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Comparison Between The Homotopy Analysis And Homotopy Pad's Method In Solve Nonlinear Ordinary Differential Equation

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Abstract

In this paper, we consider the homtopy analysis method (HAM)and homtopy Pad's medod for solving nonlinear ordinary differential equation with boundary conditions.. These methods are used for solving deformation equationAt homtopy analysis, the answer accuracy reduced by increasing the number of sentences

Keywords-: *homtopy analysis; homtopy Pad's; differential equation; boundary conditions.*

I. INTRODUCTION

Most problems in science and engineering are nonlinear.Thus, it is important to develop efficient methods to solve them. In the past decades, with the fast development of high-quality symbolic computing software, such as Maple, Mathematica and Matlab, analytic as well as numerical techniques for nonlinear differential equations have been developed quickly. The homotopy analysis method (HAM) first proposed by Liao in his Ph.D dissertation, is an elegant method which has proved its effectiveness and efficiency in solving many types of nonlinear equations .Liao in his book proved that HAM is a generalization of some previously used techniques such as the δ-expansion method, artificial small parameter method and Adomian decomposition method. Moreover, unlike previous analytic techniques, the HAM provides a convenient way to adjust and control the region and rate of convergence . There exist some techniques to accelerate the convergence of a given series.Among them, the so-called Pad´*e* method is widely applied.
In recent years, considerable interest in differential equations has been stimulated due to their numerous applications in physics and ngineering. In this paper, we employ the HAM and Pad's medod to solve nonlinear differential equations. Some examples areused to illustrate the effectiveness of this methods . It is shown that the answer accuracy reduced by increasing the number of sentences in the homotopy analysis method But the Pade method has an acceptable accuracy.

This paper is organized as follows. Details of the homotopy analysis and homotopy Pad´*e* methods in section 2. The results of the homotopy analysis and Homotopy Pad´*e* methods are compared for two example. Finally, we discuss and analyse the results in section 3.

II. MODEL DETAILS

1. Homotopy Analysis Method

For convenience of the readers, we will first present a brief description of the standard HAM. To achieve our goal, let us assume the nonlinear system of differential equations be in the form of

$$N(u(x)) = 0 \quad x \in [a,b] \tag{1}$$

where $\mathcal{N}$ is a nonlinear differential operator of second order, x is independent variable and u is an unknown function. For this problem the homotopy equation can be written as follows:

$$(1-q)\tau(\phi(x,q) - u_0(x)) = qN(\phi(x,q)) \tag{2}$$

where x϶ [a,b],q϶ [0,1] is an embedding parameter, u_0 is an initial guess of u,$\mathcal{L}$ is linear differential operator of second order, and φ(x,q) is the homotopy series

$$\phi(x,q) = u_0(x) + \sum_{m=1}^{\infty} u_m(x) q^m \tag{3}$$

That is assumed to be convergent on [0,1]. It easily deduces that

$$u_m(x) = \frac{1}{m!}\frac{\partial^m}{\partial q^m}\phi(x,q)\Big|_{q=0} \tag{4}$$

When q=1, it holds

$$\phi(x,1) = u_0(x) + \sum_{m=1}^{\infty} u_m(x) = u(x) \tag{5}$$

that is a solution of (2), and consequently the solution of (1), see [3]. The initial guess $u_0(x)$ satisfies two boundary conditions of (1), see [3,4,8]. Referring the homotopy literature [3-8], one can find the eformation equations of order *m* is:

$$\tau[\mathrm{u}_m(x) - \chi_m u_{m-1}(x)] = R_{m-1} \quad m \geq 1 \tag{6}$$

Where

$$R_k = \frac{1}{k!}\frac{\partial^k}{\partial q} N(\phi(x,q))\Big|_{q=0} \; k = 1,2,.., m-1 \tag{7}$$

and

$$\chi_m = \begin{cases} 0 & m \le 1 \\ 1 & m > 1 \end{cases} \quad (8)$$

These equations can be easily solved by symbolic computation softwaressuch as Maple and Mathematica .

2. Homotopy Pad's Method

A Padé approximant is the ratio of two polynomials constructed from the coefficients of the Taylor series expansion of a function $u(x)$. The $[L/M]$ Padé approximants to a function $y(x)$ are given by Boyd (1997), Momani (2007)

$$\left[\frac{L}{M}\right] = \frac{P_L(x)}{Q_M(x)} \quad (9)$$

where $P_L(x)$ is polynomial of degree at most L and $Q_M(x)$ is a polynomial of degree at most M.the formal power series

$$y(x) = \sum_{i=1}^{\infty} a_i x^i \quad (10)$$

$$y(x) - \frac{P_L(x)}{Q_M(x)} = O(x^{L-M+1}) \quad (11)$$

determine the coefficients of $P_L(x)$ and $Q_M(x)$ by the equation. Since we can clearly multiply the numerator and denominator by a constant and leave $[L/M]$ unchanged, we imposed thenormalization condition

$$Q_M(0) = 1.0 \quad (12)$$

Finally, we require that $P_L(x)$ and $Q_M(x)$ have non-common factors. If we write the coefficient of $P_L(x)$ and $Q_M(x)$ as

$$\begin{cases} P_L(x) = p_0 + p_1 x + p_2 x^2 + ..p_L x^L \\ Q(x) = q_0 + q_0 x + q_2 x^2 + ..+ q_M x^M \end{cases} \quad (13)$$

Then by (12) and (13), we may multiply (11) by$Q_M(x)$, which linearizes the coefficient equations.We can write out (8) in more details as

$$\begin{cases} a_{L+1} + a_L q_1 + ..+ a_{L-M} q_M = 0 \\ q_{L+2} + q_{L+1} q_1 + ...+ a_{L-M+2} q_M = 0 \\ \vdots \\ a_{L+M} + a_{M+L-1} q_1 + ...+ a_L q_M = 0 \end{cases} \quad (14)$$

$$\begin{cases} a_0 = p_0 \\ a_0 + a_0 q_1 + ... = p_1 \\ \vdots \\ a_L + a_{L-1} q_1 + ...+ a_0 q_L = p_L \end{cases} \quad (15)$$

To solve these equations, we start with equation (14), which is a set of linear equations for all the unknown q's. Once the q's are known, then equation (15) gives and explicit formula fro the unknown p's, which complete the solution. If equations (14) and (15) are nonsingular, then we can solve them directly and obtain equation (16) [1-2], where equation (16) holds, and if the lower index on a sum exceeds the upper, the sum is replaced by zero:

$$\left[\frac{L}{M}\right] = \frac{\det\begin{bmatrix} a_{L-M+1} & a_{L-M+2} & \cdots & a_{L+1} \\ \vdots & & & \\ a_L & a_{L+2} & \cdots & a_{L-M} \\ \sum_{j=M}^{L} a_{j-M} x^j & \sum_{j=M-1}^{L} a_{j-M+1} x^j & \cdots & \sum_{j=0}^{L} a_j x^j \end{bmatrix}}{\det\begin{bmatrix} a_{L-M+1} & a_{L-M+2} & \cdots & a_{L+1} \\ \vdots & & & \\ a_L & a_{L+2} & \cdots & a_{L-M} \\ x^M & x^{M-1} & \cdots & 1 \end{bmatrix}} \quad (16)$$

we can use the symbolic calculus software, Mathematica or Maple.

III. RESULTS

Example:

Following differential equation is considered:

$$f''' + \frac{m}{2} ff' + m(f'^2 - 1) = 0$$

$$\lim_{n\to\infty} f'(\eta) = 1$$

$$f(0) = 0$$

$$f'(0) = \alpha \qquad n = \frac{3}{2}$$

Numerical results by homotopy analysis and homotopy Pad's method s are shown in table1 and table 2 .The graph of solution is presented in Figures[1 -6].

Table 1 Numerical results by homotopy analysis ***for M=20 and n=2/3.***

η	f	f_p	F_s
0	0	0	1.030765
0.732233	0.221028	0.540229	0.506214
2.108913	1.285539	0.921816	0.127471
3.272542	2.416076	1.001965	0.025917

4.727516	3.879356	0.998115	-0.0195330
0	4.150637	0.993293	-0.014049

Table 2 Numerical results by and homotopy Pad's *for M=20 and n=2/3.*

η	f	f_p	F_s
0	0	0	0.927174
0.732233	0.1986426	0.485595	0.456193
2.108913	1.160735	0.839793	0.131183
3.272542	2.205018	0.941099	0.0574412
4.727516	3.623526	1.003469	0.034680
0	3.898227	1.012757	0.335845

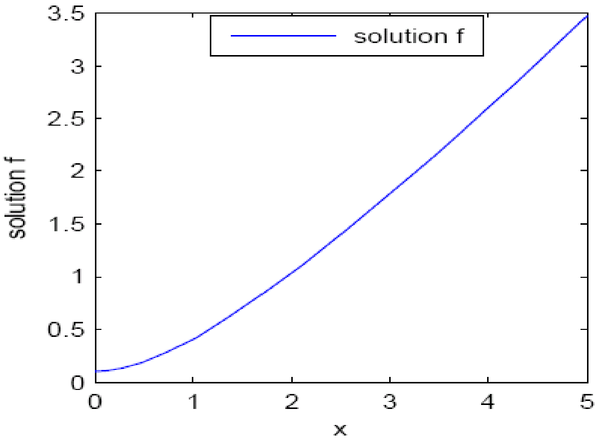

Fig 1. The result of homotopy analysis for f

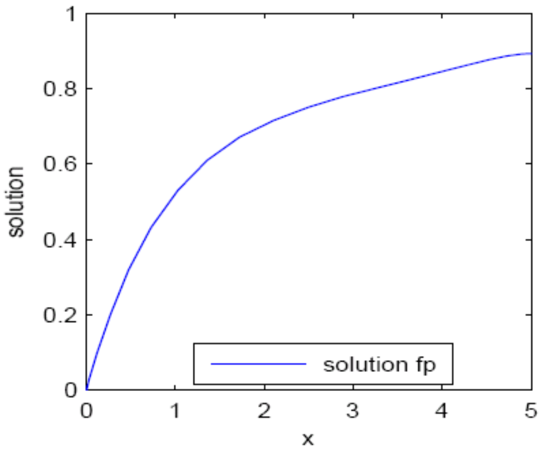

Fig 2. The result of homotopy analysis for f_p

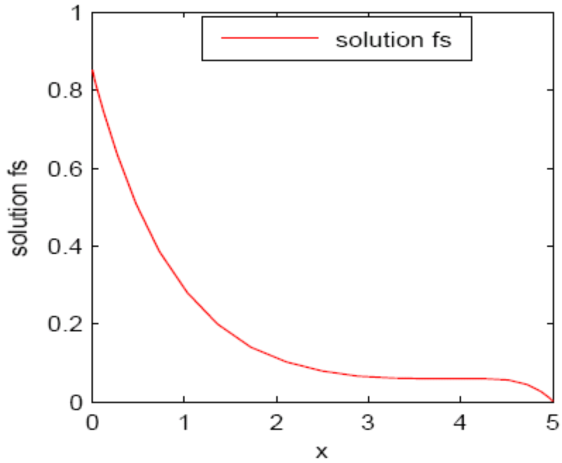

Fig 3. The result of homotopy analysis for f_s

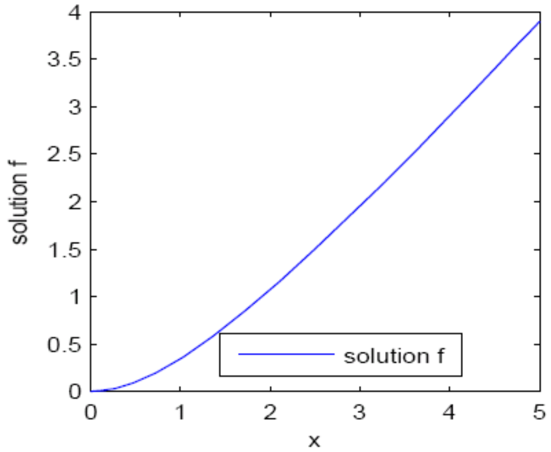

Fig 4. The result of homotopy Pad's for f

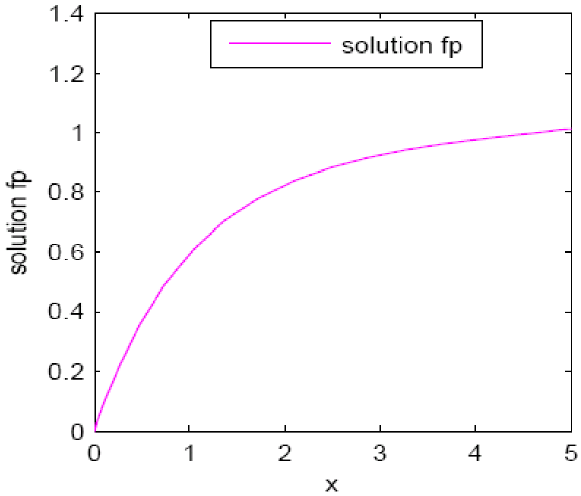

Fig 5. The result of homotopy Pad's for f_p

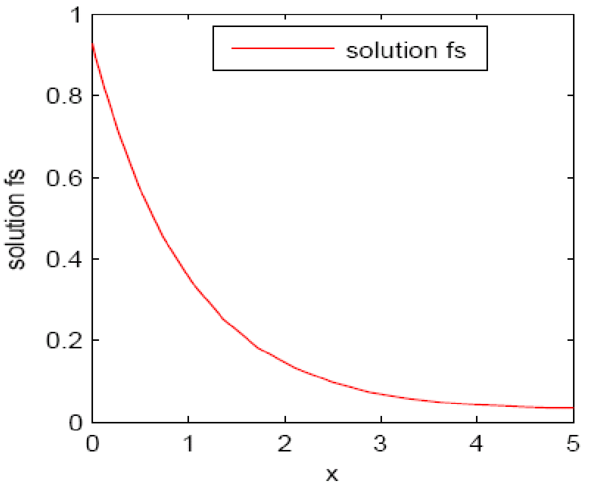

Fig 6. The result of homotopy Pad's for f_s

III. Conclusion

In this paper, the HAM and Pad's were applied to obtain the analytic solution of nonlinear ordinary differential equation . We studied the effciency of HAM and pad's in solving differential equation. Pad's method is more accurate than the homotopy analysis method .

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