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Impact Of Mesh Quality Parameters On Elements Such As Beam, Shell And 3D Solid In Structural Analysis

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Abstract.

This study compares the performance of linear and quadratic tetrahedral elements and hexahedral elements in various structural problems. The problems selected demonstrate different types of behaviour, namely, bending, shear, torsional and axial deformations. It was observed that the results obtained with quadratic tetrahedral elements and hexahedral elements were equivalent in terms of accuracy. The comparison is done for linear static problems, modal analyses and nonlinear analyses involving large deflections, contact and plasticity. The advantages and disadvantages are shown using tetrahedral and hexahedral elements. Some recommendations and general rules are given for finite element users in choosing the element shape.

Keywords – Linear and Quadratic Element, Bending, Shear, Torsional and Axial Deformation

1. Introduction

Finite element analysis has reached a state of maturity in which 3-D applications are commonplace. Most analysts, as well as most commercial codes (MSC/Nastran, etc.), use solid elements based on the iso-parametric formulation--or variations of it for 3D analyses [1-4]. For simple geometries, or for applications in which it is possible to build a mesh "by hand", analysts have relied heavily on the 8-node hexahedral element commonly known as "brick" or "hexa" [5]. For more complex geometries, however, the analyst must rely on automatic (or semi-automatic) mesh generators. In general, automatic mesh generators produce meshes made of tetrahedral elements, rather than hexahedral elements. The reason is that a general 3-D domain cannot always be decomposed into an assembly of bricks. However, it can always be represented as a collection of tetrahedral elements. As the demand for analyses of more complex configurations has grown, coupled with the increasing popularity of automatic mesh generators, the need to understand better the relative merits of tetrahedral and hexahedral elements has become apparent. It is known, for example, that linear tetrahedral elements do not perform very well--as expected because they are constant-strain elements; thus, too many elements are required to achieve a satisfactory accuracy. What remains unclear, however, is whether brick elements perform better or worse than quadratic tetrahedra, that is, tetrahedral elements including mid-side nodes. Specifically, for a given number of nodes (or degrees of freedom), the analyst needs to know under what circumstances it is better to use bricks instead of quadratic tetrahedra. This amounts to investigating the accuracy and efficiency of such elements under a variety of problems characterized by different deformation patterns, such as, bending, shear, torsion and axial behaviour. In addition, if a mesh made of linear tetrahedral elements does not yield a result within acceptable error, it is useful to know what strategy to follow: (a) decrease the size of the elements while keeping them linear, or (b) make the elements quadratic by introducing additional (mid-side) nodes. Previous authors have proposed some useful benchmark tests for individual elements or simple arrays of elements [6-8]. However, no study comparing tetrahedra with hexahedra in a more general setting seems to be available. While it is difficult to give a final answer to all the issues involved, the aim of this study is to shed some light on this problem by investigating the performance of tetrahedral and hexahedral elements in a number of problems that have known analytical solutions. These findings are expected to be useful for finite elements analysts.

Today, the some finite element method is not only applied to mechanical problems by some specialists anymore who know every single finite element and its function. The finite element method has become a standard numerical method for the virtual product development and is also applied by designers who are not permanent users and have less detailed understanding of the element functionality. With the rapid development in hardware performance and easy-to-use finite element software, the finite element method is not used only for simple problems any more. Today finite element models are often so complex that a mapped mesh with hexahedral shaped elements is often not economically feasible. Experience shows that the most efficient and common way is to perform the analysis using quadratic tetrahedral elements. As a consequence of that, the total number of the degrees of freedom for a complex model increases dramatically. Finite element models containing several millions degrees of freedom are regularly solved. Typically iterative equation solvers are used for solving the linear equations. Figure1shows typical models meshed with tetrahedra and hexahedra elements.

With modern finite element tools it is not difficult to represent results as color pictures. However, the correctness of the results are actually the cornerstone of the simulation. The correctness of the numerical results crucially depends on the element quality itself. There are no general rules which can be applied just to decided which element shape should be preferred but there do exist some basic principles and also certain experiences from applications which can be very helpful in avoiding simulation errors and in judging the validity of the results. In this paper we compare some analytic solutions and experimental results with finite element results coming from a mesh of tetrahedra and hexahedra. We also compare the solutions on tetrahedra and hexahedra for complex models, performing linear and nonlinear static and dynamic analyses.

2. Method

2.1 Bending

Consider a cantilever beam oriented in the y-direction and loaded in the z-direction (see Fig. 1). The beam has a rectangular cross-section and it deforms under the action of a load per unit of length equal to 0.01. The beam dimensions are as follows: L (length)= 8, b (width) = 1 and h (height) --- 1. The material properties are: E (Young's modulus) = 1000 and u (Poisson's ratio) = 0.15. The analytical expression for the vertical displacement at the free end of the beam centre-line, including both bending and shear deformations (although bending is the dominant effect in this case), yields a value of 0.0625 [9].

2.2 Shear

Consider a short shear beam deforming under a unit distributed load (load per unit of length) as depicted in Fig. 2. The beam is oriented in the y-direction and loaded in the z-direction. The beam dimensions are: L = 1, b = 0.6 and h = 1. The material properties are: E = 1000 and u = 0.15. The vertical displacement at the free end of the beam center-line, considering both bending and shear deformations (which in this case are dominant) is 0.00538[10].

2.3 Torsion

Consider a beam with a square cross-section oriented along the y-axis. The beam dimensions are: L = 16, b = 1 and h = 1. Material properties: E = 1000 and u = 0.15. Displacements in the x- and z-directions are fixed at one end. At the other end, which is free, a rotation of 0.03 radians is applied (this corresponds to a 0.1146 torsional moment). The maximum value of the shear stress occurs at the mid-points of the cross-section sides. A solution based on a series expansion gives a value of 0.551 for the maximum shear stress [11].This solution allows warping of the cross-section.

2.4 Axial behaviour

Consider a short beam clamped at both ends and oriented in the y-direction (see Fig. 4). L = 4, b = 1 and h = 1. In addition, E = 1000, v = 0.0 and p (mass density per unit of volume) = 1. The natural frequency corresponding to the first axial mode is 3.953 Hz [12]. This problem was chosen because it involves a non uniform axial displacement field.

3. Analysis

The finite element analyses were performed using Nastran, a general-purpose finite element code for structural analysis [13]. Three solid elements were tested:

(a) C3D4, a 4-node tetrahedral element. This element was included only for comparison purposes; its performance was not expected to be good since it is a constant-strain element. One integration point is used.

(b) C3D10, a second-order 10-node tetrahedral element. In this study, the "intermediate" nodes were located exactly halfway between the corner nodes. Four integration points are used.

(c) C3D8, an 8-node isoparametric hexahedral element. This is a trilinear element. In this case "full" Gauss integration was employed in the stiffness matrix determination. This means that the Gauss scheme used integrates the stiffness matrix terms exactly if

(i) the material properties are constant throughout the element and

(ii) the Jacobian of the mapping from the isoparametric coordinates to the physical coordinates is constant and diagonal throughout the element.

Each problem was solved using four different models (four different meshes), described as Follows:

Mesh 1. This is a regular mesh made of linear tetrahedral elements (C3D4).

Mesh 2. This is a regular mesh made of quadratic tetrahedra (C3D10) obtained by adding mid-side nodes to Mesh 1. This represents an attempt to improve the accuracy of the results obtained with the first mesh.

Mesh 3. This mesh corresponds to another attempt to improve the results obtained with

Mesh 1, but in this case decreasing the size of the linear tetrahedra (C3D4). This mesh Obviously has more nodes than the mesh employed in the first model, but exactly the same Number of nodes as Mesh 2. This is to make the second and third model comparable in terms of the size (same number of degrees of freedom) and therefore address the issue of what strategy is better if one wants to improve the accuracy of the results given by a mesh of linear tetrahedra (Mesh 1): to increase the order of the interpolation (Mesh 2) or to reduce the size of the elements (Mesh 3).

Mesh 4. This is a regular mesh of brick elements. Again, the number of nodes is the same as in Mesh 2. This is to compare the performance of two meshes with the same number of degrees of freedom, one made with bricks and the other made with quadratic tetrahedra. (Notice that nodal coordinates in Mesh 2 coincide with those of Mesh 4).

4. Results

The four problems described before were solved using the four different meshes. The analysis was performed on CAE lab, Pune using Nastran. The models were setup with Hypermesh, a geometric modeller that has a parametric solid object representation and is integrated with an automatic mesh generator and a Nastran pre-processor. Regular meshes were employed in all cases. In each case, the error was computed by comparing the result given by the finite element model against the analytical solution.

Tables 1-4 summarize the results. The nomenclature is as follows: N is the number of nodes in the mesh (including mid-side nodes when quadratic tetrahedra are used); E is the number of elements in the mesh; Δx, Δy, Δz denote the node spacing in the x-, y- and z-direction (in meshes made of quadratic tetrahedra, the spacing is determined by the distance between corner nodes);

Table 1- Result for Bending

Mesh Type	E	N	Δx	Δy	Δz	Vertical Displacement $\times 10^{-2}$ (R)	% Error
Mesh 1	576	225	0.5	0.333	0.5	3.822	38.9
Mesh 2	576	1225	0.5	0.333	0.5	6.210	0.7
Mesh 3	4800	1225	0.25	0.26	0.25	5.334	14.7
Mesh 4	960	1225	0.25	0.26	0.25	6.264	0.2

The analytical solution gives the vertical displacement at the end of the beam centre line
$R= 6.254 \times 10^{-2}$

It is important to know that for bending dominated problems only linear hexahedra elements lead to good results if extra shape functions or enhanced strain formulations are used. Linear tetrahedrons tend to be too stiff in bending problems. By increasing the number of the elements in depth the structure is still too stiff. The quadratic mid-side node tetrahedron element shows the exact analytic solution for pure bending dominated problems even with a coarse mesh with only one element in depth.It is obvious that using a linear tetrahedron element yields unacceptable approximations. The user should not use it for bending dominated problems. On the other hand quadratic mid-side node tetrahedra elements are good for bending dominated problems.

To illustrate the difference in mapping the stiffness of a structure in a correct manner using different element types we perform a modal analysis of a cantilever beam. The first two frequencies and mode shapes are computed. We take the solution of quadratic hexahedra elements as a reference solution and compare the results with a mesh of quadratic tetrahedra and linear tetrahedra with a coarse and a fine mesh respectively.

A good agreement in modelling the stiffness of the structure correctly is just obtained if quadratic tetrahedra elements are taken. Even the fine mesh of linear tetrahedra elements does not result in a good approximation of the solution.

Table 2- Result for Shear

Mesh Type	E	N	Δx	Δy	Δz	Vertical Displacement $\times 10^{-3}$(R)	% Error
Mesh 1	2475	768	0.3	0.15	0.15	4.687	12.8
Mesh 2	2475	4123	0.3	0.15	0.15	4.808	9.2
Mesh 3	3420	4123	0.2	0.033	0.033	4.739	11.8
Mesh 4	19620	4123	0.2	0.033	0.033	4.756	11.5

The analytical solution gives the vertical displacement at the end of the beam centre line
$R= 5.375\times 10^{-3}$

Table 3- Result for Torsion

Mesh Type	E	N	Δx	Δy	Δz	R	% Error
Mesh 1	384	153	0.5	1.00	0.5	0.3859	30
Mesh 2	384	825	0.5	1.00	0.5	0.5799	5.2
Mesh 3	3200	825	0.25	0.5	0.25	0.4736	14.1
Mesh 4	640	825	0.25	0.5	0.25	0.5300	3.8

The analytical solution gives maximum shear stress on the cross section of the beam
R = 0.5511.

Table 4- Result for Axial Deformation

Mesh Type	E	N	Δx	Δy	Δz	R	% Error
Mesh 1	1536	425	0.25	0.25	0.25	3.848	2.7
Mesh 2	1536	2673	0.25	0.25	0.25	3.845	2.8
Mesh 3	10752	2673	0.125	0.125	0.125	3.793	4.2
Mesh 4	1792	2673	0.125	0.125	0.125	3.743	4.2

The analytical solution gives the fundamental frequency (Hz) for axial vibrations R = 3.953..

5. Tetrahedral and hexahedral element solution in nonlinearities

Now we will compare the tetrahedra elements with the hexahedral elements in nonlinear applications. A nonlinear contact simulation has been performed first to compare the local stress coming from a quadratic tetrahedra discretization with the results from a quadratic hexahedra discretization. The material behaviour is linear. Geometric nonlinearities have been ignored.
The advantage for hexahedra is, one can achieve the good stress result, without having very fine mesh.

6. Discussion

The main goal of this analysis was to investigate the performance of hexahedral elements versus quadratic tetrahedra under similar conditions. This has been achieved by comparing the results given by Mesh 2 and Mesh 4. The location of the nodes is identical in both meshes. Thus, the number of active degrees of freedom is exactly the same. This is necessary to make a meaningful comparison. In addition, the element aspect ratio in both meshes is equivalent (the ratio between the node spacing in the x-, y- and z-direction is the same in both meshes). It can be observed that the results obtained with bricks and quadratic tetrahedra, in terms of both accuracy, are roughly equivalent. This is significant because it indicates that analysts who rely on automatic mesh generators (which in general generate meshes made of tetrahedral elements) do not have a disadvantage compared to those analysts who use bricks. In other words, the trilinear brick element—a long-time favourite of many finite element practitioners--appears not to have a substantial advantage compared to the quadratic tetrahedron. A second conclusion is concerned with what is the best approach to take if a model made of linear tetrahedra does not give satisfactory results (Mesh 1). These analyses (Mesh 2 versus Mesh 3) suggest that, in general, it seems better to increase the order of the elements rather than refining the mesh with smaller linear elements. Except for Problem 4, in which Mesh 2 and Mesh 3 give approximately the same result, the quadratic tetrahedra do better than the linear tetrahedra, for the same number of nodes. In terms of CPU time, the advantage of quadratic tetrahedra is more clear-there is a threefold penalty, in all cases, for using linear tetrahedra. This is because Mesh 3 includes many more elements than Mesh 2 and consequently the CPU time required to generate the stiffness matrix and mass matrices increases, as does the time for solving the resulting linear system of equations.

7. The quality of tetrahedra elements in thin-walled structures

Now we will investigate the quality of quadratic tetrahedral elements when used for simulating the mechanical behaviour of thin-walled structures. We investigate the stiffness of the plate by performing a modal analysis and compare the numerical results with the analytic solution for the first frequencies. Because of the nature of thin-walled structure (no stiffness normal to plane) usually Kirchhof-Love or Reissner-Mindlin based shell elements are used for the finite element simulation instead of classical displacement based solid elements. The geometric modelling effort to be able to use finite shell elements might be expensive nowadays since for shell applications the user typically needs a mid-surface model. However, most of the CAD models are 3D solid models and the user must work on the solid model to obtain a mid-surface model which is usually not an easy task. For very complicated 3D solid models it is very difficult and maybe even impossible to get the mid-surface in an efficient way. It follows that more and more thin-walled 3D solid models are meshed and calculated using quadratic tetrahedral elements. Caution must be taken in using tetrahedral elements for thin-walled structure since the structural behaviour could be much too stiff in bending, if the element size comparing to the thickness is not properly chosen. This also might result in numerically ill-conditioned stiffness matrices.

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