control techniques by introducing linguistic information can provide a good solution for these problems. The most difficult aspect in the design of fuzzy controller is the construction of the rule base [9]. The construction of fuzzy rules has been mainly based on the operator's control experience or actions. Also tuning of the fuzzy rule membership functions is an important task in the design of fuzzy system.

A PID controller is a widely used feedback controller. It is the most commonly used feedback controller and calculates an "error" value as the difference between a measured process variable and a desired set point [1], [2]. The controller attempts to reduce the error by adjusting the process control inputs.

Fuzzy inference systems (FIS) are rule-based systems with concepts and operations associated with fuzzy set theory and fuzzy logic [3]. These systems are mappings from an input space to an output space; therefore, they allow constructing structures that can be used to generate responses to certain inputs, based on stored knowledge on how the responses and inputs related. The knowledge is stored in the form of a set of rules that express the relations between inputs of the system and expected outputs. Controllers based on fuzzy logic theory not only try to mimic the behavior of an expert operator but also do not demand model identification [4]. In terms of inference process there are two main classes of FIS: the Mamdani-type FIS [3] and the Takagi-Sugeno- Kang (TSK) type FIS. Henceforth, the TSK FIS will be just called Sugeno FIS. In this paper we are using Mamdani type FIS. Fuzzy logic controller in different configuration is applied to inverted pendulum for validation of the control problem aspects.

II. MODELING OF INVERTED PENDULUM

In this section, the model of the single inverted pendulum [2], [5] is established and the derivations of dynamical equations of the system is shown.

Fig.1 shows the schematic drawing of the inverted pendulum hinged on a cart system. The cart is able to move on a limited horizontal rail length. Fig.2 shows the free body diagram of the system.

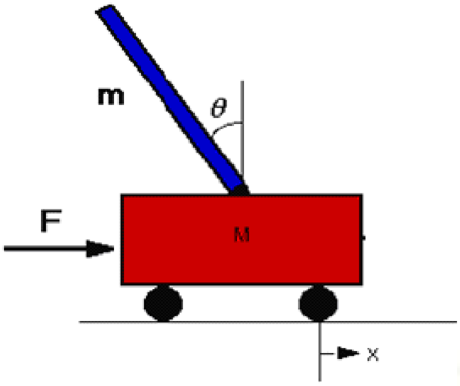

Fig.1Cart pole system

Certain assumptions are made that the initial conditions are zero so that the system will start in a state of equilibrium and the pendulum will move only a few degrees away from the y direction to meet the requirement of a linear model.

Parameters of the Cart and Pendulum

M – Mass of cart 0.6kg
m – Mass of pendulum 0.23kg
b – Friction on cart 0.11$Nm^{-1}s^{-1}$
l– Length to pendulum centre of mass 0.3m
I – Inertia of the pendulum 0.0056 kg m^2
F – Force applied to cart
x – Cart position co-ordinate
θ – Pendulum angle from the vertical position

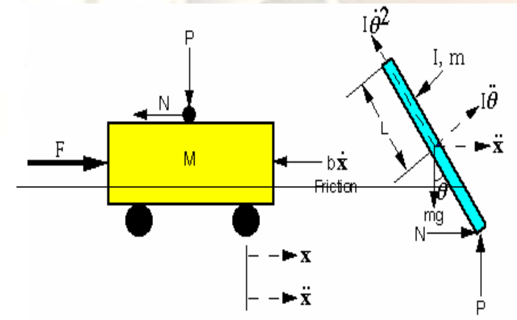

Fig. 2 Free Body Diagram

By combining the forces of the cart in the horizontal direction

$$M x + b x + N = F \qquad (1)$$

Adding the forces of the pendulum in the horizontal direction,

$$M \ddot{x} + ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2 \sin\theta = N \qquad (2)$$

Putting (2) in (1)

$$(M + m)x + b x + ml\theta\cos\theta - ml\theta\sin\theta = F \qquad (3)$$

Sum of the forces perpendicular to the pendulum

$$P\sin\theta + N\cos\theta - mg\sin\theta = ml\theta + m x\cos\theta \qquad (4)$$

To eliminate the P and N terms in the (4), and adding the moments around the centroid of the pendulum

$$-Pl\sin\theta - Nl\cos\theta = I\theta \qquad (5)$$

Combining (4) and (5)

$$(I + ml^2)\ddot{\theta} + mgl\sin\theta = -ml\ddot{x}\cos\theta$$

The set of equations should be linearized about $\theta = \pi$

Assuming that $\theta = \pi + \varphi$ (φ Represents a small angle from the vertical upward direction).

Therefore, $\cos\theta = -1, \sin\theta = -\varphi$ and d2 θ/dt2=0 (7)

After linearization (3) and (6) and F in (3) equals to u

$$(I + ml^2)\varphi - mgl\,\varphi = ml\,x \qquad (8)$$

$$(M + m)x + b x - ml\,\varphi = u \qquad (9)$$

Laplace transforms of the system equations are as below

$$(I + ml^2)\varphi(s)s^2 - mgl\varphi(s) = -mlX(s)s^2 \qquad (10)$$

$$(M + m)X(s)s^2 + bX(s)s - ml\varphi(s)s^2 = U(s) \qquad (11)$$

Since the angle φ is the output of interest, solving (9) for X(s)

$$X(s) = \left[\frac{(I + ml^2)}{ml} - \frac{g}{s^2}\right]\phi(s) \qquad (12)$$

Substituting (12) into (11) and Re-arranging, the transfer function is

$$\frac{\phi(s)}{U(s)} = \frac{\frac{ml}{q}s^2}{s^4 + \frac{b(I+ml^2)}{q}s^3 - \frac{(M+m)mgl}{q}s^2 - \frac{bmgl}{q}s} \qquad (13)$$

III. PID Controller

PID is a feedback controller whose output, a control variable (CV), is generally based on the error (e) between some user-defined set point (SP) and some measured process variable (PV). Each element of the PID controller refers to a particular action taken on the error [1].

Two cases are considered in this control technique , in the first case, set value zero with impulse disturbance are taken in plant and in second case, set value without disturbance in plant is taken

A. Controller with disturbance in plant

Since the objective is to make the pendulum return to the vertical position after the initial disturbance, applied to the cart is added as an impulse disturbance. It is shown in block diagram in fig.3

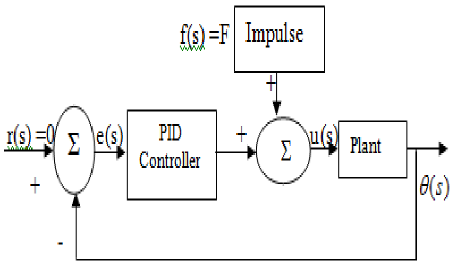

Fig.3. closed-loop system with Disturbance

For the PID Controller the parameter values inside the block are (P=100, I=1, D=21).

Pulse generator parameters are set to

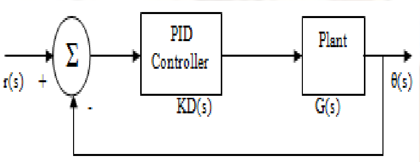

Fig. 5 closed-loop system with r(s)

as angle (radian) set value

The closed-loop transfer function of the fig. 5 can be found as shown by the (15)

$$\theta(s) = \frac{KD(s)G(s)}{1 + KD(s)G(s)} R(s) \quad (15)$$

The closed-loop transfer function in (15) can be implemented into Matlab by using the m-file code and also implemented in simulink.Fig.6 shows Simulink model of closed loop system with PID controller.

IV. FUZZY LOGIC CONTROLLER

It is more difficult to set the controller gains for fuzzy controllers compared to proportional-integral- derivative (PID) controllers as they are nonlinear . The fuzzy logic controller shown in fig.7 was first implemented by Mamdani and Assilian based on the fuzzy logic system generalized from fuzzy set theory introduced by Zadeh.

As it is already discussed in section III two cases for controlling the inverted pendulum by PID controller, here fuzzy controller is discussed.

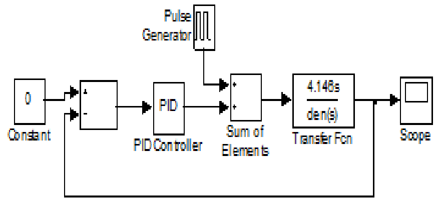

Fig. 4 The inverted pendulum system with PID Controller in simulink

period=10,duty cycle=0.01%, amplitude=1000. Pulse generator gives an impulse disturbance.

B. Controller without disturbance in plant

Fig.5 shows the inverted pendulum with PID controller for step input r(s).

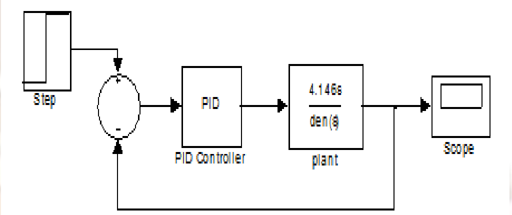

Fig. 6 Simulink model of closed-loop system with step input

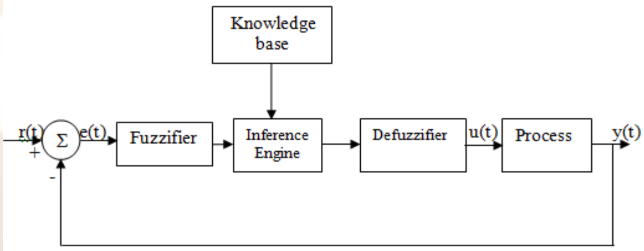

Fig. 7 Block diagram of fuzzy controller

In this paper the stages of development of the fuzzy controller for an inverted pendulum are presented by developing a two- input, one output Mamdani type system. The fuzzy sets of two-input (e, edot) and one output (u) are designed in fig.8. The output or the defuzzification stage converts the combined result back into a specific control output value using the Centroid method [7].

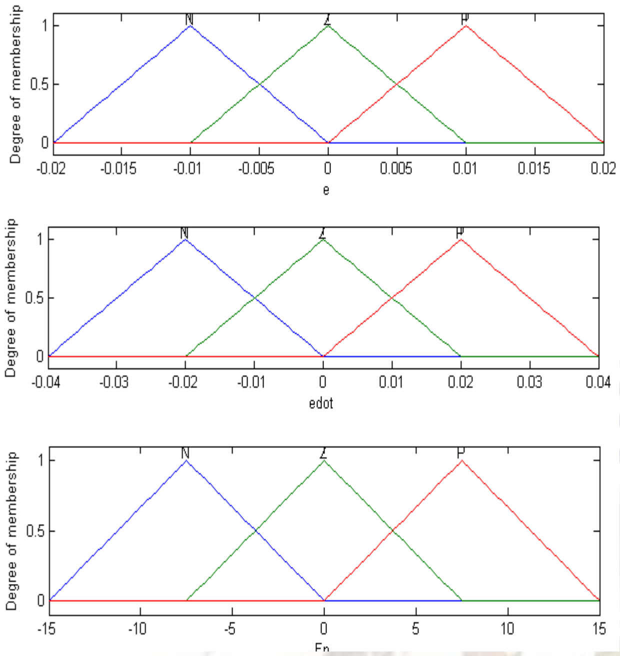

Fig.8 membership function for e,edot and Fp

Note that we are using "NS" as an abbreviation for "negative small in size" and so on for the other variables. Such abbreviations help keep the linguistic descriptions short yet precise:
"NB" to represent "negative big" "NM" to represent "negative medium" "ZE" to represent zero" "PS" to represent "positive small". "PM" to represent "positive medium" "PB" to represent "positive big"
The choice of membership functions (MFs) and rule base of the fuzzy controller will affect the performance of the system. The control performance is greatly improved by adopting suitable rule base according to the angle of the pole with the vertical. In this paper 7 inputs & 7 output membership function are described in 49 Fuzzy–if-then rules (which are represented in table 1) gives the better response.

TABLE: I Fuzzy Rule Base

ec / e	NB	NM	NS	ZE	PS	PM	PB
NB	NB	NB	NB	NB	NB	NM	ZE
NM	NB	NB	NB	NB	NM	ZE	PM
NS	NB	NB	NB	NM	ZE	PM	PB
ZE	NB	NB	NM	ZE	PM	PB	PB
PS	NB	NM	ZE	PM	PB	PB	PB
PM	NM	ZE	PM	PB	PB	PB	PB
PB	ZE	PM	PB	PB	PB	PB	PB

Fig.9 shows the simulink model of fuzzy PD controller with disturbance in plant. It improves the performance index in comparison to conventional PID controller which simulink model is shown in fig.4.

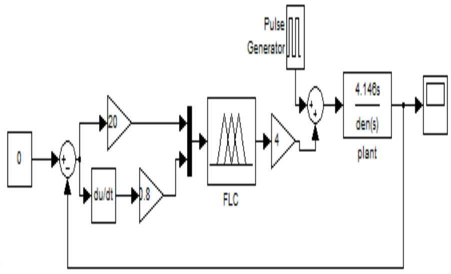

Fig.9 Simulink model of fuzzy PD Controller with disturbance in plant

Fig.10 shows the simulink model of fuzzy PD controller without disturbance in plant. It gives the better performance in comparison to conventional PID controller which simulink model shown in fig.6.

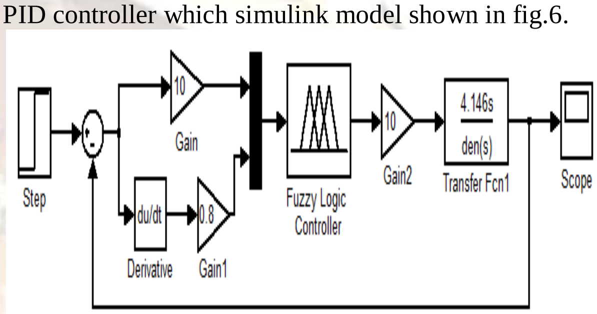

Fig.10 Simulink model of fuzzy PD Controller without disturbance in plant

Fig. 11shows the simulink model of fuzzy PD+I controller that improves the performance index such as maximum overshoot , settling time, and study state error in comparison to simulink model of fuzzy PD controller without disturbance in plant which is shown in fig.10.

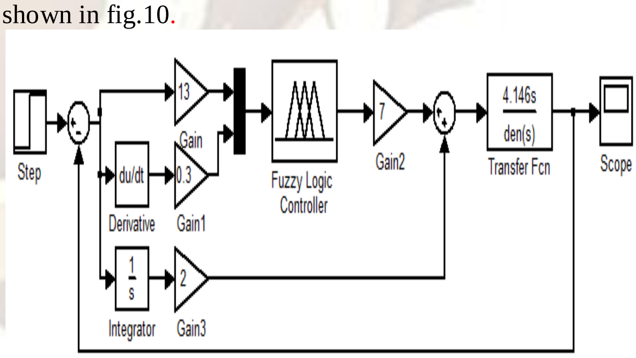

Fig.11 Simulink model of fuzzy PD+I Controller

A high level fuzzy controller works on the level below that of the human operator. It improves the control performance [6]. Normally, control systems based on PID controllers are capable of controlling the process when the operation is steady and close to normal conditions. However, if sudden changes occur, or if the process enters abnormal states, then the configuration in Fig.12 may be applied to bring the process back to normal

operation as fast as possible [6].
Fig.12 shows the schematic diagram of fuzzy in association with PID control. It improves the settling time.

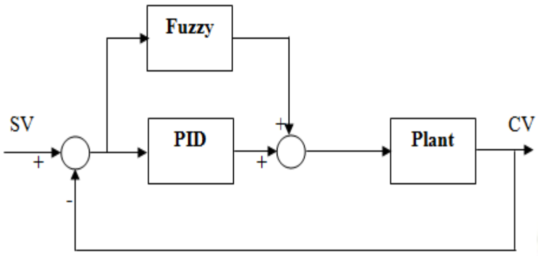

Fig.12 Schematic diagram of Fuzzy adds to PID Control

Fig.13 shows the fuzzy PD+PID control in simulink. It gives better performance index in comparison to all other configuration of fuzzy controller which is developed and discussed above.

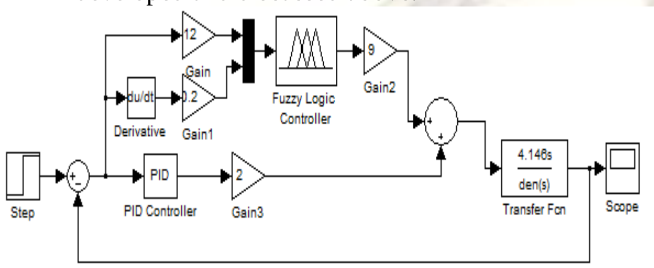

Fig.13 fuzzy PD+ PID Control in simulink

V. SIMULATION RESULT AND DISCUSSION

This section presents the results obtained from the open loop system without controller, and the response of the Inverted pendulum cart system with controllers such as PID and FLC . The pendulum angle PID controller and cart position PID controller are tuned by the Ziegler-Nichols criteria. Gain values of these PID controllers are shown in Table III below.

TABLE II PID Controller Gains

Gain	For Pendulum angle controller	For Cart Position Controller
Kp	40	2
Ki	38	.03
Kd	28	0.5

Fig.14 shows the step response of pendulum angle without controller.

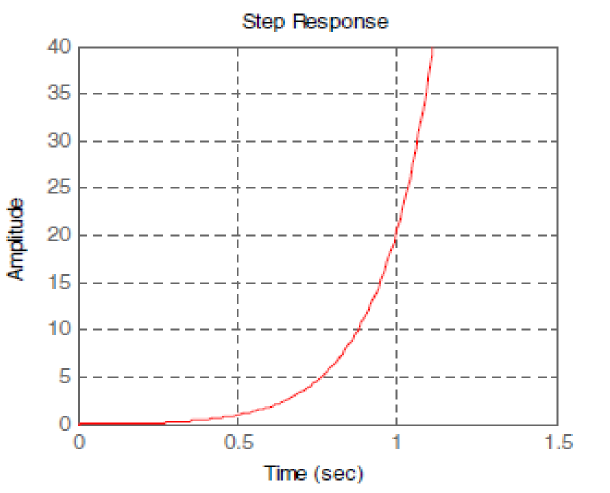

Fig.14 Step response of pendulum angle without controller.

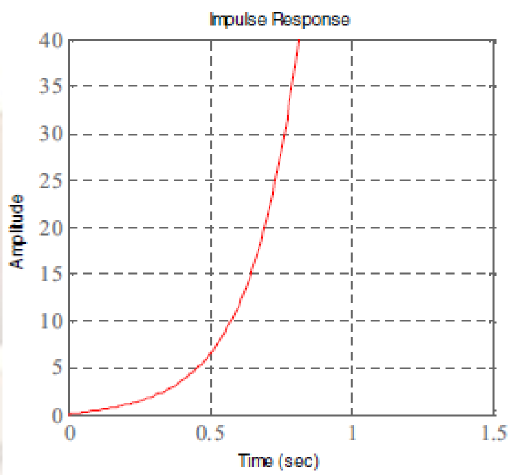

Fig.15 shows the impulse response of pendulum angle without controller

From fig.14 and fig.15 it is noticed that the inverted pendulum system is unstable without controller.

Fig.16 shows the Impulse response of both conventional(PID) & Artificial Intelligence (AI) base controller.
Impulse response of different controllers

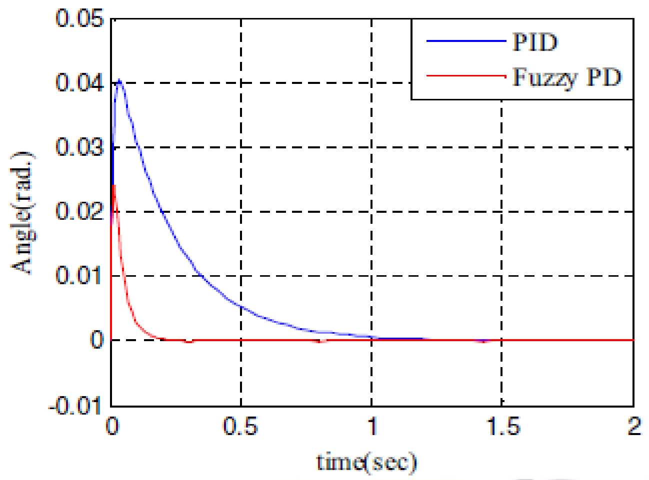

Fig.16 Impulse response of PID & fuzzy PD Controller

The table 2 shows the comparison of these controllers as applied to the inverted pendulum system.

TABLE III

Performance Index of Controllers for impulse response

Controller type	Max Overshoot (%)	Settling Time(s)	Steady state Error
PID Fuzzy PD	4.02 2.44	1.13 0.38	0.02% 0.012%

As shown in table 2, the different performance index of PID and fuzzy PD control such as Maximum Overshoot, Settling time and Study state error were evaluated and found to be improved by 40%, 68% and 40% respectively in fuzzy PD controller. Hence it is concluded that fuzzy PD comparatively better than PID controller. It may also be applied to other system. Fig.17 and fig. 18 shows the step response of various controllers (both conventional & AI base).

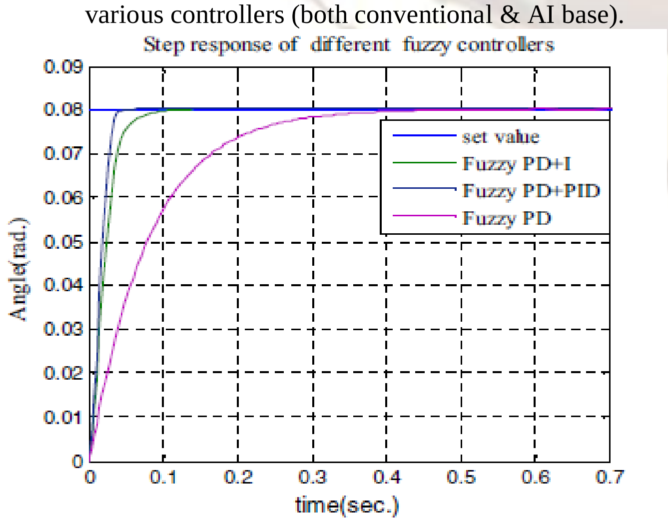

Fig.17 Step response of different fuzzy Controller

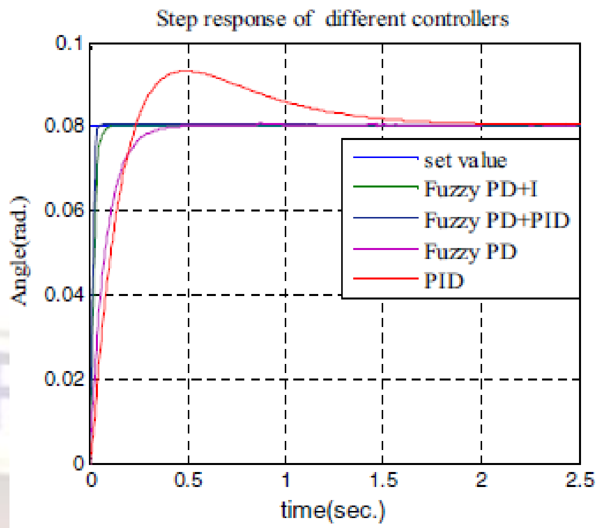

Fig.18 Step response of different Controller

The table 3 shows the comparison on different controllers as applied to the inverted pendulum system.

TABLE IV

Performance Index of Controllers for step input

Controller Type	Max overshoot (%)	Settling Time(s)	Steady state Error
PID	16.45	2.3	0.5%
Fuzzy PD	0.925	0.50	0.42%
Fuzzy PD + I	0	0.32	0.312%
Fuzzy PD+PID	0	0.20	0.315%

As shown in table 3 fuzzy PD+I and fuzzy PD+PID both have approximate same study state error but the settling time in fuzzy PD+PID controller improves by 38%.The simulation structure of the inverted pendulum for PID and fuzzy controller is designed in the simulink environment which results shown in fig.16 to fig.18. These results show that the inverted pendulum has been controlled

Fig 19 to Fig 22 are the graphical results of PID and Fuzzy Logic controller with disturbance.

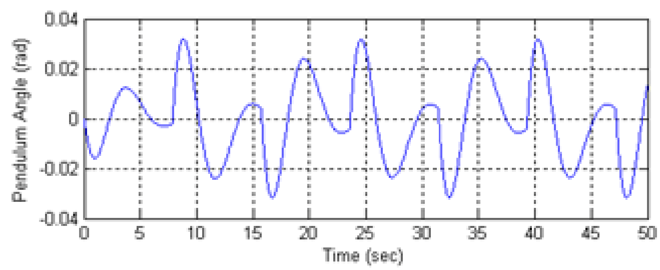

Fig. 19 PID Response for disturbance 1 unit

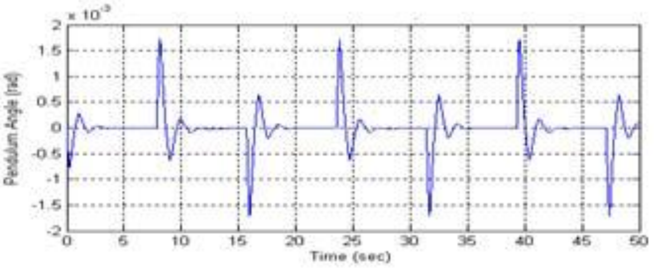

Fig. 20 Fuzzy Response for disturbance 1 unit

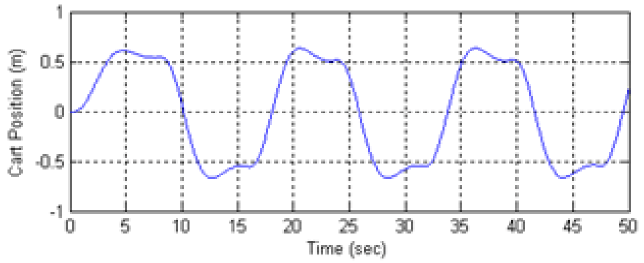

Fig. 21 PID Response for disturbance 1 unit

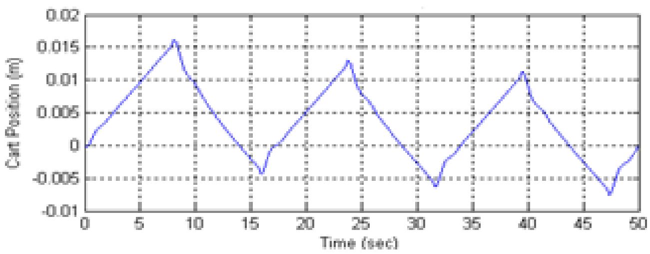

Fig. 22 Fuzzy Response for disturbance 1 unit

VI. CONCLUSION

A PID and fuzzy controller based control scheme for inverted pendulum-cart system has been proposed in this paper. A nonlinear model of the inverted pendulum-cart system has been considered to develop the controllers. The fuzzy controller is proved to be effective and feasible in angle control of pendulum at upright position. Hence fuzzy PD controller is better than conventional PID controller and Fuzzy PD+PID controller gives better performance as compared to the other combination of fuzzy based controller for inverted pendulum. These responses are shown in Fig16 to Fig18. It is clear from simulation results that response using PID and fuzzy controller has stabilized pendulum angle. Responses of the system with disturbance with fuzzy logic controller have been compared with the responses obtained using the PID controller. These responses are shown in Fig 19 to Fig 22 It is clear from the simulation studies that Fuzzy Logic control scheme works more effectively compared to the PID Control scheme. Hence it may be applied to other nom linear system.

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