Analysis of Heat Transfer and Flow due to Natural Convection in Air around Heated Square Cylinders of Different Sizes inside an Enclosure

Ravi Kumar Singh*, K. B. Sahu, Thakur Debasis Mishra*****
[1](Department of Mechanical Engineering, KIIT University, Bhubaneswar- 751024, India)

ABSTRACT

In this paper, analysis of heat transfer and flow due to natural convection in air around heated square cylinders of different sizes inside an enclosure has been computed. The square cylinder is at higher temperature and the vertical walls of the enclosure are at lower temperature with insulated horizontal walls. The various governing equations such as continuity equation, momentum equation, and energy conservation equation are calculated by finite elements using Galerkin method. Assuming constant properties of air, the Rayleigh number depends on the size of square enclosure for fixed high and low temperatures. The sizes of enclosures are taken as 20 mm, 40 mm, and 80 mm for different Rayleigh numbers. The size of the cylinder is taken accordingly for aspect ratio of 0.2, 0.3, and 0.4. Results are displayed in the form of Isotherms, streamline and velocity vector diagrams and the heat transfer and flow around the cylinder is analyzed.

Keywords - *Aspect ratio, Finite Element Method, Fluent, Galerkin method*

NOMENCLATURE

L Size of square enclosure filled with air, m
a Size of heated square cylinder, m
AR (In dimensionless form) Aspect Ratio (a/L)
Nu, Pr, Ra (In dimensionless form) Nusselt number, Prandtl number, Rayleigh Number (air)
Q (In dimensionless form) Heat flux of air
T Temperature of the wall, K
X, Y (In dimensionless form) Co-ordinate in x, y-direction

g Gravitational force of air, m/s^2
k Thermal conductivity of air, W/m.K
x, y Co-ordinate in x, y-direction, m
u, v Velocity of air constituents along x, y-direction, m/s
u^* v^* (In dimensionless form) Velocity of air constituents along x, y-direction

Greek Symbol

ν Kinematic viscosity of air, m^2/s
ρ Density of air, kg/m^3 (1.007)
α Thermal diffusivity of air, m^2/s
β Volumetric co-efficient of thermal expansion of air, K^{-1}
θ (In dimensionless form)Temperature of the wall
ω, ω^* Vorticity of enclosure(s^{-1}), (In dimensionless form) Vorticity of enclosure
ψ, ψ^* Stream function of enclosure (m^2/s), (In dimensionless form) Stream function

Subscripts

H Hot wall of heated square cylinder
C Cold wall of enclosure filled with air

1. INTRODUCTION

The curiosity of natural convection in enclosures filled with air having cold vertical walls and adiabatic horizontal walls has been theme of research over the past years. The natural convection issue is of concern in various engineering and technology uses such as solar energy collectors, cooling of electronics components etc. A. Dalal et al. [1] state that natural convection occur in the vicinity of tilted square cylinder in the range of ($0 \leq \theta \leq 45°$) inside an enclosure having horizontal adiabatic wall and cold vertical wall figure out by cell-centered finite volume method, which is used to reckoned two dimension Navier strokes equation for incompressible laminar flow. And taken the value of Rayleigh number is Ra= 10^5, Pr= 0.71. G. De Vahl Davis [2] expresses that differentially heated side walls of square cavity are figure out for precise solution of the equations. It has taken the Rayleigh numbers in the range of $10^3 \leq Ra \leq 10^6$. N. C. Markatos [3] deal with Buoyancy-driven laminar and turbulent flow is reckoned by computational method. It has taken the Rayleigh numbers in the range of $10^3 \leq Ra \leq 10^{16}$. Donor-cell differencing is applied for this problem. The Rayleigh numbers is more than 10^6 for turbulence model. Demirdzic et al. [5] state that out of four issue of heat transfer and fluid flow only two orthogonal, boundary-fitted grids are utilize. They have reckoned the issue of this work by using multigrid finite volume method. It has apply 320×320 grid for the control volume, a found the error approximately to be less than 0·1%. D. G. Roychowdhury et al. [6] deal with convective flow and heat transfer in the area of heated cylinder situated in square enclosure is reckoned by non-orthogonal grid based finite volume technique.

In the current paper, natural convection in air around heated square cylinders of different sizes inside an enclosure having adiabatic horizontal and cold vertical walls. The continuity, momentum and energy conservation equation are reckoned using finite element method. The impacts of aspect ratio at different Rayleigh number on the air flow pattern, temperature distribution and heat transfer have been debated.

1. PROBLEM STATEMENT

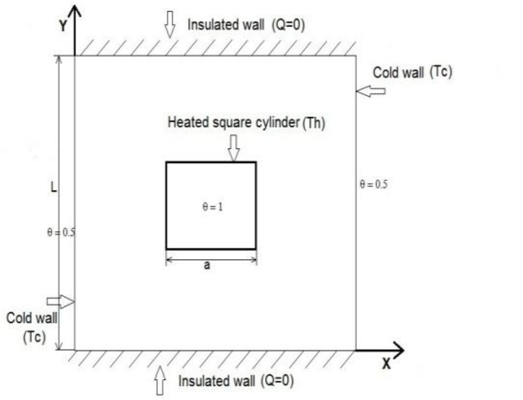


Figure1. Computational domain

1.1. MODEL

An air is filled in the square enclosure of size (L) is considered. The heated square cylinder (T_h) of different sizes (a) placed inside the centre of the enclosure filled with air with the aspect ratio of 0.2, 0.3, and 0.4. The left and right walls are cold wall (T_c) and top and bottom walls are adiabatic wall (Q=0). There is no slip condition (velocity) at all the walls.

1.2. GOVERNING EQUATIONS

The governing Navier-stroke's equation for steady laminar flow of a boussinesq air in stream function and vorticity form is given below

$$\nabla^2\psi = -\omega \qquad (1)$$

$$u\frac{\partial\omega}{\partial x} + v\frac{\partial\omega}{\partial y} = \nu\nabla^2\omega + g\,\beta\frac{\partial T}{\partial x} \qquad (2)$$

For constant air properties in the absence of internal heat generation and insignificant viscous dissipation, the conservation equation can be represented as,

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) \qquad (3)$$

The boundary conditions are

$\Psi = 0$, at all walls (4)

Boundary conditions in dimension form are

Left wall $T = T_C$, at $x = 0$; $0 \le y \le L$ (5)

Right wall $T = T_C$, at $x = L$; $0 \le y \le L$ (6)

Top and bottom wall $\frac{\partial T}{\partial y} = 0$; at $y = 0, L$; $0 \le x \le L$ (7)

Left solid wall $T = T_H$, at $x = \frac{L-a}{2}$; $\frac{L-a}{2} \le y \le \frac{L+a}{2}$ (8)

Right solid wall $T = T_H$, at $x = \frac{L+a}{2}$; $\frac{L-a}{2} \le y \le \frac{L+a}{2}$ (9)

Bottom solid wall $T = T_H$, at $y = \frac{L-a}{2}$; $\frac{L-a}{2} \le x \le \frac{L+a}{2}$ (10)

Top solid wall $T = T_H$, at $y = \frac{L+a}{2}$; $\frac{L-a}{2} \le x \le \frac{L+a}{2}$ (11)

The following parameters are used for representing the governing equations in dimension- less form.

$X = \frac{x}{L}$ if $x = \frac{L-a}{2}$ then $X = \frac{1-AR}{2}$ where, (AR = a/L)

$X = \frac{x}{L}$ if $x = \frac{L+a}{2}$ then $X = \frac{1+AR}{2}$

$Y = \frac{y}{L}$ if $y = \frac{L-a}{2}$ then $Y = \frac{1-AR}{2}$

$Y = \frac{y}{L}$ if $y = \frac{L+a}{2}$ then $Y = \frac{1+AR}{2}$

$\Psi^* = \frac{\psi}{\alpha}$, $\omega^* = \frac{\omega}{(\alpha/L^2)}$, $u^* = \frac{u}{(\alpha/L)}$,

$v^* = \frac{v}{(\alpha/L)}$, $\theta = \frac{T}{T_H}$

By using the above dimensionless quantities, the various governing equation are represented as,

$$\nabla^2\psi^* = -\omega^* \qquad (12)$$

$$u^*\frac{\partial\omega^*}{\partial X} + v\frac{\partial\omega^*}{\partial Y} = Pr\nabla^2\omega + Pr\,Ra\frac{\partial\theta}{\partial X} \qquad (13)$$

$$u^*\frac{\partial\theta}{\partial X} + v^*\frac{\partial\theta}{\partial Y} = \nabla^2\theta \qquad (14)$$

The boundary conditions for all walls are

$\Psi^* = 0$, at all walls (15)

Boundary condition in dimension-less form is

$\theta = 0.5$; at $X = 0$; $0 \le Y \le 1$ where, $\theta = \frac{T_C}{T_H}$ (16)

$\theta = 0.5$; at $X = 1$; $0 \le Y \le 1$ (17)

$\frac{\partial\theta}{\partial y} = 0$; at $Y = 0, 1$; $0 \le X \le 1$ (18)

$\theta = 1$; $X = \frac{1-AR}{2}$; $\frac{1-AR}{2} \le Y \le \frac{1+AR}{2}$ (19)

$\theta = 1$; $X = \frac{1+AR}{2}$; $\frac{1-AR}{2} \le Y \le \frac{1+AR}{2}$ (20)

$\theta = 1$; $Y = \frac{1-AR}{2}$; $\frac{1-AR}{2} \le X \le \frac{1+AR}{2}$ (21)

$\theta = 1$; $Y = \frac{1+AR}{2}$; $\frac{1-AR}{2} \le X \le \frac{1+AR}{2}$ (22)

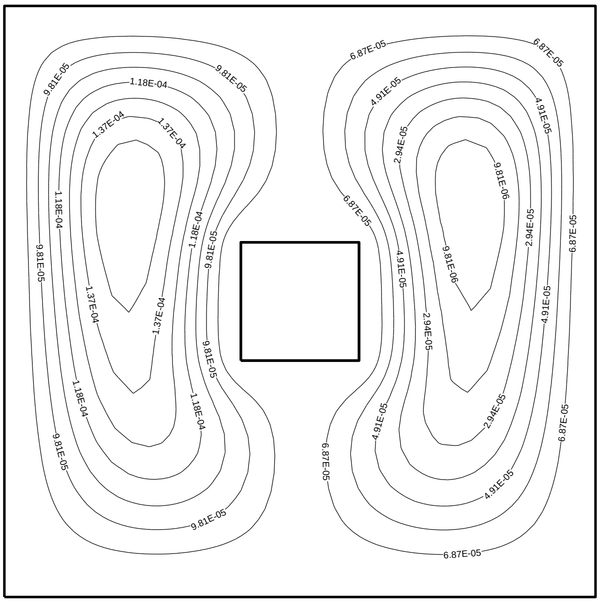
2. COMPUTATIONAL ANALYSIS

In the computational analysis first we have used standard boussinesq air properties for different length of enclosure (L) of 20mm, 40, and 80mm to reckon the value of Rayleigh number. we have depict our geometry according to given aspect ratio (AR) of 0.2, 0.3 and 0.4 at different Rayleigh numbers (Ra) of 1.93 $\times 10^4$, 1.54 $\times 10^5$ and 1.23 $\times 10^6$. Then we have done mesh and refinement at 3 in vicinity of heated square cylinder, and taken mesh size 100, 40 for enclosure walls and heated square cylinder walls respectively. After that we gave boundary condition 373.16 K for all heated square cylinder walls, 323.16K for both vertical diathermic walls of enclosure and heat flux = 0 for horizontal adiabatic walls of enclosure. Then we have made solution for this problem with respect to aspect ratio (AR) of 0.2, 0.3 and 0.4 at different Rayleigh numbers of 1.93 $\times 10^4$, 1.54 $\times 10^5$ and 1.23 $\times 10^6$ to analyze the heat transfer and flow.

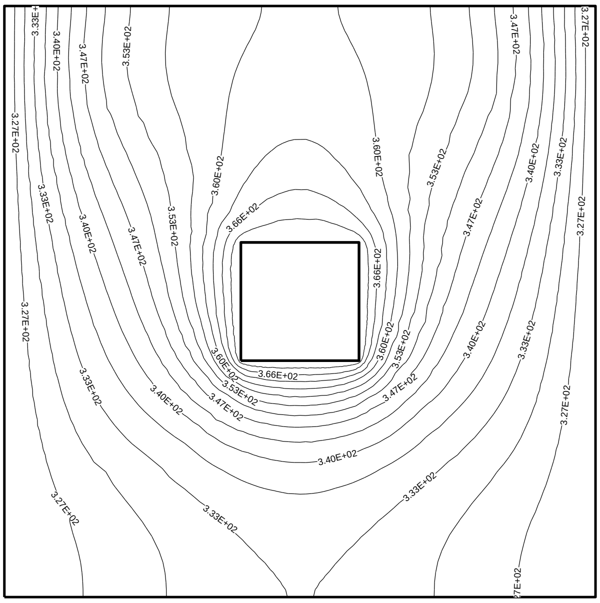
The Nusselt number of heated square cylinder walls and cold vertical walls are find out after reaches convergence by using following formula,

$$Nu_H = \frac{Q_H}{k\,(T_H - T_C)} = \frac{1}{AR\,(1-\theta_C)} \int_0^{AR} \left(\frac{\partial\theta}{\partial X}\right) \text{atX=0 dY} \quad (23)$$

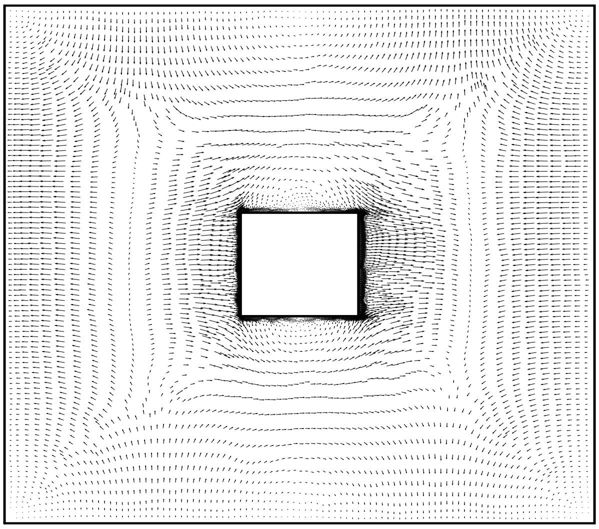
$$Nu_C = \frac{Q_C}{k\,(T_H - T_C)} = \frac{1}{AR\,(1-\theta_C)} \int_0^{AR} \left(\frac{\partial\theta}{\partial X}\right) \text{atX=1 dY} \quad (24)$$



(a)

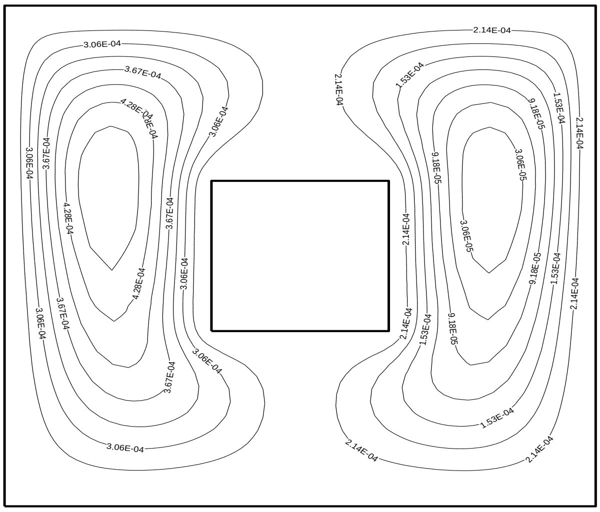


(b)

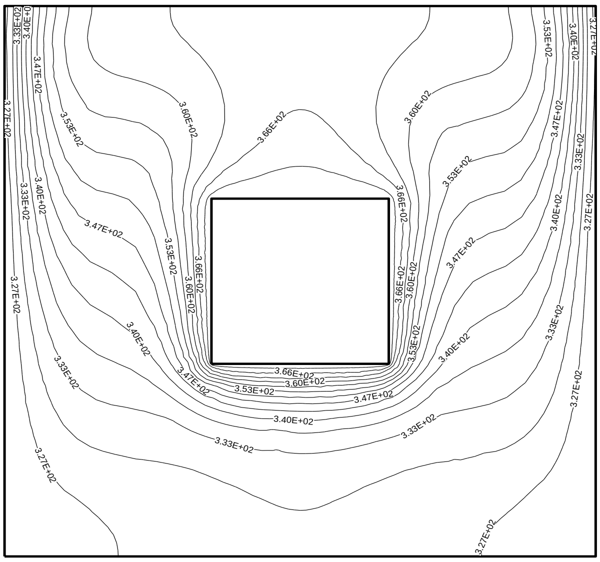


(c)

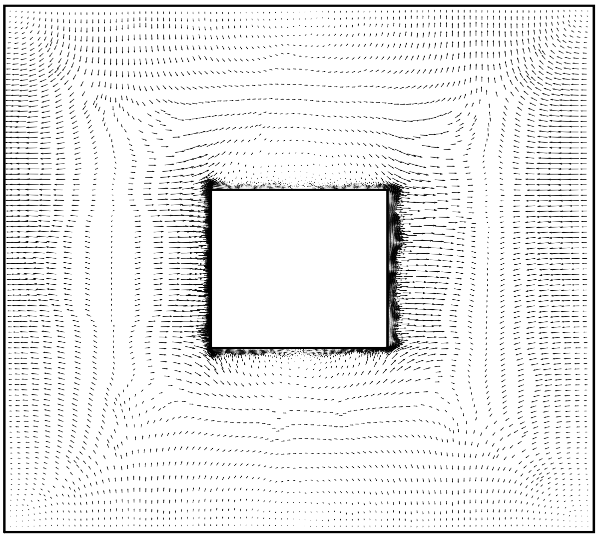
Figure2. Stream function, Isotherm and velocity vector diagram for Ra = 1.93* 10^4, AR = 0.2



(a)

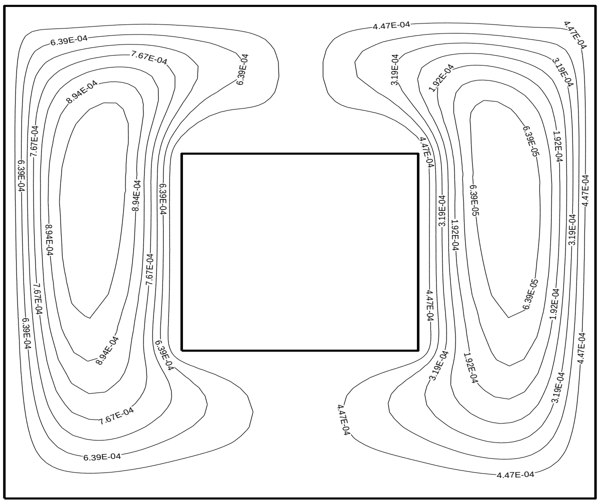


(b)

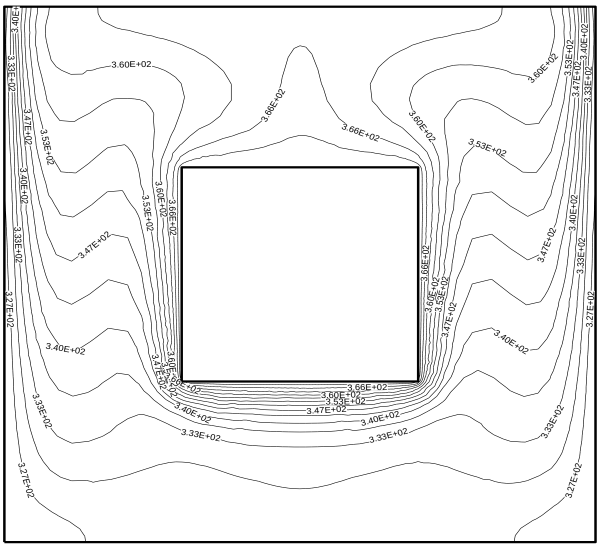


(c)

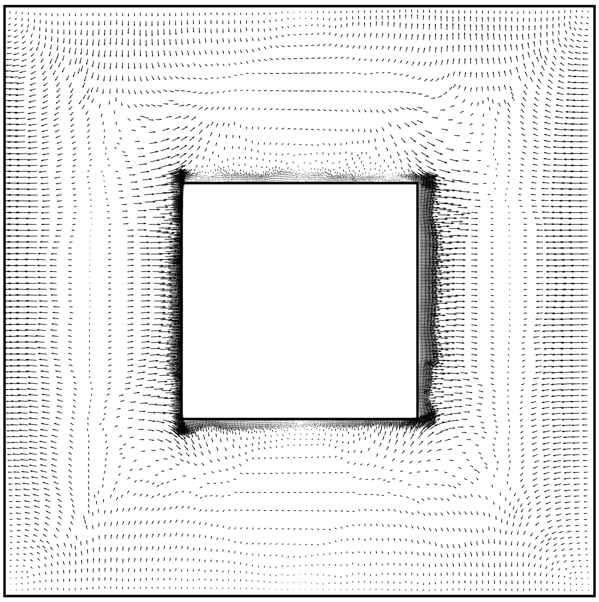
Figure3. Stream function, Isotherm and velocity vector diagram for Ra = 1.54* 10^5, AR = 0.3



(a)

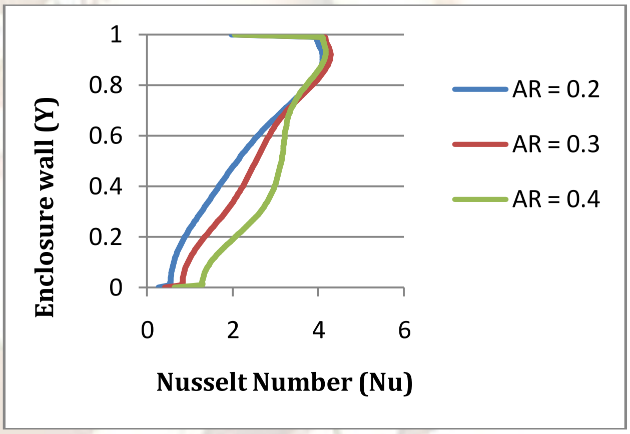


(b)

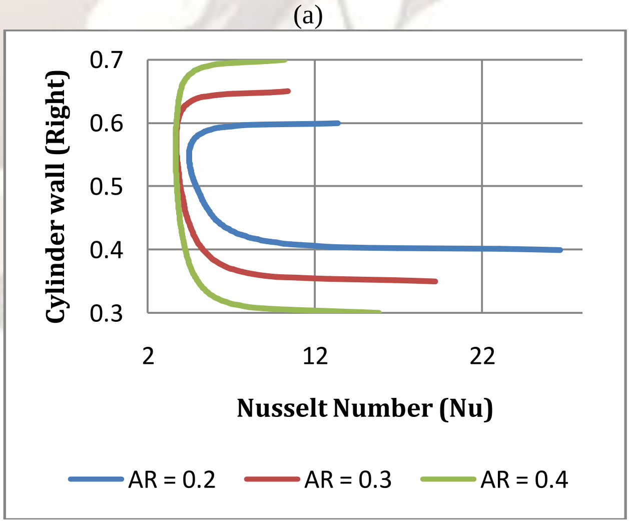


(c)

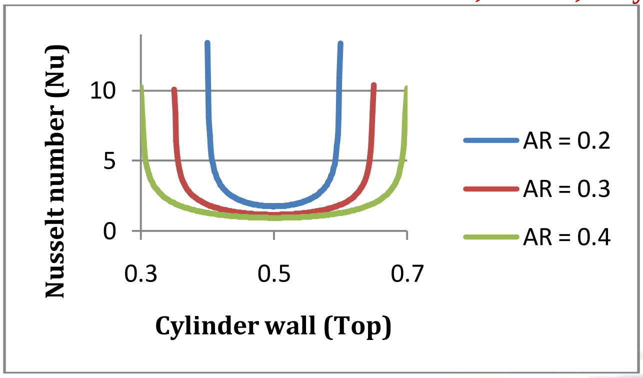
Figure4. Stream function, Isotherm and velocity vector diagram for Ra = 1.23* 10^6, AR = 0.4



(a)

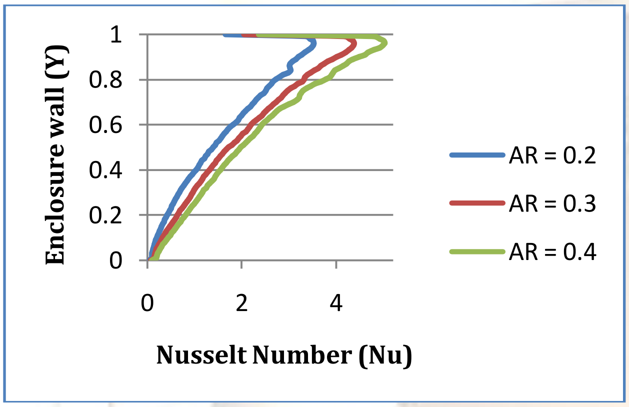


(b)

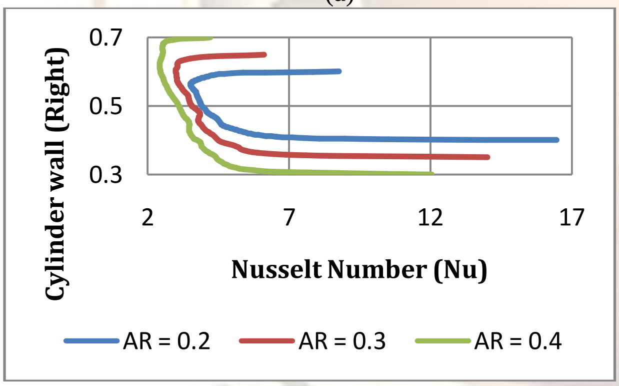


(c)

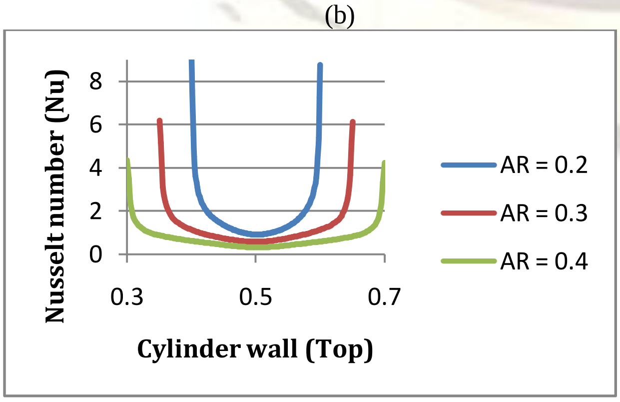
Figure5. Nusselt number variation along vertical walls of enclosure & Right and top walls of cylinder for Rayleigh number of 1.93×10^4



(a)



(b)



(c)

Figure6. Nusselt number variation along vertical walls of enclosure & Right and top walls of cylinder for Rayleigh number of 1.23×10^6

3. COMPUTER CODE VALIDATION

A common benchmark solution for natural convection in differentially heated square cavity with adiabatic horizontal walls and isothermal vertical walls with T= 1, for left wall and T= 0, for right wall [2]. In the present work, numerical predictions have been acquired for Rayleigh numbers of Ra = 10^4, 10^5. The maximum, minimum, and average nusselt number on hot wall for different Rayleigh numbers are compared in table 1.

Table1. Comparison of solution for natural convection in square cavity

Ra = 10^4

	a	b	$\frac{a-b}{a}\times100$
Nu_{max}	3.528	3.455	2.06
Nu_{min}	0.586	0.567	3.24
Nu	2.243	2.132	4.94

Ra = 10^5

	a	b	$\frac{a-b}{a}\times100$
Nu_{max}	7.117	7.215	-1.37
Nu_{min}	0.729	0.756	-3.70
Nu	4.519	4.417	2.25

a, solution of de Vahl Davis [2]; b, present solution.

4. RESULT AND ANALYSIS

The analysis has been taken place of heat transfer and flow due to natural convection in air around heated square cylinders of different sizes inside an enclosure having adiabatic horizontal and diathermic vertical walls. We have taken size of enclosure constant in range of 20mm, 40mm, and 80mm with respect to aspect ratio of 0.2, 0.3, and 0.4 to reckon the value of Rayleigh number for the constant air properties. Thus we obtain the Rayleigh numbers for that size enclosure is 1.93×10^4, 1.54×10^5 and 1.23×10^6. Then we have figure out the appropriate boundary condition for this problem which are represented above. After that we analyze this problem to acquire the outcome in dimension form of stream function, Isotherm and velocity vector diagram which are represented in fig. 2, 3and 4.

Table.2 Maximum velocity along X and Y direction & maximum stream function for Rayleigh number of 1.93×10^4, 1.54×10^5 and 1.23×10^6 with respect to aspect ratio 0.2, 0.3 and 0.4 (In dimension-less form)

Ra	AR	u_{max}	v_{max}	Ψ_{max}
1.93×10^4	0.2	13.57	20.75	5.49
	0.3	11.93	18.04	4.22
	0.4	9.68	13.93	2.82
1.54×10^5	0.2	44.59	76.81	17.99
	0.3	41.12	77.26	17.13
	0.4	42.07	74.73	14.80
1.23×10^6	0.2	131.16	205.65	32.31
	0.3	124.33	216.26	32.46
	0.4	120.08	217.52	35.79

4.1. STREAM FUNCTION, ISOTHERM AND VELOCITY VECTOR DIAGRAM

We are estimating that how heat transfer and flow occur inside an enclosure filled with air. This result has been exhibiting in dimension form of stream function Isotherm and velocity vector diagram. Figure 2, 3and 4. (a) Shows Stream lines increases with increase of Rayleigh number but decrease with rise of aspect ratio. Isotherm's curvature increases with increases of aspect ratio at different Rayleigh number. And velocity of fluid flow inside a computational domain rise with increase of Rayleigh number for aspect ratio of 0.2, 0.3, and 0.4 shown in fig. 2, 3and 4. (c). Table.2 shows that velocity along X and Y direction and stream line increasing by rising the value of Rayleigh number.

4.2. NUSSELT NUMBER

We have represented Nusselt number (Nu) in the dimension-less form of graph for the aspect ratio (AR) of 0.2, 0.3 and 0.4 at different Rayleigh numbers of 1.93×10^4 and 1.23×10^6. The figure 5, 6 shows that Nusselt number versus enclosure wall and cylinder walls (Right & Top) in dimension–less form, and indicates that Nusselt number dependent of temperature. Because by increasing sizes of heated square cylinder it gets closer to diathermic walls of enclosure and transfer more heat to that walls, so that the Nusselt number (Nu) increases with increase of aspect but decrease with increases of Rayleigh number for enclosure walls. While the nusselt number decreases with increase of aspect ratio as well as Rayleigh number for heated square cylinder walls. Hence it's proved that heated square cylinder can absorb more energy by increasing aspect ratio and transfer more heat to diathermic walls.

CONCLUSION

Heat transfer and fluid flow due to natural convection in air around heated square cylinders of different sizes inside an enclosure having adiabatic horizontal and diathermic vertical walls of size 20mm, 40mm, and 80mm with respect to aspect ratio of 0.2, 0.3, and 0.4 at different Rayleigh numbers of 1.93×10^4, 1.54×10^5 and 1.23×10^6 are analyzed, and results are exhibited in the dimension form of Isotherm, Stream function and Velocity vector diagram.

It's proved that Nusselt number dependent of temperature and heated square cylinder can absorb more energy by increasing the size and can transfer more heat to the diathermic walls for various purposes.

REFERENCES

[1] A.Dalal, V. Eswaran, G. Biswas, "Natural convection around a heated square cylinder placed in different angles inside an enclosure," Heat and mass transfer conference, 2008, Paper No.45.

[2] G. de Vahl Davis, "Natural convection of air in a square cavity: A benchmark numerical solution," International journal for numerical methods in fluids, 3, 1983, pp. 249-264.

[3] N. C. Markatos, and K. A. Perikleous, "Laminar and turbulent natural convection in an enclosed cavity," International journal of heat and mass transfer, 27 (5), 1984, pp. 755-772.

[4] S. Ostrach, "Natural convection in enclosures," Journal of heat transfer 50th anniversary issue, 110 (4-B), 1988, pp. 1175-1190.

[5] I. Demirdzic, Z. Lilek, M. Peric, "Fluid flow and heat transfer test problems for non- orthogonal grids: Bench-mark solutions," International journal for numerical methods in fluids, 15, 1992, pp. 329-354.

[6] D. G. Roychowdhury, S. K. Das, T. S. Sundararajan, " Numerical simulation of natural convective heat transfer and fluid flow around a heated cylinder inside an enclosure," Heat and mass transfer, 38, 2002, pp. 565-576.

[7] V. Eswaran, S. Prakash, "A finite volume method for navier stroke equations," Proceeding of third Asian CFD conference, Bangalore, India, 1998, PP. 127-136.

[8] A. K. De, A.Dalal, "A numerical study of natural convection around a square horizontal heated cylinder placed in an enclosure," International journal of heat and mass transfer, 49, 2006, pp. 4608-4623.

[9] P. H. Oosthuizen, "Free convective flow in an enclosure with a cooled inclined upper surface," Computational mechanics 14, 1994, pp. 420-430

[10] K. B. Sahu, S. K. Mahapatra, S. Sen, A. Sarkar, "Natural convection in differentially heated rectangular cavities of various aspect ratios," Heat and mass transfer conference, 2008, Paper No. NCH-1.