A Finite Element Model And Active Vibration Control Of Composite Beams With Distributed Piezoelectrics Using Third Order Theory

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Abstract
In this study, A finite element model based on the third order theory is presented for the active vibration control of composite beams with distributed piezoelectric sensors and actuators. For calculating the total charge on the sensor, the direct piezoelectric equation is used and the actuators provide a damping effect on the composite beam by coupling a negative velocity feedback control algorithm in a closed control loop. A modal superposition technique and a Newmark-β method are used in the numerical analysis to compute the dynamic response of composite beams. Algorithm for calculating the various values of variables is carried out using Matlab tool

Keywords- *Active vibration control, cantilever composite beam, Newmark-β method*

1. Introduction
Engineering structures work frequently under dynamic excitations. The type of excitation may vary but the results of these excitations are shown generally in the form of the vibrations. Vibrations can be attributed as an unwanted for many engineering structures due to precision losses, noise, waste of energy, etc. and should be kept under control for lightweight structures. Attempts at solving these problems have recently stimulated extensive research into smart structures and systems. A smart structure can be defined as a structure with bonded or embedded sensors and actuators with an associated control system, which enables the structure to respond simultaneously to external stimuli exerted on it and then suppress undesired effects. Smart structures have found application in monitoring and controlling the deformation of the structures in a variety of engineering systems. Advances in smart materials technology have produced much smaller actuators and sensors high integrity in structures and an increase in the application of smart materials for passive and active structural damping. Several investigators have developed analytical and numerical, linear and non-linear models for the response of integrated piezoelectric structures. These models provide platform for exploring active vibration control in smart structures. The experimental work of Bailey and Hubbard, 1985 [1] is usually cited as the first application of piezoelectric materials as actuators. They successfully used piezoelectric sensors and actuators in the vibration control of isotropic cantilever beams. P. R. Heyliger and J. N. Reddy, 1988 [2] had presented higher order beam theory for static and dynamic behaviour of rectangular beams using finite element equations. Ha, Keilers and Chang, 1992 [3] developed a modal based on the classical laminated plate theory for the dynamic and static response of laminated composites with distributed piezoelectric. H.S. Tzou and C. I. Tseng, 1990 [4] had presented finite element formulation of distributed vibration control and identification of coupled piezoelectric systems. Chandrashekhara and Varadarajan, 1997 [5] gave a finite element model based on higher order shear deformation theory for laminated composite beams with integrated piezoelectric actuators. Woo-seok Hwang, Woonbong Hwang and Hyun Chul Park, 1994 [6] had derived vibration control of laminated composite plate with piezoelectric sensor/actuator using active and passive control methods. X. Q. Peng, K. Y. Lam and G. R. Liu,1998 [7] gave finite element model based on third order theory for the active position and vibration control of composite beams with distributed piezoelectric sensors and actuartors. Bohua Sun and Da Huang, 2001 [8] had derived analytical solution for active vibration control and suppression of smart laminated composite beam with piezoelectric damping layer. V. Balamuragan and S. Narayanan, 2002 [9] proposed finite element formulation and active vibration control on beams using smart constrained layer damping treatment (SCLD). S. Narayanan and V. Balamuragan, 2003 [10] gave finite element modal of piezolaminated smart structures for active vibration control with distributed sensors and actuators.

2. Mathematical modelling of beam
2.1 Piezoelectric equations
Assuming that a beam consists of a number of layers (including piezoelectric layers) and each layer possesses a plane of material

symmetrically parallel to the x-y plane and a linear piezoelectric coupling between the elastic field and the electric field the constitutive equations for the layer can be written as, [12]

$$\begin{Bmatrix} D_1 \\ D_3 \end{Bmatrix}_k = \begin{bmatrix} 0 & e_{15} \\ e_{31} & 0 \end{bmatrix}_k \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_5 \end{Bmatrix}_k + \begin{bmatrix} \bar{\varepsilon}_{11} & 0 \\ 0 & \bar{\varepsilon}_{33} \end{bmatrix}_k \begin{Bmatrix} E_1 \\ E_3 \end{Bmatrix}_k \quad (1)$$

$$\begin{Bmatrix} \sigma_1 \\ \sigma_5 \end{Bmatrix}_k = \begin{bmatrix} Q_{11} & 0 \\ 0 & Q_{55} \end{bmatrix}_k \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_5 \end{Bmatrix}_k - \begin{bmatrix} 0 & e_{31} \\ e_{15} & 0 \end{bmatrix}_k \begin{Bmatrix} E_1 \\ E_3 \end{Bmatrix}_k \quad (2)$$

The thermal effects are not considered in the analysis.

The piezoelectric constant matrix $[e]$ can be expressed in terms of piezoelectric strain constant matrix $[d]$ as

$$[e]=[d][Q] \quad (3)$$

Where,

$$[d]=\begin{bmatrix} 0 & d_{15} \\ d_{31} & 0 \end{bmatrix} \quad (4)$$

2.2 Displacement field of third order theory

The displacement field based on the third order beam theory of Reddy [2] given by

$$u(x,z,t)=u_0(x,t)+z\phi_x(x,t)-\alpha z^3\left(\phi_x+\frac{\partial w_0}{\partial x}\right) \quad (5)$$

$$w(x,z,t)=w_0(x,t) \quad (6)$$

Where, $\alpha=\dfrac{4}{3h^2}$ and h is the total thickness of the beam.

The displacement functions are approximated over each finite element by [7]

$$u_0(x,t)=\sum_{i=1}^{2} u_i(t)\psi_i(x) \quad (7)$$

$$\phi_x(x,t)=\sum_{i=1}^{2} \phi_i(t)\psi_i(x) \quad (8)$$

$$w_0(x,t)=\sum_{i=1}^{4} \Delta_i(t)\varphi_i(x) \quad (9)$$

Where, ψ_i are the linear Lagrangian interpolation polynomials and the φ_i are the cubic Hermit interpolation polynomials. Δ_1 and Δ_3 represent nodal values of w_0, whereas Δ_2 and Δ_4 represent nodal values of $\dfrac{\partial w_0}{\partial x}$.

We define

$$\{u\}=\{u \quad w\}^T$$

$$\{\bar{u}\}=\{u_1 \quad \phi_1 \quad \Delta_1 \quad \Delta_2 \quad u_2 \quad \phi_2 \quad \Delta_3 \quad \Delta_4\}^T \quad (10)$$

Using finite element equations (5) and (6) can be expressed as

$$\{u\}=[N]\{\bar{u}\} \quad (11)$$

The strain-displacement relations are given by

$$\{\varepsilon\}=\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_5 \end{Bmatrix}=[B]\{\bar{u}\} \quad (12)$$

Where, $[N]$ and $[B]$ are as follows

$$[N]=\begin{bmatrix} \psi_1 & 0 \\ (z-\alpha z^3)\psi_1 & 0 \\ -\alpha z^3\dfrac{\partial\varphi_1}{\partial x} & \varphi_1 \\ -\alpha z^3\dfrac{\partial\varphi_2}{\partial x} & \varphi_2 \\ \psi_2 & 0 \\ (z-\alpha z^3)\psi_2 & 0 \\ -\alpha z^3\dfrac{\partial\varphi_3}{\partial x} & \varphi_3 \\ -\alpha z^3\dfrac{\partial\varphi_4}{\partial x} & \varphi_4 \end{bmatrix}^T \quad (13)$$

$$[B]=\begin{bmatrix} \dfrac{\partial\psi_1}{\partial x} & 0 \\ (z-\alpha z^3)\dfrac{\partial\psi_1}{\partial x}+(1-3\alpha z^2)\psi_1 & 0 \\ -\alpha z^3\dfrac{\partial^2\varphi_1}{\partial x^2}-3\alpha z^2\dfrac{\partial\varphi_1}{\partial x} & \dfrac{\partial\varphi_1}{\partial x} \\ -\alpha z^3\dfrac{\partial^2\varphi_2}{\partial x^2}-3\alpha z^2\dfrac{\partial\varphi_2}{\partial x} & \dfrac{\partial\varphi_2}{\partial x} \\ \dfrac{\partial\psi_2}{\partial x} & 0 \\ (z-\alpha z^3)\dfrac{\partial\psi_2}{\partial x}+(1-3\alpha z^2)\psi_2 & 0 \\ -\alpha z^3\dfrac{\partial^2\varphi_3}{\partial x^2}-3\alpha z^2\dfrac{\partial\varphi_3}{\partial x} & \dfrac{\partial\varphi_3}{\partial x} \\ -\alpha z^3\dfrac{\partial^2\varphi_4}{\partial x^2}-3\alpha z^2\dfrac{\partial\varphi_4}{\partial x} & \dfrac{\partial\varphi_4}{\partial x} \end{bmatrix}^T \quad (14)$$

2.3 Dynamic Equations

The dynamic equations of a piezoelectric structure can be derived by Hamilton's principle [7].

$$\delta\int_{t_1}^{t_2}[T-U+W]dt=0 \quad (15)$$

Where T is the kinetic energy, U is the strain energy, and W is the work done by the applied forces.
In this, the kinetic energy at elemental level is

$$T^e = \frac{1}{2}\int_{V_e} \rho \{\dot{u}\}^T \{\dot{u}\} dV \qquad (16)$$

Where, V_e is the volume of the beam element.
The strain energy at elemental level is

$$U^e = \frac{1}{2}\int_{V_e} \{\varepsilon\}^T \{\sigma\} dV \qquad (17)$$

And work done by the external forces is

$$W^e = \int_{V_e} \{u\}^T \{f_b\} dV + \int_{S_1} \{u\}^T \{f_s\} dS + \{u\}^T \{f_c\} \qquad (18)$$

Where, $\{f_b\}$ is the body force, S_1 is the surface area of the beam element, $\{f_s\}$ is the surface force and $\{f_c\}$ is the concentrated load.

2.4 Equation of motion
To develop the equation of motion of the system, consider the dynamic behavior of the system. These equations also provide coupling between electrical and mechanical terms. The equation of motion in the matrix form can be written as, [7]

$$[M^e]\{\ddot{\bar{u}}\}^e + [K^e]\{\bar{u}\}^e = \{F\}^e + [K_{uv}^e]V^e \qquad (19)$$

Where,

$$[M^e] = \int_{V_e} \rho [N]^T [N] dV \qquad (20)$$

$$[K^e] = \int_{V_e} [B]^T [Q][B] dV \qquad (21)$$

$$[K_{uv}^e] = \int_{V_e} [B]^T [e]^T [B_v] dV \qquad (22)$$

$$\{F^e\} = \int_{V_e} [N]^T \{f_b\} dV + \int_{S_1} [N]^T \{f_s\} dS + [N]^T \{f_c\} \qquad (23)$$

In order to include the damping effects, Rayleigh damping is assumed. So equation (19) modified as,

$$[M^e]\{\ddot{\bar{u}}\}^e + [C^e]\{\dot{\bar{u}}\}^e + [K^e]\{\bar{u}\}^e = \{F^e\} + [K_{uv}^e]V^e \qquad (24)$$

Where, $[C^e]$ is the damping matrix, which is

$$[C^e] = a[M^e] + b[K^e] \qquad (25)$$

Where, a and b are the constants can be determined from experiments [11].
Assembling all elemental equations gives global dynamic equation is as follows:

$$[M]\{\ddot{\bar{u}}\} + [C]\{\dot{\bar{u}}\} + [K]\{\bar{u}\} = \{F\} + \{F_V\} \qquad (26)$$

Where, $\{F\}$ is the external mechanical force vector and $\{F_V\}$ is the electrical force vector.

$$\{F_v\} = [K_{uv}]\{V\} \qquad (27)$$

2.5 Sensor equations
Since no external electrical field is applied to the sensor layer and a charge is collected only in the thickness direction, only the electric displacement is of interest and can be written as [7]

$$D_3 = e_{31}\varepsilon_1 \qquad (28)$$

Assuming that the sensor patch covers several elements, the total charge developed on the sensor surface is

$$q(t) = \sum_{j=1}^{N_s} \frac{1}{2}\left[\int_{S_j} \left([B_1]_{(z=z_k)} + [B_1]_{(z=z_{k+1})}\right) e_{31} dS \{\bar{u}_j\}\right] \qquad (29)$$

Where, $[B_1]$ is the first row of $[B]$.

The distributed sensor generates a voltage when the structure is oscillating; and this signal is fed back into distributed actuator using a control algorithm, as shown in fig. 1. The actuating voltage under a constant gain control algorithm can be expressed as,

$$V^e = G_i V_s = G_i G_c \frac{dq}{dt} \qquad (30)$$

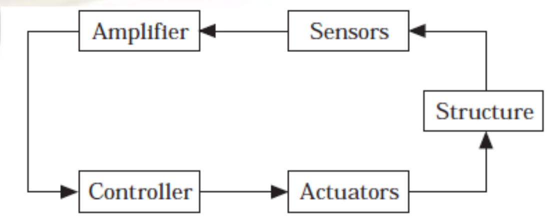

Fig.1 Block diagram of feedback control system

The system actuating voltages can be written as

$$\{V\}=[G][K_v]\{\dot{\bar{u}}\} \quad (31)$$

Where, $[G]$ is the control gain matrix and $G=G_iG_c$.

In the feedback control, the electrical force vector $\{F_v\}$ can be regarded as a feedback force. Substituting equation (31) into equation (27) gives,

$$[F_v]=[K_{uv}][G][K_v]\{\dot{\bar{u}}\} \quad (32)$$

We define,

$$[C^*]=-[K_{uv}][G][K_v] \quad (33)$$

Thus, the system equation of motion equation (26) becomes

$$[M]\{\ddot{\bar{u}}\}+([C]+[C^*])\{\dot{\bar{u}}\}+[K]\{\bar{u}\}=\{F\} \quad (34)$$

As shown in equation (34), the voltage control algorithm equation (30) has a damping effect on the vibration suppression of a distributed system.

For obtaining a dynamic response under a given external loading condition, a modal analysis is used, and the nodal displacement $\{\bar{u}\}$ is represented by [11]

$$\{\bar{u}\}=[\Phi]\{x\} \quad (35)$$

Where $\{x\}$ are referred to as the generalized displacements. $[\Phi]$ is the modal matrix and has the orthogonal properties as:

$$[\Phi]^T[K][\Phi]=[\Omega^2], \quad [\Phi]^T[M][\Phi]=[I] \quad (36)$$

Where, $[\Omega^2]$ is a diagonal matrix that stores the squares of the natural frequencies ω_i.

Substituting equation (35) into (34) and then multiplying equation (34) gives

$$[\Phi]^T[M][\Phi]\{\ddot{x}\}+[\Phi]^T([C]+[C^*])[\Phi]\{\dot{x}\}+[\Phi]^T[K][\Phi]\{x\}=[\Phi]^T\{F\} \quad (37)$$

Substituting equation (36) into (37) gives

$$\{\ddot{x}\}+(2\xi\omega+[\Phi]^T[C^*][\Phi])\{\dot{x}\}+\omega^2\{x\}=[\Phi]^T\{F\} \quad (38)$$

The initial condition on $\{x\}$ is

$$\{x_0\}=[\Phi]^T[M]\{\bar{u}_0\} \quad (39)$$

$$\{\dot{x}_0\}=[\Phi]^T[M]\{\dot{\bar{u}}_0\} \quad (40)$$

3. Material properties of cantilever composite beam

A cantilever composite beam with both upper and lower surfaces symmetrically bonded by piezoelectric ceramics, shown in fig. 2

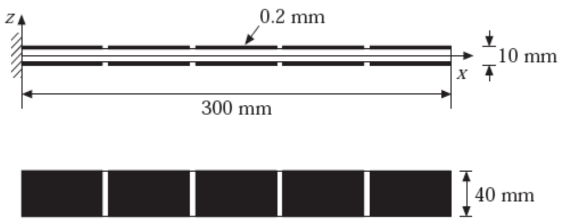

Fig. 2 A cantilever composite beam

The beam is made of T300/976 graphite/epoxy composites and the piezoceramic is PZT G1195N. The adhesive layers are neglected. The material properties are given in table 1. The stacking sequence of composite beam is $[-45^0/45^0]_s$. The total thickness of the composite beam is 9.6 mm and each layer has the same thickness (2.4 mm). and the thickness of each piezo-layer is 0.2 mm. The lower piezoceramics serve as sensors and the upper ones as actuators. The relative sensors and actuators form sensor/actuator pairs through closed control loops.

Table 1. Material properties of PZT and T300/976 graphite/epoxy composite

Properties	Symbol	PZT	T300/976
Young moduli (GPa)	E_{11} $E_{22}=E_{33}$	63.0 63.0	150.0 9.0
Poisson's ratio	$v_{12}=v_{13}$ v_{23}	0.3 0.3	0.3 0.3
Shear moduli (GPa)	$G_{12}=G_{13}$ G_{23}	24.2 24.2	7.10 2.50
Density (Kg/mg^3)	P	7600	1600
Piezoelectric constants (m/V)	$d_{31}=d_{32}$	254×10^{-12}	
Electrical permittivity (F/m)	$\varepsilon_{11}=\varepsilon_{22}$ ε_{33}	15.3×10^{-9} 15.0×10^{-9}	

4. Active vibration suppression

Here size of the beam is considered as 600 mm long, 40 mm width and 9.6 mm thick, also thickness of each piezo layer is 0.2 mm.

In the analysis, the composite beam is divided into 30 elements. The piezoelectric composite beam given in figure 2 is considered to simulate the active vibration suppression through a simple S/A active control algorithm (negative velocity feedback). The piezoceramics on the lower surface serve as sensors,and on the upper surface are actuators. In the analysis the beam is divided into 30 elements so each S/A pairs covers six elements.

First, modal superposition technique is used to decrease the size of the problem. the different modes are used in the modal space analysis and transient response of the cantilever beam is computed by the Newmark direct integration method [11]. The parameters α and β are from Rayleigh method.

Results obtain after these methods are as under

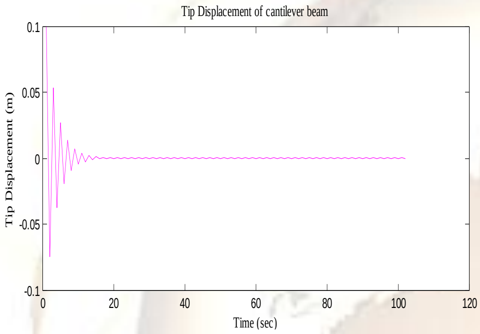

Fig. 3 The effect of negative velocity gain on the beam subjected to first mode vibration. (Gain=-2 V/A)

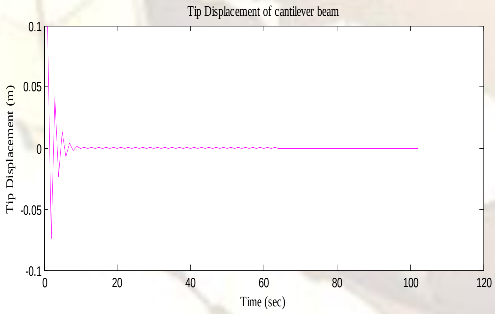

Fig. 4 The effect of negative velocity gain on the beam subjected to first mode vibration. (Gain=-5 V/A)

Figure 3 and 4 shows the vibration control performance for PZT fully covered beam with control gain -2 V/A and -5 V/A for first value of alpha and beta of Rayleigh damping.

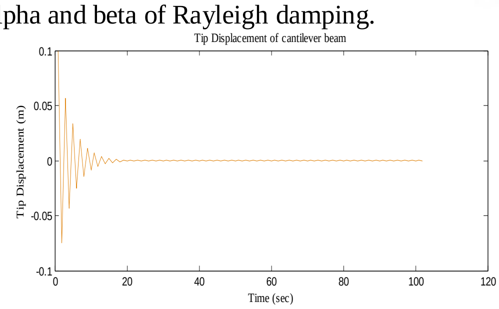

Fig. 5 The effect of negative velocity gain on the beam subjected to second mode vibration. (Gain=-1 V/A)

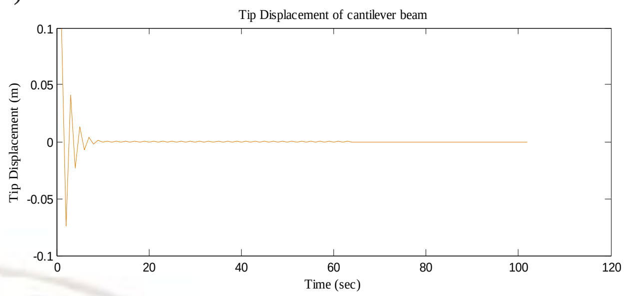

Fig. 6 The effect of negative velocity gain on the beam subjected to second mode vibration. (Gain=-5 V/A)

Figure 5 and 6 shows the vibration control performance for PZT fully covered beam with control gain -1 V/A and -5 V/A for the second value of alpha and beta of Rayleigh damping.

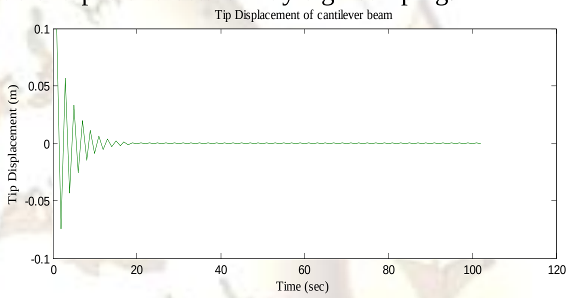

Fig. 7 The effect of negative velocity gain on the beam subjected to third mode vibration. (Gain=-1 V/A)

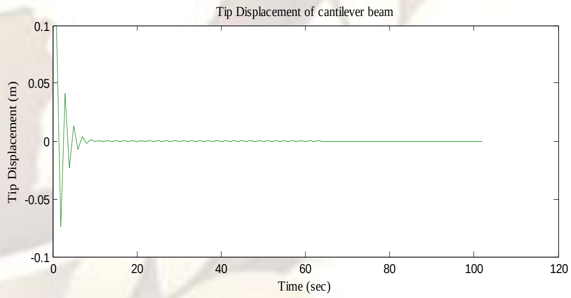

Fig. 8 The effect of negative velocity gain on the beam subjected to third mode vibration. (Gain=-5 V/A)

Figure 7 and 8 shows the vibration control performance for PZT fully covered beam with control gain -1 V/A and -5 V/A for the second value of alpha and beta of Rayleigh damping.

As shown in figures, the vibrations decay more quickly when higher control gains are applied. However, it must be noted that the gains should be limited for the sake of the breakdown voltage of the piezoelectrics.

5. Conclusion

A finite element model and computer codes in Matlab, based on third order laminate theory, are developed for the cantilever composite beam with distributed piezoelectric ceramics. Equation of motion of composite beam is derived by using Hamilton's principle.

- The S/A pairs must be placed in high strain regions and away from the area of low strain regions for maximum effectiveness.
- More S/A pairs can generally induce more efficiency on the active vibration suppression. So the number of S/A pairs has a great effect on the performance of smart structures.
- As the feedback gain increases, the damping ratios are large and as the feedback gain decreases, the damping ratios are smaller.
- Changes in stiffness and the location and the number of S/A pairs also affect the dynamic response of the composite structure.

So, based on this work, active vibration control with finite element model is reliable and fail-safe.

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