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A Study On (I,V)- Fuzzy Rw-Open Maps, (I,V)-Fuzzy Rw-Closed Maps And (I,V)-Fuzzy Rw-Homeomorphisms In (I,V)-Fuzzy Topological Space

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ABSTRACT:

In this paper, we study some of the properties of interval valued fuzzy rw-open maps, interval valued fuzzy rw-closed maps and interval valued fuzzy rw-homeomorphism in interval valued fuzzy topological spaces and prove some results on these.Note interval valued is denoted as (i,v).

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KEY WORDS: (i,v)-fuzzy subset, (i,v)-fuzzy topological spaces, (i,v)-fuzzy rw-closed, (i,v)-fuzzy rw-open, (i,v)-fuzzy rw-continuous maps, (i,v)-fuzzy rw-irresolute maps, (i,v)-fuzzy rw-open maps, (i,v)-fuzzy rw-closed maps, (i,v)-fuzzy rw-homeomorphism.

INTRODUCTION:

The concept of a fuzzy subset was introduced and studied by L.A.Zadeh [22] in the year 1965. The subsequent research activities in this area and related areas have found applications in many branches of science and engineering. The following papers have motivated us to work on this paper. C.L.Chang [5] introduced and studied fuzzy topological spaces in 1968 as a generalization of topological spaces. Many researchers like R.H.Warren [21], K.K.Azad [2], G.Balasubramanian and P.Sundaram [3, 4], S.R.Malghan and S.S.Benchalli [13, 14] and many others have contributed to the development of fuzzy topological spaces. Tapas kumar mondal and S.K.Samanta [19] have introduced the topology of interval valued fuzzy sets. We introduce the concept of interval valued fuzzy rw-open maps, interval valued fuzzy rw-closed maps and interval valued fuzzy rw-homeomorphism in interval valued fuzzy topological spaces and established some results.

1.PRELIMINARIES:

1.1 Definition:[19] Let X be any nonempty set. A mapping $\bar{A} : X \to D[0,1]$ is called an interval valued fuzzy subset (briefly, (i,v)-fuzzy subset) of X, where D[0,1] denotes the family of all closed subintervals of [0,1] and $\bar{A}(x) = [A^-(x), A^+(x)]$ for all $x \in X$, where A^- and A^+ are fuzzy subsets of X such that $A^-(x) \le A^+(x)$ for all $x \in X$. Thus $\bar{A}(x)$ is an interval (a closed subset of [0,1]) and not a number from the interval [0,1] as in the case of fuzzy subset.

1.1 Remark: Let D^X be the set of all interval valued fuzzy subset of X.

1.2 Definition: Let $\bar{A} = \{ \langle x, \bar{\mu}_A(x) \rangle / x \in X \}$, $\bar{B} = \{ \langle x, \bar{\mu}_B(x) \rangle / x \in X \} \in D^X$. We define the following relations and operations:

(i) $\bar{A} \subseteq \bar{B}$ if and only if $\bar{\mu}_A(x) \le \bar{\mu}_B(x)$, for all x in X.

(ii) $\bar{A} = \bar{B}$ if and only if $\bar{\mu}_A(x) = \bar{\mu}_B(x)$, for all x in X.

(iii) $(\bar{A})^c = \bar{1} - \bar{A} = \{ \langle x, \bar{1} - \bar{\mu}_A(x) \rangle / x \in X \}$.

(iv) $\bar{A} \cap \bar{B} = \{ \langle x, \min\{ \bar{\mu}_A(x), \bar{\mu}_B(x) \} \rangle / x \in X \}$.

(v) $\bar{A} \cup \bar{B} = \{ \langle x, \max\{ \bar{\mu}_A(x), \bar{\mu}_B(x) \} \rangle / x \in X \}$.

1.3 Definition:[19] Let X be a set and $\Im$ be a family of (i,v)-fuzzy subsets of X. The family $\Im$ is called an (i,v)-fuzzy topology on X if and only if $\Im$ satisfies the following axioms

(i) $\bar{0}, \bar{1} \in \Im$,

(ii) If $\{ \bar{A}_i ; i \in I\} \subseteq \Im$, then $\bigcup_{i \in I} \bar{A}_i \in \Im$,

(iii) If $\bar{A}_1, \bar{A}_2, \bar{A}_3, \ldots \bar{A}_n \in \Im$, then $\bigcap_{i=1}^{i=n} \bar{A}_i \in \Im$.

The pair (X, $\Im$) is called an (i,v)-fuzzy topological space. The members of $\Im$ are called (i,v)-fuzzy open sets in X. An (i,v)-fuzzy set $\bar{A}$ in X is said to be (i,v)-fuzzy closed set in X if and only if $(\bar{A})^c$ is an (i,v)-fuzzy open set in X.

1.4 Definition: Let (X, $\Im$) be an (i,v)-fuzzy topological space and $\bar{A}$ be (i,v)-fuzzy set in X. Then $\bar{A}$ is said to be

(i) (i,v)-fuzzy semiopen if and only if there exists an (i,v)-fuzzy open set $\bar{V}$ in X such that $\bar{V} \subseteq \bar{A} \subseteq cl(\bar{V})$.

(ii) (i,v)-fuzzy semiclosed if and only if there exists an (i,v)-fuzzy closed set $\bar{V}$ in X such that $int(\bar{V}) \subseteq \bar{A} \subseteq \bar{V}$.

(iii) (i,v)-fuzzy regular open set of X if int(cl($\bar{A}$)) = $\bar{A}$.
(iv) (i,v)-fuzzy regular closed set of X if cl(int($\bar{A}$)) = $\bar{A}$.
(v) (i,v)-fuzzy regular semiopen set of X if there exists an (i,v)-fuzzy regular open set $\bar{V}$ in X such that $\bar{V} \subseteq \bar{A} \subseteq$ cl($\bar{V}$). We denote the class of (i,v)-fuzzy regular semiopen sets in (i,v)-fuzzy topological space X by IVFRSO(X).

1.5 Definition: A mapping f : X⟶Y from an (i,v)-fuzzy topological space X to an (i,v)-fuzzy topological space Y is called
(i) (i,v)-fuzzy continuous if f^{-1}($\bar{A}$) is (i,v)-fuzzy open in X for each (i,v)-fuzzy open set $\bar{A}$ in Y.
(ii) (i,v)-fuzzy generalized continuous (ivfg-continuous) if f^{-1}($\bar{A}$) is (i,v)-fuzzy generalized closed in X for each (i,v)-fuzzy closed set $\bar{A}$ in Y.

1.6 Definition: A mapping f : X⟶Y from an (i,v)-fuzzy topological space X to an (i,v)-fuzzy topological space Y is called (i,v)-fuzzy open mapping if f($\bar{A}$) is (i,v)-fuzzy open in Y for every (i,v)-fuzzy open set in $\bar{A}$ in X.

1.7 Definition: Let (X, $\Im$) be an (i,v)-fuzzy topological space. An (i,v)-fuzzy set $\bar{A}$ of X is called (i,v)-fuzzy regular w-clsoed(briefly, (i,v)-fuzzy rw-closed) if cl($\bar{A}$) $\subseteq$ $\bar{U}$ whenever $\bar{A} \subseteq \bar{U}$ and $\bar{U}$ is (i,v)-fuzzy regular semiopen in (i,v)-fuzzy topological space X.

NOTE: We denote the family of all (i,v)-fuzzy regular w-closed sets in (i,v)-fuzzy topological space X by IVFRWC(X).

1.8 Definition: An (i,v)-fuzzy set $\bar{A}$ of an (i,v)-fuzzy topological space X is called an (i,v)-fuzzy regular w-open (briefly, (i,v)-fuzzy rw-open) set if its complement $\bar{A}^C$ is an (i,v)-fuzzy rw-closed set in (i,v)-fuzzy topological space X.

NOTE: We denote the family of all (i,v)-fuzzy rw-open sets in (i,v)-fuzzy topological space X by IVFRWO(X).

1.9 Definition: Let X and Y be (i,v)-fuzzy topological spaces. A map f : X⟶ Y is said to be (i,v)-fuzzy rw-continuous if the inverse image of every (i,v)-fuzzy open set in Y is (i,v)-fuzzy rw-open in X.

1.10 Definition: Let X and Y be (i,v)-fuzzy topological spaces. A map f : X⟶Y is said to be an (i,v)-fuzzy rw-irresolute map if the inverse image of every (i,v)-fuzzy rw-open set in Y is an (i,v)-fuzzy rw-open set in X.

1.11 Definition: Let (X, $\Im$) be an (i,v)-fuzzy topological space and $\bar{A}$ be an (i,v)-fuzzy set of X. Then (i,v)-fuzzy rw-interior and (i,v)-fuzzy rw-closure of $\bar{A}$ are defined as follows.
ivfrwcl($\bar{A}$) = $\cap$\{ $\bar{K}$: $\bar{K}$ is an (i,v)-fuzzy rw-closed set in X and $\bar{A} \subseteq \bar{K}$ \}.
ivfrwint($\bar{A}$) = $\cup$ \{ $\bar{G}$: $\bar{G}$ is an (i,v)-fuzzy rw-open set in X and $\bar{G} \subseteq \bar{A}$ \}.

1.2 Remark: It is clear that $\bar{A} \subseteq$ ivfrwcl($\bar{A}$) $\subseteq$ cl($\bar{A}$) for any (i,v)-fuzzy set $\bar{A}$.

1.12 Definition: Let (X, $\Im_1$) and (Y, $\Im_2$) be two (i,v)-fuzzy topological spaces. A map f : (X, $\Im_1$) ⟶ (Y, $\Im_2$) is called (i,v)-fuzzy rw-open if the image of every (i,v)-fuzzy open set in X is (i,v)-fuzzy rw-open in Y.

1.13 Definition: Let (X, $\Im_1$) and (Y, $\Im_2$) be two (i,v)-fuzzy topological spaces. A map f : (X, $\Im_1$) ⟶ (Y, $\Im_2$) is called (i,v)-fuzzy rw-closed if the image of every (i,v)-fuzzy closed set in X is an (i,v)-fuzzy rw-closed set in Y.

1.14 Definition: Let X and Y be (i,v)-fuzzy topological spaces. A bijection map f : (X, $\Im_1$) ⟶ (Y, $\Im_2$) is called (i,v)-fuzzy rw-homeomorphsim if f and f^{-1} are (i,v)-fuzzy rw-continuous.

NOTE: The family of all (i,v)-fuzzy rw-homeomorphism from (X, $\Im$) onto itself is denoted by IVFRW-H(X, $\Im$).

1.15 Definition: A bijection map f : (X, $\Im_1$) ⟶ (Y, $\Im_2$) is called an (i,v)-fuzzy rwc-homemorphism if f and f^{-1} are (i,v)-fuzzy rw-irresolute. We say that spaces (X, $\Im_1$) and (Y, $\Im_2$) are (i,v)-fuzzy rwc-homeomorphism if there exsit an (i,v)-fuzzy rwc-homeomorphism from (X, $\Im_1$) onto (Y, $\Im_2$).

NOTE: The family of all (i,v)-fuzzy rwc-homeomorphism from (X, $\Im$) onto itself is denoted by IVFRWC-H(X, $\Im$).

2. SOME PROPERTIES:

2.1 Theorem: Every (i,v)-fuzzy open map is an (i,v)-fuzzy rw-open map.

Proof: Let f : (X, $\Im_1$) ⟶ (Y, $\Im_2$) be an (i,v)-fuzzy open map and $\bar{A}$ be an (i,v)-fuzzy open set in (i,v)-fuzzy topological space X. Then f($\bar{A}$) is an (i,v)-fuzzy open set in (i,v)-fuzzy topological space Y. Since every (i,v)-fuzzy open set is (i,v)-

fuzzy rw-open, f($\bar{A}$) is an (i,v)-fuzzy rw-open set in (i,v)-fuzzy topological space Y. Hence f is an (i,v)-fuzzy rw-open map.

2.1 Remark: The converse of the above theorem need not be true in general.

2.1 Example: Let X = Y = { 1, 2, 3 } and the (i,v)-fuzzy sets $\bar{A}$, $\bar{B}$, $\bar{C}$: X $\longrightarrow$D [0, 1] be defined as $\bar{A}$ = { <1, [1, 1]>, <2, [0, 0]>, <3, [0, 0]> }, $\bar{B}$ = { <1, [1, 1]>, <2, [1, 1]>, <3, [0, 0] > }, $\bar{C}$ = { <1, [1, 1] >, < 2, [0, 0] >, <3, [1, 1] > }. Consider $\Im_1$ = { $\bar{0}$, $\bar{1}$, $\bar{A}$, $\bar{B}$, $\bar{C}$ } and $\Im_2$ = { $\bar{0}$, $\bar{1}$, $\bar{A}$ }. Then (X, $\Im_1$) and (Y, $\Im_2$) are (i,v)-fuzzy topological spaces. Let f : (X, $\Im_1$) $\longrightarrow$ (Y, $\Im_2$) be defined as f(1) = f(2) = 1 and f(3) = 3. Then this function is (i,v)-fuzzy rw-open but it is not (i,v)-fuzzy open, since the image of the (i,v)-fuzzy open set $\bar{C}$ in X is the (i,v)-fuzzy set $\bar{C}$ in Y which is not (i,v)-fuzzy open.

2.2 Theorem: Let f : (X, $\Im_1$) $\longrightarrow$ (Y, $\Im_2$) and g : (Y, $\Im_2$) $\longrightarrow$ (Z, $\Im_3$) be two (i,v)-fuzzy rw-open maps. Show that g•f need not be (i,v)-fuzzy rw-open.

Proof: Consider the following example, let X=Y={ 1, 2, 3 } and the (i,v)-fuzzy sets $\bar{A}$, $\bar{B}$, $\bar{C}$, $\bar{D}$: X$\longrightarrow$ D [0, 1] be defined as $\bar{A}$ ={<1, [1, 1]>, <2, [0, 0]>, <3, [0, 0]>},
$\bar{B}$ = { <1, [0, 0]>, <2, [1, 1]>, <3, [0, 0]> }, $\bar{C}$ = { <1, [1, 1]>, <2, [1, 1]>, <3, [0, 0]> },
$\bar{D}$ = { <1, [0, 0] >, < 2, [1, 1] >, <3, [1, 1] > }. Consider $\Im_1$={ $\bar{0}$, $\bar{1}$, $\bar{A}$, $\bar{D}$} and $\Im_2$= { $\bar{0}$, $\bar{1}$, $\bar{A}$} and $\Im_3$ ={ $\bar{0}$, $\bar{1}$, $\bar{A}$, $\bar{B}$, $\bar{C}$ }. Then (X, $\Im_1$), (Y, $\Im_2$) and (Z, $\Im_3$) are (i,v)-fuzzy topological spaces. Let f : (X, $\Im_1$) $\longrightarrow$ (Y, $\Im_2$) and g : (Y, $\Im_2$) $\longrightarrow$ (Z, $\Im_3$) be the identity maps. Then f and g are (i,v)-fuzzy rw-open maps but their composition g•f : (X, $\Im_1$) $\longrightarrow$ (Z, $\Im_3$) is not (i,v)-fuzzy rw-open as $\bar{D}$:X$\longrightarrow$ [0, 1] is (i,v)-fuzzy open in X but (g•f)($\bar{D}$) = $\bar{D}$ is not (i,v)-fuzzy rw-open in Z.

2.3 Theorem: If f : (X, $\Im_1$) $\longrightarrow$ (Y, $\Im_2$) is (i,v)-fuzzy open map and g : (Y, $\Im_2$) $\longrightarrow$ (Z, $\Im_3$) is (i,v)-fuzzy rw-open map, then their composition g•f : (X, $\Im_1$) $\longrightarrow$ (Z, $\Im_3$) is (i,v)-fuzzy rw-open map.

Proof: Let $\bar{A}$ be (i,v)-fuzzy open set in (X, $\Im_1$). Since f is (i,v)-fuzzy open map, f($\bar{A}$) is an (i,v)-fuzzy open set in (Y, $\Im_2$). Since g is an (i,v)-fuzzy rw-open map, g(f($\bar{A}$)) is (i,v)-fuzzy rw-open set in (Z, $\Im_3$). But g (f($\bar{A}$)) = (g•f)($\bar{A}$). Thus g•f is an (i,v)-fuzzy rw-open map.

2.2 Remark: Every (i,v)-fuzzy w-open map is (i,v)-fuzzy rw-open but converse may not be true.

Proof: Consider the example let X = { a, b}, Y = {x, y } and the (i,v)-fuzzy set $\bar{A}$ and $\bar{B}$ are defined as follows $\bar{A}$ = { <a, [0.7, 0.7] >, <b, [0.8, 0.8] > }, $\bar{B}$ = { <x, [0.7, 0.7]>, <y, [0.6, 0.6] > }. Then $\Im$ = { $\bar{0}$, $\bar{1}$, $\bar{A}$ } and σ = { $\bar{0}$, $\bar{1}$, $\bar{B}$ } be (i,v)-fuzzy topologies on X and Y respectively. Then, the mapping f : (X, $\Im$) → (Y, σ) defined by $f(a)$ = x and $f(b)$ = y is (i,v)-fuzzy rw-open but it is not (i,v)-fuzzy w-open.

2.4 Theorem: A mapping f : (X, $\Im$) → (Y, σ) is (i,v)-fuzzy rw-open if and only if for every (i,v)-fuzzy set $\bar{A}$ of X, $f (int(\bar{A})) \subseteq$ rwint(f($\bar{A}$)).

Proof: Let f be an (i,v)-fuzzy rw-open mapping and $\bar{A}$ is an (i,v)-fuzzy open set in X. Now int($\bar{A}$) $\subseteq$ $\bar{A}$ which implies that f (int($\bar{A}$) $\subseteq f(\bar{A})$. Since f is an (i,v)-fuzzy rw-open mapping, $f(int(\bar{A}))$ is (i,v)-fuzzy rw-open set in Y such that $f(int(\bar{A})) \subseteq f(\bar{A})$ therefore $f(int(\bar{A})) \subseteq$ rwint $f(\bar{A})$.
For the converse suppose that $\bar{A}$ is an (i,v)-fuzzy open set of X. Then f ($\bar{A}$) = f (int($\bar{A}$)) $\subseteq$ rwint $f(\bar{A})$. But wint(f($\bar{A}$)) $\subseteq f$ ($\bar{A}$). Consequently f ($\bar{A}$) = wint($\bar{A}$) which implies that $f(\bar{A})$ is an (i,v)-fuzzy rw-open set of Y and hence f is an (i,v)-fuzzy rw-open.

2.5 Theorem: If f : (X, $\Im$) → (Y, σ) is an (i,v)-fuzzy rw-open map then int (f^{-1}($\bar{A}$)) $\subseteq$ f^{-1}(rwint($\bar{A}$) for every (i,v)-fuzzy set $\bar{A}$ of Y.

Proof: Let $\bar{A}$ is an (i,v)-fuzzy set of Y. Then int f^{-1}($\bar{A}$) is an (i,v)-fuzzy open set in X. Since f is (i,v)-fuzzy rw-open f ($int\ f^{-1}$($\bar{A}$)) is (i,v)-fuzzy rw-open in Y and hence $f(int\ f^{-1}(\bar{A})) \subseteq$ rwint(f(f^{-1}($\bar{A}$)) $\subseteq$ rwint(f (f^{-1}($\bar{A}$))) $\subseteq$ rwint($\bar{A}$). Thus int f^{-1}($\bar{A}$) $\subseteq$ f^{-1}(rwint($\bar{A}$)).

2.6 Theorem: A mapping f : (X, $\Im$) → (Y, σ) is (i,v)-fuzzy rw-open if and only if for each (i,v)-fuzzy set $\bar{A}$ of Y and for each (i,v)-fuzzy closed set $\bar{U}$ of X containing f^{-1}($\bar{A}$) there is a (i,v)-fuzzy rw-closed $\bar{V}$ of Y such that $\bar{A} \subseteq \bar{V}$ and f^{-1}($\bar{V}$) $\subseteq$ $\bar{U}$.

Proof: Suppose that f is an (i,v)-fuzzy rw-open map. Let $\bar{A}$ be the (i,v)-fuzzy closed set of Y and $\bar{U}$ is an (i,v)-fuzzy closed set of X such that f^{-1}($\bar{A}$) $\subseteq$ $\bar{U}$. Then $\bar{V}$ = (f^{-1}($\bar{U}^c$) $)^c$ is (i,v)-fuzzy rw-closed set of Y such that f^{-1}($\bar{V}$) $\subseteq$ $\bar{U}$.
For the converse suppose that $\bar{B}$ is an (i,v)-fuzzy open set of X. Then f^{-1}((f($\bar{B}$) $)^c \subseteq \bar{B}^c$ and $\bar{B}^c$ is (i,v)-fuzzy closed set in X. By hypothesis there is an (i,v)-fuzzy rw-closed set $\bar{V}$ of Y such that $(f (\bar{B}))^c \subseteq \bar{V}$ and f^{-1}($\bar{V}$) $\subseteq \bar{B}^c$. Therefore $\bar{B} \subseteq (f^{-1}(\bar{V}))^c$.

Hence $\bar{V}^c \subseteq f(\bar{B}) \subseteq f((f^{-1}(\bar{V}))^c) \subseteq \bar{V}^c$ which implies $f(\bar{B}) = \bar{V}^c$. Since $\bar{V}^c$ is (i,v)-fuzzy rw-open set of Y. Hence $f(\bar{B})$ is (i,v)-fuzzy rw-open in Y and thus f is (i,v)-fuzzy rw-open map.

2.7 Theorem: Let $f : (X, \Im_1) \longrightarrow (Y, \Im_2)$ be (i,v)-fuzzy closed map. Then f is an (i,v)-fuzzy rw-closed map.

Proof: Let $\bar{A}$ be (i,v)-fuzzy open set in $(X, \Im_1)$. Since f is (i,v)-fuzzy closed map, $f(\bar{A})$ is an (i,v)-fuzzy closed set in $(Y, \Im_2)$. Since every (i,v)-fuzzy closed set is (i,v)-fuzzy rw-closed, $f(\bar{A})$ is an (i,v)-fuzzy rw-closed set in $(Y, \Im_2)$. Hence f is an (i,v)-fuzzy rw-closed map.

2.3 Remark: The converse of the above theorem need not be true in general.

2.2 Example: Let X = [0, 1] and Y = [0, 1]. The (i,v)-fuzzy sets $\bar{A}$, $\bar{B}$: $X \longrightarrow D[0, 1]$ is defined as

$$\bar{A}(x) = \begin{cases} [0.6, 0.6] & \text{if } x = 1/3 \\ [1, 1] & \text{otherwise} \end{cases}$$

$$\bar{B}(x) = \begin{cases} [0.8, 0.8] & \text{if } x = 1/3 \\ [1, 1] & \text{otherwise.} \end{cases}$$

Consider $\Im_1 = \{\bar{0}, \bar{1}, \bar{A}\}$ and $\Im_2 = \{\bar{0}, \bar{1}, \bar{B}\}$. Then $(X, \Im_1)$ and $(Y, \Im_2)$ are (i,v)-fuzzy topological spaces. Let $f : (X, \Im_1) \longrightarrow (Y, \Im_2)$ be the identity map. Then f is an (i,v)-fuzzy rw-closed map but it is not an (i,v)-fuzzy closed map, since the image of the (i,v)-fuzzy closed set $\bar{A}^C$ in X is not an (i,v)-fuzzy closed set in Y.

2.8 Theorem: Show that the composition of two (i,v)-fuzzy rw-closed maps need not be an (i,v)-fuzzy rw-closed map.

Proof: Consider the (i,v)-fuzzy topological spaces $(X, \Im_1)$, $(Y, \Im_2)$ and $(Z, \Im_3)$ and mappings defined in example in Theorem 2.2. The maps f and g are (i,v)-fuzzy rw-closed but their composition is not (i,v)-fuzzy rw-closed, as $\bar{A} : X \longrightarrow D[0, 1]$ is an (i,v)-fuzzy closed set in X but $(g \bullet f)(\bar{A}) = \bar{A}$ is not (i,v)-fuzzy rw-closed in Z.

2.9 Theorem: If $f : (X, \Im_1) \longrightarrow (Y, \Im_2)$ and $g : (Y, \Im_2) \longrightarrow (Z, \Im_3)$ be two maps. Then $g \bullet f : (X, \Im_1) \longrightarrow (Z, \Im_3)$ is (i,v)-fuzzy rw-closed map if f is (i,v)-fuzzy closed and g is (i,v)-fuzzy rw-closed.

Proof: Let $\bar{A}$ be an (i,v)-fuzzy closed set in $(X, \Im_1)$. Since f is an (i,v)-fuzzy closed map, $f(\bar{A})$ is an (i,v)-fuzzy closed set in $(Y, \Im_2)$. Since g is an (i,v)-fuzzy rw-closed map, $g(f(\bar{A}))$ is an (i,v)-fuzzy rw-closed set in $(Z, \Im_3)$. But $g(f(\bar{A})) = (g \bullet f)(\bar{A})$. Thus $g \bullet f$ is (i,v)-fuzzy rw-closed map.

2.10 Theorem: A map $f : X \longrightarrow Y$ is (i,v)-fuzzy rw-closed if for each (i,v)-fuzzy set $\bar{D}$ of Y and for each (i,v)-fuzzy open set $\bar{E}$ of X such that $\bar{E} \supseteq f^{-1}(\bar{D})$, there is an (i,v)-fuzzy rw-open set $\bar{A}$ of Y such that $\bar{D} \subseteq \bar{A}$ and $f^{-1}(\bar{A}) \subseteq \bar{E}$.

Proof: Suppose that f is (i,v)-fuzzy rw-closed. Let $\bar{D}$ be an (i,v)-fuzzy subset of Y and $\bar{E}$ is an (i,v)-fuzzy open set of X such that $f^{-1}(\bar{D}) \subseteq \bar{E}$. Let $\bar{A} = \bar{1} - f(\bar{1} - \bar{E})$ is (i,v)-fuzzy rw-open set in (i,v)-fuzzy topological space Y. Note that $f^{-1}(\bar{D}) \subseteq \bar{E}$ which implies $\bar{D} \subseteq \bar{A}$ and $f^{-1}(\bar{A}) \subseteq \bar{E}$.

For the converse, suppose that $\bar{E}$ is an (i,v)-fuzzy closed set in X. Then $f^{-1}(\bar{1} - f(\bar{E})) \subseteq \bar{1} - \bar{E}$ and $\bar{1} - \bar{E}$ is (i,v)-fuzzy open. By hypothesis, there is an (i,v)-fuzzy rw-open set $\bar{A}$ of Y such that $\bar{1} - f(\bar{E}) \subseteq \bar{A}$ and $f^{-1}(\bar{A}) \subseteq \bar{1} - \bar{E}$. Therefore $\bar{E} \subseteq \bar{1} - f^{-1}(\bar{A})$. Hence $\bar{1} - \bar{A} \subseteq f(\bar{E})$, $f(\bar{1} - f^{-1}(\bar{A})) \subseteq \bar{1} - \bar{A}$ which implies $f(\bar{E}) = \bar{1} - \bar{A}$. Since $\bar{1} - \bar{A}$ is (i,v)-fuzzy rw-closed, $f(\bar{E})$ is (i,v)-fuzzy rw-closed and thus f is (i,v)-fuzzy rw-closed.

2.11 Theorem: Let $f : (X, \Im_1) \longrightarrow (Y, \Im_2)$ be (i,v)-fuzzy irresolute and $\bar{A}$ be (i,v)-fuzzy regular semiopen in Y. Then $f^{-1}(\bar{A})$ is (i,v)-fuzzy regular semiopen in X.

Proof: Let $\bar{A}$ be (i,v)-fuzzy regular semiopen in Y. To prove $f^{-1}(\bar{A})$ is (i,v)-fuzzy regular semiopen in X. That is to prove $f^{-1}(\bar{A})$ is both (i,v)-fuzzy semiopen and (i,v)-fuzzy semi-closed in X. Now $\bar{A}$ is (i,v)-fuzzy semiopen in Y. Since f is (i,v)-fuzzy irresolute, $f^{-1}(\bar{A})$ is (i,v)-fuzzy semiopen in X. Now $\bar{A}$ is (i,v)-fuzzy semi-closed in Y, as (i,v)-fuzzy regular semiopen set is (i,v)-fuzzy semi-closed. Then $\bar{1} - \bar{A}$ is (i,v)-fuzzy semiopen in Y. Since f is (i,v)-fuzzy irresolute, $f^{-1}(\bar{1} - \bar{A})$ is (i,v)-fuzzy semiopen in X. But $f^{-1}(\bar{1} - \bar{A}) = \bar{1} - f^{-1}(\bar{A})$ is (i,v)-fuzzy semiopen in X and so $f^{-1}(\bar{A})$ is (i,v)-fuzzy semi-closed in X. Thus $f^{-1}(\bar{A})$ is both (i,v)-fuzzy semiopen and (i,v)-fuzzy semi-closed in X and hence $f^{-1}(\bar{A})$ is (i,v)-fuzzy regular semiopen in X.

2.12 Theorem: If a map $f : (X, \Im_1) \longrightarrow (Y, \Im_2)$ is (i,v)-fuzzy irresolute and (i,v)-fuzzy rw-closed and $\bar{A}$ is (i,v)-fuzzy rw-closed set of X, then $f(\bar{A})$ is an (i,v)-fuzzy rw-closed set in Y.

Proof: Let $\bar{A}$ be an (i,v)-fuzzy closed set of X. Let $f(\bar{A}) \subseteq \bar{E}$, where $\bar{E}$ is (i,v)-fuzzy regular semiopen inY. Since f is (i,v)-fuzzy irresolute, $f^{-1}(\bar{E})$

is an (i,v)-fuzzy regular semiopen in X, by Theorem 2.11 and $\overline{A} \subseteq f^{-1}(\overline{E})$. Since $\overline{A}$ is an (i,v)-fuzzy rw-closed set in X, cl($\overline{A}$) $\subseteq f^{-1}(\overline{E})$. Since f is (i,v)-fuzzy rw-closed, f(cl($\overline{A}$)) is an (i,v)-fuzzy rw-closed set contained in the (i,v)-fuzzy regular semiopen set $\overline{E}$, which implies cl(f(cl($\overline{A}$))) $\subseteq \overline{E}$ and hence cl(f($\overline{A}$)) $\subseteq \overline{E}$. Therefore f($\overline{A}$) is an (i,v)-fuzzy rw-closed set in Y.

2.13 Theorem: If a map f : (X, $\Im_1$) $\longrightarrow$ (Y, $\Im_2$) is (i,v)-fuzzy irresolute and (i,v)-fuzzy closed and $\overline{A}$ is an (i,v)-fuzzy rw-closed set in (i,v)-fuzzy topological space X, then f($\overline{A}$) is an (i,v)-fuzzy rw-clsoed set in (i,v)-fuzzy topological space Y.

Proof: The proof follows from the theorem 2.12 and the fact that every (i,v)-fuzzy closed map is an (i,v)-fuzzy rw-closed map.

2.14 Theorem: Let f : X$\longrightarrow$ Y and g : Y$\longrightarrow$ Z be two mappings such that g•f : X$\longrightarrow$ Z is an (i,v)-fuzzy rw-closed map, then

(i) if f is (i,v)-fuzzy continuous and surjective, then g is (i,v)-fuzzy rw-closed and

(ii) if g is (i,v)-fuzzy rw-irresolute and injective, then f is (i,v)-fuzzy rw-closed.

Proof: (i) Let $\overline{E}$ be an (i,v)-fuzzy closed set in Y. Since f is (i,v)-fuzzy continuous, $f^{-1}(\overline{E})$ is an (i,v)-fuzzy closed set in X. Since g•f is an (i,v)-fuzzy rw-closed map, (g•f)($f^{-1}(\overline{E})$) is an (i,v)-fuzzy rw-closed set in Z. But (g•f)($f^{-1}(\overline{E})$) = g($\overline{E}$), as f is surjective. Thus g is (i,v)-fuzzy rw-closed.

(ii) Let $\overline{B}$ be an (i,v)-fuzzy closed set of X. Then (g•f)($\overline{B}$) is an (i,v)-fuzzy rw-closed set in Z, since g•f is an (i,v)-fuzzy rw-closed map. Since g is (i,v)-fuzzy rw-irresolute, g^{-1}(g•f)($\overline{B}$)) is (i,v)-fuzzy rw-closed in Y. But g^{-1}(g•f)($\overline{B}$)) = f($\overline{B}$), as g is injective. Thus f is (i,v)-fuzzy rw-closed map.

2.15 Theorem: If *f* : (X, ℑ)→ (Y, σ) is (i,v)-fuzzy almost irresolute and (i,v)-fuzzy rw-closed map and $\overline{A}$ is an (i,v)-fuzzy w-closed set of X, then *f* ($\overline{A}$) is (i,v)-fuzzy rw-closed.

Proof: Let *f*($\overline{A}$)$\subseteq \overline{O}$ where $\overline{O}$ is an (i,v)-fuzzy regular semi open set of Y. Since *f* is (i,v)-fuzzy almost irresolute therefore $f^{-1}(\overline{O})$ is an (i,v)-fuzzy semi open set of X such that $\overline{A} \subseteq f^{-1}(\overline{O})$. Since $\overline{A}$ is (i,v)-fuzzy w-closed of X which implies that cl($\overline{A}$) $\subseteq f^{-1}(\overline{O})$ and hence *f(cl(* $\overline{A}$*))*$\subseteq \overline{O}$ which implies that cl(f (cl($\overline{A}$)) .) $\subseteq \overline{O}$ therefore cl(f($\overline{A}$)) $\subseteq \overline{O}$ whenever *f(* $\overline{A}$ *)*$\subseteq \overline{O}$ where $\overline{O}$ is an (i,v)-fuzzy regular semi open set of Y. Hence *f(* $\overline{A}$*)* is an (i,v)-fuzzy rw-closed set of Y.

2.16 Theorem: Every (i,v)-fuzzy homeomorphism is (i,v)-fuzzy rw- homeomorphism.

Proof: Let a map f : (X, $\Im_1$) $\longrightarrow$ (Y, $\Im_2$) be an (i,v)-fuzzy homeomorphism. Then f and f^{-1} are (i,v)-fuzzy continuous. Since every (i,v)-fuzzy continuous map is (i,v)-fuzzy rw-continuous, f and f^{-1} are (i,v)-fuzzy rw-continuous. Therefore f is (i,v)-fuzzy rw-homeomorphism.

2.4 Remark: The converse of the above theorem need not be true.

2.3 Example: Let X=Y= { 1, 2, 3 } and the (i,v)-fuzzy sets $\overline{A}$, $\overline{B}$, $\overline{C}$: X$\longrightarrow$ D [0, 1] be defined as $\overline{A}$ = { < 1, [1, 1] >, < 2, [0, 0]>, <3, [0, 0] > }, $\overline{B}$ = { <1, [1, 1] >, <2, [1, 1] >, < 3, [0, 0] > } and $\overline{C}$ = { <1, [1, 1] >, <2, [0, 0] >, <3, [1, 1] > }. Consider $\Im_1$ = { $\overline{0}$, $\overline{1}$, $\overline{A}$, $\overline{C}$ } and $\Im_2$ = { $\overline{0}$, $\overline{1}$, $\overline{B}$ }. Then (X, $\Im_1$) and (Y, $\Im_2$) are (i,v)-fuzzy topological spaces. Define a map f : (X, $\Im_1$) $\longrightarrow$ (Y, $\Im_2$) by f(1) = 1, f(2) = 3 and f(3) = 2. Here the function f is an (i,v)-fuzzy rw-homeomorphism but it is not an (i,v)-fuzzy homeomorphism, as the image of an (i,v)-fuzzy open set $\overline{A}$ in (X, $\Im_1$) is $\overline{A}$ which is not an (i,v)-fuzzy open set in (Y, $\Im_2$).

2.17 Theorem: Let X and Y be (i,v)-fuzzy topological spaces and f : (X, $\Im_1$) $\longrightarrow$ (Y, $\Im_2$) be a bijective map. Then the following statements are equivalent.

(a) f^{-1} is (i,v)-fuzzy rw-continuous
(b) f is an (i,v)-fuzzy rw-open map
(c) f is an (i,v)-fuzzy rw-closed map.

Proof: (a) $\Longrightarrow$(b). Let $\overline{A}$ be any (i,v)-fuzzy open set in X. Since f^{-1} is (i,v)-fuzzy rw-continuous, $(f^{-1})^{-1}(\overline{A})$ = f($\overline{A}$) is (i,v)-fuzzy rw-open in Y. Hence f is an (i,v)-fuzzy rw-open map.

(b) $\Longrightarrow$(c). Let $\overline{A}$ be any (i,v)-fuzzy closed set in X. Then $\overline{1}-\overline{A}$ is (i,v)-fuzzy rw-open in X. Since f is an (i,v)-fuzzy rw-open map, f($\overline{1}-\overline{A}$) is (i,v)-fuzzy rw-open in Y. But f($\overline{1}-\overline{A}$) = $\overline{1}$–f($\overline{A}$), as f is a bijection map. Hence f($\overline{A}$) is (i,v)-fuzzy rw-closed in Y. Therefore f is (i,v)-fuzzy rw-closed.

(c)$\Longrightarrow$(a). Let $\overline{A}$ be any (i,v)-fuzzy closed set in X. Then f($\overline{A}$) is an (i,v)-fuzzy rw-closed set in Y. But $(f^{-1})^{-1}(\overline{A})$ = f($\overline{A}$). Therefore f^{-1} is (i,v)-fuzzy rw-continuous.

2.18 Theorem: Let f : (X, $\Im_1$) $\longrightarrow$ (Y, $\Im_2$) be a bijection and (i,v)-fuzzy rw-continuous map. Then the following statements are equivalent.

(a) f is an (i,v)-fuzzy rw-open map
(b) f is an (i,v)-fuzzy rw-homeomorphism
(c) f is an (i,v)-fuzzy rw-closed map.

Proof: (a) $\Longrightarrow$(b). By hypothesis and assumption f is an (i,v)-fuzzy rw-homeomorphism.

(b) $\Longrightarrow$(c). Since f is an (i,v)-fuzzy rw-homeomorphism; it is (i,v)-fuzzy rw-open. So by the above theorem 2.17, it is an (i,v)-fuzzy rw-closed map.

(c) $\Longrightarrow$(a). Let $\overline{B}$ be an (i,v)-fuzzy open set in X, so that $\overline{1}-\overline{B}$ is an (i,v)-fuzzy closed set and f being (i,v)-fuzzy rw-clsoed, f($\overline{1}-\overline{B}$) is (i,v)-fuzzy rw-closed in Y. But f($\overline{1}-\overline{B}$) = $\overline{1}-$ f($\overline{B}$) thus f($\overline{B}$) is (i,v)-fuzzy rw-open in Y. Therefore f is an (i,v)-fuzzy rw-open map.

2.19 Theorem: Every (i,v)-fuzzy rwc-homeomorphism is (i,v)-fuzzy rw-homeomorphism but not conversely.

Proof: The proof follows from the fact that every (i,v)-fuzzy rw-irresolute map is (i,v)-fuzzy rw-continuous but not conversely.

2.20 Theorem: Let (X, $\Im_1$), (Y, $\Im_2$) and (Z, $\Im_3$) be (i,v)-fuzzy topological spaces and f : (X, $\Im_1$) $\longrightarrow$ (Y, $\Im_2$), g : (Y, $\Im_2$) $\longrightarrow$ (Z, $\Im_3$) be (i,v)-fuzzy rwc-homeomorphisms. Then their composition g•f : (X, $\Im_1$) $\longrightarrow$ (Z, $\Im_3$) is an (i,v)-fuzzy rwc-homeomorphism.

Proof: Let $\overline{A}$ be an (i,v)-fuzzy rw-open set in (Z, $\Im_3$). Since g is an (i,v)-fuzzy rw-irrresolute, g^{-1}($\overline{A}$) is an (i,v)-fuzzy rw-open set in (Y, $\Im_2$). Since f is an (i,v)-fuzzy rw-irresolute, $f^{-1}(g^{-1}$($\overline{A}$)) is an (i,v)-fuzzy rw-open set in (X, $\Im_1$). But f^{-1}(g^{-1}($\overline{A}$)) = (g•f $)^{-1}$($\overline{A}$). Therefore g•f is (i,v)-fuzzy rw-irresolute.

To prove that (g•f $)^{-1}$ is (i,v)-fuzzy rw-irresolute. Let $\overline{B}$ be an (i,v)-fuzzy rw-open set in (X, $\Im_1$). Since f^{-1} is (i,v)-fuzzy rw-irresolute, $(f^{-1})^{-1}$($\overline{B}$) is an (i,v)-fuzzy rw-open set in (Y, $\Im_2$). Also $(f^{-1})^{-1}$($\overline{B}$) = f($\overline{B}$). Since g^{-1} is (i,v)-fuzzy rw-irresolute, $((g^{-1})^{-1}$)(f($\overline{B}$)) is an (i,v)-fuzzy rw-open set in (Z, $\Im_3$). That is ($(g^{-1})^{-1}$)(f($\overline{B}$)) = g(f($\overline{B}$)) = (g•f)($\overline{B}$) = ((g•f $)^{-1}$ $)^{-1}$($\overline{B}$). Therefore (g•f $)^{-1}$ is (i,v)-fuzzy rw-irresolute. Thus g•f and (g•f $)^{-1}$ are (i,v)-fuzzy rw-irresolute. Hence g•f is (i,v)-fuzzy rwc-homoeomorphism.

2.21 Theorem: The set IVFRWC-H(X, $\Im$) is a group under the composition of maps.

Proof: Define a binary operation * : IVFRWC-H(X, $\Im$) × IVFRWC-H(X, $\Im$) $\longrightarrow$ IVFRWC-H(X, $\Im$) by f*g = g•f, for all f, g∈IVFRWC-H(X, $\Im$) and • is the usual operation of composition of maps. Then by theorem 2.20, g•f ∈IVFRWC-H(X, $\Im$). We know that the composition of maps is associative and the identity map I:(X, $\Im$) $\longrightarrow$ (X, $\Im$) belonging to IVFRWC-H(X, $\Im$) serves as the identity element. If f ∈IVFRWC-H(X, $\Im$), then f^{-1}∈IVFRWC-H(X, $\Im$) such that f•f^{-1} = f^{-1}•f = I and so inverse exists for each element of IVFRWC-H(X, $\Im$). Therefore (IVFRWC-H(X, $\Im$), •) is a group under the operation of composition of maps.

2.22 Theorem: Let f : (X, $\Im_1$) $\longrightarrow$ (Y, $\Im_2$) be an (i,v)-fuzzy rwc-homeomorphism. Then f induces an isomorphism from the group IVFRWC-H(X, $\Im_1$) on to the group IVFRWC-H(Y, $\Im_2$).

Proof: Using the map f, we define a map Ψ_f : IVFRWC-H(X, $\Im_1$)$\longrightarrow$IVFRWC-H(Y, $\Im_2$) by Ψ_f(h) = f•h•f^{-1}, for every h ∈IVFRWC-H(X, $\Im_1$). Then Ψ_f is a bijection. Further, for all h_1h_2∈IVFRWC-H(X, $\Im_1$), $\Psi_f(h_1•h_2)$ = f•$(h_1•h_2)$•f^{-1} = (f•h_1•f^{-1})• (f•h_2•f^{-1}) = $\Psi_f(h_1)$•$\Psi_f(h_2)$. Therefore Ψ_f is a homeomorphism and so it is an isomorphism induced by f.

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