An EOQ Model With Stock Dependent Demand Rate And Variable time

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Abstract

Inventories are being considered to be an important aspect for any organization. EOQ model is defined as a controller of quantity and total cost per unit. This paper throws light on the Optimal Economic Order Quantity with stock dependent demand rate and consider variable time‘t’ which is used for two constants values α and β because after long period it changes time to time. This model tries to overcome the limitations of the model given by Gupta and Vrat who have used functional relationship in which α and β were constant but in present model we used time‘t’ to make them varies. Here, we used instantaneous case of replenishment and finite case of replenishment for different functional relationship to get optimum value of stock quantity and total cost per unit time. It is easy to use EOQ models to get an optimal order quantity level. In addition, we make different conditions and propose economic interpretation.

Keywords: EOQ Model, A (Assumption), Instantaneous Demand,Replenishment rate, time (t).

Introduction

In past inventory model it is assumed that demand rate does not depend upon inventory level. But it is not true in the cases [for many commodities] where the consumption pattern is influenced by the inventory maintained.

In general situation the demand may increase if the level of commodities [inventory or stock] is high and may decrease if the level of commodities [inventory or stock] is low, because the customer may be attracted with thepresence of large pile of inventory or stock in the market. This is called “Stock dependent demand rate”. In such cases the total cost should be include the direct purchase cost of the items as certainly the inventory process does affect the direct purchase cost of product. So we should include direct purchase cost in total inventory cost to get actual or accurate result of Inventory model.

Gupta and Vrat presented an EOQ model where stock level affects the consumption rate. They have given different functional relationship for demand and stock or supply which are as follows:

(I) Firstly, with instantaneous rate of replenishment and no shortages allowed

(II) Secondly, with finite rate of replenishment with no shortages allowed and linear demand rate

In this EOQ model it is derived with “Stock Dependent Demand Rate”. Here for different functional relationship between demand and supply we have to derive EOQ model with instantaneous rate of replenishment [commodities] with no shortages allow and finite rate of replenishment also with no shortages and with time period [t]. The analysis is given below:

Analysis

Condition: - EOQ model with instantaneous replenishment with no shortages.

In prior EOQ model which is assumed that demand rate (r) is constant. For this the total cost of the model per unit time is given by:

Calculation of EOQ Model

$$E(q)=r(t)C_p+\frac{Sr(t)}{q}+\frac{C_pC_cq}{2} \quad \text{.........} \ (1)$$

Where q = the order quantity
r = replenishment rate or demand rate
C_p = the unit purchase cost
S = setup cost per order
C_c = unit carrying charges per unit per unit time

EOQ model has been derived for the different functional relationships which are given below:

Case: - (a) $r(t)=\alpha t+\beta t\log q$

Substituting the value of r in equation (1) we get,

$$E(q)=(\alpha t+\beta t\log q)C_p+\frac{S(\alpha t+\beta t\log q)}{q}+\frac{C_pC_cq}{2}$$

To get the stationary value of q we take $\frac{dE(q)}{dq}=0$

$$\Rightarrow \frac{\beta t C_p}{q} - \left(\frac{S\alpha t}{q^2}\right) + \frac{S\beta t}{q^2} - \frac{S\beta t \log q}{q^2} + \frac{C_p C_c}{2} = 0$$

………… (2)

By simplification of this result we get,

$$\Rightarrow C_p C_c q^2 + 2\beta t C_p q - 2S\beta t \log q - 2s\, t\,(\alpha - \beta) = 0$$

………… (3)

The above equation no (2) can be solved by taking second derivation to obtain the value of q

$$\therefore \frac{d^2E(q)}{dq^2} = \frac{St[\beta(\log q^2 - 3) + 2\alpha] - \beta t C_p q}{q^3} > 0$$

……….. (4)

This is positive for all values of q given by,

$$\Rightarrow \frac{\beta(\log q^2 - 3) + 2\alpha}{q} > \frac{\beta C_p}{S} \qquad \text{……… (5)}$$

The equation (3) gives an optimal value of q under the condition (5)

Case: - (b) $r(t) = \alpha t + \beta t a^q$

Putting the value of r in to equation (1), we get,

$$E(q) = (\alpha t + \beta t a^q)C_p + \frac{S(\alpha t + \beta t a^q)}{q} + \frac{C_p C_c q}{2}$$

……..…. (6)

By busing first derivation we get stationary values of q which is given below.

$$\therefore \beta t C_p a^q \log_e a - S\beta t\left(\frac{1}{q^2}\right) + \frac{S\beta t q a^q \log_e a}{q^2} - \frac{S\beta t a^q}{q^2} + \frac{C_p C_c}{2} = 0$$

…………… (7)

$$\Rightarrow 2\beta t C_p (\log_e a) a^q q^2 - 2S\beta t + 2S\beta t(\log_e a) a^q q - 2S\beta t a^q + C_p C_c q^2 = 0$$

……....... (8)

Now to get optimum solution we can use second derivation.

$$\Rightarrow \frac{\beta t C_p (\log_e a)^2 a^q q^3 + 2S\beta t + S\beta t(\log_e a)^2 a^q q^2 - 2S\beta t(\log_e a) a^q q + 2S\beta t a^q}{q^3}$$

….. (9)

Now to get the optimal value of q we take $\frac{d^2E(q)}{dq^2} > 0$ because q is always positive and non zero.

$$\Rightarrow \beta C_p (\log_e a)^2 a^q q^3 + 2S\beta\,[1 + (\log_e a)^2 a^q q^2 - (\log_e a) a^q q + a^q)] > 0$$

……… (10)

Solving equation (10) we get optimal value of q.

Case: - (c) $r(t) = \alpha t + \beta t a^{-q}$

Putting this value of demand rate r into equation (1), we get

$$E(q) = (\alpha t + \beta t a^{-q})C_p + \frac{S}{q}(\alpha + \beta a^{-q}) + \frac{C_p C_c q}{2}$$

$$\therefore E(q) = \alpha t C_p + \beta t C_p a^{-q} + \frac{S\alpha}{q} + \frac{S\beta a^{-q}}{q} + \frac{C_p C_c q}{2}$$

... (11)

Now to get stationery value of q, we used first derivative and take $\frac{d\,E(q)}{d\,q} = 0$

$$\Rightarrow -\beta t C_p a^{-q} \log_e a - \frac{S\alpha}{q^2} - \frac{S\beta a^{-q}\log_e a}{q} - \frac{A\beta a^{-q}}{q^2} + \frac{C_p C_c}{2} \qquad \text{.. (12)}$$

$$\Rightarrow -2\beta t C_p a^{-q} \log_e a.q^2 - 2S\alpha - 2S\beta a^{-q}\log_e a.q - 2S\beta a^{-q} + C_p C_c q^2 \qquad \text{.. (13)}$$

Now to get optimum value of q we can use second derivation. From equation(12), we get

$$\Rightarrow 2\beta t C a^{-q}(\log_e a)^2 q^3 + 2A\{\alpha + \beta a^{-q}\} + A\beta a^{-q}.q.\log_e a\{2 + q.\log_e a\} > 0$$

…… (14)

Case: - (d) $r(t) = \alpha t + \beta t q + \gamma t q^2$

Putting r (t) in to equation (1), we get,

$$E(q) = (\alpha t + \beta t q + \gamma t q^2)C_p + \frac{S(\alpha t + \beta t q + \gamma t q^2)}{q} + \frac{C_p C_c q}{2}$$

……….. (15)

Now to get stationery value of q we take $\frac{d\,E(q)}{d\,q} = 0$, from equation (15), we get,

$$\Rightarrow q^2[C_p C_c + 2\beta t C_P + 2S\gamma t + 4\gamma t C_p q] - 2S\alpha t = 0$$

……. (16)

Equation (16) can be solved by using second order derivation to get an optimal value of q.

$$\Rightarrow q > \pm \sqrt[3]{\frac{2S\alpha t}{\gamma t C_p}} \quad \text{……..…………… (17)}$$

Here q is positive, so it gives an optimum quantity.

Case: - (e) $r(t) = \alpha t + \beta t q^{\gamma}$

Putting r (t) into equation (1), we get,

$$E(q) = (\alpha t + \beta t q^{\gamma})C_p + \frac{S(\alpha t + \beta t q^{\gamma})}{q} + \frac{C_p C_c q}{2}$$

…..… (18)

Now to get stationery value of q, we take $\frac{d\,E(q)}{d\,q} = 0$, from equation (18), we get,

$$\Rightarrow \beta t q^{\gamma}[C_p \gamma q + S\gamma - S] + \frac{C_p C_c q^2}{2} - S\alpha t = 0$$

……. (19)

Now to obtain the optimal value of q, we take $\frac{d^2 E(q)}{dq^2} > 0$, from equation (19), we get

$$\Rightarrow \beta t C_p \gamma^2 q^{\gamma} + S\beta t \gamma^2 q^{\gamma - 1} - S\beta t \gamma q^{\gamma - 1} + \beta t C_p \gamma q^{\gamma} + C_p C_c q$$

To get minimum cost for this model, we must insert two conditions which are given below:

(a) If $\gamma > 1$ then $\frac{d^2 E(q)}{dq^2} > 0$ *is positive* $\forall q$, *and*

(b) If $0 \le \gamma \le 1$ then $\frac{d^2 E(q)}{dq^2} > 0$ is positive $\forall q > \frac{S\,t\,(1-\gamma)}{C_p\,t\,(1+\gamma)}$

If α, β and γare restricted to be positive constant then a marginally different form of functional relationship similar to $r(t) = \alpha t + \beta t q^{\gamma}$ can be expressed as

$$r(t) = \alpha t - \beta t q^{-\gamma} \quad \text{……………... [A-1]}$$

Substituting [A-1] in equation (1), we get,

$$E(q) = (\alpha t - \beta t q^{-\gamma})C_p + \frac{S(\alpha t - \beta t q^{-\gamma})}{q} + \frac{C_p C_c q}{2}$$

Now to get stationery value of q, we take $\frac{d\,E(q)}{d\,q} = 0$

$$\Rightarrow [2\beta\gamma C_p t q^{-\gamma+1} + 2S\beta\gamma t q^{-\gamma} + 2S\beta t q^{-\gamma}] + \frac{C_p C_c q^2}{2} - S\alpha t = 0$$

…..... (20)

$$\Rightarrow 1 \quad \text{…....... (21)}$$

Now to get optimal value of q, we take

$$\frac{d^2E(q)}{dq^2}$$

$$\Rightarrow -\beta t\gamma^2 C_p t q^{-\gamma} - S\beta\gamma^2 t q^{-\gamma-1} - S\beta t\gamma q^{-\gamma-1} + \beta\gamma C_p t q^{-\gamma} + C_p C_c q. \quad \text{.......... (22)}$$

Insert equation after confirming,

$\frac{d^2E(q)}{dq^2}$ is positive when

$$C_p t q(1-\gamma) - S t(1+\gamma) \geq 0$$

$$\therefore q \geq \frac{St\,(1+\gamma)}{C_p\, t(1-\gamma)}$$

$$\therefore 0 \leq \gamma \leq 1$$

It implies that here value of q is non negative which is define optimum solution.

Case: - (f) $r(t) = \alpha t + \beta t e^q$

Substituting r (t) into equation (1), we get,

$$E(q) = [\alpha t + \beta t e^q]C_p + \frac{S(\alpha t + \beta t e^q)}{q} + \frac{C_p C_c q}{2} \quad \text{... (23)}$$

Now to get stationery value of q we take $\frac{d\,E(q)}{d\,q}$ = 0 from equation (23), we get

$$\Rightarrow \beta t e^q[C_p q^2 + Sq - S] + \frac{C_p C_c q^2}{2} - S\alpha t = 0. \quad (24)$$

Now to get optimum value of q we take

$$\frac{d^2E(q)}{dq^2} > 0$$

From equation (24), we get,

$$\therefore q > -\left(\frac{C_p C_c + S\beta t e^q + 2\beta C_p t e^q}{\beta t e^q}\right) \quad \text{.... (25)}$$

Here the value of q become optimum because it is greater than 0.

For positive values of □and□a marginally different forms for functional relationship can be expressed as

$$r(t) = \alpha t - \beta t e^{-q} \quad \text{.......... [A-2]}$$

Substituting [A-2] into equation (1), we get,

$$E(q) = \alpha t C_p - \beta t C_p e^{-q} + \frac{S\alpha t}{q} - \frac{St\beta e^{-q}}{q} + \frac{C_p C_c q}{2}$$

Now to get stationery value of q, we take $\frac{d\,E(q)}{d\,q}$ =0

$$\Rightarrow \beta t e^{-q}[C_p q^2 + Sq + S] + \frac{C_p C_c q^2}{2} - S\alpha t = 0 \quad \text{....... (26)}$$

Now to get optimum value of q, we take

$$\frac{d^2E(q)}{dq^2} > 0$$

From equation (26), we get,

$$\Rightarrow q > \frac{C_p C_c - S\beta t e^{-q} + 2\beta C_p t e^{-q}}{\beta t e^{-q}} \quad \text{.......... (27)}$$

Thus equation (29) gives an optimum solution of q.

Analysis:

Finite Replenishment case

Here EOQ model has been derived under finite rate of replenishment and for this for this following functional relationship will be used.

(a) $r'(t) = r + \frac{\beta t}{q}$

(b) $r'(t) = r + \beta t q^{\gamma}$

(c) $r'(t) = r + \beta t e^{q}$

Where β and γ are positive constants and r' (t) is the consumption rate during depletion time. The total cycle time T consists of two parts:

T_{rc} = the time during which replenishment come and

T_d = the time during which stock depletion takes place

Thus, $T = T_{rc} + T_d$

The finite replenishment rate and the consumption rate are taken k and r respectively such that k > r. obviously net rate of procuring of items in inventory is k– r.

Case (a): $r'(t) = r + \dfrac{\beta t}{q}$

Following Gupta and Vrat the inventory carrying cost per cycle is given by

$$\frac{C_p C_c q^2}{2r}\left(1-\frac{r}{k}\right)$$, where

$$\frac{1}{r^*} = \frac{1}{k} + \frac{1}{r'}\left(1-\frac{r}{k}\right)$$

The total cost per unit time of the system is

$$E(q) = \frac{1}{T}\left[A + \frac{C_p C_c q^2}{2r^*}\left(1-\frac{r}{k}\right)\right]$$

$$= \frac{Ak\beta t}{q(\beta t + kg)} + \frac{Akr}{(\beta t + kg)} + \frac{C_p C_c q}{2}\left(1-\frac{r}{k}\right) \quad \text{...........(28)}$$

For which optimum condition is

$$\frac{dE(q)}{dq} = -\frac{Ak\beta t}{q^2(\beta t + kg)} - \frac{Ak^2\beta t}{q(\beta t + kg)^2} - \frac{Ak^2\lambda}{(\beta t + kg)^2}$$

$$+ \frac{C_p C_c}{2}\left(1-\frac{r}{k}\right) = 0$$

This equation can be solved for q by taking second derivation.

Clearly $\dfrac{d^2E(q)}{dq^2} > 0$ for all values of q

$$\Rightarrow q > \frac{\left(\dfrac{2Ak\beta t}{q^2(\beta t + kq)} + \dfrac{2Ak^2\beta t}{q(\beta t + kq)^2} + \dfrac{2Ak^3\beta t}{(\beta t + kq)^3}\right)}{\dfrac{2Ak^3 r}{(\beta t + kq)^3}} \quad . (29)$$

Here clearly q > 0 for all values of q which is called optimum quantity.

Case (b) $r'(t) = r + \beta t q^{\gamma}$

Here the total cost per unit time is given by

$$E(q) = \frac{Akr}{q(k + \beta t q^{\gamma})} + \frac{Ak\beta t q^{\gamma-1}}{q(k + \beta t q^{\gamma})} + \frac{C_p C_c q}{2}\left(1-\frac{r}{k}\right)$$

Condition to obtain value of q is $\dfrac{dE(q)}{dq} = 0$

$$\Rightarrow \left(-\frac{Akr}{q^2(k + \beta t q^{\gamma})} - \frac{Akr\beta t\gamma q^{\gamma-1}}{q(k + \beta t q^{\gamma})^2} + \frac{Ak\beta t(\gamma-1)q^{\gamma-2}}{(k + \beta t q^{\gamma})}\right)$$

$$-\frac{Ak\beta^2 t^2 \gamma q^{2\gamma-2}}{(k + \beta t q^{\gamma})^2} + \frac{C_p C_c}{2}\left(1-\frac{r}{k}\right) = 0 \quad \text{.......... (30)}$$

Equation (30) can be solved by using second derivation for optimum value of q.

$$\frac{d^2E(q)}{dq^2} = \frac{2Akr}{q^3(k + \beta t q^{\gamma})} + \frac{Ak\beta t(\gamma-1)(\gamma-2)q^{\gamma-3}}{(k + \beta t q^{\gamma})} +$$

$$\frac{Ak\beta t\gamma q^{\gamma-3}[r(3-\gamma) - 3\beta t(\gamma-1)q^{\gamma}]}{(k + \beta t q^{\gamma})^2}$$

$$+ \frac{2Ak(\beta t\gamma)^2 q^{2\gamma-3}(r + \beta t q^{\gamma})}{(k + \beta t q^{\gamma})^3} \quad \text{…………….. (31)}$$

Now $\dfrac{d^2E(q)}{dq^2} > 0$, if

$r(3-\gamma) - 3\beta t(\gamma-1)q^{\gamma} > 0$ or

$$q < \left[\frac{r(3-\gamma)}{3\beta t(\gamma-1)}\right]^{\frac{1}{\gamma}} \; and\; 2 \le \gamma \le 3 \quad \text{.................... (32)}$$

Hence solution of equation (31) under condition of equation (32) gives an optimal solution of q

Case: - (c) $r'(t) = r + \beta t e^{q}$

In this case the total cost per unit time of the system is:

$$E(q)=\frac{Ak(r+\beta te^{q})}{q(k+\beta te^{q})}+\frac{C_pC_cq}{2}\left(1-\frac{r}{k}\right)$$

Condition to obtain value of q is $\dfrac{dE(q)}{dq}=0$

$$\Rightarrow\left(\frac{Ak\beta te^{q}}{q(k+\beta te^{q})}-\frac{Ak(r+\beta te^{q}}{q^{2}(k+\beta te^{q})}-\frac{Ak\beta t(re^{q}+\beta te^{2q})}{q(k+\beta te^{q})^{2}}\right)$$

$$+\frac{C_pC_c}{2}\left(1-\frac{r}{k}\right)=0 \qquad \text{………………..} (33)$$

To obtain optimum solution, we use second derivation:

$$\frac{d^2E(q)}{dq^2}=\frac{Ak\beta te^{q}[(q-1)+1}{q^{3}(k+\beta te^{q})}+\frac{2Akr}{q^{3}(k+\beta te^{q})}+$$

$$\frac{2Ak\beta t(re^{q}+\beta te^{2q})}{q^{2}(k+\beta te^{q})^{2}}$$

$$+\frac{Ak\beta te^{q}(\phi-r)(k-\beta te^{q})}{q(k+\beta tew^{q})^{3}} \qquad \text{…………} (34)$$

It is clear that

$$\frac{d^2E(q)}{dq^2}>0, if\ (k-\beta te^{q})>0\ or\ q<\log\frac{k}{\beta t} \quad \text{……..} (35)$$

Equation (34) gives an optimum value of order quantity q under condition of equation (35).

Conclusion:

In EOQ model derived here, not allowing shortages, demand rate depends upon the stocks i.e. more the stock, the consumption or demand is also more i.e. if stock increases, the demand will also be increased. First model is discussed for instantaneous case of replenishment and then after for finite replenishment case for assumed demand rates. A solution procedure is explained to get optimum value of stock quantity and substituting this in cost equation the total cost of the system per unit time can be forecasted.

An EOQ model formulated in second case matches with the corresponding EOQ model given by Gupta and (36) Vrat under specific conditions. Some more functional relations between the demand rate and stock quantity can be constructed to obtain the new results.

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