P.Kiran Kumar, J.V.Subrahmanyam, P.RamaLakshmi / International Journal of Engineering Research and Applications (IJERA) ISSN: 2248-9622 www.ijera.com
Vol. 3, Issue 1, January-February 2013, pp.181-207

A Review on Non-Linear Vibrations of Thin Shells

P.Kiran Kumar [1], J.V.Subrahmanyam [2], P.RamaLakshmi [3]
1,3 Assistant Professor, Department of Mechanical Engineering, Chaitanya Bharathi Institute of Technology, Gandipet, Hyderabad – 500 075,
2 Professor, Department of Mechanical Engineering, Vasavi College of Engineering, Ibrahimbagh, Hyderabad – 500 0075,

Abstract

This review paper aims to present an updated review of papers, conference papers, books, dissertations dealing with nonlinear vibrations of circular cylindrical thin shells. This paper surveyed mathematically, experimentally, analytically, numerically analyzed vibrations of cylindrical shells. This includes shells of open type, closed type, with and without fluid interactions; shells subjected to free and forced vibrations, radial harmonic excitations, seismically excitations; perfect and imperfect shell structures of various materials with different boundary conditions. This paper presented 210 reference papers in alphabetical order.

This paper is presented as Geometrically nonlinear shell theories, Free and forced vibrations under radial harmonic excitation, Imperfect shells, shells subjected to seismic excitations, References.

1. INTRODUCTION

Most of the structural components are generally subjected to dynamic loadings in their working life. Very often these components may have to perform in severe dynamic environment where in the maximum damage results from the resonant vibrations. Susceptibility to fracture of materials due to vibration is determined from stress and frequency. Maximum amplitude of the vibration must be in the limited for the safety of the structure. Hence vibration analysis has become very important in designing a structure to know in advance its response and to take necessary steps to control the structural vibrations and its amplitude.The non-linear or large amplitude vibration of plates has received considerable attention in recent years because of the great importance and interest attached to the structures of low flexural rigidity. These easily deformable structures vibrate at large amplitudes. The solution obtained based on the lineage models provide no more than a first approximation to the actual solutions. The increasing demand for more realistic models to predict the responses of elastic bodies combined with the availability of super computational facilities have enabled researchers to abandon the linear theories in favor of non-linear methods of solutions.

2. Literature Review

2.1. GEOMETRICALLY NONLINEAR SHELL THEORIES

A short overview of some theories for geometrically nonlinear shells will now be given.

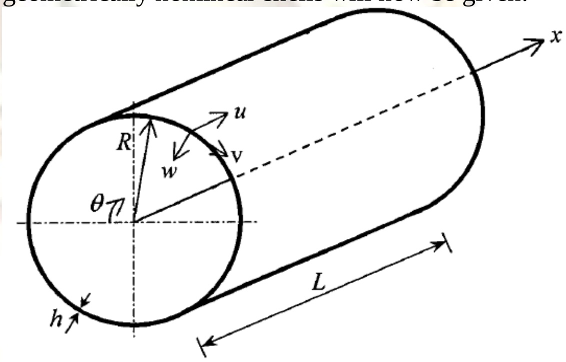

Figure 2.1 shows a circular cylindrical shell with the co-ordinate system and the displacements of the middle surface.

Shell geometry and coordinate system

Donnell (1934) established the nonlinear theory of circular cylindrical shells under the simplifying shallow-shell hypothesis. Due to its relative simplicity and practical accuracy, this theory has been widely used. The most frequently used form of Donnell's nonlinear shallowshell theory (also referred as Donnell-Mushtari-Vlasov theory) introduces a stress function in order to combine the three equations of equilibrium involving the shell displacements in the radial, circumferential and axial directions into two equations involving only the radial displacement w and the stress function F:

The fundamental investigation on the stability of circular cylindrical shells is due to Von Karman and Tsien (1941), who analyzed the static stability (buckling) and the postcritical behavior of axially loaded shells. In this study, it was clarified that discrepancies between forecasts of linear models and experimental results were due to the intrinsic simplifications of linear models; indeed, linear analyses are not able to predict the actual buckling phenomenon observed in experiments; conversely, nonlinear analyses show that the bifurcation path is strongly subcritical, therefore, safe design information can be obtained with a nonlinear analyses only. After this important contribution, many other studies have been published on static and dynamic stability of shells.

Mushtari and Galimov (1957) presented nonlinear theories for moderate and large deformations of thin elastic shells in their book. In the book of Vorovich (1999) the nonlinear theory of shallow shells is also discussed.

Sanders (1963) and Koiter (1966) developed a more refined nonlinear theory of shells, expressed in tensorial form; the same equations were obtained by them around the same period, leading to the designation of these equations as the Sanders-Koiter equations. Later, this theory has been reformulated in lines-of-curvature coordinates, *i.e*. in a form that can be more suitable for applications; see Budiansky (1968). According to the Sanders-Koiter theory, all three displacements are used in the equations of motion.

Changes in curvature and torsion are linear according to both the Donnell and the Sanders-Koiter nonlinear theories (Yamaki 1984). The Sanders-Koiter theory gives accurate results for vibration amplitudes significantly larger than the shell thickness for thin shells.

Details on the above-mentioned nonlinear shell theories may be found in Yamaki's (1984) book, with an introduction to another accurate theory, called the modified Flügge nonlinear theory of shells, also referred to as the Flügge-Lur'e-Byrne nonlinear shell theory (Ginsberg 1973). The Flügge-Lur'e-Byrne theory is close to the general large deflection theory of thin shells developed by Novozhilov (1953) and differs only in terms for change in curvature and torsion. Additional nonlinear shell theories were formulated by Naghdi and Nordgren (1963),using the Kirchhoff hypotheses, and Libai and Simmonds (1988).

2.2. Free and Forced (Radial Harmonic Excitation) Vibrations of Shells

The first study on vibrations of circular cylindrical shells is attributed to Reissner (1955), who isolated a single half-wave (lobe) of the vibration mode and analyzed it for simply supported shells; this analysis is therefore only suitable for circular panels. By using Donnell's nonlinear shallow-shell theory for thin-walled shells, Reissner found that the nonlinearity could be either of the hardening or softening type, depending on the geometry of the lobe. Almost at the same time, Grigolyuk (1955) studied large-amplitude free vibrations of circular cylindrical panels simply supported at all four edges. He used the same shell theory as Reissner (1955) and a two-mode expansion for the flexural displacement involving the first and third longitudinal modes. He also developed a single mode approach. Results show a hardening type nonlinearity. Chu (1961) continued with Reissner's work, extending the analysis to closed cylindrical shells. He found that nonlinearity in this case always leads to hardening type characteristics, which, in some cases, can become quite strong.

Cummings (1964) confirmed Reissner's analysis for circular cylindrical panels simply supported at the four edges; he also investigated the transient response to impulsive and step functions, as well as dynamic buckling. Nowinski (1963) confirmed Chu's results for closed circular shells. He used a single-degree-of-freedom expansion for the radial displacement using the linear mode excited, corrected by a uniform displacement that was introduced to satisfy the continuity of the circumferential displacement. All of the above expansions for the description of shell deformation, except for Grigolyuk's, employed a single mode based on the linear analysis of shell vibrations.

Evensen (1963) proved that Nowinski's analysis was not accurate, because it did not maintain a zero transverse deflection at the ends of the shell. Furthermore, Evensen found that Reissner's and Chu's theories did not satisfy the continuity of in-plane circumferential displacement for closed circular shells. Evensen (1963) noted in his experiments that the nonlinearity of closed shells is of the softening type and weak, as also observed by Olson (1965). Indeed, Olson (1965) observed a slight nonlinearity of the softening type in the experimental response of a thin seamless shell made of copper; the measured change in resonance frequency was only about 0.75 %, for a vibration amplitude equal to 2.5 times the shell thickness. The shell ends were attached to a ring; this arrangement for the boundary conditions gave some kind of constraint to the axial displacement and rotation. Kaña *et al*. (1966; see also Kaña 1966) also found experimentally a weak softening type response for a simply supported thin circular cylindrical shell.

To reconcile this most important discrepancy between theory and experiments, Evensen (1967) used Donnell's nonlinear shallow-shell theory but with a different form for the assumed flexural displacement *w*, involving more modes. Specifically, he included the companion mode in the analysis, as well as an axisymmetric contraction having twice the frequency of the mode excited.

Evensen's assumed modes are not moment-free at the ends of the shell, as they should be for classical simply supported shells, and the homogeneous solution for the stress function is neglected; however, the continuity of the circumferential displacement is exactly satisfied. Evensen studied the free vibrations and the response to a modal excitation without considering damping and discussed the stability of the response curves. His results are in agreement with Olson's (1965) experiments. Evensen's study is an extension of his very well-known study on the vibration of rings (1964, 1965, 1966), wherein he proved theoretically and experimentally that thin circular rings display a softening-type nonlinearity. The other classical work on the vibration of circular rings is due to Dowell (1967a) who confirmed Evensen's results and removed the assumption of zero mid-surface circumferential strain. Evensen (1968) also extended his work to infinitely long shells vibrating in a mode in which the generating lines of the cylindrical surface remain straight and parallel, by using the method of harmonic balance and without considering the companion mode. Evensen (2000) published a paper, in which he studied the influence of pressure and axial loading on large-amplitude vibrations of circular cylindrical shells by an approach similar to the one that he used in his previous papers, but neglecting the companion mode; consequently only backbone curves, pertaining to free vibrations, were computed.

It is important to note that, in most studies, the assumed mode shapes (those used by Evensen, for example) are derived in agreement with the experimental observation that, in large amplitude vibrations, (i) the shell does not spend equal time-intervals deflected outwards and deflected inwards, and (ii) inwards maximum deflections, measured from equilibrium, are larger than outwards ones. However, it is important to say that these differences are very small if compared to the vibration amplitude. Hence, the original idea for mode expansion was to add to asymmetric linear modes an axisymmetric term (mode) giving a contraction to the shell.

Mayers and Wrenn (1967) analyzed free vibrations of thin, complete circular cylindrical shells. They used both Donnell's nonlinear shallow-shell theory and the Sanders-Koiter nonlinear theory of shells. Their analysis is based on the energy approach and shows that nonperiodic (more specifically, quasiperiodic) motion is obtained for free nonlinear vibrations. In their analysis based on Donnell's theory, the same expansion introduced by Evensen, without the companion mode, was initially applied; the backbone curves (pertaining to free vibrations) of Evensen were confirmed almost exactly. A second expansion with an additional degree of freedom was also applied for shells with many axial waves; finally, an expansion with more axisymmetric terms was introduced, but the corresponding backbone curves were not reported. In their analysis based on the Sanders-Koiter theory, an original expansion for the three shell displacements (i.e. radial, circumferential and axial) was used, involving 7 degrees of freedom. However, only one term was used for the flexural displacement, and the axisymmetric radial contraction was neglected; axisymmetric terms were considered for the in-plane displacements. The authors found that the backbone curves for a mode with two circumferential waves predict a hardening-type nonlinearity which increases with shell thickness.

Matsuzaki and Kobayashi (1969a, b) studied theoretically simply supported circular cylindrical shells (1969a), then studied theoretically and experimentally clamped circular cylindrical shells (1969b). Matsuzaki and Kobayashi (1969a) based their analysis on Donnell's nonlinear shallow-shell theory and used the same approach and mode expansion as Evensen (1967), with the opposite sign to the axisymmetric term because they assumed positive deflection outwards. In addition, they discussed the effect of structural damping and studied in detail travelling-wave oscillations. In the paper, considering clamped shells, Matsuzaki and Kobayashi (1969b) modified the mode expansion in order to satisfy the different boundary conditions and retained both the particular and the homogeneous solutions for the stress function. The analysis found softening type nonlinearity also for clamped shells, in agreement with their own experimental results. They also found amplitude-modulated response close to resonance and identified it as a beating phenomenon due to frequencies very close to the excitation frequency.

Dowell and Ventres (1968) used a different expansion and approach in order to satisfy exactly the out-of-plane boundary conditions and to satisfy "on the average" the in-plane boundary conditions. They studied shells with restrained in-plane displacement at the ends and obtained the particular and the homogeneous solutions for the stress function. Their interesting approach was followed by Atluri (1972) who found that some terms were missing in one of the equations used by Dowell and Ventres; Dowell *et al*. (1998) corrected these omissions. The boundary conditions assumed both by Dowell and Ventres and by Atluri constrain the axial displacement at the shell extremities to be zero, so that they are different from the classical constraints of a simply supported shell (zero axial force at both ends). Atluri (1972) found that the nonlinearity is of the hardening type for a closed circular cylindrical shell, in contrast to what was found in experiments. Varadan *et al*. (1989) showed that hardening-type results based on the theory of Dowell and Ventres and Atluri are due to the choice of the axisymmetric term. More recently, Amabili *et al*. (1999b, 2000a) showed that at least the first and thirdaxisymmetric modes (axisymmetric modes with an even number of longitudinal half-waves do not give any contribution) must be included in the mode expansion (for modes with a single longitudinal half-wave), as well as using both the driven and companion modes, to correctly predict the trend of nonlinearity with sufficiently good accuracy.

Sun and Lu (1968) studied the dynamic behaviour of conical and cylindrical shells under sinusoidal and ramp-type temperature loads. The equations of motion are obtained by starting with strain-displacement relationships that, for cylindrical shells, are those of Donnell's nonlinear shallow-shell theory. All three displacements were used, instead of introducing the stress function *F*. Therefore, the three displacements were expanded by using globally six terms, without considering axisymmetric terms in the radial displacement; these six terms were related together to give a single-degree-of-freedom system with only cubic nonlinearity, and the shell thus exhibited hardening type nonlinearity.

Leissa and Kadi (1971) studied linear and nonlinear free vibrations of shallow shell panels, simply supported at the four edges without in-plane restraints. Panels with two different. curvatures in orthogonal directions were studied. This is the first study on the effect of curvature of the generating lines on large-amplitude vibrations of shallow shells. Donnell's nonlinear shallow-shell theory was used in a slightly modified version to take into account the meridional curvature. A single mode expansion of the transverse displacement was used. The Galerkin method was used; the compatibility equation was exactly satisfied, and in-plane boundary conditions were satisfied on the average. Results were obtained by numerical integration and non-simple harmonic oscillations were found. For cylindrical and spherical panels, phase plots show that, during vibrations, inward deflections are larger that outward deflections, as previously found by Reissner (1955) and Cummings (1964); the converse was found for the hyperbolic paraboloid. The analysis shows that for all shells, excluding the hyperbolic paraboloid but including the circular cylindrical panel, a softening nonlinearity is expected, changing to hardening for very large deflections (in particular for vibration amplitude equal to 15 times the shell thickness for a circular cylindrical panel).

El-Zaouk and Dym (1973) investigated free and forced vibrations of closed circular shells having a curvature of the generating lines by using an extended form of Donnell's nonlinear shallow-shell theory to take into account curvature of the generating lines and orthotropy. They also obtained numerical results for circular cylindrical shells; for this special case, their study is almost identical to Evensen's (1967), but the additional effects of internal pressure and orthotropy are taken into account. Their numerical results showed a softening type nonlinearity, which becomes hardening only for very large displacements, at about 30 times the shell thickness. Stability of the free and forced responses was also investigated.

Ginsberg used a different approach to solve the problem of asymmetric (Ginsberg 1973) and axisymmetric (Ginsberg 1974) large-amplitude vibrations of circular cylindrical shells. In fact, he avoided the assumptions of mode shapes and employed instead an asymptotic analysis to solve the nonlinear boundary conditions for a simply supported shell. The solution is reached by using the energy of the system to obtain the Lagrange equations, which are then solved via the harmonic balance method. Both softening and hardening nonlinearities were found, depending on some system parameters. Moreover, Ginsberg used the more accurate Flügge-Lur'e-Byrne nonlinear shell theory, instead of Donnell's nonlinear shallow-shell theory which was used in all the work discussed in the foregoing (excluding the two papers allowing for curvature of the generating lines). The Flügge-Lur'e-Byrne theory is the same referred as the modified Flügge theory by Yamaki (1984). Ginsberg (1973) studied the damped response to an excitation and its stability, considering both the driven and companion modes.

Chen and Babcock (1975) used the perturbation method to solve the nonlinear equations obtained by Donnell's nonlinear shallow-shell theory, without selecting a particular deflection solution. They solved the classical simply supported case and studied the driven mode response, the companion mode participation, and the appearance of a travelling wave. A damped response to an external excitation was found. The solution involved a sophisticated mode expansion, including boundary layer terms in order to satisfy the boundary conditions. They also presented experimental results in good agreement with their theory, showing a softening nonlinearity. Regions with amplitude-modulated response were also experimentally detected.

It is important to state that Ginsberg's (1973) numerical results and Chen and Babcock's (1975) numerical and experimental results constitute fundamental contributions to the study of the influence of the companion mode on the nonlinear forced response of circular cylindrical shells. Moreover, in these two studies, a multi-mode approach involving also modes with $2n$ circumferential waves was used. However, Ginsberg (1973) and Chen and Babcock (1975), in order to obtain numerical results, condensed their models to a two-degree-of-freedom one by using a perturbation approach with some truncation error.

Vol'mir *et al*. (1973) studied nonlinear oscillations of simply supported, circular cylindrical panels and plates subjected to an initial deviation from the equilibrium position (response of the panel to initial conditions) by using Donnell's nonlinear shallow-shell theory. Results were calculated by numerical integration of the equations of motion obtained by Galerkin projection, retaining three or five modes in the expansion. The results reported do not show the trend of nonlinearity but only the time response.

Radwan and Genin (1975) derived nonlinear modal equations by using the Sanders- Koiter nonlinear theory of shells and the Lagrange equations of motion, taking into account imperfections. However, the equations of motion were derived only for perfect, closed shells, simply supported at the ends. The nonlinear coupling between the linear modes, that are the basis for the expansion of the shell displacements, was neglected. As previously observed, this singlemode approach gives the wrong trend of nonlinearity for a closed circular shell. The numerical results give only the coefficients of the Duffing equation, obtained while solving the problem. Radwan and Genin (1976) extended their previous work to include the nonlinear coupling with axisymmetric modes; however, only an approximate coupling was proposed. Most of the numerical results still indicated a hardening type nonlinearity for a closed circular shell; however, a large reduction of the hardening nonlinearity was obtained relative to the case without coupling.

Raju and Rao (1976) employed the Sanders-Koiter theory and used the finite element method to study free vibrations of shells of revolution. They found hardening-type results for a closed circular cylindrical shell, in contradiction with all experiments available. Their paper was discussed by Evensen (1977), Prathap (1978a, b), and then again by Evensen (1978a, b). In particular, Evensen (1977) commented that the authors ignored the physics of the problem: i.e., that thin shells bend more readily than they stretch.

Ueda (1979) studied nonlinear free vibrations of conical shells by using a theory equivalent to Donnell's shallow-shell theory and a finite element approach. Each shell element was reduced to a nonlinear system of two degrees of freedom by using an expansion similar to Evensen's (1967) analysis but without companion mode participation. Numerical results were carried out also for complete circular cylindrical shells and are in good agreement with those obtained by Olson (1965) and Evensen (1967).

Yamaki (1982) performed experiments for large-amplitude vibrations of a clamped shell made of a polyester film. The shell response of the first eight modes is given and the nonlinearity is of softening type for all these modes with a decrease of natural frequencies of 0.6, 0.7, 0.9 and 1.3 % for modes with n = 7, 8, 9 and 10 circumferential waves (and one longitudinal half-wave), respectively, for vibration amplitude equal to the shell thickness (rms amplitude equal to 0.7 times the shell thickness). He also proposed two methods of solution of the problem by using the Donnell's nonlinear shallow-shell theory: one is a Galerkin method and the second is a perturbation approach. No numerical application was performed.

Maewal (1986a, b) studied large-amplitude vibrations of circular cylindrical and axisymmetric shells by using the Sanders-Koiter nonlinear shell theory. The mode expansion used contains the driven and companion modes, axisymmetric modes and terms with twice the number of circumferential waves of the driven modes. However, it seems that axisymmetric modes with three longitudinal half-waves were neglected; as previously discussed, these modes are essential for predicting accurately the trend of nonlinearity. It is strongly believed that the difference between the numerical results of Maewal (1986b) and those obtained by Ginsberg (1973) may be attributed to the exclusion of the axisymmetric modes with three longitudinal half-waves in the analysis. Results were obtained by asymptotic analysis and by a speciallydeveloped finite element method.

Nayfeh and Raouf (1987a, b) studied vibrations of closed shells by using plane-strain theory of shells and a perturbation analysis; thus, their study is suitable for rings but not for supported shells of finite length. They investigated the response when the frequency of the axisymmetric mode is approximately twice that of the asymmetric mode (two-to-one internal resonance). The phenomenon of saturation of the response of the directly excited mode was observed. Raouf and Nayfeh (1990) studied the response of the shell by using the same shell theory previously used by Nayfeh and Raouf (1987a, b), retaining both the driven and companion modes in the expansion, finding amplitude-modulated and chaotic solutions. Only two degrees of freedom were used in this study; an axisymmetric term and terms with twice the number of circumferential waves of the driven and companion modes were obtained by perturbation analysis and added to the solution without independent degrees of freedom. The method of multiple scales was applied to obtain a perturbation solution from the equations of motion. By using a similar approach, Nayfeh *et al*. (1991) investigated the behaviour of shells, considering the presence of a two-to-one internal resonance between the axisymmetric and asymmetric modes for the problem previously studied by Nayfeh and Raouf (1987a, b).

Gonçalves and Batista (1988) conducted an interesting study on fluid-filled, complete circular cylindrical shells. A mode expansion that can be considered a simple generalisation of Evensen's (1967) was introduced by Varadan *et al.* (1989), along with Donnell's nonlinear shallow-shell theory, in their brief note on shell vibrations. This expansion is the same as that used by Watawala and Nash (1983); as previously observed, it is not moment-free at the ends of the shell. The results were compared with those obtained by using the mode expansion proposed by Dowell and Ventres (1968) and Atluri (1972). Varadan *et al*. (1989) showed that the expansion of Dowell and Ventres and Atluri gives hardening-type results, as previously discussed; the results obtained with the expansion of Watawala and Nash correctly display a softening type nonlinearity.

Koval'chuk and Podchasov (1988) introduced a travelling-wave representation of the radial deflection, in order to allow the nodal lines to travel in the circumferential direction over time. This expansion is exactly the same as that introduced by Kubenko *et al*. (1982) to study free vibrations; however, here forced vibrations were studied. It should be recognized that travelling-wave responses were also obtained in all the other previous work that included the companion mode; however, the expansion used was different.

Andrianov and Kholod (1993) used an analytical solution for shallow cylindrical shells and plates based on Bolotin's asymptotic method, but they obtained results only for a rectangular plate. A modified version of the nonlinear Donnell shallow-shell equations was used. Large amplitude vibrations of thin, closed circular cylindrical shells with wafer, stringer or ring stiffening were studied by Andrianov *et al*. (1996) by using the Sanders-Koiter nonlinear shell theory. The solution was obtained by using an asymptotic procedure and boundary layer terms to satisfy the shell boundary conditions. Only the free vibrations (backbone curve) were investigated. The single numerical case investigated showed a softening type nonlinearity.

Chiba (1993a) studied experimentally large-amplitude vibrations of two cantilevered circular cylindrical shells made of polyester sheet. He found that almost all responses display a softening nonlinearity. He observed that for modes with the same axial wave number, the weakest degree of softening nonlinearity can be attributed to the mode having the minimum natural frequency. He also found that shorter shells have a larger softening nonlinearity than longer ones. Travelling wave modes were also observed.

Koval'chuk and Lakiza (1995) investigated experimentally forced vibrations of large amplitude in fiberglass shells of revolution. The boundary conditions at the shell bottom simulated a clamped end, while the top end was free (cantilevered shell). One of the tested shells was circular cylindrical. A weak softening nonlinearity was found, excluding the beam-bending mode for which a hardening nonlinearity was measured. Detailed responses for three different excitation levels were obtained for the mode with four circumferential waves (second mode of the shell). Travelling wave response was observed around resonance, as well as the expected weak softening type nonlinearity.

Kobayashi and Leissa (1995) studied free vibrations of doubly curved thick shallow panels; they used the nonlinear first-order shear deformation theory of shells in order to study thick shells. The rectangular boundaries of the panel were assumed to be simply supported at the four edges without in-plane restraints, as previously assumed by Leissa and

Kadi (1971). A single mode expansion was used for each of the three displacements and two rotations involved in the theory; in-plane and rotational inertia were neglected. The problem was then reduced to one of a single degree of freedom describing the radial displacement. Numerical results were obtained for circular cylindrical, spherical and paraboloidal panels. Except for hyperbolic paraboloid shells, a softening behaviour was found, becoming hardening for vibration amplitudes of the order of the shell thickness. However, increasing the radius of curvature, i.e. approaching a flat plate, the behaviour changed and became hardening. The effect of the shell thickness was also investigated.

Thompson and de Souza (1996) studied the phenomenon of escape from a potential well for two-degree-of-freedom systems and used the case of forced vibrations of axially compressed shells as an example. A very complete bifurcation analysis was performed.

Ganapathi and Varadan (1996) used the finite element method to study large-amplitude vibrations of doubly- curved composite shells. Numerical results were given for isotropic circular cylindrical shells. They showed the effect of including the axisymmetric contraction mode with the asymmetric linear modes, confirming the effectiveness of the mode expansions used by many authors, as discussed in the foregoing. Only free vibrations were investigated in the paper, using Novozhilov's theory of shells. A four-node finite element was developed with five degrees of freedom for each node. Ganapathi and Varadan also pointed out problems in the finite element analysis of closed shells that are not present in open shells. The same approach was used to study numerically laminated composite circular cylindrical shells (Ganapathi and Varadan 1995).

Selmane and Lakis (1997a) applied the finite element method to study free vibrations of open and closed orthotropic cylindrical shells. Their method is a hybrid of the classical finite element method and shell theory. They used the refined Sanders-Koiter nonlinear theory of shells. The formulation was initially general but in the end, to simplify the solution, only a single linear mode was retained. As previously discussed, this approximation gives erroneous results for a complete circular shell. In fact, numerical results for free vibrations of the same closed circular cylindrical shell, simply supported at the ends, investigated by Nowinski (1963) and Kanaka Raju and Venkateswara Rao (1976) showed a hardening type nonlinearity. The criticism raised by Evensen (1977) on Nowinski's and Kanaka Raju and Venkateswara Rao's results should be recalled to explain the hardening type of nonlinearity. Lakis *et al*. (1998) presented a similar solution for closed circular cylindrical shells.

A finite element approach to nonlinear dynamics of shells was developed by Sansour *et al*. (1997, 2002). They implemented a finite shell element based on a specifically developed nonlinear shell theory. In this shell theory, a linear distribution of the transverse normal strains was assumed, giving rise to a quadratic distribution of the displacement field over the shell thickness. They developed a time integration scheme for large numbers of degrees of freedom; in fact, problems can arise in finite element formulations of nonlinear problems with a large number of degrees of freedom due to the instability of integrators. Chaotic behaviour was found for a circular cylindrical panel simply supported on the straight edges, free on the curved edges and loaded by a point excitation having a constant value plus a harmonic component. The constant value of the load was assumed to give three equilibrium points (one unstable) in the static case.

Amabili *et al*. (1998) investigated the nonlinear free and forced vibrations of a simply supported, complete circular cylindrical shell, empty or fluid-filled. Donnell's nonlinear shallowshell theory was used. The boundary conditions on the radial displacement and the continuity of circumferential displacement were exactly satisfied, while the axial constraint was satisfied on the average. Galerkin projection was used and the mode shape was expanded by using three degrees of freedom; specifically, two asymmetric modes (driven and companion modes), plus an axisymmetric term involving the first and third axisymmetric modes (reduced to a single term by an artificial constraint), were employed. The time dependence of each term of the expansion was general. Different tangential constraints were imposed at the shell ends. An inviscid fluid was considered. Solution was obtained both numerically and by the method of normal forms. Numerical results were obtained for both free and forced vibrations of empty and water-filled shells. Some additions to this paper were given by Amabili *et al*. (1999a). Results showed a softening type nonlinearity and travelling-wave response close to resonance. Numerical results are in quantitative agreement with those of Evensen (1967), Olson (1965), Chen and Babcock (1975) and Gonçalves and Batista (1988).

In a series of four papers, Amabili, Pellicano and Païdoussis (1999b, c, 2000a, b) studied the nonlinear stability and nonlinear forced vibrations of a simply supported circular cylindrical shell with and without flow by using Donnell's nonlinear shallow-shell theory. The Amabili *et al*. (1999c, 2000a) papers deal with large-amplitude vibrations of empty and fluid-filled circular shells, also investigated by Amabili *et al*. (1998), but use an improved model for the solution expansion. In Amabili et *al*. (1999c) three independent axisymmetric modes with an odd number of longitudinal half-waves were added to the driven and companion modes. Therefore, the model can be considered to be an extension of the three-degree-of-freedom one developed by Amabili *et al*. (1998), in which the artificial kinematic constraint between the first and third axisymmetric modes, previously used to reduce the number of degrees of freedom, was removed. Results showed that the first and third axisymmetric modes are fundamental for predicting accurately the trend of nonlinearity and that the fifth axisymmetric mode only gives a small contribution.

Driven modes with one and two longitudinal half-waves were numerically computed. Periodic solutions were obtained by a continuation technique based on the collocation method. Amabili *et al*. (2000a) added, to the expansion of the radial displacement of the shell, modes with twice the number of circumferential waves vis-à-vis the driven mode and up to three longitudinal half-waves, in order to check the convergence of the solution with different expansions. Results for empty and water-filled shells were compared, showing that the contained dense fluid largely enhances the weak softening type nonlinearity of empty shells; a similar conclusion was previously obtained by Gonçalves and Batista (1988). Experiments on a water-filled circular cylindrical shell were also performed and successfully compared to the theoretical results, validating the model developed. Experiments showed a reduction of the resonance frequency by about 2 % for a vibration amplitude equal to the shell thickness. These experiments are described in detail in Amabili, Garziera and Negri (2002), where additional experimental details and results are given. It is important to note that, in this series of papers (Amabili *et al*. 1999b, c; 2000a, b), the continuity of the circumferential displacements and the boundary conditions for the radial deflection were exactly satisfied.

Pellicano, Amabili and Païdoussis (2002) extended the previous studies on forced vibrations of shells by using a mode expansion where a generic number of modes can be retained. Numerical results with up to 23 modes were obtained and the convergence of the solution discussed. In particular, it is shown that the model developed by Amabili *et al*. (2000a) is close to convergence. In that study a map is also given, for the first time, showing whether the nonlinearity is softening or hardening as a function of the shell geometry.

If the shell is not very short ($L/R > 0.5$) and not thick ($h/R < 0.045$) the nonlinearity is of the softening type, excluding the case of long ($L/R > 5$) thin shells. It is interesting to note that the boundary between softening and hardening regions has been found to be related to internal resonances (i.e. an integer relationship between natural frequencies) between the driven mode and axisymmetric modes.

Large-amplitude (geometrically nonlinear) vibrations of circular cylindrical shells with different boundary conditions and subjected to radial harmonic excitation in the spectral neighbourhood of the lowest resonances have been investigated by Amabili (2003b). In particular, simply supported shells with allowed and constrained axial displacements at the edges have been studied; in both cases the radial and circumferential displacements at the shell edges are constrained. Elastic rotational constraints have been assumed; they allow simulating any condition from simply supported to perfectly clamped, by varying the stiffness of this elastic constraint. The Lagrange equations of motion have been obtained by energy approach, retaining damping through the Rayleigh's dissipation function. Two different nonlinear shell theories, namely Donnell's and Novozhilov's theories, have been used to calculate the elastic strain energy. The formulation is valid also for orthotropic and symmetric cross-ply laminated composite shells; geometric imperfections have been taken into account. The large-amplitude response of circular cylindrical shells to harmonic excitation in the spectral neighbourhood of the lowest natural frequency has been computed for three different boundary conditions. Circular cylindrical shell with restrained axial displacement at the shell edges display stronger softening type nonlinearity than simply supported shells. Both empty and fluid-filled shells have been investigated by using a potential fluid model.

A problem of internal resonance (one-to-one-to-one-to-two) was studied by Amabili, Pellicano and Vakakis (2000c) and Pellicano *et al*. (2000) for a water-filled shell. In these papers, both modal and point excitations were used.

Autoparametric resonances for free flexural vibrations of infinitely long, closed shells were investigated by McRobie *et al*. (1999) and Popov *et al*. (2001) by using the energy formulation previously developed by the same authors (Popov *et al*. 1998a). A simple two-mode expansion, derived from Evensen (1967) but excluding the companion mode, was used. An energy approach was applied to obtain the equations of motion, which were transformed into action-angle co-ordinates and studied by averaging. Dynamics and stability were extensively investigated by using the methods of Hamiltonian dynamics, and chaotic motion was detected. Laing *et al.* (1999) studied the nonlinear vibrations of a circular cylindrical panel under radial point forcing by using the standard Galerkin method, the nonlinear Galerkin method and the post-processed Galerkin method. The system was discretized by the finite-difference method, similarly to the study of Foale *et al*. (1998). The authors conclude that the post-processed Galerkin method is often more efficient than either the standard Galerkin or nonlinear Galerkin methods.

Kubenko and Koval'chuk (2000b) used Donnell's nonlinear shallow-shell theory with Galerkin projection to study nonlinear vibrations of simply supported circular cylindrical shells. Driven and companion modes were included, but axisymmetric terms were neglected, giving a hardening type response. Interaction between two modes with different numbers of circumferential waves was discussed; this kind of interaction arises in correspondence to internal resonances. Results show interesting interactions between these modes. Another study considering the interaction between two modes with different numbers of circumferential waves was developed by Amiro and Prokopenko (1999), who also did not consider axisymmetric terms in the mode expansion.

Yamaguchi and Nagai (1997) studied vibrations of shallow cylindrical panels with a rectangular boundary, simply supported for transverse deflection and with in-plane elastic support at the boundary. The shell was excited by an acceleration having a constant value plus a harmonic component. Donnell's nonlinear shallow-shell theory was utilized with the Galerkin projection along with a multi-mode expansion of flexural displacement. Initial deflection (imperfection) was taken into account in the theoretical formulation but not in the calculations. The harmonic balance method and direct integration were used. The response of the panel was of the softening type over the whole range of possible stiffness values of the in-plane springs (elastic support), becoming hardening for a vibration amplitude of the order of the shell thickness; in-plane constraints reduce the softening nonlinearity, which turns to hardening for smaller vibration amplitudes. The objective of this study was to investigate regions of chaotic motion; these regions were identified by means of Poincare maps and Lyapunov exponents. It was found that, when approaching the static instability point (due to the constant acceleration load), chaotic shell behaviour may be observed. In a previous study, Nagai and Yamaguchi (1995) investigated shallow cylindrical panels without in-plane elastic support, via an approach similar to that used by Yamaguchi and Nagai (1997). Also in this case, an accurate investigation of the regions of chaotic motion was performed.

Using a similar approach, Yamaguchi and Nagai (2000) studied oscillations of a circular cylindrical panel, simply supported at the four edges, having an elastic radial spring at the center. The panel was excited by an acceleration with a constant term plus a harmonic component, and regions of chaotic motion were identified. Nagai *et al*. (2001) also studied theoretically and experimentally chaotic vibrations of a simply supported circular cylindrical panel carrying a mass at its center. A single-mode approximation was used by Shin (1997) to study free vibration of doubly-curved, simply supported panels. Backbone curves and response to initial conditions were studied. Free vibrations of doubly-curved, laminated, clamped shallow panels, including circular cylindrical panels, were studied by Abe *et al*. (2000). Both first-order shear deformation theory and classical shell theory (analogous to Donnell's theory) were used. Results obtained neglecting in-plane and rotary inertia are very close to those obtained retaining these effects. Only two modes were considered to interact in the nonlinear analysis. Free vibrations and parametric resonance of complete, simply supported circular cylindrical shells were studied by Mao and Williams (1998a, b). The shell theory used is similar to Sanders', but the axial inertia is neglected. A single-mode expansion was used without considering axisymmetric contraction. Han *et al*. (1999) studied the axisymmetric motion of circular cylindrical shells under axial compression and radial excitation. A region of chaotic response was detected.

Extensive experiments on large-amplitude vibrations of clamped, circular cylindrical shells were performed by Gunawan (1998). Two almost perfect aluminum shells and two with axisymmetric imperfection were tested. Contact-free acoustic excitation and vibration measurement were used.

Mostly of the experiments were performed for the mode with 11 and 9 circumferential waves for perfect and imperfect shells, respectively. These modes were selected in order to have a good frequency separation from other modes and avoid nonlinear interaction among different modes. The effect of axial load on the shell response was investigated. Softening type nonlinearity and traveling wave response were observed for all the shells. Nonstationary responses appeared at relatively large vibration amplitudes.

Mikhlin (2000) studied vibrations of circular cylindrical shells under a radial excitation and an axial static load, using Donnell's nonlinear shallow-shell theory with Galerkin projection and two different two-mode expansions. One included the first circumferential harmonic of the driven mode and the other was derived from Evensen's (1967) expansion; in both cases, the companion mode was not considered. Stability and energy "pumping" from one mode to the other were discussed. Lee and Kim (1999) used a single-mode approach to study nonlinear free vibration of rotating cylindrical shells. The equation of motion shows only a cubic nonlinearity, thus explaining the appearance of hardening nonlinearity. Moussaoui *et al*. (2000) studied free, large-amplitude vibrations of infinitely long, closed circular cylindrical shells, neglecting the motion in the longitudinal direction and assuming that the generating lines of the shell remain straight after deformation. Thus, their model is suitable for rings but it is not adequate for real shells of finite length. The system was discretised by using a multi-mode expansion that excluded all axisymmetric terms, which are fundamental. Therefore, an erroneous hardening type nonlinearity was obtained. Nayfeh and Rivieccio (2000) used a single-mode expansion of the displacements to study forced vibrations of simply supported, composite, complete circular cylindrical shells. This approach, as previously discussed, is inadequate for predicting the correct trend of nonlinearity; in fact, hardening type results were erroneously obtained and only a cubic nonlinearity was found in the equation of motion. However, due to the simple expansion, the equation of motion was obtained in closed-form.

Jansen (2002) studied forced vibrations of simply supported, circular cylindrical shells under harmonic modal excitation and static axial load. Donnell's shallow-shell theory was used, but the in-plane inertia of axisymmetric modes was taken into account in an approximate way. In-plane boundary conditions were satisfied on the average. A four-degree-of-freedom expansion of the radial displacement was used with Galerkin's method. Numerical results display a softening type response with traveling wave in the vicinity of resonance. The narrow frequency range in which there are amplitude modulations in the response was deeply investigated. In another paper, Jansen (2001) presented a perturbation approach for both the frequency and the shell coordinates. The resulting boundary value problem was numerically approached by using a parallel shooting method. Results are in good agreement with those obtained by the same author (Jansen 2002) via Galerkin's method.

Amabili *et al*. (2003) experimentally studied large-amplitude vibrations of a stainless steel circular cylindrical panels supported at the four edges. The nonlinear response to harmonic excitation of different magnitude in the neighbourhood of three resonances was investigated. Experiments showed that the curved panel tested exhibited a relatively strong geometric nonlinearity of softening type. In particular, for the fundamental mode, the resonance was reached at a frequency 5 % lower than the natural (linear) frequency, when the vibration amplitude was equal to 0.55 times the shell thickness. This is particularly interesting when these results are compared to the nonlinearity of the fundamental mode of complete (closed around the circumference), simply supported, circular cylindrical shells of similar length and radius, which display much weaker nonlinearity.

Finite-amplitude vibrations of long shells in beam-bending mode were studied by Hu and Kirmser (1971). Birman and Bert (1987) and Birman and Twinprawate (1988) extended the study to include elastic axial constraints, shells on an elastic foundation, and uniformly distributed static loads. Single-mode and two-mode approximations were used. Results show a softening nonlinearity for long shells without restrained axial displacements at the ends, which becomes hardening for long shells with restrained displacements; other boundary conditions are those of a clamped shell.

Circular cylindrical shells carrying a concentrated mass were investigated by Likhoded (1976). Large-amplitude free vibrations of non-circular cylindrical shells on Pasternak foundations were studied by Paliwal and Bhalla (1993) by using the approach developed by Sinharay and Banerjee (1985, 1986).

2.3 Imperfect shells

A point that was still waiting for an answer was the accuracy of the different nonlinear shell theories available. For this reason, Amabili (2003a) computed the large-amplitude response of perfect and imperfect, simply supported circular cylindrical shells subjected to harmonic excitation in the spectral neighbourhood of the lowest natural frequency, by using five different nonlinear shell theories: (i) Donnell's shallow-shell, (ii) Donnell's with in-plane inertia, (iii) Sanders-Koiter, (iv) Flügge-Lur'e-Byrne and (v) Novozhilov's theories. Except for the first theory, the Lagrange equations of motion were obtained by an energy approach, retaining damping through Rayleigh's dissipation function. The formulation is also valid for orthotropic and symmetric cross-ply laminated composite shells. Both empty and fluid-filled shells were investigated by using a potential fluid model. The effect of radial pressure and axial load was also studied. Results from the Sanders-Koiter, Flügge-Lur'e-Byrne and Novozhilov's theories were extremely close, for both empty and water-filled shells. For the thin shell numerically investigated by Amabili (2003a), for which h/R □ □ 288, there is almost no difference among them. Appreciable difference, but not particularly large, was observed between the previous three theories and Donnell's theory with in-plane inertia. On the other hand, Donnell's nonlinear shallow-shell theory turned out to be the least accurate among the five theories compared. It gives excessive softening type nonlinearity for empty shells. However, for water-filled shells, it gives sufficiently precise results, also for quite large vibration amplitude. The different accuracy of the Donnell's nonlinear shallow-shell theory for empty and water-filled shell can easily be explained by the fact that the in-plane inertia, which is neglected in Donnell's nonlinear shallowshell theory, is much less important for a water-filled shell, which has a large radial inertia due to the liquid, than for an empty shell. Contained liquid, compressive axial loads and external pressure increase the softening type nonlinearity of the shell. A minimum mode expansion necessary to capture the nonlinear response of the shell in the neighbourhood of a resonance was determined and convergence of the solution was numerically investigated.

Geometric imperfections of the shell geometry (e.g. out-of-roundness) were considered in buckling problems since the end of the 1950s; *e.g.*, see Agamirov and Vol'mir (1959), Koiter (1963) and Hutchinson (1965). However, there is no trace of their inclusion in studies of largeamplitude vibrations of shells until the beginning of the '70s. Research on this topic was probably introduced by Vol'mir (1972). Kil'dibekov (1977) studied free vibrations of imperfect, simply supported, complete circular shells under pressure and axial loading by using a simple mode expansion involving two degrees of freedom: the driven mode and an axisymmetric term having the same spatial form of the one introduced by Evensen. Imperfections having the same shape as this expansion were assumed. Donnell's nonlinear shallow-shell theory was used. Softening and hardening nonlinearity were found depending on shell geometry. Results show, in contrast with other studies, that initial imperfections change the linear frequency only in the presence of an axial load. Koval'chuk and Krasnopol'skaia (1980) studied forced vibrations of closed, simply supported circular cylindrical shells using Donnell's nonlinear shallow-shell theory. The same expansion of the radial displacement introduced by Evensen (1967) was used. Geometric imperfections having the same shape as the driven mode were considered. Both radial and longitudinal (parametric) excitations were examined. Some unstable zones with nonstationary vibrations and travelling waves were detected, due to differences between the driven- and companion-mode natural frequencies, caused by the geometric imperfections. The authors found softening radial responses. Kubenko *et al.* (1982) used a two-mode travellingwave expansion, taking into account axisymmetric and asymmetric geometric imperfections; the rest of the model for the complete circular shell was similar to those proposed by Koval'chuk and Krasnopol'skaia (1980). Only free vibrations were studied, but the effect of the number of circumferential waves on the nonlinear behaviour was investigated. In particular, the results showed that the nonlinearity is of the softening type and that it increases with the number of circumferential waves. In the studies of Koval'chuk and Krasnopol'skaia (1980) and Kubenko *et al.* (1982) the effect of axisymmetric and asymmetric imperfections was to increase the natural frequency; this is in contrast with results obtained in other studies.

Watawala and Nash (1983) studied the free and forced conservative vibrations of closed circular cylindrical shells using the Donnell's nonlinear shallow-shell theory. Empty and liquidfilled shells with a free-surface and a rigid bottom were studied. A mode expansion that may be considered as a simple generalization of Evensen's (1967) was introduced; therefore, it is not moment-free at the ends of the shell. A single-term was used to describe the shell geometric imperfections. Cases of (i) the circumferential wave pattern of the shell response being the same as that of the imperfection and (ii) the circumferential wave pattern of the response being different from that of the imperfection were analyzed. Numerical results showed softening type nonlinearity. The imperfections lowered the linear frequency of vibrations when the circumferential wave pattern of shell response was the same as that of the imperfection, affecting the observed nonlinearity. Results for forced vibrations were obtained only for beam-bending modes, for which Donnell's nonlinear shallow-shell theory is not appropriate; these results indicated a hardening ype nonlinearity. Hui (1984) studied the influence of imperfections on free vibrations of simply supported circular cylindrical panels; he used Donnell's nonlinear shallow-shell theory and a single-mode expansion; the imperfection was assumed to have the same shape as his single-mode expansion. These imperfections changed the linear vibration frequency (increase or decrease according with the number of circumferential half-waves) and influenced the nonlinearity of the panel.

Jansen (1992) studied large-amplitude vibrations of simply supported, laminated, complete circular cylindrical shells with imperfections. He used Donnell's nonlinear shallowshell theory and the same mode expansion as Watawala and Nash (1983); the boundary conditions at the shell ends were not fully satisfied. The results showed a softening-type nonlinearity becoming hardening only for very large amplitude of vibrations (generally larger than ten times the shell thickness). Imperfections having the same shape as the asymmetric mode analyzed gave a less pronounced softening behaviour, changing to hardening for smaller amplitudes. Moreover, the linear frequency of the imperfect shell may be considerably lower than the frequency of the perfect shell. In a subsequent study Jansen (2002) developed his model further by using a four-degree-of-freedom expansion. However, numerical results are presented only for a perfect shell. In a third paper by the same author (Jansen 2001), results for composite shells with axisymmetric and asymmetric imperfections are given.

Chia (1987a, b) studied nonlinear free vibrations and postbuckling of symmetrically and asymmetrically laminated circular cylindrical panels with imperfections and different boundary conditions. Donnell's nonlinear shallow-shell theory was used. A single-mode analysis was carried out, and the results showed a hardening nonlinearity. In a subsequent study Chia (1988b) also investigated doubly-curved panels with rectangular base by using a similar shell theory, and a single-mode expansion in all the numerical calculations, for both vibration shape and initial imperfection. However, in this study, numerical results for circular cylindrical panels and doubly curved shallow shells generally show a softening nonlinearity, which becomes hardening only for very large vibrations, as expected. The equations of motion are obtained by Gakerkin's method and are studied by using the harmonic balance method. Only the backbone curves are given. Iu and Chia (1988a) studied antisymmetrically laminated cross-ply circular cylindrical panels by using the Timoshenko-Mindlin kinematic hypothesis, which is an extension of Donnell's nonlinear shallow-shell theory. Effects of transverse shear deformation, rotary inertia and geometrical imperfections were included in the analysis. The solution was obtained by the harmonic balance method after Galerkin projection. Fu and Chia (1989) extended this analysis to include a multi-mode approach. Iu and Chia (1988b) used Donnell's nonlinear shallow-shell theory to study free vibrations and post-buckling of clamped and simply supported, asymmetrically laminated cross-ply circular cylindrical shells. A multi-mode expansion was used without considering companion mode, as only free vibrations were investigated; radial geometric imperfections were taken into account. The homogeneous solution of the stress function was retained, but the dependence on the axial coordinate was neglected, differently to what was done by Fu and Chia (1989). The equations of motion were obtained by using the Galerkin method and were studied by the harmonic balance method. Three asymmetric and three axisymmetric modes were used in the numerical calculations. Numerical results were compared to those obtained by El-Zaouk and Dym (1973) for a simply supported, glass-epoxy orthotropic circular cylindrical shell, showing some differences. In a later paper, Fu and Chia (1993) included in their model nonuniform boundary conditions around the edges. Softening or hardening type nonlinearity was found, depending on the radius-to-thickness ratio. Only undamped free vibrations and buckling were investigated in all this series of studies.

Elishakoff *et al*. (1987) conducted a nonlinear analysis of small vibrations of an imperfect cylindrical panel around a static equilibrium position. Librescu and Chang (1993) investigated geometrically imperfect, doubly-curved, undamped, laminated composite panels. The nonlinear theory of shear-deformable shallow panels was used. The nonlinearity was due to finite deformations of the panel due to in-plane loads and imperfections. Only small-amplitude free vibrations superimposed on this finite initial deformation were studied. A single-mode expansion was used to describe the free vibrations and the initial imperfections. The authors found that imperfections of the panels, of the same shape as the mode investigated, lowered the vibration frequency significantly. They also described accurately the post-buckling stability. In fact, curved panels are characterized by an unstable post-buckling behaviour, in the sense that they are subject to a snap-through-type instability. Librescu *et al*. (1996), Librescu and Lin (1997a) and Hause *et al*. (1998) extended this work to include thermomechanical loads and nonlinear elastic foundations (Librescu and Lin 1997b, 1999). Cheikh *et al*. (1996) studied nonlinear vibrations of an infinite circular cylindrical shell with geometric imperfection of the same shape as the mode excited. Plane-strain theory of shells was assumed, which is suitable only for rings (generating lines remain straight and parallel to the shell axis) and a three-degree-of-freedom model, subsequently reduced to a single-degree-of-freedom one, was developed. Results exhibit a hardening type nonlinearity, which is not reasonable for the shell geometry investigated.

Amabili (2003c) included geometric imperfections in the model previously developed by Pellicano *et al*. (2002) and performed in depth experimental investigations on large-amplitude vibrations of an empty and water-filled, simply supported circular cylindrical shell subject to harmonic excitations. The effect of geometric imperfections on natural frequencies was investigated. In particular, it was found that: (i) axisymmetric imperfections do not split the double natural frequencies associated with each couple of asymmetric modes; outward (with respect with the center of curvature) axisymmetric imperfections increase natural frequencies; small inward (with respect with the center of curvature) axisymmetric imperfections decrease natural frequencies; (ii) ovalisation has a small effect on natural frequencies of modes with several circumferential waves and it does not split the double natural frequencies; (iii)imperfections of the same shape as the resonant mode decrease both frequencies, but much more the frequency of the mode with the same angular orientation; (iv) imperfections with the twice the number of circumferential waves of the resonant mode decrease the frequency of the mode with the same angular orientation and increase the frequency of the other mode; they have a larger effect on natural frequencies than imperfections of the same shape as the resonant mode. The split of the double natural frequencies, which is present in almost all real shells due to manufacturing imperfections, changes the traveling wave response. Good agreement betweentheoretical and experimental results was obtained. All the modes investigated show a softening type nonlinearity, which is much more accentuated for the water-filled shell, except in a case displaying internal resonance (one-to-one) among modes with different number of circumferential waves. Travelling wave and amplitude-modulated responses were observed in the experiments.

Results for imperfect shells obtained by Donnell's nonlinear shell theory retaining inplaneinertia and Sanders-Koiter nonlinear shell theory are given by Amabili (2003a).

Kubenko and Koval'chuk (1998) published an interesting review on nonlinear problems of shells, where several results were reported about parametric vibrations; in such review the limitations of reduced order models were pointed out. Babich and Khoroshun (2001) presented results obtained at the S.P. Timoshenko Institute of Mechanics of the National Academy of Sciences of Ukraine over 20 years of research; the authors focused the attention on the variational–difference methods; more than 100 papers were cited.

Kubenko and Koval'chuk (2004) analyzed the stability and nonlinear dynamics of shells, following the historical advancements in this field, about 190 papers were deeply commented; they suggested, among the others, the effect of imperfections as an important issue to be further investigated.

The fundamental investigation on the stability of circular cylindrical shells is due to Von Karman and Tsien (1941), who analyzed the static stability (buckling) and the postcritical behavior of axially loaded shells. In this study, it was clarified that discrepancies between forecasts of linear models and experimental results were due to the intrinsic simplifications of linear models; indeed, linear analyses are not able to predict the actual buckling phenomenon observed in experiments; conversely, nonlinear analyses show that the bifurcation path is strongly subcritical, therefore, safe design information can be obtained with a nonlinear analyses only. After this important contribution, many other studies have been published on static and dynamic stability of shells.

2.4 Shells Subjected to Periodic axial loads

Koval (1974) used the Donnell's nonlinear shallow-shell theory to study the effect of a longitudinal resonance in the parametric transversal instability of a circular shell. He found that, combined parametric resonances give rise to complex regions of parametric instability. However, this kind of phenomenon was found at very high frequency and no damping was included in the analysis.

Hsu (1974) used the Donnell's linear shallow-shell theory to analyzethe parametric instability of a circular cylindrical shell: a uniform pressure load and an axial dynamic load were considered, the former was added in order to eliminate the Poisson's effect in the in-plane stresses. The same problem was studied by Nagai and Yamaki (1978) using the Donnell's linear shallow-shell theory, considering different boundary conditions; in this work, the effect of axisymmetric bending vibrations, induced by the axial load and essentially due to the Poisson's effect, was considered. The classical membrane approach for the in-plane stresses was found inaccurate when the vibration amplitude of axisymmetric modes is not negligible.

Koval'chuck and Krasnopol'skaya (1979) considered the role of imperfections in the resonances of shells subjected to combined lateral and axial loads. They used the Donnell's nonlinear shallow shell theory and a three terms expansion for investigating the onset of travelling waves and nonstationary response; moreover, they gave an initial explanation about discrepancies between theoretical stability boundaries and experiments, addressing such discrepancies to the geometrical imperfections. A further contribution toward such discrepancies is due to Koval'chuck et al. (1982) using the same theory and similar modeling; they claimed that: the theoretical analyses give narrower instability region with respect to experiments if imperfections are not considered. Imperfections cause splitting of conjugate mode frequencies and give rise to doubling of the instability regions.

Bert and Birman (1988) studied the parametric instability of thick circular shells, they developed a special version of the Sanders– Koiter thin-shell theory for thick shells. A single mode analysis, giving rise to a Mathieu type equation of motion, allowed to determine the instability regions.

Argento (1993) used the Donnell's linear theory and the classical lamination theory to study the dynamic stability of composite circular clamped–clamped shells under axial and torsional loading. The linear equations of motion, obtained from discretization, were analyzed by means of the harmonic balance method and the linear instability regions were found. Argento and Scott (1993) studied the dynamic stability of layered anisotropic circular cylindrical shells.

Popov et al. (1998) analyzed the parametric stability and thepostcritical behavior of an infinitely long circular cylindrical shell, dropping the boundary conditions. A three-mode expansion was used, without the inclusion of the companion mode. Membrane theory was used to evaluate the in-plane stresses due to the axial load. The effect of internal resonances between asymmetric modes was analyzed in detail.

Goncalves and Del Prado (2000, 2002) analyzed the dynamic buckling of a perfect circular cylindrical shell under axial static and dynamic loads. The Donnell's nonlinear shallow-shell theory was used and the membrane theory was considered to evaluate the in-plane stresses. The partial differential operator was discretized through the Galerkin technique, using a relatively large modal expansion. However, no companion mode participation was considered and the boundary conditions were dropped by assuming an infinitely long shell. Escape from potential well was analyzed in detail and a correlation of this phenomenon with the parametric resonance was given.

In Pellicano and Amabili (2003), parametric instabilities of circular shells (containing an internal fluid) due to combined static and dynamic axial loads were studied by means of the Donnell's nonlinear shallow shell theory for the shell and potential flow theory for the fluid. Among the other findings it was found that the membrane stress assumptions can lead to wrong forecasting of the dynamics when the dynamics of axisymmetric modes cannot be neglected (this is in agreement with Nagai and Yamaki (1978)), the presence of an internal fluid greatly changes the dynamic response.

In Catellani et al. (2004) a multimode modelling of shells subjected to axial static and periodic loads was proposed considering the Donnell's nonlinear shallow shell theory. The effect of combined static and periodic loads was analyzed in detail considering geometric imperfections. It was found a great influence of imperfections on the instability boundaries and low influence on the postcritical dynamic behaviour for moderate imperfections.

Goncalves and Del Prado (2005) presented a low order model based on the Donnell's nonlinear shallow shell theory, the nonlinear dynamics and stability of circular cylindrical shells was analyzed.

They pointed out that suitable reduced order models could be able to reproduce correctly the shell response. An analytical simplified approach was developed by Jansen (2005), in order to simulate dynamic step and periodic axial loads acting on isotropic and anisotropic shells; he showed that simple periodic responses can be simulated through low dimensional models.

The effect of in-plane inertia was included in the Donnell-type equations: it was found that, neglecting the in-plane inertia gives rise to a moderate underestimation of the instability region. The nonlinear dynamics and chaos of shells subjected to an axial load with or without the presence of a contained fluid was presented in Pellicano and Amabili (2006); both nonlinear Donnell's shallow shell and nonlinear Sanders–Koiter theories were used; a multimode expansion, able to model both postcritical buckling and nonlinear vibrations, was considered for reducing the original nonlinear partial differential equations (PDE) to a set of ordinary differential equations (ODE). The effect of a contained fluid and as analyzed and an accurate analysis of the postcritical dynamics was carried out, including a deep study on the chaotic properties of the response.

Goncalves et al. (2007) studied steady state and transient instabilityof circular shells under periodic axial loads using the Donnell's nonlinear shallow shell theory and a low dimensional Galerkin model. The study clarified the complexity of the basins of attraction of low vibration responses and the escape from the potential well due to the combination of periodic and static compressive loads; moreover, they pointed out that shells are highly sensitive to geometric imperfections.

Darabi et al. (2008) analyzed the dynamic stability of functionally graded circular cylindrical shells using Donnell shallow shell theory. They used a displacement expansion without axisymmetric modes, the Galerkin approach for reducing the PDE to ODE and the Bolotin method for solving the latter one. Only hardening results were obtained, probably due to neglect axisymmetric modes. A detailed bibliographic analysis can be found in such paper.

Del Prado et al. (2009) analyzed the instabilities occurring to a shell conveying an internal heavy fluid and loaded with compressive and periodic axial loads. The Donnell's nonlinear shallow shell theory was used in combination with a low order Galerkin approach. They showed that the combination of a flowing fluid with axial loads can lead the shell to large amplitude of vibration and the response is highly sensitive to geometric imperfections. In Pellicano (2009) the dynamic stability of strongly compressed shells, subjected to periodic axial loads, having geometric imperfections, was investigated using the nonlinear Sanders–Koiter theory, a multimode expansion was considered in the modelling. Such paper clarifies that, the role of imperfections becomes important in the case of combined axial compression and periodic loads; indeed, dynamic instabilities interacts with the static potential well. Kochurov and Avramov (2010) presented an analytical study based on the Donnell's nonlinear shallow shell theory, a low order Galerkin approach and a perturbation procedure. They proved the coexistence of nonlinear normal modes and travelling waves. They pointed out the high modal density of shell like structures and the need of multi-mode models for an accurate analysis.

Ilyasov (2010) developed a theoretical model for analyzing the dynamic stability of shells having viscoelastic behavior. This kind of models can be particularly useful in the case of polymeric materials.

2.5 Shells Subjected to Seismic excitation

Vijayarachavan and Evan-Iwanowski (1967) analyzed, both analytically and experimentally, the parametric instabilities of a circular shell under seismic excitation. The cylinder position was vertical and the base was axially excited by using a shaker. In this problem, the in-plane inertia is variable along the shell axis and, when the base is harmonically excited, it gives rise to a parametric excitation. Instability regions were found analytically and compared with experimental results.

Bondarenko and Galaka (1977) investigated the parametric instabilities of a composite shell under base seismic motion and free top end. They identified principal instability regions of several modes and also secondary regions; they observed that, the transition from stable to unstable regions is accompanied by a ''bang'' that can lead the shell to the collapse.

Bondarenko and Telalov (1982) studied experimentally the dynamic instability and nonlinear oscillations of shells; the frequency response was hardening in the region of the main parametric resonance for circumferential wave number $n = 2$ and softening for $n > 2$.

Trotsenko and Trotsenko (2004), studied vibrations of circular cylindrical shells, with attached rigid bodies, by means of a mixed expansion based on trigonometric functions and Legendre polynomials; they considered only linear vibrations.

Pellicano (2005) presented experimental results about violent vibration phenomena appearing in a shell with base harmonic excitation and carrying a rigid mass on the top. When the first axisymmetric mode is in resonance conditions the top mass undergoes to large amplitude of vibration and a huge out of plane shell vibration is detected (more than 2000g), with a relatively low base excitation (about 10g).

Pellicano and Avramov (2007) published a paper concerning the nonlinear dynamics of a shell with base excitation and a top disk. The work was mainly theoretical, i.e. only some experimental results concerning the linear dynamics were presented; the shell was modelled using the nonlinear Sanders–Koiter theory and a reduced order model was used; the analysis was mainly focused on the principal parametric resonance due to high frequency excitations.A similar analysis was presented also in Avramov and Pellicano (2006).

Pellicano (2007) developed a new method, based on the nonlinear Sanders–Koiter theory, suitable for handling complex boundary conditions of circular cylindrical shells and large amplitude of vibrations. The method was based on a mixed expansions considering orthogonal polynomials and harmonic functions. Among the others, the method showed good accuracy also in the case of a shell connected with a rigid body; this method is the starting point for the model developed in the present research.

Mallon et al. (2008) studied isotropic circular cylindrical shells under harmonic base excitation; they used the Donnell's nonlinear shallow shell theory; a multimode expansion, suitable for analyzing the nonlinear resonance and dynamic instability of a specific mode, was used; they presented comparisons between the semianalytical procedure and a FEM model in the case of linear vibrations or buckling. Using the nonlinear semi-analytical model they found beating and chaotic responses, severe out of plane vibration and sensitivity to imperfections.

Kubenko and Koval'chuk (2009) published an experimental work focused on shells made of composite materials; they pointed out the inadequateness of the linear viscous damping models. Axial loads (base excitation, and free top end of the shell) and combined loads were considered. The analysis of the principal parametric instability was carried out (probably with sub harmonic response, but no information are give about). Dynamic instability regions were determined experimentally: a disagreement between previous theoretical models (narrower region) and experiments (wider) was found, the conjecture made by such scientists was that the disagreement was due to shells geometric imperfections. Such paper summarizes some of experimental results published on a previous book (Kubenko et al., 1984)

Mallon et al. (2010) studied circular cylindrical shells made of orthotropic material, the Donnell's nonlinear shallow shell theory was used with a multimode expansion for discretization (PDE to ODE). They presented also experimental results. The theoretical model considered also the shaker-shell interaction (it is to note that one of the first works concerning the interaction between an electromechanical shaker and a mechanical system is due to Krasnopol'skaya (1976)). Mallon et al. (2010) investigated the imperfections sensitivity and found a reduction of the instability threshold. No good quantitative match between theory and experiments was found: saturation phenomena, beating and chaos were found numerically. However, this can be considered a seminal work due to the intuition that some complex phenomena can be due to the shaker shell interaction.

In Pellicano(2011), experiments are carried out on a circular cylindrical shell, made of a polymeric material (P.E.T.) and clamped at the base by gluing its bottom to a rigid support. The axis of the cylinder is vertical and a rigid disk is connected to the shell top end. In Pellicano (2007) this problem was fully analyzed from a linear Point ofview. Here nonlinear phenomena are investigated by exciting the shell using a shaking table and a sine excitation. Shaking the shell from the bottom induces a vertical motion of the top disk that causes axial loads due to inertia forces. Such axial loads generally give rise to axial-symmetric deformations; however, in some conditions it is observed experimentally that a violent resonant phenomenon takes place, with a strong energy transfer from low to high frequencies and huge amplitude of vibration. Moreover, an interesting saturation phenomenon is observed: the response of the top disk was completely flat as the excitation frequency was changed around the first axisymmetric mode resonance.

Farshidianfar A. and Farshidianfar M.H.(2011) were carried out both theoretical and experimental analyses on a long circular cylindrical shell. A total of 18 modes, consisting of all the three main mode groups (axisymmetric, beam-like and asymmetric) were found under a frequency range of 0–1000 Hz, by only applying acoustical excitation. Acoustical excitation results were compared with those obtained from contact excitation. It was discovered that if one uses contact methods, several exciting points are required to obtain all modes; whereas with acoustical excitation only one acoustical source location for the excitation is needed. Furthermore, acoustical excitation produced much better results, compared with contact excitation, in the frequency band 0–3200 Hz.

Raydin Salahifar(2010) analysed cylindrical shells based on the variational form of Hamilton's principle, the field equations and boundary conditions are formulated for circular cylindrical thin shells under general in-phase and out-of-phase harmonic loads. The resulting field equations are solved in a closed form for general loading and boundary conditions. Through Fourier decomposition of the body forces in the longitudinal direction, a technique for developing the particular solution for general harmonic loading was developed. Through four examples, the results based on the current formulation are shown to be in consistent agreement with those based on established shell models in Abaqus.

Liu and Xing (2012) presented an exact procedure and closed-form solutions with analytically determined coefficients for free vibrations of thin orthotropic circular cylindrical shells with classical boundary conditions. The Donnell–Mushtari thin shell theory and the separation of variables method are employed in the derivation. The proposed method has been verified through comparing its results against results available in literature and of the highly accurate semi-analytical differential quadrature finite element method (S-DQFEM) developed by the authors. The characteristics of the eigenvalues are also examined. It may be known that the Donnell–Mushtari shell theory is the simplest thin shell theory and its results for the lowest frequencies of a closed cylinder may not be as accurate.

Sun and Chu (2012) has been successfully applied the Fourier expansion method to analyze the vibration of a thin rotating cylindrical shell. Validation of this method has been checked by comparisons with previous results in the literature. The method presented makes it possible to derive an analytical solution for a thin rotating cylinder with classical boundary conditions of any type. And, the frequencies of travelling wave can be obtained easily from corresponding frequency determinants. The order of frequency determinant may vary from one to eight depending on the boundary conditions to be satisfied, while the frequency determinant by previous studies [13,26] is always eight-by-eight. One can easily obtain the frequency determinant of a thin rotating cylindrical shell with any given boundary conditions by deleting corresponding rows and columns from frequency determinant of a thin rotating cylindrical shell with ‘‘AD-S-S’’ boundary, and thus complexity of reconstructing frequency determinants for various boundary conditions is avoided.

The literature shows that, in the past, several methods were developed for investigating: i) nonlinear vibration and stability; ii) the role of imperfections; iii) fluid structure interaction. However, there are few experimental studies about dynamic instabilities and the comparisons between theory and experiments are not yet satisfactory.

Therefore, a contribution toward the knowledge of complex dynamic phenomena on shells is welcome; in particular, there is need of quantitative experimental results and effective modelling tools.

References

[1]. ABE, Y. KOBAYASHI and G. YAMADA 2000 *Journal of Sound and Vibration* **234**, 405-426.Non-linear vibration characteristics of clamped laminated shallow shells.

[2]. V. L. AGAMIROV and A. S. VOL'MIR 1959 *Izvestiya Akademii Nauk SSSR, Ordelenie Tekhnicheskykh Nauk, Mekhanika i Mashinostroeni* **3**, 78-83. Behaviour of cylindrical shells under dynamic loading by hydrostatic pressure or by axial loading (in Russian). It was published in English in 1961 by *ARS Journal (Journal of the American Rocket Society)* **31**, 98-101.

[3]. M. AMABILI 2008 Nonlinear Vibrations and Stability of Shells and Plates. New York, USA: Cambridge University Press.

[4]. Amabili, M., Pai¨doussis, M.P., 2003. Review of studies on geometrically nonlinear vibrations and dynamics of circular cylindrical shells and panels, with and without fluid-structure interaction. Applied Mechanics Reviews 56, 349–381.

[5]. M. AMABILI 2003a *Journal of Sound and Vibration*, Vol. 264, No. 5, pp. 1091-1125. Comparison of shell theories for large-amplitude vibrations of circular cylindrical shells: Lagrangian approach.

[6]. M. AMABILI 2003b *AIAA Journal*, Vol. 41, No. 6, 1119-1130. Nonlinear vibrations of circular cylindrical shells with different boundary conditions.

[7]. M. AMABILI 2003c *Journal of Sound and Vibration* Vol. 262, No. 4, pp. 921-975. Theory and experiments for large-amplitude vibrations of empty and fluid-filled circular cylindrical shells with imperfections.

[8]. M. AMABILI, R. GARZIERA and A. NEGRI 2002 *Journal of Fluids and Structures* **16**, 213-227. Experimental study on large-amplitude vibrations of water-filled circular cylindrical shells.

[9]. M. AMABILI and M. P. PAÏDOUSSIS 2003 *Applied Mechanics Reviews*, Vol. 56, No. 4, 349-381. Review of studies on geometrically nonlinear vibrations and dynamics of circular cylindrical shells and panels, with and without fluid-structure interaction.

[10]. M. AMABILI, M. PELLEGRINI AND M. TOMMESANI 2003 *Journal of Sound and Vibration*, Vol. 260, No. 3, pp. 537-547. Experiments on large-amplitude vibrations of a circular cylindrical panel.

[11]. M. AMABILI, F. PELLICANO and M. P. PAÏDOUSSIS 1998 *Journal of Fluids and Structures* **12**, 883-918. Nonlinear vibrations of simply supported, circular cylindrical shells, coupled to quiescent fluid.

[12]. M. AMABILI, F. PELLICANO and M. P. PAÏDOUSSIS 1999a *Journal of Fluids and Structures* **13**, 785-788. Addendum to "Nonlinear vibrations of simply supported, circular cylindrical shells, coupled to quiescent fluid".

[13]. M. AMABILI, F. PELLICANO and M. P. PAÏDOUSSIS 1999b *Journal of Sound and Vibration* **225**, 655-699. Non-linear dynamics and stability of circular cylindrical shells containing flowing fluid. Part I: stability.

[14]. M. AMABILI, F. PELLICANO and M. P. PAÏDOUSSIS 1999c *Journal of Sound and Vibration* **228**, 1103-1124. Non-linear dynamics and stability of circular cylindrical shells containing flowing fluid, Part II: large-amplitude vibrations without flow.

[15]. M. AMABILI, F. PELLICANO and M. P. PAÏDOUSSIS 1999d *Journal of Fluids and Structures* **13**, 159-160. Further comments on the nonlinear vibrations of cylindrical shells.

[16]. M. AMABILI, F. PELLICANO and M. P. PAÏDOUSSIS 2000a *Journal of Sound and Vibration* **237**, 617-640. Non-linear dynamics and stability of circular cylindrical shells containing flowing fluid, Part III: truncation effect without flow and experiments.

[17]. M. AMABILI, F. PELLICANO and M. P. PAÏDOUSSIS 2000b *Journal of Sound and Vibration* **237**, 641-666. Non-linear dynamics and stability of circular cylindrical shells containing flowing fluid, Part IV: large-amplitude vibrations with flow.

[18]. M. AMABILI, F. PELLICANO and A. F. VAKAKIS 2000c *ASME Journal of Vibration and Acoustics* **122**, 346-354. Nonlinear vibrations and multiple resonances of fluid-filled, circular shells, Part 1: equations of motion and numerical results.

[19]. Y. AMIRO and N. Y. PROKOPENKO 1999 *International Applied Mechanics* **35**, 134-139. Study of nonlinear vibrations of cylindrical shells with allowance for energy dissipation.

[20]. V. ANDRIANOV and E. G. KHOLOD 1993 *Journal of Sound and Vibration* **165**, 9-17. Non-linear free vibration of shallow cylindrical shell by Bolotin's asymptotic method.

[21]. V. ANDRIANOV, E. G. KHOLOD and V. I. OLEVSKY 1996 *Journal of Sound and Vibration* **194**, 369-387. Approximate non-linear boundary value problems of reinforced shell dynamics.

[22]. Argento., 1993. Dynamic stability of a composite circular cylindrical shell subjected to combined axial and torsional loading. Journal of Composite Materials 27, 1722–1738.

[23]. ARGENTO and R. A. SCOTT 1993b *Journal of Sound and Vibration* **162**, 323-332. Dynamic instability of layered anisotropic circular cylindrical shells, Part II: numerical results.

[24]. S. Ashley 2001. *Scientific American.* Warp/drive Underwater.

[25]. S. ATLURI 1972 *International Journal of Solids and Structures* **8**, 549-569. A perturbation analysis of non-linear free flexural vibrations of a circular cylindrical shell.

[26]. Avramov, K.V., Pellicano, F. 2006. Dynamical instability of cylindrical shell with big mass at the end, Reports of the National Academy of Science of Ukraine, (in Russian), 5, pp. 41–46.

[27]. Babcock, C.D., 1983. Shell stability. Journal of Applied Mechanics 50, 935–940.

[28]. Babich, D.V., Khoroshun, L.P., 2001. Stability and natural vibrations of shells

with variable geometric and mechanical parameters. International Applied Mechanics. 37 (7), 837–869.

[29]. Bert, C.W., Birman, V., 1988. Parametric instability of thick, orthotropic, circular cylindrical shells. Acta Mechanica 71, 61–76.

[30]. BIRMAN V. and BERT C. W. 1987 *International Journal of Non-Linear Mechanics* **22**, 327-334.

[31]. Non-linear beam-type vibrations of long cylindrical shells.

[32]. V. BIRMAN and P. TWINPRAWATE 1988 *Journal of Applied Mathematics and Physics (ZAMP)* **39**, 768-775. Free nonlinear vibrations of statically loaded long cylindrical shells.

[33]. M. N. BISMARCK-NASR 1992 *ASME Applied Mechanics Reviews* **45**, 461-482. Finite element analysis of aeroelasticity of plates and shells.

[34]. Bondarenko, A.A., Galaka, P.I., 1977. Parametric instability of glass-plastic cylindrical shells. Soviet Applied Mechanics, 13, 411–414. Institute of Mechanics, Academy of Sciences of the Ukrainian SSR, Kiev. Translated from Prikladnaya Mekhanika, 13(4), 124–128.

[35]. Bondarenko, A.A., Telalov, A. I, 1982. Dynamic instability of cylindrical shells under longitudinal kinematics perturbation. Soviet Applied Mechanics, 18(1), 45–49. Institute of Mechanics, Academy of Sciences of the Ukrainian SSR, Kiev. Translated from Prikladnaya Mekhanika, 18(1), 57–61.

[36]. BUDIANSKY 1968 *ASME Journal of Applied Mechanics* **35**, 393-401. Notes on nonlinear shell theory.

[37]. Calladine, C.R., 1995. Understanding imperfection-sensitivity in the buckling of thin-walled shells. Thin-Walled Structures 23, 215–235.

[38]. Catellani, G., Pellicano, F., Dall'Asta, D., Amabili, M., 2004. Parametric instability of a circular cylindrical shell with geometric imperfections. Computers & Structures 82, 2635–2645.

[39]. L. CHEIKH, C. PAUCHON, C.-H. LAMARQUE, A. COMBESCURE and R. J. GIBERT 1996 *Mechanics Research Communications* **23**, 151-164. Nonlinear stability of a defective cylindrical shell. J. C. CHEN and C. D. BABCOCK 1975 *AIAA Journal* **13**, 868-876. Nonlinear vibration of cylindrical shells.

[40]. C. Y. CHIA 1987a *International Journal of Solids and Structures* **23**, 1123-1132. Non-linear free vibration and postbuckling of symmetrically laminated orthotropic imperfect shallow cylindrical panels with two adjacent edges simply supported and the other edges clamped.

[41]. C. Y. CHIA 1987b *International Journal of Engineering Sciences* **25**, 427-441. Nonlinear vibration and postbuckling of unsymmetrically laminated imperfect shallow cylindrical panels with mixed boundary conditions resting on elastic foundation.

[42]. C. Y. CHIA 1988a *ASME Applied Mechanics Reviews* **41**, 439-451. Geometrically nonlinear behaviour of composite plates: a review.

[43]. C. Y. CHIA 1988b *Ingenieur Archiv* **58**, 252-264. Nonlinear analysis of doubly curved symmetrically laminated shallow shells with rectangular platform.

[44]. M. CHIBA 1993a *International Journal of Non-Linear Mechanics* **28**, 591-599. Non-linear hydroelastic vibration of a cantilever cylindrical tank. I Experiment (Empty case).

[45]. M. CHIBA 1993b *International Journal of Non-Linear Mechanics* **28**, 601-612. Non-linear hydroelastic vibration of a cantilever cylindrical tank. II Experiment (Liquid-filled case).

[46]. M. CHIBA 1993c *ASME Journal of Pressure Vessel Technology* **115**, 381-388. Experimental studies on a nonlinear hydroelastic vibration of a clamped cylindrical tank partially filled with liquid.

[47]. M. Chiba, N. Yamaki and J. Tani 1985 *Thin-walled structures* **3**, 1-14. Free vibration of a clamped-free circular cylindrical shell partially filled with liquid. Part III: Experimental results.

[48]. M. Chiba, N. Yamaki and J. Tani 1984 *Thin-walled structures* **2**, 265-284. Free vibration of a clamped-free circular cylindrical shell partially filled with liquid. Part I: Theoretical analysis.

[49]. H.-N. CHU 1961 *Journal of Aerospace Science* **28**, 602-609. Influence of large-amplitudes on flexural vibrations of a thin circular cylindrical shell.

[50]. B. E. CUMMINGS 1964 *AIAA Journal* **2**, 709-716. Large-amplitude vibration and response of curved panels.

[51]. Darabi, M., Darvizeh, M., Darvizeh, A., 2008. Non-linear analysis of dynamic stability for functionally graded cylindrical shells under periodic axial loading. Composite Structures 83, 201–211.

[52]. Del Prado, Z.J.G.N., Goncalves, P.B., Paı¨doussis, M.P., 2009. Nonlinear vibrations and imperfection sensitivity of a cylindrical shell containing axial fluid flow. Journal of Sound and Vibration 327, 211–230.

[53]. E. J. DOEDEL, A. R. CHAMPNEYS, T. F. FAIRGRIEVE, Y. A. KUZNETSOV, B. SANDSTEDE and X. WANG 1998 *AUTO 97: Continuation and Bifurcation Software for Ordinary Differential Equations (with HomCont).* Montreal, Canada: Concordia University.

[54]. L. H. DONNELL 1934 *Transactions of the ASME* **56**, 795-806. A new theory for the buckling of thin cylinders under axial compression and bending.

[55]. E. H. DOWELL 1967a *AIAA Journal* **5**, 1508-1509. On the nonlinear flexural vibrations of rings.

[56]. E. H. DOWELL 1970a *AIAA Journal* **8**, 385-399. Panel flutter: a review of the aeroelastic stability of plates and shells.

[57]. E. H. DOWELL 1978 *Journal of Sound and Vibration* **60**, 596-597. Comments on non-linear flexural vibrations of a cylindrical shell.

[58]. E. H. DOWELL 1998 *Journal of Fluids and Structures* **12**, 1087-1089. Comments on the nonlinear vibrations of cylindrical shells.

[59]. E. H. DOWELL and C. S. VENTRES 1968 *International Journal of Solids and Structures* **4**, 975-991. Modal equations for the nonlinear flexural vibrations of a cylindrical shell.

[60]. E. H. DOWELL, C. S. VENTRES and D. TANG 1998 *Duke University Research Report*, DT 98-1. Modal equations for the nonlinear flexural vibrations of a cylindrical shell.

[61]. ELISHAKOFF, V. BIRMAN and J. SINGER 1987 *Journal of Sound and Vibration* **114**, 57-63.Small vibrations of an imperfect panel in the vicinity of a non-linear static state.

[62]. B. R. EL-ZAOUK and C. L. DYM 1973 *Journal of Sound and Vibration* **31**, 89-103. Non-linear vibrations of orthotropic doubly-curved shallow shells.

[63]. D. A. EVENSEN 1963 *AIAA Journal* **1**, 2857-2858. Some observations on the nonlinear vibration of thin cylindrical shells.

[64]. D. A. EVENSEN 1964 *Non-Linear Flexural Vibrations of Thin Circular Rings*. Ph.D. thesis, California Institute of Technology, Pasadena, CA, USA.

[65]. D. A. EVENSEN 1965 *A Theoretical and Experimental Study of Nonlinear Flexural Vibrations of Thin Circular Rings*, NASA TR R-227. Washington, DC: Government Printing Office.

[66]. D. A. EVENSEN 1966 *ASME Journal of Applied Mechanics* **33**, 553-560. Nonlinear flexural vibrations of thin circular rings.

[67]. D. A. EVENSEN 1967 Nonlinear flexural vibrations of thin-walled circular cylinders. NASA TN D-4090.

[68]. D. A. EVENSEN 1968 *AIAA Journal* **6**, 1401-1403. Nonlinear vibrations of an infinitely long cylindrical shell.

[69]. D. A. EVENSEN 1974 Nonlinear vibrations of circular cylindrical shells. In *Thin-Shell Structures:*

[70]. *Theory, Experiment and Design*, pp. 133-155 (eds Y. C. Fung & E. E. Sechler). Englewood Cliffs, NJ: Prentice-Hall.

[71]. D. A. EVENSEN 1977 *Journal of Sound and Vibration* **52**, 453-454. Comment on “Largeamplitude asymmetric vibrations of some thin shells of revolution”.

[72]. D. A. EVENSEN 1999 *Journal of Fluids and Structures* **13**, 161-164. Non-linear vibrations of cylindrical shells – Logical rationale.

[73]. D. A. EVENSEN 2000 In *Nonlinear Dynamics of Shells and Plates* (eds. M. P. Païdoussis, M. Amabili and P. B. Gonçalves), *AMD Vol. 238*, pp. 47-59, ASME, New York. The influence of initial stress and boundary restraints on the nonlinear vibrations of cylindrical shells.

[74]. D. A. EVENSEN and R. E. FULTON 1967 *Proceedings of the International Conference on Dynamic Stability of Structures*, Evanston, IL, 18-20 October 1965, pp. 237-254, Pergamon Press. Some studies on the nonlinear dynamic response of shell-type structures.

[75]. Farshidianfar A. and Farshidianfar M.H , 2011, Vibration analysis of long cylindrical shells using acoustical excitation. Journal of Sound and Vibration 330,3381-3399.

[76]. S. FOALE, J. M. T. THOMPSON and F. A. MCROBIE 1998 *Journal of Sound and Vibration* **215**, 527-545. Numerical dimension-reduction methods for non-linear shell vibrations.

[77]. Y. M. FU and C. Y. CHIA 1989 *International Journal of Non-Linear Mechanics* **24**, 365-381. Multi-mode non-linear vibration and postbuckling of anti-symmetric imperfect angle-ply cylindrical thick panels.

[78]. Y. M. FU and C. Y. CHIA 1993 *International Journal of Non-Linear Mechanics* **28**, 313-327. Non-linear vibration and postbuckling of generally laminated circular cylindrical thick shells with non-uniform boundary conditions.

[79]. M. GANAPATHI and T. K. VARADAN 1995 *Composite Structures* **30**, 33-49. Nonlinear free flexural vibrations of laminated circular cylindrical shells.

[80]. M. GANAPATHI and T. K. VARADAN 1996 *Journal of Sound and Vibration* **192**, 1-14. Largeamplitude vibrations of circular cylindrical shells.

[81]. J. H. GINSBERG 1973 *ASME Journal of Applied Mechanics* **40**, 471-477. Large-amplitude forced vibrations of simply supported thin cylindrical shells.

[82]. J. H. GINSBERG 1974 *ASME Journal of Applied Mechanics* **41**, 310-311. Nonlinear axisymmetric free vibration in simply supported cylindrical shells.

[83]. P. B. GONÇALVES and R. C. BATISTA 1988 *Journal of Sound and Vibration* **127**, 133-143. Nonlinear vibration analysis of fluid-filled cylindrical shells.

[84]. Goncalves, P.B., Del Prado, Z.J.G.N., 2000. The role of modal coupling on the nonlinear response of cylindrical shells subjected to dynamic axial loads. In: Proceedings of the Symposium on Nonlinear Dynamics of Shells and Plates. ASME International Mechanical Engineering Congress and Expo. (AMD vol. 238), Orlando, USA, pp. 105–116.

[85]. Goncalves, P.B., Del Prado, Z.J.G.N., 2002. Nonlinear oscillations and stability of parametrically excited cylindrical shells. Meccanica 37, 569–597.

[86]. Goncalves, P.B., Del Prado, Z.J.G.N., 2005. Low-dimensional Galerkin models for nonlinear vibration and instability analysis of cylindrical shells. Nonlinear Dynamics 41, 129–145.

[87]. Goncalves, P.B., Silva, F.M.A., Del Prado, Z.J.G.N., 2007. Transient and steady state stability of cylindrical shells under harmonic axial loads. International Journal of Non-Linear Mechanics 42, 58–70.

[88]. E. I. GRIGOLYUK 1955 *Prikladnaya Matematika i Mekhanika* **19**, 376-382. Vibrations of circular cylindrical panels subjected to finite deflections (in Russian).

[89]. L. GUNAWAN 1998 *Experimental study of nonlinear vibrations of thin-walled cylindrical shells*. Ph.D. Thesis, Technische Universiteit Delft, The Netherlands.

[90]. Hairer, E., Wanner, G., 1996. Solving Ordinary Differential Equations II. Stiff and Differential-Algebraic Problems, Second revised ed.. Springer Series in Comput. Mathematics, vol. 14 Springer-Verlag.

[91]. Q. HAN, H. HU and G. YANG 2000 *European Journal of Mechanics A: Solids* **18**, 351-360. A study of chaotic motion in elastic cylindrical shells.

[92]. T. HAUSE, L. LIBRESCU and T. F. JOHNSON 1998 *International Journal of Non-Linear Mechanics* **33**, 1039-1059. Non-linear response of geometrically imperfect sandwich curved panels under thermomechanical loading.

[93]. Hsu, C., 1974. On parametric excitation and snap-through stability problems of shells. In: Fung, Y.C., Sechler, E.E. (Eds.), Thin-Shell Structures. Theory Experiments and Design. Prentice-Hall, Englewood Cli?s, Prentice-Hall, New Jersey, pp. p 103–131.

[94]. K. K. HU and P. G. KIRMSER 1971 *ASME Journal of Applied Mechanics* **38**, 461-466. Non-linear beam-type vibrations of long cylindrical shells.

[95]. D. HUI 1984 *ASME Journal of Applied Mechanics* **51**, 383-390. Influence of geometric imperfections and in-plane constraints on nonlinear vibrations of simply supported cylindrical panels.

[96]. J. HUTCHINSON 1965 *AIAA Journal* **3**, 1461-1466. Axial buckling of pressurized imperfect cylindrical shells.

[97]. Ilyasov, M.H., 2010. Parametric vibrations and stability of viscoelastic shells. Mechanics of Time-Dependent Materials 14, 153–171.

[98]. Jansen, E., 2005. Dynamic stability problems of anisotropic cylindrical shells via a simplified analysis. Nonlinear Dynamics 39, 349–367.

[99]. E. L. JANSEN 1992 *Proceedings of the 17th International Seminar on Modal Analysis, Katholieke Universiteit Leuven, Leuven*, Vol. **3**, 1463-1479. Nonlinear vibrations of imperfect composite cylindrical shells.

[100]. E. L. JANSEN 2001 *Proceedings of the 42th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics and Materials Conference*, Seattle, WA, pp. 1-10 (AIAA paper 2001-1314). Nonlinear vibration analysis of composite cylindrical shells using semi-analytical formulation.

[101]. E. L. JANSEN 2002 *International Journal of Non-Linear Mechanics* **37**, 937-949. Non-stationary flexural vibration behaviour of a cylindrical shell.

[102]. D. D. KAÑA 1966 in *The Dynamic Behaviour of Liquids in Moving Containers*, editor H. N. Abramson, NASA-SP-106, Washington, DC: Government Printing Office, pp. 303-352. Interaction between liquid propellants and the elastic structure.

[103]. D. D. KAÑA, U. S. LINDHOLM and H. N. ABRAMSON 1966 *Journal of Spacecraft and Rockets* **3**, 1183-1188. Experimental study of liquid instability in a vibrating elastic tank.

[104]. T. VON KÁRMÁN and H.-S. TSIEN 1941 *Journal of the Aeronautical Sciences* **8**, 303-312. The buckling of thin cylindrical shells under axial compression.

[105]. G. KIL'DIBEKOV 1977 *Prikladnaya Mekhanika* **13(11)**, 46-52. Investigation of the characteristic nonlinear oscillations of a cylindrical shell (in Russian). English version in *Soviet Applied Mechanics* **13**, 1109- 1114.

[106]. Y. KOBAYASHI and A. W. LEISSA 1995 *International Journal of Non-Linear Mechanics* **30**, 57-66. Large-amplitude free vibration of thick shallow shells supported by shear diaphragms.

[107]. Kochurov, R., Avramov, K.V., 2010. Nonlinear modes and travelling waves of parametrically excited cylindrical shells. Journal of Sound and Vibration 329, 2193–2204.

[108]. W. T. KOITER 1963 *Proceedings Koninklijke Nederlandse Akademie van Wetenschappen* **B 66**, 265-279. The effect of axisymmetric imperfections on the buckling of cylindrical shells under axial compression.

[109]. W. T. KOITER 1966 *Proceedings Koninklijke Nederlandse Akademie van Wetenschappen* **B 69**, 1- 54. On the nonlinear theory of thin elastic shells. I, II, III.

[110]. P. S. KOVAL'CHUK and T. S. KRASNOPOL'SKAYA 1980 *Soviet Applied Mechanics* **15**, 867-872. Resonance phenomena in nonlinear vibrations of cylindrical shells with initial imperfections.

[111]. P. S. KOVAL'CHUK and V. D. LAKIZA 1995 *International Applied Mechanics* **31**, 923-927. Experimental study of induced oscillations with large deflections of fiberglass shells of revolution.

[112]. P. S. KOVAL'CHUK and N. P. PODCHASOV 1988 *Soviet Applied Mechanics* **24**, 1086-1090. Nonlinear flexural waves in cylindrical shells with periodic excitation.

[113]. Koval'chuck, P.S., Krasnopol'skaya, Podchsov, T.S.N.P., 1982. Dynamic instability of circular cylindrical shells with initial camber. Institute of Mechanics, Academy of Sciences of the Ukranian SSR, Kiev. Translated from Prikladnaya Mekhanika 18 (3), 28–33.

[114]. Koval'chuck, P.S., Krasnopol'skaya, T.S., 1979. Resonance phenomena in nonlinear vibrations for cylindrical shells with initial imperfections. Institute of Mechanics, Academy of Sciences of the Ukranian SSR, Kiev. Translated from Prikladnaya Mekhanika 15 (9), 100–107.

[115]. Koval, L.R., 1974. Effect of longitudinal resonance on the parametric stability of an axially excited cylindrical shell. Journal of Acoustic Society of America 55 (1), 91–97.

[116]. Krasnopol'skaya, T.S., 1976. Self-excitations by an electrodynamic

vibrator. Kiev State University, Translated from Prikladnaya Mekhanika 13 (2), 108–113.

[117]. V. D. KUBENKO and P. S. KOVAL'CHUK 1998 *International Applied Mechanics* **34**, 703-728. Nonlinear problems of the vibration of thin shells (review).

[118]. V. D. KUBENKO and P. S. KOVAL'CHUK 2000a *Prikladnaya Mekhanika* **36**(4), 3-34. Nonlinear problems of the dynamics of elastic shells partially filled with a liquid (in Russian). English version in *International Applied Mechanics* **36**, 421-448.

[119]. V. D. KUBENKO and P. S. KOVAL'CHUK 2000b In *Nonlinear Dynamics of Shells and Plates* (eds. M. P. Païdoussis, M. Amabili and P. B. Gonçalves), *AMD Vol. 238*, pp. 147-153, ASME, New York. Problems of nonlinear multiple-mode vibrations of thin elastic shells of revolution.

[120]. V. D. KUBENKO, P. S. KOVAL'CHUK and T. S. KRASNOPOL'SKAYA 1982 *Soviet Applied Mechanics* **18**, 34-39. Effect of initial camber on natural nonlinear vibrations of cylindrical shells.

[121]. Kubenko, V.D., Koval'chuk, P.S., 2004. Influence of initial geometric imperfections on the vibrations and dynamic stability of elastic shells. International Applied Mechanics 40 (8), 847–877.

[122]. Kubenko, V.D., Koval'chuk, P.S., 2009. Experimental studies of the vibrations and dynamic stability of laminated composite shells. International Applied Mechanics 45 (5), 514–533.

[123]. Kubenko, V.D., Koval'chuck P.S., Krasnopol'skaya. T.S., 1984. Nonlinear interaction of the flexural modes of vibration of cylindrical shells (in Russian). Nauk. Dumka, Kiev.

[124]. C. R. LAING, A. MCROBIE and J. M. T. THOMPSON 1999 *Dynamics and Stability of Systems* **14(**2), 163-181. The post-processed Galerkin method applied to non-linear shell vibrations.

[125]. Y-S. LEE and Y-W. KIM 1999 *Computers and Structures* **70**, 161-168. Nonlinear free vibration analysis of rotating hybrid cylindrical shells.

[126]. W. LEISSA 1973 *Vibration of Shells*, NASA SP-288. Washington, DC: Government Printing Office. Now available from The Acoustical Society of America (1993).

[127]. W. LEISSA and A. S. KADI 1971 *Journal of Sound and Vibration* **16**, 173-187. Curvature effects on shallow shell vibrations.

[128]. Leissa, A.W., 1993. Vibration of shells, NASA SP-288. Government Printing Office. Now available from The Acoustical Society of America, Washington, DC.

[129]. LIBAI and J. G. SIMMONDS 1988 *The Nonlinear Theory of Elastic Shells*. London, UK: Academic Press (2nd edition 1998).

[130]. L. LIBRESCU and M.-Y. CHANG 1993 *Acta Mechanica* **96**, 203-224. Effects of geometric imperfections on vibration of compressed shear deformable laminated composite curved panels.

[131]. L. LIBRESCU and W. LIN 1997a *International Journal of Solids and Structures* **34**, 2161-2181. Vibration of thermomechanically loaded flat and curved panels taking into account geometric imperfections and tangential edge restraints.

[132]. L. LIBRESCU and W. LIN 1997b *International Journal of Non-Linear Mechanics* **326**, 211-225. Postbuckling and vibration of shear deformable flat and curved panels on a non-linear elastic foundation.

[133]. L. LIBRESCU and W. LIN 1999 *International Journal of Solids and Structures* **36**, 4111-4147. Non-linear response of laminated plates and shells to thermomechanical loading: implications of violation of interlaminar shear traction continuity requirement.

[134]. L. LIBRESCU, W. LIN, M. P. NEMETH and J. H. STARNES JR. 1996 *Journal of Spacecraft and Rockets* **33**, 285-291. Vibration of geometrically imperfect panels subjected to thermal and mechanical loads.

[135]. LIKHODED 1976 *Mechanics of Solids* **11**, 127-132. Nonlinear vibrations of shells of revolution with concentrated masses.

[136]. G. Lin, B. Balachandran and E. Abed 2006 *Proceedings of the ASME International Mechanics Engineering Congress and Exposition, Chicago, IL.*

Bifurcation behaviour of a supercavitating vehicle.

[137]. Liu B.,Xing Y.F., Qatu M.S., 2012. Exact characteristic equations for free vibrationsnof thin orthotropic circular cylindrical shells. Composite Structures94,484-493.

[138]. MAEWAL 1986a *International Journal of Non-Linear Mechanics* **21**, 433-438. Miles' evolution equations for axisymmetric shells: simple strange attractors in structural dynamics.

[139]. MAEWAL 1986b *Computer Methods in Applied Mechanics and Engineering* **58**, 37-50. Finite element analysis of steady nonlinear harmonic oscillations of axisymmetric shells.

[140]. Mallon, N.J., 2008. Dynamic Stability if Thin-Walled Structures: A Semi-Analytical and Experimental Approach. PhD Thesis, Eindhoven University of Technology Library, ISBN 978-90-386-1374-1.

[141]. Mallon, N.J., Fey, R.H.B., Nijmeijer, H., 2008. Dynamic stability of a thin cylindrical shell with top mass subjected to harmonic base-acceleration. International Journal of Solids and Structures 45 (6), 1587–1613.

[142]. Mallon, N.J., Fey, R.H.B., Nijmeijer, H., 2010. Dynamic stability of a base-excited thin orthotropic cylindrical shell with top mass: simulations and experiments.Journal of Sound and Vibration 329, 3149–3170.

[143]. R. MAO and F. W. WILLIAMS 1998a *Journal of Sound and Vibration* **210**, 307-327. Post-critical behaviour of orthotropic circular cylindrical shells under time dependent axial compression.

[144]. R. MAO and F. W. WILLIAMS 1998b *International Journal of Solids and Structures* **35**, 2151- 2171. Nonlinear analysis of cross-ply thick cylindrical shells under axial compression.

[145]. Y. MATSUZAKI and S. KOBAYASHI 1969a *Journal of the Japan Society for Aeronautical and Space Sciences* **17**, 308-315. An analytical study of the nonlinear flexural vibration of thin circular cylindrical shells (in Japanese).

[146]. Y. MATSUZAKI and S. KOBAYASHI 1969b *Transactions of the Japan Society for Aeronautical and Space Sciences* **12**, 55-62. A theoretical and experimental study of the nonlinear flexural vibration of thin circular cylindrical shells with clamped ends.

[147]. J. MAYERS and B. G. WRENN 1967 On the nonlinear free vibrations of thin circular cylindrical shells. *Developments in Mechanics, Proceedings of the 10th Midwestern Mechanics Conference*, 819-846. New York: Johnson Publishing Co.

[148]. Y. V. MIKHLIN 2000 In *Nonlinear Dynamics of Shells and Plates* (eds. M. P. Païdoussis, M. Amabili and P. B. Gonçalves), *AMD Vol. 238*, pp. 95-103, ASME, New York. Stability of regular or chaotic post-buckling vibration of elastic shells.

[149]. F. MOUSSAOUI, R. BENAMAR and R. G. WHITE 2000 *Journal of Sound and Vibration* **232**, 917- 943. The effects of large vibration amplitudes on the mode shapes and natural frequencies of thin elastic shells, Part I: coupled transverse-circumferential mode shapes of isotropic circular cylindrical shells of infinite length.

[150]. KH. M. MUSHTARI and K. Z. GALIMOV 1957 *Non-Linear Theory of Thin Elastic Shells*. Academy of Sciences (Nauka), Kazan'; English version, NASA-TT-F62 in 1961.

[151]. Nagai, K., Yamaki, N., 1978. Dynamic stability of circular cylindrical shells under periodic compressive forces. Journal of Sound and Vibration 58 (3), 425–441.

[152]. K. NAGAI and T. YAMAGUCHI 1995 ASME PVP Vol. 297, pp. 107-115. Chaotic oscillations of a shallow cylindrical shell with rectangular boundary under cyclic excitation.

[153]. K. NAGAI, T. YAMAGUCHI and T. MURATA 2001 *Proceedings of the 3rd International Symposium on Vibration of Continuous Systems*, 49-51, Grand Teton National Park, Wyoming, USA, July 23-27. Chaotic oscillations of a cylindrical shell-panel with concentrated mass under gravity and cyclic load.

[154]. P. M. NAGHDI and R. P. NORDGREN 1963 *Quarterly of Applied Mathematics* **21**, 49-59. On the nonlinear theory of elastic shells under the Kirchhoff hypothesis.

[155]. H. NAYFEH and R. A. RAOUF 1987a *International Journal of Solids and Structures* **23**, 1625- 1638. Non-linear oscillation of circular cylindrical shells.

[156]. H. NAYFEH and R. A. RAOUF 1987b *ASME Journal of Applied Mechanics* **54**, 571-577. Nonlinear forced response of infinitely long circular cylindrical shells.

[157]. H. NAYFEH, R. A. RAOUF and J. F. NAYFEH 1991 *ASME Journal of Applied Mechanics* **58**, 1033-1041. Nonlinear response of infinitely long circular cylindrical shells to subharmonic radial loads.

[158]. J. F. NAYFEH and N. J. RIVIECCIO 2000 *Journal of Aerospace Engineering* **13**(2), 59-68. Nonlinear vibration of composite shell subjected to resonant excitations.

[159]. V. V. NOVOZHILOV 1953 *Foundations of the Nonlinear Theory of Elasticity*. Graylock Press, Rochester, USA (now available from Dover, N.Y., USA).

[160]. J. NOWINSKI 1963 *AIAA Journal* **1**, 617-620. Nonlinear transverse vibrations of orthotropic cylindrical shells.

[161]. M. D. OLSON 1965 *AIAA Journal* **3**, 1775-1777. Some experimental observations on the nonlinear vibration of cylindrical shells.

[162]. M. P. PAÏDOUSSIS 2002 *Fluid-Structure Interactions: Slender Structures and Axial Flow*. Vol. 2. London, UK: Academic Press.

[163]. M. P. PAÏDOUSSIS, M. AMABILI and P. B. GONÇALVES (editors) 2000 *Nonlinear Dynamics of Shells and Plates*, AMD-Vol. 238, Proceedings of the ASME International Mechanical Engineering Congress and Exposition, ASME, New York.

[164]. M. P. PAÏDOUSSIS and J.-P. DENISE 1972 *Journal of Sound and Vibration* **20**, 9-26. Flutter of thin cylindrical shells conveying fluid.

[165]. D. A. PALIWAL and V. BHALLA 1993 *International Journal of Pressure Vessels and Piping* **54**, 387-398. Large amplitude free vibrations of cylindrical shell on Pasternak foundations.

[166]. D. A. PALIWAL and V. BHALLA 1993 *Journal of Vibration and Acoustics* **115**, 70-74. Large amplitude free vibration of shallow spherical shell on a Pasternak foundation.

[167]. F. PELLICANO, M. AMABILI and A. F. VAKAKIS 2000 *ASME Journal of Vibration and Acoustics* **122**, 355-364. Nonlinear vibrations and multiple resonances of fluid-filled, circular shells, Part 2: perturbation analysis.

[168]. F. PELLICANO, M. AMABILI and M. P. PAÏDOUSSIS 2002 *International Journal of Non-Linear Mechanics* **37**, 1181-1198. Effect of the geometry on the non-linear vibration of circular cylindrical shells.

[169]. Pellicano, F., Amabili, M., 2003. Stability and vibration of empty and fluid-filled circular cylindrical shells subjected to dynamic axial loads. International Journal of Solids and Structures 40, 3229–3251.

[170]. Pellicano, F., 2005. Experimental analysis of seismically excited circular cylindrical shells. In: Proceedings of ENOC-2005, Fifth EUROMECH Nonlinear Dynamics Conference, Eindohven, The Netherlands, August 7–12.

[171]. Pellicano, F., Amabili, M., 2006. Dynamic instability and chaos of empty and fluidfilled circular cylindrical shells under periodic axial loads. Journal of Sound and Vibration 293 (1–2), 227–252. Pellicano, F., Avramov, K.V., 2007. Linear and nonlinear dynamics of a circular cylindrical shell connected to a rigid disk. Communications in Nonlinear Science and Numerical Simulation 12 (4), 496–518.

[172]. F. PELLICANO 2007 *Journal of Sound and Vibration* **303**, 154–170. Vibrations of circular cylindrical shells: Theory and experiments.

[173]. Pellicano, F., 2009. Dynamic stability and sensitivity to geometric imperfections of strongly compressed circular cylindrical shells under dynamic axial loads. Communications in Nonlinear Science and Numerical Simulations 14 (8), 3449– 3462.

[174]. POPOV, J. M. T. THOMPSON and F. A. MCROBIE 1998a *Journal of Sound and Vibration* **209**, 163-186. Low dimensional models of shell vibrations. Parametrically excited vibrations of cylindrical shells.

[175]. POPOV, J. M. T. THOMPSON and F. A. MCROBIE 2001 *Journal of Sound and Vibration* **248**, 395-411. Chaotic energy exchange through auto-parametric resonance in cylindrical shells.

[176]. G. PRATHAP 1978a *Journal of Sound and Vibration* **56**, 303-305. Comments

on the large amplitude asymmetric vibrations of some thin shells of revolution.

[177]. G. PRATHAP 1978b *Journal of Sound and Vibration* **59**, 295-297. On the large amplitude vibration of circular cylindrical shells.

[178]. H. RADWAN and J. GENIN 1975 *International Journal of Non-Linear Mechanics* **10**, 15-29. Nonlinear modal equations for thin elastic shells.

[179]. K. K. RAJU and G. V. RAO 1976 *Journal of Sound and Vibration* **44**, 327-333. Large-amplitude asymmetric vibrations of some thin shells of revolution.

[180]. R. A. RAOUF and A. H. NAYFEH 1990 *Computers and Structures* **35**, 163-173. One-to-one autoparametric resonances in infinitely long cylindrical shells.

[181]. Raydin Salahifar,2010, Analysis of circular cylindrical shells under harmonic forces. Thin-walled Structures 48,528-539.

[182]. E. REISSNER 1955 Nonlinear effects in vibrations of cylindrical shells. Ramo-Wooldridge Corporation Report AM5-6.

[183]. J. L. SANDERS JR. 1963 *Quarterly of Applied Mathematics* **21**, 21-36. Nonlinear theories for thin shells.

[184]. SANSOUR, W. WAGNER, P. WRIGGERS and J. SANSOUR 2002 *International Journal of Non- Linear Mechanics* **37**, 951-966. An energy-momentum integration scheme and enhanced strain finite elements for the non-linear dynamics of shells.

[185]. SANSOUR, P. WRIGGERS and J. SANSOUR 1997 *Nonlinear Dynamics* **13**, 279-305. Nonlinear dynamics of shells: theory, finite element formulation, and integration schemes.

[186]. SELMANE and A. A. LAKIS 1997a *International Journal for Numerical Methods in Engineering* **40**, 1115-1137. Influence of geometric non-linearities on free vibrations of orthotropic open cylindrical shells.

[187]. K. SHIN 1997 *Computers and Structures* **62**, 35-49. Large amplitude free vibration behavior of doubly curved shallow open shells with simply-supported edges.

[188]. G. C. SINHARAY and B. BANERJEE 1985 *International Journal of Non-Linear Mechanics* **20**, 69- 78. Large-amplitude free vibrations of shallow spherical shell and cylindrical shell – a new approach.

[189]. G. C. SINHARAY and B. BANERJEE 1986 *AIAA Journal* **24**, 998-1004. Large-amplitude free vibrations of shells of variable thickness – a new approach.

[190]. Snyder, M.A., 1966. Chebyshev Methods in Numerical Approximation. Prentice-Hall, London.

[191]. Teng, J.G., 1996. Buckling of thin shells: recent advances and trends. Applied Mechanics Reviews 49 (4), 263–274.

[192]. L. SUN and S. Y. LU 1968 *Nuclear Engineering Design* **7**, 113-122. Nonlinear dynamic behaviour of heated conical and cylindrical shells.

[193]. Sun S.,Chu S.,2012, Vibration characteristics of thin rotating cylindrical shells with various boundary conditions. Journal of Sound and Vibration, 331, 4170- 4186.

[194]. J. M. T. THOMPSON and J. R. DE SOUZA 1996 *Proceedings of the Royal Society of London Series A* **452**, 2527-2550. Suppression of escape be resonant modal interactions: in shell vibration and heave-roll capsize.

[195]. Trotsenko, V.A., Trotsenko, Yu.V., 2004. Methods for calculation of free vibrations of a cylindrical shell with attached rigid body. Nonlinear Oscillations 7 (2), 262– 284.

[196]. T. UEDA 1979 *Journal of Sound and Vibration* **64**, 85-95. Non-linear free vibrations of conical shells.

[197]. T. K. VARADAN, G. PRATHAP and H. V. RAMANI 1989 *AIAA Journal* **27**, 1303-1304. Nonlinear free flexural vibration of thin circular cylindrical shells.

[198]. Vijayarachavan, A., Evan-Iwanowski, R.M., 1967. Parametric instability of circular cylindrical shells. Journal of Applied Mechanics 34, 985–990.

[199]. S. VOL'MIR 1956 *Flexible Plates and Shells* (in Russian). Gittl, Moscow.

[200]. S. VOL'MIR 1972 *Nonlinear Dynamics of Plates and Shells* (in Russian). Academy of Sciences (Nauka), Moscow.

[201]. S. VOL'MIR, A. A. LOGVINSKAYA and V. V. ROGALEVICH 1973 *Soviet Physics – Doklady* **17**, 720-721. Nonlinear natural vibrations of rectangular plates and cylindrical panels.

[202]. VOROVICH 1999 *Nonlinear Theory of Shallow Shells*. Springer-Verlag, New York.
[203]. L. WATAWALA and W. A. NASH 1983 *Computers and Structures* **16**, 125-130. Influence of initial geometric imperfections on vibrations of thin circular cylindrical shells.
[204]. S. WOLFRAM 1999 *The Mathematica Book*, 4th edition. Cambridge, UK: Cambridge University Press.
[205]. Xu, X., Pavlovskaia, E., Wiercigroch, M., Romeo, F., Lenci, S., 2007. Dynamic interactions between parametric pendulum and electro-dynamical shaker.
[206]. T. YAMAGUCHI and K. NAGAI 1997 *Nonlinear Dynamics* **13**, 259-277. Chaotic vibration of a cylindrical shell-panel with an in-plane elastic-support at boundary.
[207]. T. YAMAGUCHI and K. NAGAI 2000 In *Nonlinear Dynamics of Shells and Plates* (eds. M. P. Païdoussis, M. Amabili and P. B. Gonçalves), *AMD Vol. 238*, pp. 85-94, ASME, New York. Chaotic oscillations of a shallow cylindrical shell-panel with a concentrated elastic-support.
[208]. N. YAMAKI 1982 In *Developments in Thin-Walled Structures* **1** (eds. J. Rhodes and A. C. Walker), pp. 81-118, Applied Science Publishers, London. Dynamic behaviour of a thinwalled circular cylindrical shell.
[209]. N. YAMAKI 1984 *Elastic Stability of Circular Cylindrical Shells*. Amsterdam: North-Holland.
[210]. J. C. YAO 1963 *AIAA Journal* **1**, 1391-1396. Dynamic stability of cylindrical shells under static and periodic axial and radial loads.