

## Mathematical Models for drying behaviour of green beans

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### ABSTRACT

Drying is the oldest methods for prevention of the Agricultural products such as fruits and vegetables. Green Beans have a significant share in vegetables production in the world. It is also important raw material for many food products. Temperature velocity and relative humidity of drying air are important parameters for hot air drying process. Drying characteristics of green beans were examined for average moisture content from  $90.53 \pm 0.5\%$  to  $14 \pm 0.3\%$  using hot air of the temperature range of 50 OC. The experimental drying curves obtained were fitted to a number of semi-theoretical models, namely Handerson and Pabis, Lewis and page models. Comparing the determination of coefficient, reduced chi-square and root mean square values of three models, it was concluded that the page model represents drying characteristics better than others. The effective diffusivity coefficient of moisture obtained as  $2.641 * 10^{-9} \text{ m}^2 / \text{s}$  over the temperature range.

**Keywords:** *Drying rate; Modelling; Effective diffusivity, Equilibrium Moisture content.*

### 1. Introduction

Drying involves the application of heat to vaporize the volatile substances and some means of removing water vapour after its separation from the solid. The drying process is a heat and mass transfer phenomenon where water migrates from the interior of the drying product on to the surface from which it evaporates.

The major objective in drying agricultural products is the reduction of the moisture content to a level, which allows safe storage over an extended period. Also, it brings about substantial reduction in weight and volume, minimizing packaging, storage and transportation costs (1, Mujumdar, 1995; Okos, Narsimhan, Singh, & Witnauer, 1992).

The main objective of drying is as follows:

A dry product is less susceptible to spoilage caused by growth odd bacteria, mold and insects. The activity of many microorganisms and insects is inhibited in an environment in which the equilibrium relative humidity is below 70%. Many favourable qualities and nutritional values of food may be enhanced by drying. Palatability is improved.

The study of drying behaviour of various vegetables has been subject of interest of different researchers. There have been many studies on the drying behaviour of various vegetables such as

soybeans and white beans (2,Hutchinson & Otten, 1983; Kitic & Viollaz, 1984), green beans (3,Senadeera, Bhandari, Young, & Wije- singhe, 2003), red pepper (4,Akpinar, Bicer, & Yildiz, 2003; Doymaz & Pala, 2002), carrot (5,Doymaz, 2004), eggplant (6,Ertekin & Yaldiz, 2004), and pumpkin, green pepper, stuffed pepper, green bean and onion (7,Yaldiz & Ertekin, 2001).

### 1.1. Mathematical modelling

Mathematical modelling has been widely and effectively used for analysis of drying of agricultural products. Many mathematical models have described the drying process, of them thin layer drying models have been widely in use. These models are categories, namely theoretical, semi-theoretical and empirical. Among semi-theoretical thin layer drying models, namely the Handerson and Pabis model, the Lewis model and the Page model are used widely. These models are generally derived by simplifying general series solution of Fick's second law. The Henderson and Pabis model is the first term of a general series solution of Fick's second law. This model was used successfully to model drying of corn (8, Henderson & Pabis, 1961). There are many statistical based models correlating experimentally obtained moisture ratio values in terms of time. In these models the moisture ratio is termed as  $MR = (M - Me) / (Mo - Me) \text{-----} 1$  M is the moisture content at any time, Mo is the initial moisture content, Me is the equilibrium moisture content.

The Lewis model is a special case of the Henderson and Pabis model where intercept is unity. It was used to describe drying of black tea (9,Panchariya, Popovic, & Sharma, 2002).The Page model is an empirical modification of the Lewis model to correct for its shortcomings. This model was also used to fit the experimental data of soybean, white bean, green bean and corn ( 10,Doymaz & Pala, 2003; )

### 1.2. The statistical modelling procedure

The correlation coefficient (R<sup>2</sup>) is one of the primary criteria for selecting the best equation to define the drying curves. in addition to R<sup>2</sup>, reduced chi-square ( $\chi^2$ ) and the root mean square error (RMSE) were used to determine the quality of fit.

**2. Materials and methods**

Fresh green samples were purchased from the near markets which are suitable for our experiments. Green bean samples are stored at a temperature range of 3-40C for about one day in a refrigerator. They were stored at desired temperature to allow them to attain equilibrium of moisture. Average diameter of bean samples was measured as 16.2cm.then washed and the top and bottom parts of the samples are cut in the form of slices of 4cm length with a knife. The average moisture content of the bean sample was about 90.53% ( on wet weight basis).

**2.1. Drying experiments**

Drying experiments were performed in a cabinet laboratory scale hot air dryer, The air drying temperatures were maintained at 50 OC and the relative humidity at 25% The relative humidity of air was determined using wet and dry bulb temperatures obtained from the psychometric chart. Air passed perpendicular to drying surfaces of the samples. Drying process started when drying conditions were achieved constant air temperatures. The green beans samples were placed on a tray in a single layer and the measurement started from this point. Experiments were conducted with 125± 0.3 g of green beans. The samples were dried until moisture content reached approximately 14 ± 0.3% (w/w). The product was cooled and packed in low density polyethylene bags, which were sealed with heat. The drying data from the different drying runs, were expressed as drying rate versus drying time and moisture ratio versus drying time.

**3. Results and discussion**

Drying rate is defined as the amount of water removed and time is shown in Fig. 1 for bean samples during thin layer drying at 50 oC. It is apparent that drying rate decreases continuously with improving drying time. In this curves, there was not constant-rate period but it seen to occur the falling-rate period. The results indicated that diffusion is the most likely physical mechanism governing moisture movement in the bean samples. As expected from Fig. 1, increasing the air temperature increases the drying rate (consequently decreases drying time). The experimental results were showed that air temperature is considered as the most important factor affected drying rate. Drying rate curve for given beans at 50oC temperature with 1.0m/s air velocity from fig1.Increasing the air temperature increases the drying rate. The experimental results were showed that air temperature is considered as the most important factor for drying rate. Experimental results of moisture ratio with drying time were fitted with by using models like

Henderson and Pabis, Lewis Model and Page Model

Chi square Error :

The chi square error values were calculated for all the three models using the formula

$$\chi^2 = \sum_{Ni=1}^N (MR_{\text{experimental } i} - MR_{\text{Predicted } i})^2 / N - Z$$

Where N = number of observations = 32

Z = no of constants

Handerson and Pabis Model:

For Z = 2; N = 32

$$\chi^2 = [(1.00 - 0.986)^2 + (0.9535 - 0.943)^2 + \dots + (0.002 - 0.374)^2] / (32-2)$$

$$\chi^2 \text{ (Handerson Model)} = 0.00062$$

Lewis Model:

For Z = 1

$$\chi^2 = [(1.00 - 0.95)^2 + (0.9537 - 0.909)^2 + \dots + (0.0021 - 0.352)^2] / (32-1)$$

$$\chi^2 \text{ (Lewis Model)} = 0.00074$$

Page Model:

For Z = 2

$$\chi^2 = [(1.00 - 0.992)^2 + \dots + (0.0021 - 0.440)^2] / (32-2)$$

$$\chi^2 \text{ (Page Model)} = 0.00019$$

Root Mean Square Error:

Root mean square error (RMSE) values for all the three models were calculated using the formula

$$RMSE = (1/N \sum_{Ni=1}^N (MR_{\text{experimental } i} - MR_{\text{Predicted } i})^2)^{1/2}$$

RMSE for Handerson & Pabis Model:

$$RMSE = [1/32 (1.00 - 0.95)^2 + (0.9535 - 0.943)^2 + \dots + (0.0021 - 0.374)^2]^{1/2}$$

$$RMSE = 0.0964$$

RMSE for LEWIS Model:

$$RMSE = (1/N \sum_{Ni=1}^N (MR_{\text{experimental } i} - MR_{\text{Predicted } i})^2)^{1/2}$$

$$RMSE = [1/32(1.00 - 0.992)^2 + \dots + (0.0021 - 0.044)^2]^{1/2}$$

$$RMSE = 0.1732$$

RMSE for PAGE Model:

$$RMSE = (1/N \sum_{Ni=1}^N (MR_{\text{experimental } i} - MR_{\text{Predicted } i})^2)^{1/2}$$

$$RMSE = [1/32(1.00 - 0.992)^2 + \dots + (0.0021 - 0.044)^2]^{1/2}$$

$$RMSE = 0.05401$$

Determination of effective diffusivity coefficient:

$$(\partial M / \partial t) = Deff (\partial^2 M / \partial X^2)$$

Where MR = M - Me / Mo - Me

$$= 8/\pi^2 \sum_{n=1}^{\infty} (1/2n - 1)^2 \exp(-n^2 \pi^2 Deff / 4L^2)$$

Simplifying the above equation only the first term of series solution gives the equation

$$MR = 8/\pi^2 \exp(-\pi^2 Deff t / 4L^2)$$

The slope between ln(MR) and time gives value of ko

$$Ko = \pi^2 Deff / 4L^2 = 0.018$$

$$Deff = (0.0018 * 4 * 10^{-2}) / (3.414)^2 = 2.4709 * 10^{-8} \text{ m}^2/\text{s}$$

Conclusion:

The influence of drying air temperature of 50oC and 1.0m/s of air velocity for green beans was studied. The value of calculated effective diffusivity coefficient was  $2.641 \times 10^{-9}$  m<sup>2</sup>/s. The drying rate and effective diffusivity increases as the air temperature increases.

Page empirical Model showed a good fit curve than Handerson and Pabis, Lewis Models. Drying rate curves indicated that drying takes place mostly in the falling rate period expect very short unsteady state initial and constant rate periods. When the temperature was increased the velocity decreased, the effective diffusion coefficients generally increase. The consistency of the model is evident but R<sup>2</sup> values for constants are low.

Table1: observation data of weight of sample with time

S.No	Time	weight
1	0	125
2	3	121
3	6	119
4	9	116
5	12	112
6	15	110
7	18	108
8	21	105
9	24	101
10	27	98
11	30	95
12	33	91
13	36	88
14	39	86
15	42	82
16	45	79
17	48	75
18	51	72
19	54	70
20	57	68
21	60	66
22	63	63
23	66	61
24	69	59
25	72	56
26	78	54
27	81	52
28	84	50
29	87	48
30	90	45
31	93	43
32	96	42
33	99	42
34	100	42

S.No	Time	Moisture content	Drying rate
1	0	69	1.33
2	3	66.94	1.32
3	6	66.38	1.30
4	9	65.51	1.24
5	12	64.285	1.23
6	15	63.636	1.20
7	18	62.96	1.19
8	21	61.904	1.17
9	24	60.396	1.16
10	27	59.1836	1.14
11	30	57.894	1.11
12	33	56.043	1.08
13	36	54.56	1.04
14	39	53.49	1.00
15	42	51.2195	0.97
16	45	49.367	0.92
17	48	46.666	0.91
18	51	44.42	0.89
19	54	42.86	0.80
20	57	41.26	0.74
21	60	39.93	0.70
22	63	37.507	0.68
23	66	35.841	0.62
24	69	32.403	0.58
25	72	29.671	0.54
26	75	27.925	0.48
27	78	26.01	0.46
28	81	24.39	0.38
29	84	22.21	0.26
30	87	20.01	0.20
31	90	17.34	0.14
32	93	12.01	0.041
33	96	9.46	0.002
34	99	8.02	0.001



Table2: Calculated values of moisture contents and drying rates.

S.No	Moisture Ratio	Predicted values of different models		
	Experimental values	Henderson & Pabis Model	Lewis Model	Page Model
1	1.00	0.9860	0.95	0.992
2	0.9535	0.943	0.909	0.982
3	0.9502	0.9321	0.898	0.9714
4	0.9376	0.9241	0.890	0.9604
5	0.9158	0.907	0.873	0.946
6	0.9042	0.906	0.874	0.929
7	0.8732	0.894	0.8619	0.904
8	0.8463	0.862	0.8310	0.894
9	0.8247	0.841	0.8108	0.876
10	0.7889	0.801	0.772	0.864
11	0.7421	0.7620	0.734	0.860
12	0.7230	0.734	0.707	0.821
13	0.6824	0.693	0.668	0.800
14	0.6494	0.689	0.664	0.798
15	0.6011	0.670	0.645	0.770
16	0.5610	0.664	0.640	0.720
17	0.5332	0.620	0.597	0.690
18	0.5001	0.609	0.587	0.643
19	0.4808	0.594	0.5726	0.621
20	0.4375	0.568	0.5476	0.604
21	0.4078	0.549	0.5293	0.598
22	0.3469	0.520	0.5013	0.582
23	0.2976	0.511	0.4296	0.579
24	0.2665	0.499	0.4811	0.560
25	0.2321	0.484	0.466	0.538
26	0.2033	0.480	0.4627	0.530
27	0.1644	0.462	0.442	0.512
28	0.0977	0.454	0.4377	0.502
29	0.0721	0.390	0.3760	0.498
30	0.042	0.387	0.379	0.482
31	0.0021	0.374	0.352	0.440

Table3: Experimental and predicted values of three mathematical models.

Temperature 0C	Models and Constants	R2	$\chi^2$	RMSE
50	Handerson and Pabis ( a = 1.0372 ; k = 0.0056)	0.9971	0.00062	0.0964
50	Lewis ( k = 0.0054)	0.9963	0.00074	0.1732
50	PAGE ( k= 0.0023 ; y = 1.1531)	0.992	0.00019	0.05401

Table4: Curve fitting criteria for the drying models for green beans

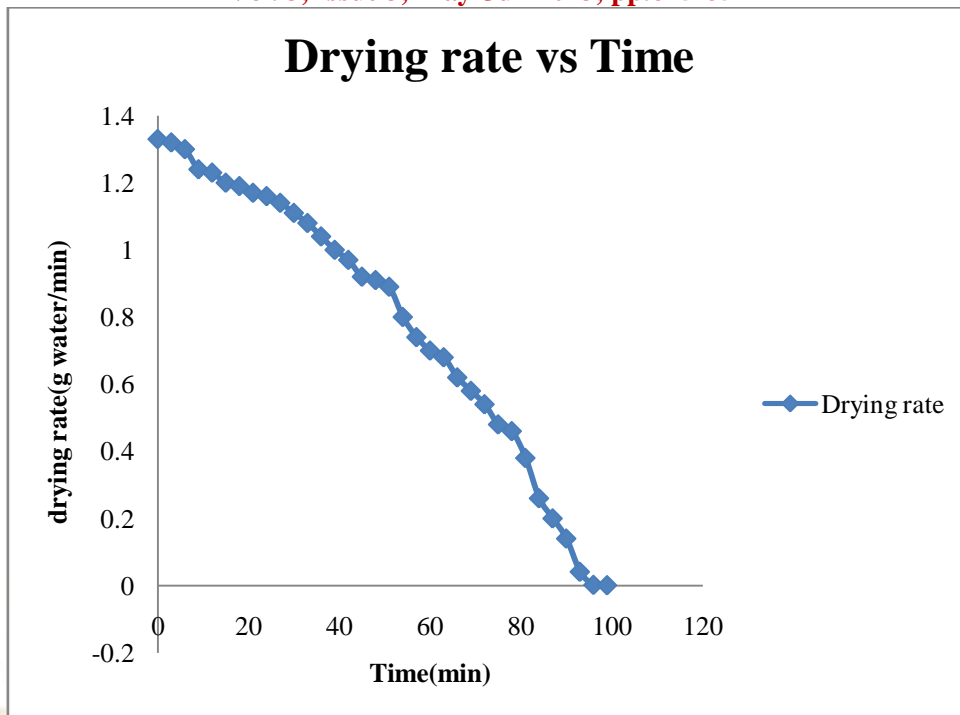


Fig1: Drying Rate curves for green beans at 50oC with 1.0 m/s air velocity

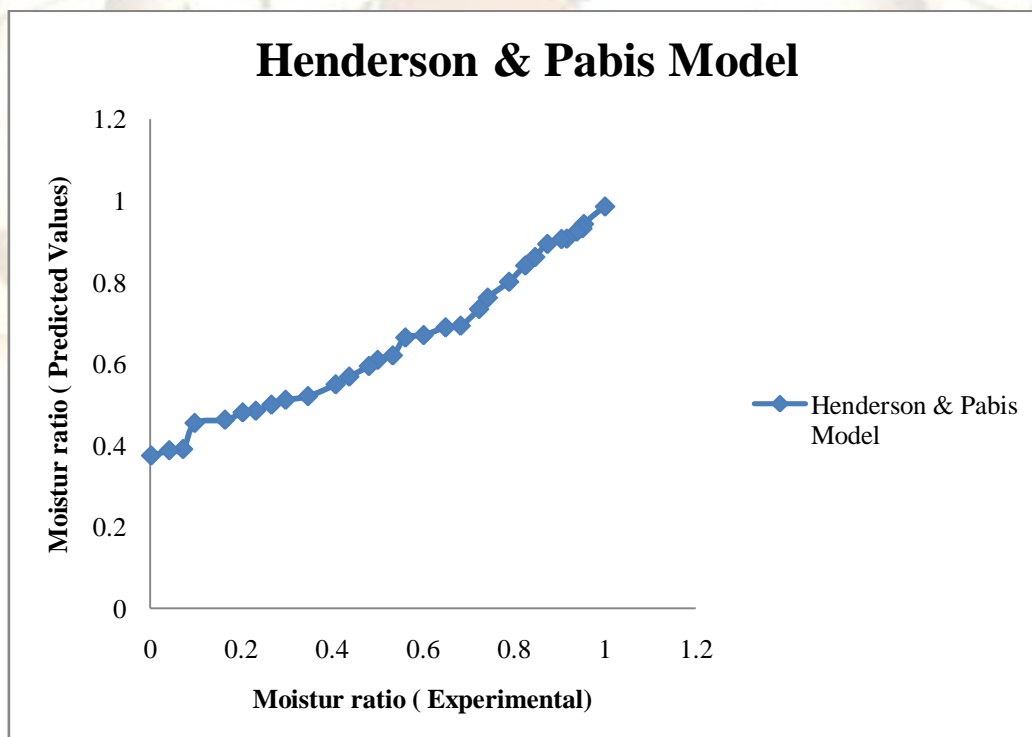


Fig2: Variation of Experimental and predicted moisture ratio by Handerson and Pabis Model.

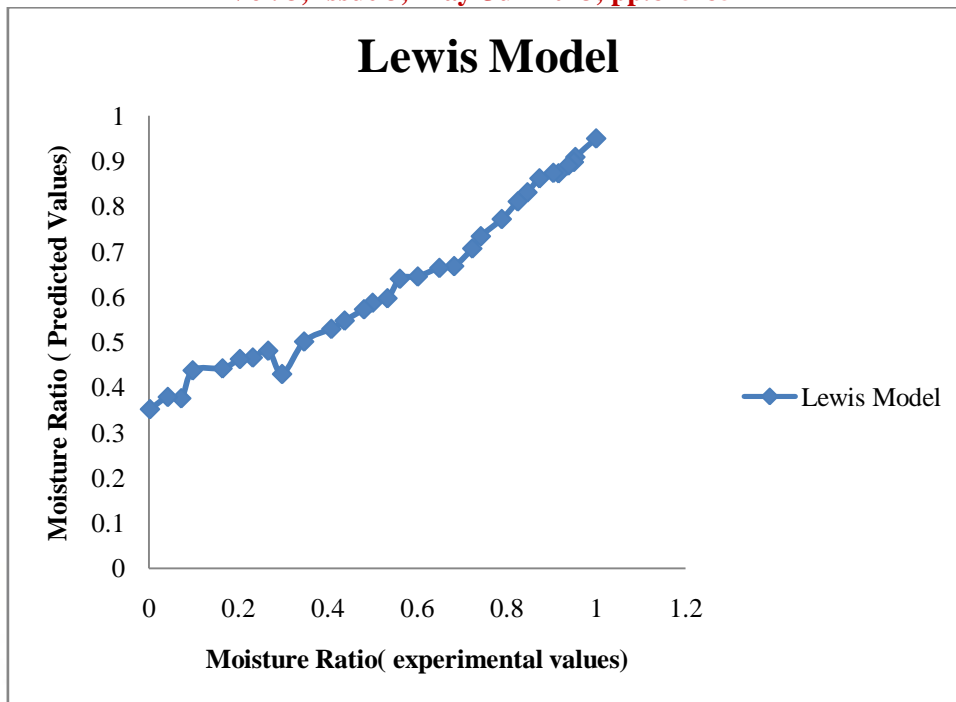


Fig3: Variation of experimental and predicted moisture ratio by Lewis model.

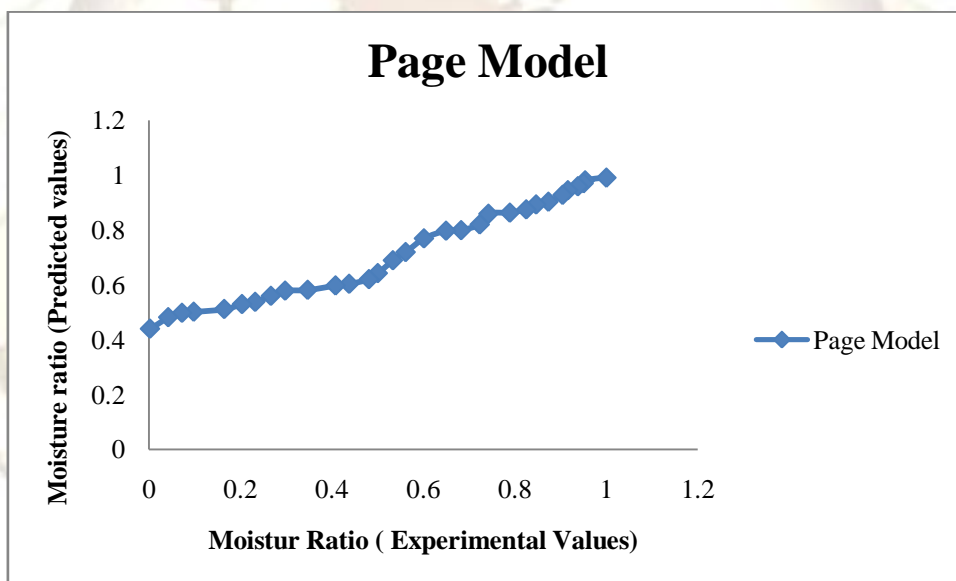


Fig4: Variation of experimental and predicted moisture ratio by Page Model

**Nomenclature**

a, k, y constants in models  
 Deff effective diffusivity coefficient m<sup>2</sup>/s  
 L half-thickness of slab (m)  
 M moisture content (kg moisture/kg dry matter)  
 Me equilibrium moisture content (kg moisture/kg dry matter)

M0 initial moisture content (kg moisture/kg dry matter)  
 N number of observations  
 n positive integer  
 R2 determination of coefficient  
 T air temperature (OC)  
 t drying time (min)  
 z number of constants  
 RMSE Root mean square error

R2 correlation coefficient  
 $\chi^2$  chi-square

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