Shape Control And Optimization Using Cantilever Beam

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Abstract

In this study, Analytical work is carried out for static shape control of cantilever beam structure with a use of laminated piezoelectric actuator (LPA). The mathematical modeling of beam element covered with LPA based on Timoshenko beam element theory and linear theory of piezoelectricity has been used. This work shows how number of actuators, actuator size, actuator location on beam and control voltage are depended on the desired shape of beam. Initial condition of beam is taken as horizontal position and three higher order polynomial curves are taken as desired shapes for beam to achieve. Here error between desired shape and achieved shape is taken as an objective to minimize, size, location and control voltage of actuators taken as variables. Genetic Algorithm for calculating optimum values of all variables is carried out using Matlab tool

Keywords – Shape control, Genetic Optimization, Cantilever beam,

1. Introduction

Now a day due to requirement of highly precise structure works considerable attention has been carried out behind shape control of structure. In some application areas like reflecting surfaces of space antennas, aerodynamic surfaces of aircraft wings, hydrodynamic surfaces of submarines and ships where highly precise surfaces required. Recently, researcher giving considerable attention on developing advanced structures having integrated control and self monitoring capability. Such structures are called smart structure. Using direct and converse effect of piezoelectric materials for sensing and control of structures, a considerable analytical and computational modeling works for smart structures have been reported in the review paper of Sunar and Rao [10], most of the past work carried out for the control of vibration characteristics of structures, but on shape control side fewer work is carried out. A review paper by H. Irschik [4] on static and dynamic shape control of structure by piezoelectric actuation show work carried out for shape control and relevant application of shape control. Agrawal and Trensor [1] show in his work analytical and experimental results for optimal location for Piezoceramic actuators on beam here size of Piezoceramic actuators are not varied only optimization carried out for optimal location of actuators and the desired shape of the curve is taken as parabola which is a second order polynomial curve. One more conference report is made available by Rui Ribeiro and Suzana Da Mota Silva [8] for the optimal design and control of adaptive structure using genetic optimization algorithm. They have been carried shape control for beam in two different boundary conditions like clamped - free and clamped – clamped on both the end of beam and carried optimization for location of piezo electric actuators and sensors. Also in paper on the application of genetic algorithm for shape control with piezo electric patches and also show comparison with experimental data by S daMota Silva and R Ribeiro [9]. Paper presented by author Osama J. Aldraihem [7] shows analytical results for optimal size and location of piezoelectric actuator on beam for various boundary conditions of beam at here desired shape of beam is considered as horizontal and single pair of actuator is used for his work. In the paper of E.P. Hadjigeorgiou, G.E. Stavroulakis, C.V. Massalas [3] work carried out for shape control of beam using piezo electric actuator. Here all the mathematical modeling is based on the Timoshenko beam theory. They have been investigated as placement of actuator near the fixed end of beam And optimization carried out for the optimal voltage of actuators. If control voltage is goes higher then actuators get damaged so for minimization of control voltage and increase actuation force of actuator, layers of laminated piezoelectric actuators (LPA) are mounted one over another so actuation energy of patches are added and with lower voltage we can get higher actuation force. This concept is used by author Y Yu, X N Zhang and S L Xie [11] in this work carried out for shape control of cantilever beam and optimization done for optimum control voltage.

From all above work carried out earlier by different authors are limited to vary only position of actuators on the beam. Here in this work number of actuators, size and location of actuators are going to be varied, and also three types of desired shape of beam having higher order rather than parabolic shape is to be considered to control.

2. Mathematical Modeling of Beam

2.1 governing equation of beam

Mathematical modeling carried out based on Timoshenko beam element theory and linear theory of piezoelectricity.

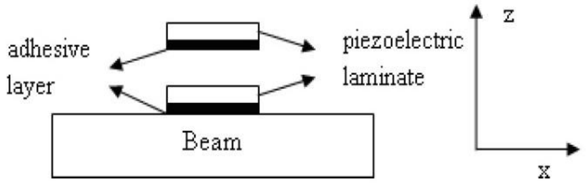

Figure 1. Beam with surface bonded LPA [11]

Considering Cartesian coordinate system as shown in Fig. 1, here analysis is restricted in x-z plane only so displacements in all three directions (using Timoshenko beam theory) are as below [3]. Where ω is transverse displacement of point on centroidal axis and ψ is the rotation of beam cross section about positive y axis.

$$u_1(x,y,z,t) \approx z\psi(x,t), \tag{1}$$

$$u_2(x,y,z,t) \approx 0\,, \tag{2}$$

$$u_3(x,y,z,t) \approx \omega(x,t), \tag{3}$$

Nonzero strain component of beam using above equations are as below:[3]

$$\varepsilon_x = z\frac{\partial\psi}{\partial x}, \qquad \gamma_{xz} = \psi + \frac{\partial\omega}{\partial x}, \tag{4}$$

The linear piezoelectric coupling between elastic field and electric field – no thermal effect is considered are as following [5].

$$\{\sigma\} = [Q]\{\varepsilon\} - [e]^T\{E\}, \tag{5}$$

$$\{D\} = [e]\{\varepsilon\} + [\xi]\{E\}, \tag{6}$$

The equation of laminated beam is derived with the use of Hamilton principle as below.

$$\delta\int_{t_1}^{t_2}(H - W_e)dt = 0 \tag{7}$$

Where (.) denote first variation operator, T kinetic energy of beam, H enthalpy of beam with laminated piezoelectric actuator and W_e external work done.
Now electric enthalpy of beam [5,6] by using above equations form (4) to (6) as

$$H = \frac{1}{2}\int_{V_b}\{\varepsilon\}^T\{\sigma\}dV + \frac{1}{2}\int_{V_p}(\{\varepsilon\}^T\{\sigma\} - \{E\}^T\{D\})dV$$

$$= \frac{1}{2}\int_{V_b}\{\varepsilon\}^T[Q]\{\varepsilon\}dV + \frac{1}{2}\int_{V_p}(\{\varepsilon\}^T[Q]\{\varepsilon\} - \{\varepsilon\}^T[e]^T\{E\} - \{E\}^T[e]\{\varepsilon\} - \{E\}^T[\xi]\{E\})dV$$

$$= \int_0^{L_e}\left[\frac{1}{2}(EI)\left(\frac{\partial\psi}{\partial x}\right)^2 + \frac{1}{2}(GA)\left(\psi + \frac{\partial\omega}{\partial x}\right)^2 - M^{el}\left(\frac{\partial\psi}{\partial x}\right) - Q^{el}\left(\psi + \frac{\partial\omega}{\partial x}\right)\right]dx - \frac{1}{2}\int_0^{L_e}\int_{s_p}(E_x^2\xi_{11} + E_z^2\xi_{33})dsdx \tag{8}$$

Where,

$$(EI) = \int_{s_b} z^2 Q_{11}ds + \int_{s_p} z^2 Q_{11p}ds\,,$$

$$(GA) = k\left(\int_{s_b} Q_{55}ds + \int_{s_p} Q_{55p}ds\right)$$

$$M^{el} = \int_{s_p} z e_{31}E_z ds,$$

$$Q^{el} = \int_{s_p} e_{15}E_x ds = 0,$$

Here $Q^{el}=0$ because electric field intensity in x direction is neglected and also value of k is taken as 5/6 [3].
Finally the work of external force is given by

$$W_e = \int_0^{L_e}(q\omega + m\psi)dx, \tag{9}$$

2.2 Finite element formulation of beam

Considering a beam element of length L_e having two degree of freedom per node one in transverse direction W_1 (or W_2) and other is rotation degree of freedom ψ_1 (or ψ_2). As shown in Fig 2.

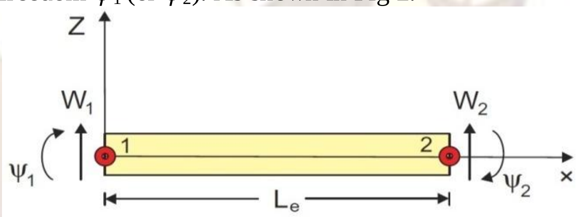

Figure 2. Beam element

The array of nodal displacement is defined as

$$\{\boldsymbol{X}^e\} = [\omega_1 \quad \psi_1 \quad \omega_2 \quad \psi_2]^T \tag{10}$$

On the basis of Timoshenko beam element theory the cubic and quadratic Lagrangian polynomial are used for transverse and rotation displacement where the polynomials are made interdependent by requiring them to satisfy the two homogeneous differential equations associated with Timoshenko's beam theory. The displacement and rotation of beam element can be expressed as

$$\begin{Bmatrix}\omega\\ \psi\end{Bmatrix} = \begin{bmatrix}[N_\omega]\\ [N_\psi]\end{bmatrix}\{\boldsymbol{X}^e\} \tag{11}$$

$$\begin{Bmatrix}\varepsilon_x\\ \gamma_{xz}\end{Bmatrix} = \begin{bmatrix}0 & z\frac{\partial}{\partial x}\\ \frac{\partial}{\partial x} & 1\end{bmatrix}\begin{Bmatrix}\omega\\ \psi\end{Bmatrix} = \begin{bmatrix}[B_u]\\ [B_\psi]\end{bmatrix}\{\boldsymbol{X}^e\} \tag{12}$$

Where, $[N_\omega]$ and $[N\psi]$ are cubic and quadratic shape functions of tiemoshenko beam element [11].
Electric potential inside element at any arbitrary position as

$$V = \left[1 - \frac{x}{L_e} \quad \frac{x}{L_e}\right] [V_1 \quad V_2]^T = \{N_V\}\{V_e\} \qquad (13)$$

And also electric field intensity E_z can be expressed as

$$E_z = -\frac{1}{nt_p}\left[1 - \frac{x}{L_e} \quad \frac{x}{L_e}\right] [V_1 \quad V_2]^T = [B_V]\{V_e\} \qquad (14)$$

Substituting equations (10) – (14) in equations (8) and (9) and then substituting them in to equation (7) the equation for shape control is obtained as bellow [11].

$$[K^e]\{\boldsymbol{X}\}^e = \{F\}^e + \{F_{el}\}^e \qquad (15)$$

$$[K^e] = [K^e_{uu}] + [K^e_{uV}][K^e_{VV}]^{-1}[K^e_{uV}]^T \qquad (16)$$

Where all matrices can be express as bellow

$$[K^e_{uu}] = \int_0^{L_e} \left[\frac{\partial [N_\psi]}{\partial x} \quad [N_\psi] + \frac{\partial [N_\omega]}{\partial x}\right] \times \begin{bmatrix} EI & 0 \\ 0 & GA \end{bmatrix} \times \begin{bmatrix} \frac{\partial [N_\psi]}{\partial x} \\ [N_\psi] + \frac{\partial [N_\omega]}{\partial x} \end{bmatrix} dx, \qquad (17)$$

$$[K^e_{uV}] = \int_0^{L_e} \int_{sp} [B_u]^T e_{31} [B_V] ds dx, \qquad (18)$$

$$[K^e_{VV}] = \int_0^{L_e} \int_{sp} [B_V]^T \xi_{33} [B_V] ds dx, \qquad (19)$$

To obtain stiffness matrix of beam without coverage of laminated piezoelectric actuator put t_p=0 in equation of $[K^e_{uu}]$.
And the external force matrix and electric force matrix are as follow,

$$\{F\}^e = \int_0^{L_e} [[N_\omega] \quad [N_\psi]] \begin{bmatrix} q \\ m \end{bmatrix} dx, \qquad (20)$$

$$\{F_{el}\}^e = \int_0^{L_e} \left[\frac{\partial [N_\psi]}{\partial x} \quad [N_\psi] + \frac{\partial [N_\omega]}{\partial x}\right] \begin{bmatrix} M^{el} \\ Q^{el} \end{bmatrix} dx, \qquad (21)$$

Table 1. Material properties of base beam and LPA

Properties	Symbol	LPA material PZT G1195N	Graphite epoxy composite material T300/976
Young modulus (GPa)	E_{11}	63	150
Poisson Ratio	ν_{12}	0.3	0.3
Shear modulus (GPa)	G_{12}	24.2	7.1
Density (Kg/m^3)	ρ	7600	1600
Piezoelectric constant (C/m^2)	e_{13}	17.584	
Electric permittivity (F/m)	ξ_{13}	15.3 x 10^{-9}	
	ξ_{15}	15.0 x 10^{-9}	

3. Shape control and optimization using genetic algorithm

Here, study carried out for static shape control, so transverse displacement vector X is time independent, in above equation (15) there is not any time dependent vector so it is directly used in static shape control problem. With fixed time independent values of electric voltage at various location of actuator one can control shape of beam.

3.1 Material properties of composite cantilever beam and Piezoceramic actuator

Material selected for cantilever beam is graphite epoxy composite because having lower density and lower thermal coefficient of expansion, which will minimize deflection due to temperature rise and also due to self weight, and material selected for actuator is PZT G1195N, properties of both the material are specified in Table 1.[3]

Here size of beam is considered as 300 mm long, 40 mm width and 9.6 mm thick, also width and thickness of LPA layer is considered as 40 mm and 0.2 mm respectively. Here length of LPA is going to be varied.

3.2 Shape control

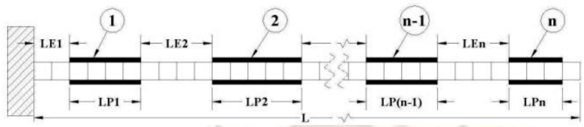

Figure 3. Cantilever beam with surface bonded LPA

Layouts of beam with laminated piezoelectric actuators (LPA) are as shown in below Fig 3. As shown in below figure beam divided in to 30 finite elements having 10 mm elemental length. And one LPA covers several elements of beam, equal amplitude voltage with an opposite sign are provided to upper and lower LPA, in generally says for upward displacement of beam upper LPA need negative voltage and lower LPA need positive voltage.

For shape control work initial condition of beam is taken as horizontal beam. Here no other effect is considered like gravity or thermal expansion of beam. And for desired shape of beam is take as three higher order polynomial curves having different curvatures as shown in figure 5.

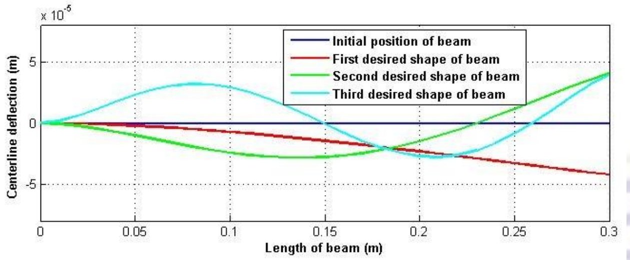

Figure 4. Desired shape of beam

Here error between desired shape and achieved shape is to be considered as an objective function to minimize and in this study number, size, location and Voltage provided to the LPA are considered as design variables. The error function used in this study as under[8].

$$Error = \sqrt{\sum_{i=1}^{n} (\gamma_i - q_i)^2} \quad (22)$$

Where γ_i is the pre-defined displacement at the ith node and q_i the achieved displacement.

3.3.1 Optimization using Genetic algorithm

For varying no of actuators on the beam here optimization is carried out in four cases as bellow table.

Table 2. Design variable for different cases.

Case	Description
1	Using single LPA
2	Using two LPA's
3	Using three LPA's
4	Using four LPA's

For Genetic algorithm population is set as 100 for all three cases and max. Number of generation is allowed as 1000.Also algorithm having stochastic in nature. Here crossover function is selected as heuristic in nature. Upper Control limit for voltage is specified as 400 V and lower limit is specified as 400 V in reverse polarity (for reverse polarity voltage shown as minus (-) sign), minimum length of LPA is considered as 30 mm because as length decreased control force (Actuation force) of LPA on beam is also decreased. And maximum length provided as full length of beam, for empty length upper limit is considered as full length of beam and lower limit is considered as zero. Constraint equation is become as the summation of all Actuators length and Empty length of beam is less than or equal to 300 mm.

3.3.2 Optimization for first desired shape of beam.

Results obtained after optimization for first desired shape are shown in bar chart as bellow. Here voltage shown in negative direction means it is in reverse polarity.

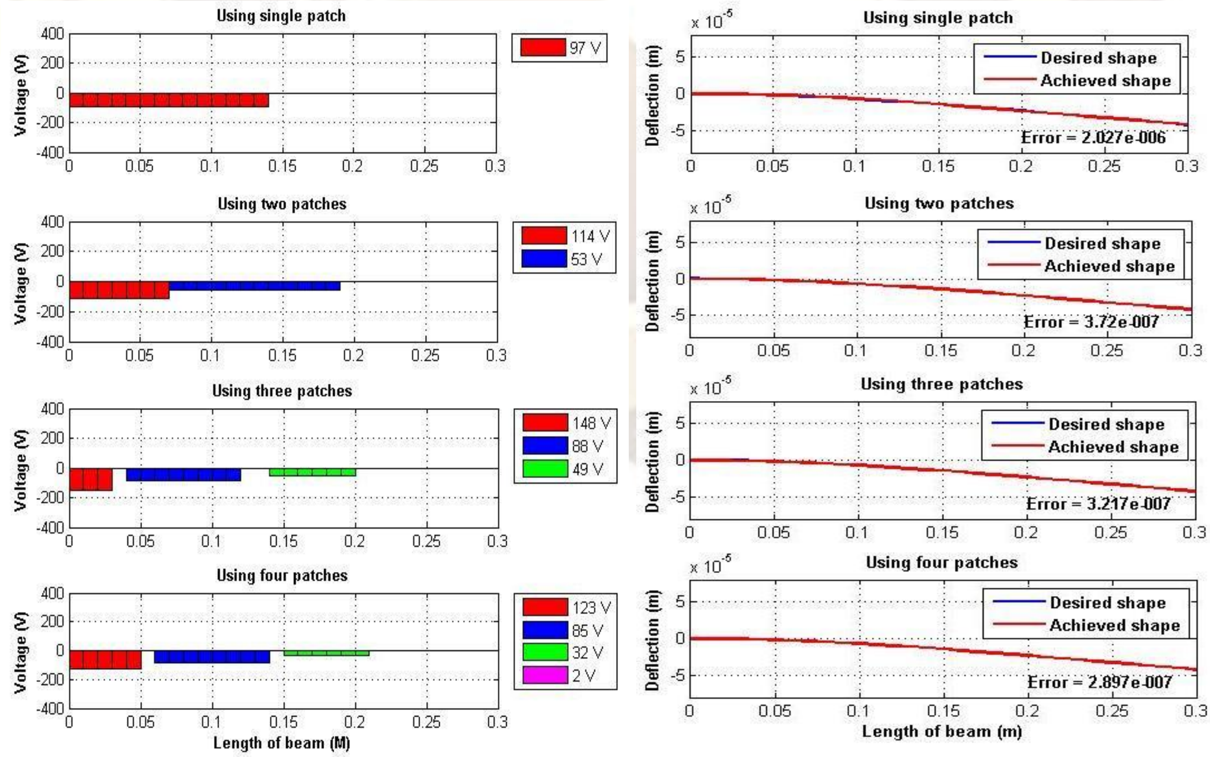

Figure 5. Optimization result for first shape

From above results we can say that To achieving such desired shape one LPA is enough for optimal control of shape, further more if number of actuators increased we can obtain better fitness means better control of shape, also it is required to place actuators at higher strain area on beam, in this case higher strain area is near the fixed end of beam.

3.3.3 Optimization for second desired shape of beam

Results obtained after optimization for second type of desired shape are as under.

For optimal control of such kind of desired shape we need minimum 2 numbers of actuators, from above results to increase number of actuators for control of such a kind of shape is meaningless, also there is two strain concentrated area in desired shape of beam one at fixed end and other at the middle position were beam is getting to deflect form lower to upper position. Here form above results we can also say that it is necessary to cover strain concentrated area on beam by actuator for optimally control the shape.

3.3.4 Optimization for third desired shape of beam.

Results obtained after optimization for third type of desired shape as following.

From above result we can say that it is not possible to achieve third kind of shape using two LPA, also further using three and four LPA and obtained results as show in below figure 10.

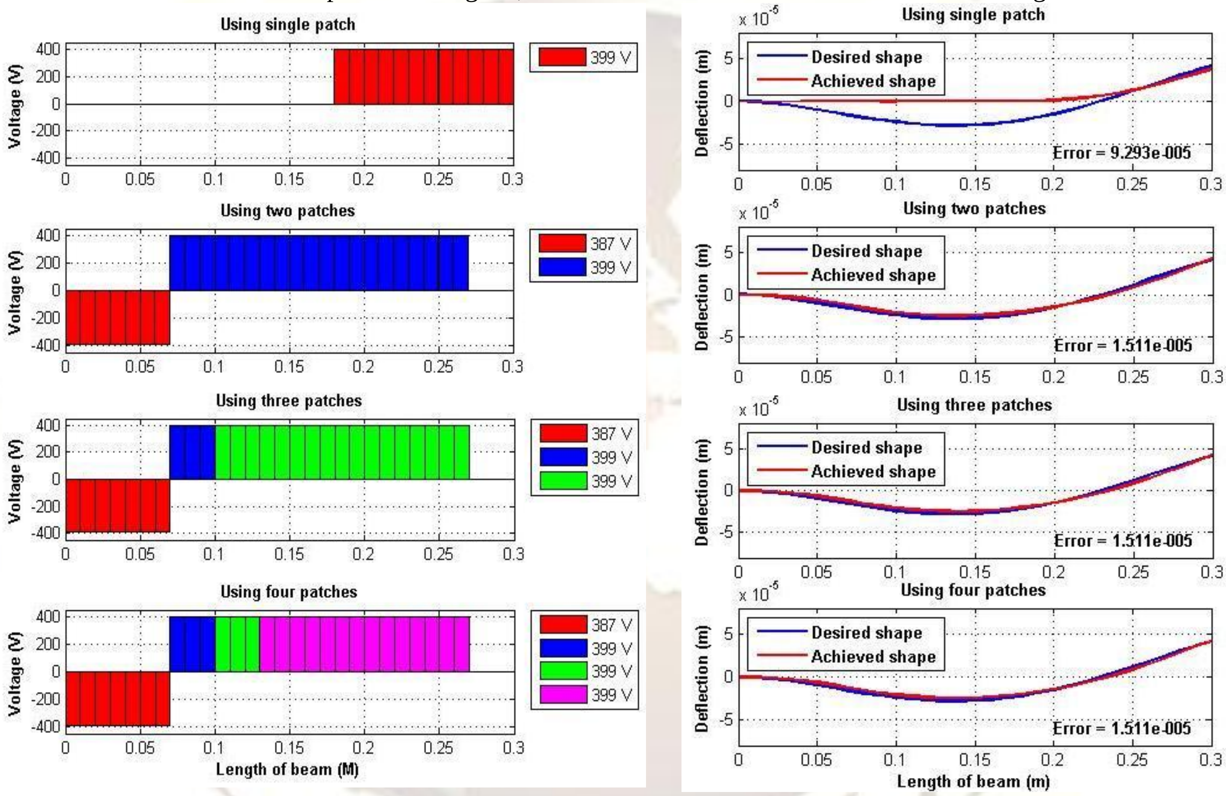

Figure 6. Optimization result for second shape

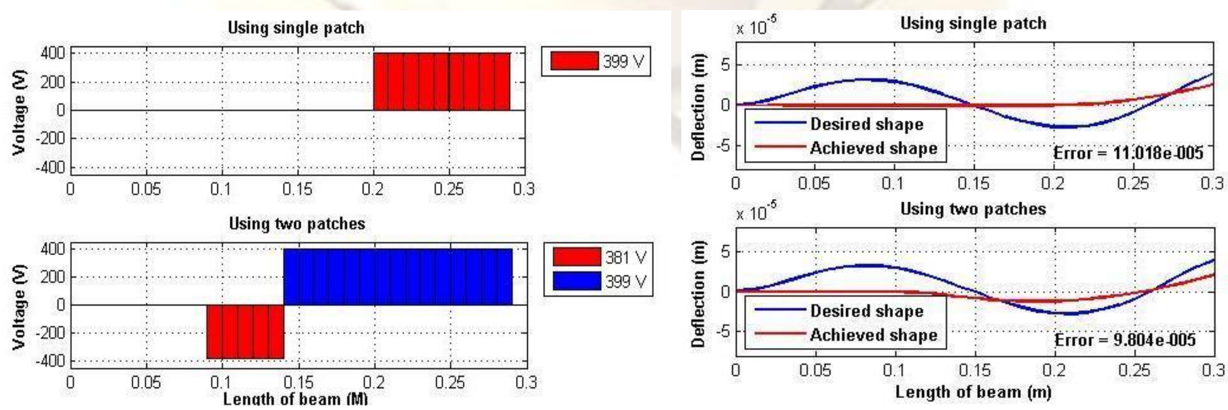

Figure 7. Optimization result using one and two patches for third shape

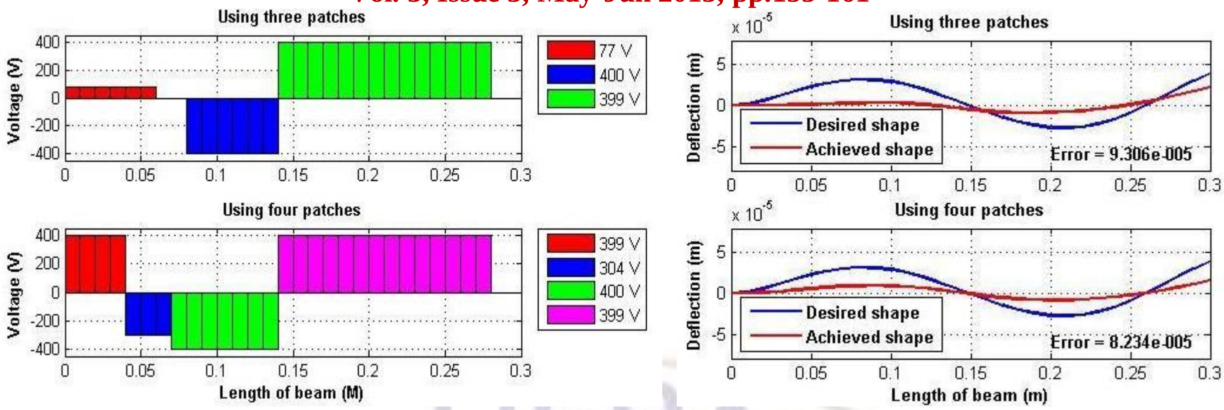

Figure 8. Optimization result using three and four patches for third shape

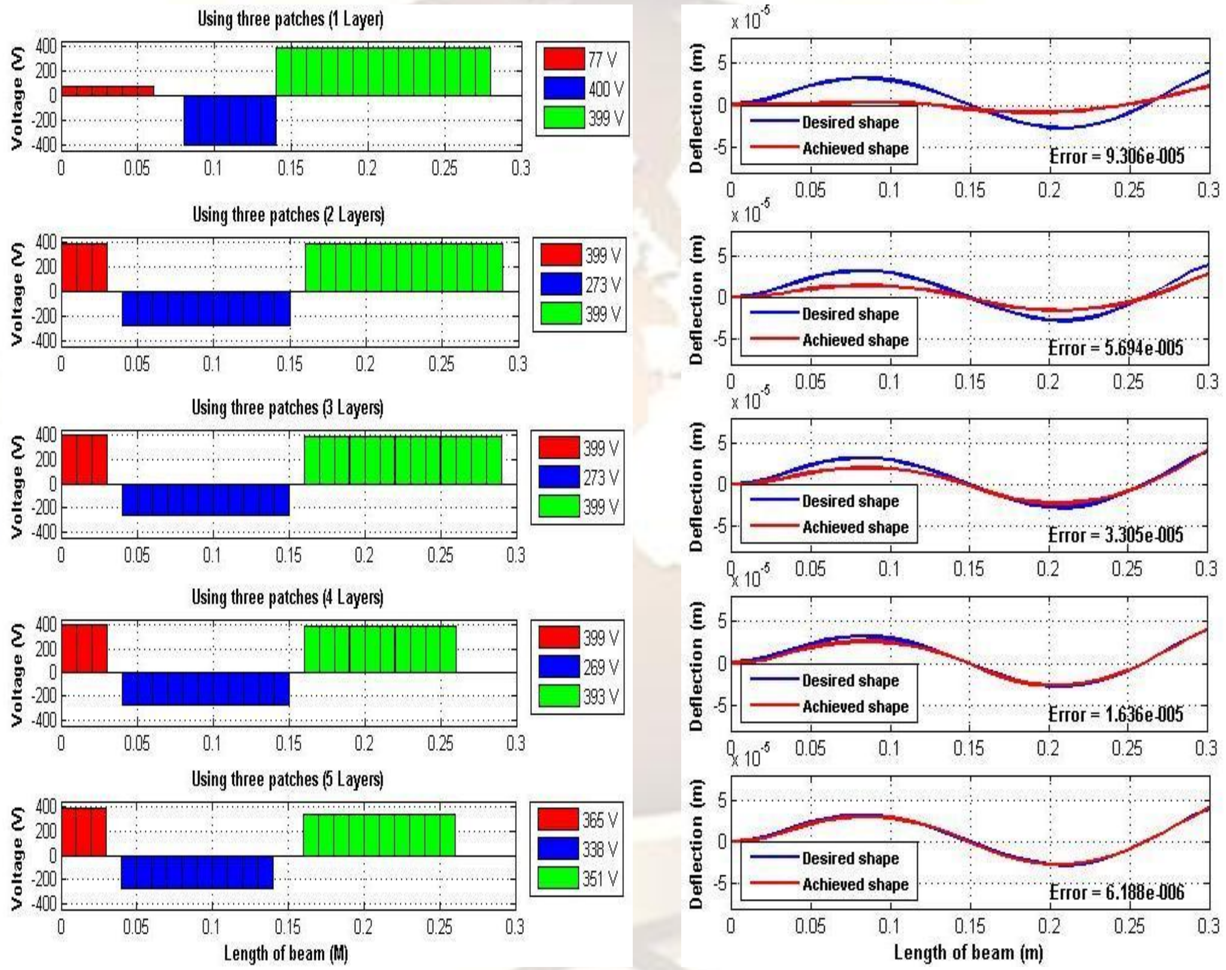

Figure 9. Optimization result using three LPA (Varying number of layers) for third curve

In this third type of shape for cantilever beam there are three strain concentrated area on the beam at various location. From the above results better control of shape obtained as compared to use of one and two LPA, but voltage limit reached for LPA so for better control of shape here multiple layers of LPA on one over another is used, further optimization being carried out for three LPA having multiple layers. And results obtained from that are as below shown in figure 11.

For Optimal shape control of third kind three LPA are enough, there is no need to increase LPA furthermore, there are three strain concentrated area on the beam, also from above charts there are two areas on beam where strain concentration is zero and so there is no need to place LPA over there.

4 Conclusion

Here mathematical modeling of cantilever beam based on Timoshenko beam element theory. To minimize control cost of any structure it is required to optimize voltage, location and size of piezoelectric patch on the structure. Optimization is carried out for the error minimization between desired and actual shape with the use of genetic optimization algorithm for different three types of higher order polynomial curves. Here number of actuators, voltage applied to the actuator, size of actuator and location of actuator on beam carried as a variable.

- Number of actuators, size of actuators, location of actuators and control voltage provided to the actuators are depending on the curvature of desired shape of beam.
- For optimum shape control, Minimum number of actuator required to control the shape of beam is greater than or equal to number of strain concentrated area formed during achieving desired shape. As number of actuators increased we get better shape control. But if number of actuators less than required then shape control not possible.
- For better shape control it is desired to cover higher strain concentrated area by actuators and there is no need to cover that portion of the beam where strain concentration is remain zero while achieving desired shape.
- In case of cantilever beam strain concentrated area is near the fixed end of beam so it is essential to cover fixed end of beam with actuator for better shape control. There is no need to place actuators at the free end side of beam.

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