

## Pde-Based Disparity Estimation From Stereo Images

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### Abstract:

Partial differential equations (PDEs) are very useful now days in image processing and computer vision. They are mainly used for smoothing and restoration purpose. PDEs are also very useful in areas such as physics and engineering sciences for a very long time. This paper gives a broad picture of mathematical image processing through one of the most recent and very successful approaches—the variational PDE (partial differential equation) method.<sup>[1]</sup> This paper discusses the basic concepts of PDE based disparity estimation for stereo images and its application in Image processing. In this paper we used PDE for measure the regularization using Euler's function.

**Index Terms**—Disparity estimation, Variational method, Partial differential Equation, Surface reconstruction

### I. Introduction

Disparity estimation is very useful technique in image-based 3D reconstruction. To apply PDE models in image processing efficiently it is important to merge the two fields, namely, numerical analysis of PDE models and image processing techniques. PDE based Disparity Estimation is required for the accurate depth recovery of image, which can solve the problems occur during the multiple image matching. Many evolution equations for restoring images can be derived as gradient descent methods for minimizing a suitable energy functional, and the restored image is given by the steady-state of this process. Typical PDE techniques for image smoothing regard the original image as initial state of a parabolic (diffusion-like) process, and extract filtered versions from its temporal evolution. The whole evolution can be regarded as a so-called scale-space, an embedding of the original image into a family of subsequently simpler, more global representations of it. Since this introduces a hierarchy into the image structures, one can use a scale-space representation for extracting semantically important information. One of the main challenges in stereo matching is to handle the disparity estimation in occluded areas. Since occluded areas are only visible in one image of a stereo image pair, there is no sufficient information

to find the correspondence between the two input images. Our work on adisparity estimation procedure is proposed to address this issue.

### II. Overview of Disparity Estimation

Many animals, including humans, have substantial binocular overlap within their visual field. In the binocular zone, each eye's viewpoint yields a slightly different image of the same part of the scene. Binocular disparity – the local differences between the images – is a powerful cue for estimating the depth structure of the scene. But before disparity can be used for depth estimation, disparity must be estimated from the images. Disparity estimation is a key part of depth estimation from binocular stereo visions<sup>[2]</sup>. It is a Wide research topic in computer vision which includes the technique aimed at inferring depth from two or more cameras. With two (or more) cameras we can infer depth, by means of triangulation, if we are able to find corresponding (homologous) points in the two images.

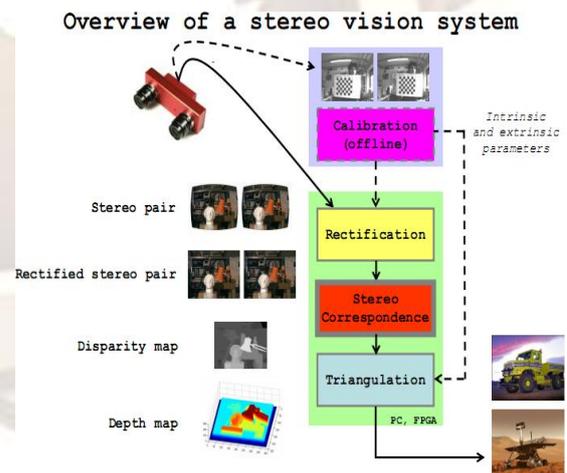


Figure: 1 Overview of a stereo system<sup>[3]</sup>

Here the figure shows the overview of stereo vision system. Assuming that we have two pinhole cameras in a canonical stereo configuration, i.e. parallel optical axes and the baseline aligned with the x-axes, we have the well-known relation that depth is inversely proportional to disparity. We define the disparity  $d$  as

$$d = x_l / x_r \quad (1)$$

Where  $x_l$  and  $x_r$  are the  $x$  coordinates corresponding to the same point in the scene for the left and right camera respectively. A wide variety of computational models have been proposed to explain how binocular disparity is computed from left-right image pairs. Binocular disparity refers to the difference in image location of an object seen by the left and right eyes, resulting from the eyes' horizontal separation (parallax). The brain uses binocular disparity to extract depth information from the two-dimensional retinal images in stereopsis. In computer vision, binocular disparity refers to the difference in coordinates of similar features within two stereo images. Here figure shows retinal images.

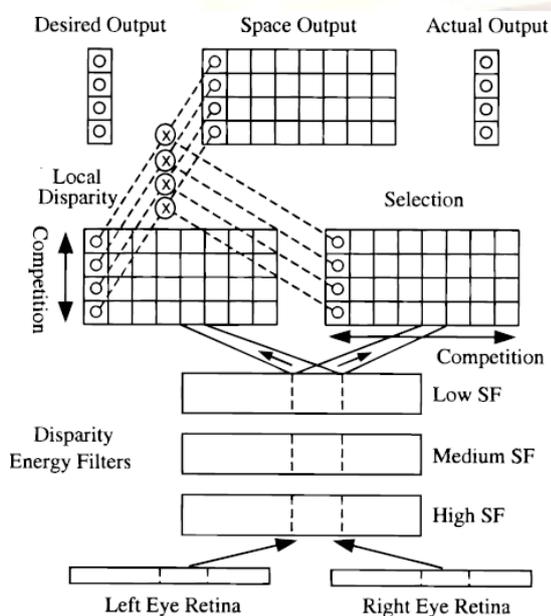


Figure: 2 Retinal Disparity

### III. PDE based Disparity Estimation

Partial differential equations (PDEs) have been successful for solving many problems in image processing and computer vision. However, designing PDEs usually requires high mathematical skills and good insight to the problems. In this paper, we focus on a framework related to disparity estimation. Many interesting problems in computer vision can be formulated as minimization problems for energy functional. In order to solve these problems, the Euler- Lagrange equations of the functional are computed, resulting in a set of necessary conditions. In effect, these conditions are partial differential equations which are introducing some basic facts on the theory of curve and surface. Algorithms for reconstruction from stereo vision can be further classified into image space methods and object space methods. While image space methods are ideal for certain scenarios, in particular camera setups with parallel optical axes.

The basics of PDE which are used in Disparity estimation is described as following:

#### Curve Evolution

As the name already suggest, curve evolution deals with the task of deforming a curve.

Let  $C_0 : I \rightarrow R^2$  be our initial curve and let  $p$  denote its parameterisation.

To describe an evolution in time, we introduce a time parameter  $t \geq 0$ .

Mathematically, a curve evolution is described by

$$\frac{\partial}{\partial t} C(p,t) = \alpha(p,t)t(p,t) + \beta(p,t)n(p,t) \quad (2)$$

The initial curve  $C(p,0) = C_0(p)$ . The movement of each curve point is written in the local coordinate system  $(t,n)$  of the curve, where  $t$  denotes the tangential vector and  $n$  denotes the normal vector.

#### Surface Evolution

Analogous to curve evolution, one can define a surface evolution on a surface

$$S_0 : D \rightarrow R^3$$

Let  $u$  and  $v$  denote the surface parameters and  $t \geq 0$  the necessary time parameter.

The surface evolution is then defined by

$$\frac{\partial}{\partial t} S(u,v,t) = \alpha^1(u,v,t)t_u(u,v,t) + \alpha^2(u,v,t)t_v(u,v,t) + \beta(u,v,t)n(u,v,t) \quad (3)$$

and  $S(u,v,0) = S_0(u,v)$ .  $t_u, t_v$  and  $n$  denote the tangential vectors in  $u$  and  $v$  direction and the surface normal, respectively. Similar as in the case of curves, the tangential components of the surface evolution act only as re-parameterisations and are therefore irrelevant for the geometric shape of the evolving surface.

A number of studies have been reported on the stereo correspondence problems over the past three decades. However most current disparity estimation algorithms includes graph based methods produce integer disparity fields which are not sufficient to find smooth depth. Figure 3 shows the example of the change in the depth for 1 pixel error in disparity according the baseline and the distance of the camera. We can see that depth errors for a 0.5 pixel disparity error inherent in integer disparity fields. For example, a 0.5 pixel error for the point at a distance of 5m leads to a depth error of 6cm, and the point at 8m leads to a 15cm depth.

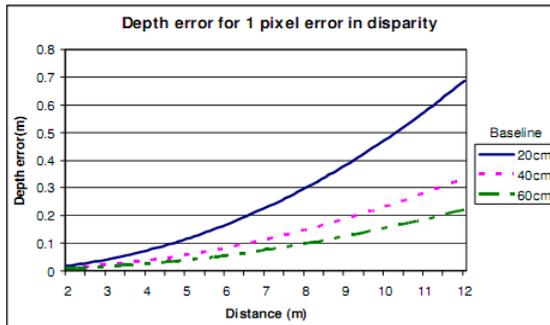


Figure: 3 Depth errors according to distance <sup>[2]</sup>

We can work on it and the correspondence problem can be solved by the PDE based Disparity estimation.

#### IV. Conclusion & Future work

Here in this paper we have provide the basic literature which focus on the disparity estimation of stereo images and how PDE is used in disparity estimation. We can develop a new algorithm to increase the precision for disparity estimation in a multi baseline variational Regularization framework, as well as an effective occlusion Detection scheme. With The experimental efforts we can improve the outputs of disparity estimation. The exemplary experiments show that the PDEs, can solve some image processing problems reasonably well. We have also presented in this paper a framework of learning PDEs from examples for image processing problems.

We expect that someday learning based PDEs, in their improved formulations, could be a general framework for designing PDEs for most image processing problems.

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