Effect Of Anisotropy On Plane Strain Deformation Of A Poroelastic Half-Space In Welded Contact With Elastic Half Space

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ABSTRACT

The Biot linearized theory for fluid saturated porous materials is used to study the plane strain deformation of an isotropic, homogeneous, poroelastic half space in welded contact with homogeneous, orthotropic elastic half space in case (a) and with transversely isotropic elastic half space in case (b) caused by a normal line-load. The analytical expressions for the displacements and stresses in the two half spaces in welded contact have been obtained by applying boundary conditions at the interface. The integrals are solved analytically for the limiting case i.e. undrained conditions in high frequency limit. To examine the effect of anisotropy, the variation of the undrained dimensionless displacements, stresses and pore pressure are shown graphically and it is found that anisotropy is affecting the deformation substantially.

Key words: - Anisotropy, Inclined line-load, Orthotropic, Plane strain, Poroelastic, Transversely Isotropic, Welded half-spaces.

1. Introduction

Poroelasticity is the mechanics of poroelastic solids with fluid filled pores. Its mathematical theory deals with the mechanical behaviour of an elastic porous medium which is either completely filled or partially filled with pore fluid and study the time dependent coupling between the deformation of the rock and fluid flow within the rock. The study of deformation by buried sources of a fluid saturated porous medium is very important because of its applications in earthquake engineering, soil mechanics, seismology, hydrology, geomechanics, geophysics etc. Biot [1], [2] developed linearized constitutive and field equations for poroelastic medium which has been used by many researchers (see e.g. Wang [3] and the references listed there in).

When the source surface is very long in one direction in comparison with the others, the use of two dimensional approximation is justified and consequently calculations are simplified to a great extent and one gets a closed form analytical solution. A very long strip-source and a very long line-source are examples of two dimensional sources. Love [4] obtained expressions for the displacements due to a line-source in an isotropic elastic medium. Maruyama [5] obtained the displacements and stress fields corresponding to long strike-slip faults in a homogeneous isotropic half-space. The two dimensional problem has also been discussed by Rudnicki [6], Rudnicki and Roeloffs[7],Singh and Rani [8], Rani and Singh [9],Singh et al.[10].

Different approaches and methods like boundary value method, displacement discontinuity method, Galerkin vector approach, displacement function approach and eigen value approach, Biot stress function approach etc. have been made to study the plane strain problem of poroelasticity. The use of eigen value approach has the advantage of finding the solutions of the governing equations in the matrix form notations that avoids the complicated nature of the problem. Kumar et al. [11], [12], Garg et al. [13], Kumar and Ailwalia [14], Selim and Ahmed [15], Selim [16],Chugh et al.[17] etc. have used this approach for solving plane strain problem of elasticity and poroelasticity.

In the present paper we examine the effect of anisotropy on plane strain deformation of a two phase medium consisting of a homogeneous, isotropic, poroelastic half space lies below a homogeneous, orthotropic elastic half space in case (a) and lies below a homogeneous, transversely isotropic elastic half space in case (b).which are in welded contact, caused by an inclined line-load in elastic half space in both cases. Using Biot stress function (Biot [18], Roeloffs[19]) and Fourier transform, we find stresses, displacement and pore pressure for unbounded poroelastic medium in integral form and using eigen value approach following Fourier transform, we find stresses and displacement for unbounded elastic medium in integral form. Then we obtain the integral expressions for the displacements and stresses in the two half spaces in welded contact from the corresponding expressions for an unbounded elastic and poroelastic medium by applying suitable boundary conditions at the interface in both cases. These integrals cannot be solved analytically for arbitrary values of the frequency. We evaluate these integrals for the limiting case i.e. undrained condition in high frequency limit. To examine the effect of anisotropy, the variation of the dimensionless displacements, stresses and pore pressure for poroelastic half space have been compared graphically and it is found that anisotropy is affecting the deformation substantially. Anisotropy provides important information about processes and mineralogy in the Earth (poroelastic medium). It is used to find oil and gas in wells. There are so many applications where we can use anisotropic effect in poroelasticity.

2. Formulation of the problem

Consider a homogeneous, isotropic, poroelastic half space lies below a homogeneous orthotropic elastic half space in case (a) and lies below a homogeneous transversely isotropic elastic half space in case (b). A rectangular Cartesian coordinate system oxyz is taken in such a way that a plane x=0 coincides with the intersecting surface of the two half spaces. We take x-axis vertically downwards in the poroelastic half space so that homogeneous, isotropic, poroelastic half space becomes the medium-I (x≥0) and orthotropic elastic half space becomes the medium- II (x≤0)in case (a) and transversely isotropic, elastic half space becomes the medium- II (x≤0) in case (b). Further a normal line-load of magnitude F_1, per unit length, is acting in the positive x-direction on the interface x=0 along z-axis in both cases. The geometry of the problem as shown in figure 1 and figure 2 and it conforms to the two dimensional approximation. Let the Cartesian coordinates be denoted by (x, y, z) ≡ (x_1, x_2, x_3) with x–axis vertically downwards. We consider a two dimensional approximation in which the displacement component (U_1, U_2, U_3) are independent of the Cartesian coordinate x_3, so that $\frac{\partial}{\partial x_3} \equiv 0$. For this two dimensional approximation the plane strain problem ($U_3 = 0$) and the antiplane strain problem ($U_1 = U_2 = 0$) get decoupled, and can therefore be solved independently. Since the antiplane deformation is not affected by pore pressure, we shall discuss plane strain problem only.

3. Solution for poroelastic half space medium-I (x≥0)

A homogeneous, isotropic, poroelastic medium can be described by five poroelastic parameters: Drained Poisson's ratio (ν), undrained Poisson's ratio (ν_μ), shear modulus (G) hydraulic diffusivity (c) and Skempton's coefficient (B). Darcy conductivity (χ) and Biot-willis coefficient α can be expressed in terms of these five parameters:

$$\chi = \frac{9c(1-\nu_\mu)(\nu_\mu - \nu)}{2GB^2(1-\nu)(1+\nu_\mu)^2} \tag{1.1}$$

$$\alpha = \frac{3(\nu_\mu - \nu)}{B(1-2\nu)(1+\nu_\mu)} \tag{1.2}$$

The two dimensional plane strain problem for an isotropic poroelastic medium can be solved in terms of Biot's stress function F (Wang [3]) as

$$\sigma_{11} = \frac{\partial^2 F}{\partial y^2}, \quad \sigma_{22} = \frac{\partial^2 F}{\partial x^2}, \quad \sigma_{12} = -\frac{\partial^2 F}{\partial x\, \partial y} \tag{1.3}$$

$$\nabla^2(\nabla^2 F + 2\eta p) = 0 \tag{1.4}$$

$$\left(c\nabla^2 - \frac{\partial}{\partial t}\right)\left[\nabla^2 F + \frac{3}{(1+\nu_\mu)B} p\right] = 0 \tag{1.5}$$

where σ_{ij} denotes the total stress in the fluid saturated porous elastic material, p the excess fluid pore pressure (compression negative) and

$$\eta = \frac{(1-2\nu)\alpha}{2(1-\nu)} \tag{1.6}$$

is the poroelastic stress coefficient.

From equations (1.4) and (1.5), we get the following decoupled equations

$$\left(c\nabla^2 - \frac{\partial}{\partial t}\right)\nabla^2 p = 0 \tag{1.7}$$

$$\left(c\nabla^2 - \frac{\partial}{\partial t}\right)\nabla^4 F = 0 \tag{1.8}$$

The general solution of equation (1.7) may be expressed as

$$p = p_1 + p_2 \tag{1.9}$$

Where p_1 and p_2 satisfies the following equations:

$$c\,\nabla^2 p_1 = \frac{\partial p_1}{\partial t} \tag{1.10}$$

$$\nabla^2 p_2 = 0 \tag{1.11}$$

Similarly, the general solution of equation (1.8) may be expressed as

$$F = F_1 + F_2 \tag{1.12}$$

where

$$c\nabla^2 F_1 = \frac{\partial F_1}{\partial t} \quad (1.13)$$

$$\nabla^4 F_2 = 0 \quad (1.14)$$

Separation of time and space variables can be made for each of the four functions p_1 , p_2, F_1 and F_2 . Assuming the time dependence as exp $(-i\omega t)$, equations (1.10), (1.11), (1.13) and (1.14) become

$$\nabla^2 p_1 + \frac{i\omega}{c} p_1 = 0 \quad (1.15)$$

$$\nabla^2 p_2 = 0 \quad (1.16)$$

$$\nabla^2 F_1 + \frac{i\omega}{c} F_1 = 0 \quad (1.17)$$

$$\nabla^4 F_2 = 0 \quad (1.18)$$

where p_1 , p_2, F_1 and F_2 are now functions of x and y only.

Fourier transforms are now used to get suitable solutions of equations (1.15)-(1.18), which on using equations (1.9) and (1.12), can be written as

$$P = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[A_1 e^{-mx} + A_2 e^{-|k|x} + A_3 e^{mx} + A_4 e^{|k|x}\right] e^{-iky}\, dk \quad (1.19)$$

$$F = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[B_1 e^{-mx} + B_4 e^{mx} + (B_2 + B_3|k|x)e^{-|k|x} + (B_5 + B_6|k|x)e^{|k|x}\right] e^{-iky}\, dk \quad (1.20)$$

For medium-I($x \geq 0$), using the relation conditions, we have

$$p = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[A_1 e^{-mx} + A_2 e^{-|k|x}\right] e^{-iky}\, dk \quad (1.21)$$

$$F = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[B_1 e^{-mx} + (B_2 + B_3|k|x)e^{-|k|x}\right] e^{-iky}\, dk \quad (1.22)$$

where B_1, B_2 , B_3, A_1 and A_2 are functions of k. From (1.4), (1.5), (1.21) and (1.22). We find

$$A_1 = \frac{i\omega}{2\eta c} B_1, \quad A_2 = \frac{2}{3}(1 + v_\mu) Bk^2 B_3, \quad m = \left(\left(\frac{ck^2 - i\omega}{c}\right)\right)^{\frac{1}{2}}, (\mathrm{Re}\, m > 0) \quad (1.23)$$

Using (1.22) in (1.3), the stresses in medium I($x \geq 0$) are obtained as

$$\sigma_{22} = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[B_1 m^2 e^{-mx} + (B_2 - 2B_3 + B_3|k|x)k^2 e^{-|k|x}\right] e^{-iky}\, dk \quad (1.24)$$

$$\sigma_{11} = -\frac{1}{2\pi}\int_{-\infty}^{\infty}\left[B_1 e^{-mx} + (B_2 + B_3|k|x)e^{-|k|x}\right] k^2 e^{-iky}\, dk \quad (1.25)$$

$$\sigma_{12} = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[B_1 m e^{-mx} + (B_2 - B_3 + B_3|k|x)|k|e^{-|k|x}\right](-ik)e^{-iky}\, dk \quad (1.26)$$

Corresponding to these stresses, the displacements are obtained as (Singh and Rani [8])

$$2GU_2 = -\frac{1}{2\pi}\int_{-\infty}^{\infty}\left[B_1 e^{-mx} + \{B_2 + B_3(2v_\mu - 2 + |k|x)\}e^{-|k|x}\right](-ik)e^{-iky}\, dk \quad 1.27)$$

$$2GU_1 = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[B_1 m e^{-mx} + \{B_2 + B_3(1 - 2v_\mu + |k|x)\}|k|e^{-|k|x}\right] e^{-iky}\, dk \quad (1.28)$$

Also from equation (1.21), we have

$$\frac{\partial p}{\partial x} = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[-mA_1 e^{-mx} - |k|A_2 e^{-|k|x}\right] e^{-iky}\, dk \quad (1.29)$$

4. Solution for orthotropic elastic solid half space, Medium-II($x \leq 0$) case (a)

The equilibrium equations in Cartesian coordinate system (x_1, x_2, x_3) in absence of body forces are

$$\frac{\partial \tau_{11}}{\partial x_1} + \frac{\partial \tau_{12}}{\partial x_2} + \frac{\partial \tau_{13}}{\partial x_3} = 0 \quad (1.30)$$

$$\frac{\partial \tau_{21}}{\partial x_1} + \frac{\partial \tau_{22}}{\partial x_2} + \frac{\partial \tau_{23}}{\partial x_3} = 0 \quad (1.31)$$

$$\frac{\partial \tau_{31}}{\partial x_1} + \frac{\partial \tau_{32}}{\partial x_2} + \frac{\partial \tau_{33}}{\partial x_3} = 0 \quad (1.32)$$

where τ_{ij} $(i, j = 1,2,3)$ are components of stress tensor.

The stress-strain relations for an orthotropic elastic medium are

$$\begin{bmatrix}\tau_{11}\\ \tau_{22}\\ \tau_{33}\\ \tau_{23}\\ \tau_{13}\\ \tau_{12}\end{bmatrix} = \begin{bmatrix} d_{11} & d_{12} & d_{13} & 0 & 0 & 0\\ d_{12} & d_{22} & d_{23} & 0 & 0 & 0\\ d_{13} & d_{23} & d_{33} & 0 & 0 & 0\\ 0 & 0 & 0 & d_{44} & 0 & 0\\ 0 & 0 & 0 & 0 & d_{55} & 0\\ 0 & 0 & 0 & 0 & 0 & d_{66}\end{bmatrix}\begin{bmatrix} e_{11}\\ e_{22}\\ e_{33}\\ 2e_{23}\\ 2e_{13}\\ 2e_{12}\end{bmatrix} \quad (1.33)$$

where e_{ij} are the components of the strain tensor and are related with displacement components (u_1, u_2, u_3) by the relations

$$e_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right), 1 \leq i, j \leq 3 \quad (1.34)$$

The two suffix quantity d_{ij} $(i, j = 1,2 - -6)$ are the elastic moduli for the orthotropic elastic medium. We shall write (x_1, x_2, x_3)=(x,y,z), (u_1, u_2, u_3)=(u,v,w)

The equilibrium equations in terms of the displacement components can be obtained from equations (1.30)-(1.32) by using (1.33) and (1.34) and for the present two dimensional problem are

$$d_{11}\frac{\partial^2 u}{\partial x^2}+d_{66}\frac{\partial^2 u}{\partial y^2}+(d_{66}+d_{12})\frac{\partial^2 v}{\partial x\,\partial y}=0 \qquad (1.35)$$

$$(d_{66}+d_{12})\frac{\partial^2 u}{\partial x\,\partial y}+d_{66}\frac{\partial^2 v}{\partial x^2}+d_{22}\frac{\partial^2 v}{\partial y^2}=0 \qquad (1.36)$$

We define Fourier transform $\bar{f}(x,k)$ of $f(x,y)$ (Debnath, [21]) as

$$\bar{f}(x,k)=F[f(x,y)]=\int_{-\infty}^{\infty} f(x,y)e^{iky}\,dy \qquad (1.37)$$

So that

$$f(x,y)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\bar{f}(x,k)e^{-iky}\,dk, \qquad (1.38)$$

where k is the transformed Fourier parameter. We know that (Sneddon, [22])

$$F\left(\frac{\partial}{\partial y}f(x,y)\right)=(-ik)\bar{f}(x,k)$$

$$F\left(\frac{\partial^2}{\partial y^2}f(x,y)\right)=(-ik)^2\bar{f}(x,k) \qquad (1.39)$$

Applying Fourier Transform as defined above on equations (1.35) and (1.36), we get

$$-d_{66}k^2\bar{u}+d_{11}\frac{d^2\bar{u}}{dx^2}-ik(d_{66}+d_{12})\frac{d\bar{v}}{dx}=0 \qquad (1.40)$$

$$-ik(d_{66}+d_{12})\frac{d\bar{u}}{dx}-d_{22}k^2\bar{v}+d_{66}\frac{d^2\bar{v}}{dx^2}=0 \qquad (1.41)$$

The equations (1.40)-(1.41) can be written in the following vector matrix differential equation as

$$A\frac{d^2N}{dx^2}+B\frac{dN}{dx}+CN=0 \qquad (1.42)$$

where

$$A=\begin{bmatrix} d_{11} & 0\\ 0 & d_{66}\end{bmatrix},\qquad B=\begin{bmatrix} 0 & -ik(d_{66}+d_{12})\\ -ik(d_{66}+d_{12}) & 0\end{bmatrix},$$

$$C=\begin{bmatrix} -k^2d_{66} & 0\\ 0 & -k^2d_{22}\end{bmatrix},\qquad N=\begin{bmatrix}\bar{u}\\ \bar{v}\end{bmatrix} \qquad (1.43)$$

We note that the matrices A, B, C are all symmetric and matrices A, B, C depends upon elastic moduli only. Applying eigen value method to solve equation (1.42), we try a solution of the matrix equation (1.42) of the form

$$N(x,k)=E(k)\,e^{sx}, \qquad (1.44)$$

where s is a parameter and E(k) is a matrix of the type 2×1.Substituting the value of N from equation (1.44) into equation (1.42),we get the following characteristic equation.

$$(d_{11}d_{66})s^4-(d_{11}d_{22}-2d_{12}d_{66}-d_{12}{}^2)k^2s^2+d_{22}d_{66}k^4=0, \qquad (1.45)$$

After solving the characteristic equation (1.45), we get the eigenvalues as

$s^2=m_1{}^2k^2, m_2{}^2k^2$ where

$$m_1{}^2=\frac{A_0+\sqrt{A_0{}^2-4B_0}}{2},\qquad m_2{}^2=\frac{A_0-\sqrt{A_0{}^2-4B_0}}{2}$$

$$A_0=\frac{(d_{11}d_{22}-2d_{12}d_{66}-d_{12}{}^2)}{(d_{11}d_{66})}=m_1{}^2+m_2{}^2$$

And $$B_0=\frac{d_{22}}{d_{11}}=m_1{}^2\,m_2{}^2 \qquad (1.46)$$

where the quantities m_1 and m_2 for an orthotropic elastic medium depends upon elastic moduli only and are independent of k. They may be real or complex. We assume that $m_1\neq m_2$ for an orthotropic elastic medium. Then the eigen values can be written as

$$s_1=m_1|k|,\quad s_2=m_2|k|,\quad s_3=-m_1|k|,\quad s_4=-m_2|k| \qquad (1.47)$$

with real parts of (m_1, m_2) as positive. The eigen vectors for an orthotropic elastic medium are obtained by solving the matrix equation

$$[s^2A+sB+C]E(K)=0 \qquad (1.48)$$

in which the matrices A, B ,C are given by equation (1.43)

The eigen vectors become

$$X_F{}^T=[P_{2F},1],\quad X_{F+2}{}^T=[-P_{2F},1],\ \text{for } F=1,2 \qquad (1.48a)$$

In which

$$P_{21}=\frac{im_1|k|}{k}\left(\frac{d_{66}+d_{12}}{d_{11}m_1{}^2-d_{66}}\right)=\frac{-ik}{m_1|k|}\left(\frac{d_{66}m_1{}^2-d_{22}}{d_{66}+d_{12}}\right)$$

$$P_{22} = \frac{im_2|k|}{k}\left(\frac{d_{66}+d_{12}}{d_{11}m_2{}^2-d_{66}}\right) = \frac{-ik}{m_2|k|}\left(\frac{d_{66}m_2{}^2-d_{22}}{d_{66}+d_{12}}\right) \quad (1.49)$$

Thus a solution of the matrix equation (1.42) for orthotropic elastic medium is

$$N(x,k) = \sum_{F=1}^{2}(c_F X_F e^{m_F|k|x} + c_{F+2}X_{F+2}e^{-m_F|k|x}) \quad (1.50)$$

Where c_1, c_2, c_3, c_4, are coefficients to be determined from boundary conditions. From equations (1.43), (1.48a), (1.49) and (1.50), we write

$$\bar{u}(x,k) = c_1P_{21}e^{m_1|k|x} + c_2P_{22}e^{m_2|k|x} - c_3P_{21}e^{-m_1|k|x} - c_4P_{22}e^{-m_2|k|x}, \quad (1.51)$$

$$\bar{v}(x,k) = c_1e^{m_1|k|x} + c_2e^{m_2|k|x} + c_3e^{-m_1|k|x} + c_4e^{-m_2|k|x} \quad (1.52)$$

Inversion of equation (1.51) and (1.52) gives the displacements in the following integral forms

$$u(x,y) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[c_1P_{21}e^{m_1|k|x} + c_2P_{22}e^{m_2|k|x} - c_3P_{21}e^{-m_1|k|x} - c_4P_{22}e^{-m_2|k|x}\right]e^{-iky}\,dk \quad (1.53)$$

$$v(x,y) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[c_1e^{m_1|k|x} + c_2e^{m_2|k|x} + c_3e^{-m_1|k|x} + c_4e^{-m_2|k|x}\right]e^{-iky}\,dk \quad (1.54)$$

For the transformed stresses for plane strain deformation parallel to xy plane of orthotropic elastic medium, using equations (1.33)-(1.34) and (1.38)-(1.39), we get

$$\bar{\tau}_{11} = (-ik)d_{12}\bar{v} + d_{11}\frac{d\bar{u}}{dx} \quad (1.55)$$

$$\bar{\tau}_{12} = (-ik)d_{66}\bar{u} + d_{66}\frac{d\bar{v}}{dx} \quad (1.56)$$

where $\bar{u}$, $\bar{v}$ used in equations (1.55)-(1.56) are Fourier Transforms of u(x, y) and v(x, y).

Putting the values of $\bar{u}$ and $\bar{v}$ from equations (1.51) and (1.52) and their derivatives into equations (1.55)-(1.56) we obtain $\bar{\tau}_{11}$, $\bar{\tau}_{12}$, as

$$\bar{\tau}_{11} = \left[c_1Q_{21}e^{m_1|k|x} + c_2Q_{22}e^{m_2|k|x} + c_3Q_{21}e^{-m_1|k|x} + c_4Q_{22}e^{-m_2|k|x}\right] \quad (1.57)$$

$$\bar{\tau}_{12} = d_{66}\left(c_1R_{21}e^{m_1|k|x} + c_2R_{22}e^{m_2|k|x} - c_3R_{21}e^{-m_1|k|x} - c_4R_{22}e^{-m_2|k|x}\right) \quad (1.58)$$

where

$$Q_{2F} = d_{11}P_{2F}m_F|k| - id_{12}k \quad (1.59)$$

$$R_{2F} = m_F|k| - iP_{2F}k, \quad \text{for } F=1,2 \quad (1.60)$$

Inversion of equations (1.57)-(1.58) gives the stresses in the following integral forms for an orthotropic elastic medium as

$$\tau_{11} = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[c_1Q_{21}e^{m_1|k|x} + c_2Q_{22}e^{m_2|k|x} + c_3Q_{21}e^{-m_1|k|x} + c_4Q_{22}e^{-m_2|k|x}\right]e^{-iky}\,dk \quad (1.61)$$

$$\tau_{12} = \frac{1}{2\pi}\int_{-\infty}^{\infty}d_{66}\left[c_1R_{21}e^{m_1|k|x} + c_2R_{22}e^{m_2|k|x} - c_3R_{21}e^{-m_1|k|x} - c_4R_{22}e^{-m_2|k|x}\right]e^{-iky}\,dk \quad (1.62)$$

The displacements and stress components for orthotropic elastic half space medium II (x≤0) case (a) are now obtained as from equations (1.53)-(1.54) and(1.61)-(1.62)

$$u(x,y) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[c_1P_{21}e^{m_1|k|x} + c_2P_{22}e^{m_2|k|x}\right]e^{-iky}\,dk \quad (1.63)$$

$$v(x,y) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[c_1e^{m_1|k|x} + c_2e^{m_2|k|x}\right]e^{-iky}\,dk \quad (1.64)$$

$$\tau_{11} = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[c_1Q_{21}e^{m_1|k|x} + c_2Q_{22}e^{m_2|k|x}\right]e^{-iky}\,dk \quad (1.65)$$

$$\tau_{12} = \frac{1}{2\pi}\int_{-\infty x}^{\infty}\left[d_{66}\left(c_1R_{21}e^{m_1|k|x} + c_2R_{22}e^{m_2|k|x}\right)\right]e^{-iky}\,dk \quad (1.66)$$

4.1 Normal Line-load

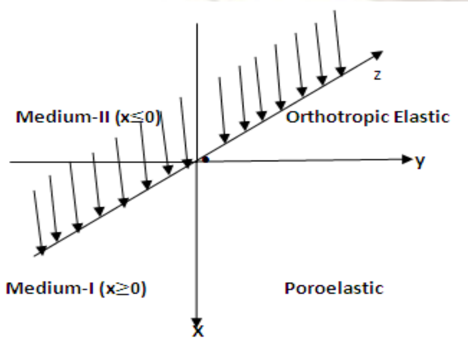


Fig.1: A Normal line-load F_1, per unit length, acting in the positive x-direction on the interface x=0 along z-axis.

Consider a normal line-load F_1, per unit length, acting in the positive x-direction on the interface x=0 along z-axis. Since the half spaces (poroelastic half space and orthotropic elastic half space) are assumed to be in welded contact along the plane x=0, the continuity of the stresses and the displacements give the following boundary conditions at x=0:

$\sigma_{12} = \tau_{12}$ (1.67)
$\sigma_{11} - \tau_{11} = -F_1\delta(y)$ (1.68)
$U_1^{\,1}(x,y) = u(x,y)$ (1.69)
$U_2^{\,1}(x,y) = v(x,y)$ (1.70)
where $\delta(y)$ in equation (1.68) is the Dirac delta function and it satisfies the following properties
$\int_{-\infty}^{\infty}\delta(y)dy = 1\,,\delta(y) = \frac{1}{2\pi}\int_{-\infty}^{\infty}e^{-iky}\,dk$ (1.71)
Also, if we assume that the interface is impermeable, the hydraulic boundary condition x=0 is
$\frac{\partial p}{\partial x} = 0$ (1.72)
Now using equations (1.25)-(1.29), (1.63)-(1.66) and (1.67)-(1.72) we get the following system of equations
$mB_1 + |k|B_2 - |k|B_3 - \frac{id_{66}R_{21}}{k}c_1 - \frac{id_{66}R_{22}}{k}c_2 = 0$ (1.73)
$B_1 + B_2 + \frac{Q_{21}}{K^2}c_1 + \frac{Q_{22}}{K^2}c_2 = \frac{F_1}{K^2}$ (1.74)
$B_1 + B_2 + (2v_\mu - 2)B_3 + \frac{2Gi}{k}c_1 + \frac{2Gi}{k}c_2 = 0$ (1.75)
$B_1 m + B_2|k| + |k|(1-2v_\mu)B_3 - 2GP_{21}c_1 - 2GP_{22}c_2 = 0$ (1.76)
$ma^1B_1 + |k|b^1B_3 = 0$ (1.77)
where
$a^1 = \frac{i\omega}{2\eta c}, b^1 = \frac{2}{3}(1+v_\mu)Bk^2$ (1.78)
Solving the system of equations (1.73)-(1.77) using (1.78) we obtain
$B_1 = \frac{-|k|b^1}{ma^1}B_3$ (1.79)
$B_2 = \frac{F_1}{|k|k^2}\left\{\frac{s}{tE-Fs}(pt + 2GP_{21}) - \frac{PR}{P-Q}(pt + 2GP_{21}) + \frac{R}{P-Q}(pu + 2GP_{22})\right\}$ (1.80)
$B_3 = \frac{F_1}{k^2 s}\left\{\frac{-ts}{tE-Fs} + \frac{tPR}{P-Q} - \frac{uR}{P-Q}\right\}$ (1.81)
$c_1 = \frac{F_1}{k^2}\left\{\frac{s}{tE-Fs} - \frac{PR}{P-Q}\right\}$ (1.82)
$c_2 = \frac{F_1}{k^2}\left\{\frac{R}{P-Q}\right\}$ (1.83)
Where
$\alpha = \frac{-|k|b^1}{ma^1}, p = m\alpha + |k|(1-2v_\mu), r = \alpha m - |k|, q = \alpha + 2v_\mu - 2,$
$E = q - \alpha,\quad F = \frac{2Gi}{k} - \frac{Q_{21}}{K^2},\quad G = \frac{2Gi}{k} - \frac{Q_{22}}{K^2},\quad L = \alpha - \frac{r}{|k|},\quad M = \frac{Q_{21}}{K^2} + \frac{id_{66}R_{21}}{k|k|},$
$N = \frac{Q_{22}}{K^2} + \frac{id_{66}R_{22}}{k|k|},\quad s = \frac{p}{|\mathbf{k}|} - q,\quad t = -\frac{2Gi}{k} - \frac{2GP_{21}}{|k|},\quad u = -\frac{2Gi}{k} - \frac{2GP_{22}}{|k|},$
$P = \frac{(uE-Gs)}{(tE-Fs)},\quad Q = \frac{(GL-EN)}{(FL-EM)},\quad R = \left(\frac{s}{tE-Fs}\right) + \left(\frac{E+L}{FL-EM}\right)$ (1.84)

4.1.1 Undrained state, $\omega \to \infty$

Putting the values of B_1, B_2 and B_3 from equations (1.79)-(1.81) into equations(1.24)-(1.29) which corresponds to the stresses ,displacements and pore pressure for poroelastic half space medium-I($x \geq 0$) and then taking limit $\omega \to \infty$ and then integrate, we get

$$\sigma_{11}^{(N)} = \frac{-F_1}{\pi}P_7\frac{x}{x^2+y^2} - \frac{F_1P_6}{\pi}\frac{x(x^2-y^2)}{(x^2+y^2)^2} \quad (1.85)$$
$$\sigma_{22}^{(N)} = \frac{F_1(P_7 - 2P_6)}{\pi}\frac{x}{x^2+y^2} + \frac{F_1P_6}{\pi}\frac{x(x^2-y^2)}{(x^2+y^2)^2} \quad (1.86)$$
$$\sigma_{12}^{(N)} = \frac{F_1}{\pi}(P_6 - P_7)\frac{y}{x^2+y^2} - \frac{2F_1P_6}{\pi}\frac{yx^2}{(x^2+y^2)^2} \quad (1.87)$$
$$2GU_2^{1(N)} = \frac{F_1}{\pi}\left(\left(P_7 + P_6(2v_\mu - 2)\right)\tan^{-1}\frac{y}{x} + \frac{F_1P_6}{\pi}\frac{xy}{x^2+y^2}\right. \quad (1.88)$$
$$2GU_1^{1(N)} = -\frac{F_1}{2\pi}(P_7 + P_6(1-2v_\mu))\log(x^2+y^2) + \frac{F_1P_6}{\pi}\frac{x^2}{x^2+y^2} \quad (1.89)$$
$$p^{(N)} = \frac{2F_1(1+v_\mu)BP_6}{3\pi}\frac{x}{x^2+y^2} \quad (1.90)$$
Where P_6, P_7 are as follow

$$P_1^{\,1} = \left\{-2Gm_2\left(\frac{d_{66}+d_{12}}{d_{11}m_2^{\,2}-d_{66}}\right) - 2G\right\}(2v_\mu - 2) - \left\{2G - d_{11}m_2^{\,2}\left(\frac{d_{66}+d_{12}}{d_{11}m_2^{\,2}-d_{66}}\right) + d_{12}\right\}(3-4v_\mu) \quad (1.91)$$

$$P_1^{11} = \left\{-2Gm_1\left(\frac{d_{66}+d_{12}}{d_{11}m_1^2-d_{66}}\right)-2G\right\}(2v_\mu-2) - \left\{2G-d_{11}m_1^2\left(\frac{d_{66}+d_{12}}{d_{11}m_1^2-d_{66}}\right)+d_{12}\right\}(3-4v_\mu) \quad (1.92)$$

$$P_1 = \frac{P_1^1}{P_1^{11}} \quad (1.93)$$

$$P_2^1 = \left\{2G-d_{11}m_2^2\left(\frac{d_{66}+d_{12}}{d_{11}m_2^2-d_{66}}\right)+d_{12}\right\} - (2v_\mu-2)\left[\left\{d_{11}m_2^2\left(\frac{d_{66}+d_{12}}{d_{11}m_2^2-d_{66}}\right)-d_{12}\right\}+d_{66}m_2\left\{1+\left(\frac{d_{66}+d_{12}}{d_{11}m_2^2-d_{66}}\right)\right\}\right] \quad (1.94)$$

$$P_2^{11} = \left\{2G-d_{11}m_1^2\left(\frac{d_{66}+d_{12}}{d_{11}m_1^2-d_{66}}\right)+d_{12}\right\} - (2v_\mu-2)\left[\left\{d_{11}m_1^2\left(\frac{d_{66}+d_{12}}{d_{11}m_1^2-d_{66}}\right)-d_{12}\right\}+d_{66}m_1\left\{1+\left(\frac{d_{66}+d_{12}}{d_{11}m_1^2-d_{66}}\right)\right\}\right] \quad (1.95)$$

$$P_2 = \frac{P_2^1}{P_2^{11}} \quad (1.96)$$

$$P_3^1 = \frac{(3-4v_\mu)}{P_1^{11}} \quad (1.97)$$

$$P_3^{11} = \frac{(2v_\mu-1)}{P_2^{11}} \quad (1.98)$$

$$P_3 = P_3^1 + P_3^{11} \quad (1.99)$$

$$P_4 = \frac{P_3}{P_1-P_2} \quad (1.100)$$

$$P_5 = P_3^1 - P_1P_4 \quad (1.101)$$

$$P_6 = \frac{P_5\left[2Gm_1\left(\frac{d_{66}+d_{12}}{d_{11}m_1^2-d_{66}}\right)+2G\right]+P_4\left[2Gm_2\left(\frac{d_{66}+d_{12}}{d_{11}m_2^2-d_{66}}\right)+2G\right]}{(3-4v_\mu)} \quad (1.102)$$

$$P_7 = \left\{-(1-2v_\mu)P_6+2Gm_1\left(\frac{d_{66}+d_{12}}{d_{11}m_1^2-d_{66}}\right)P_5+2Gm_2\left(\frac{d_{66}+d_{12}}{d_{11}m_2^2-d_{66}}\right)P_4\right\} \quad (1.103)$$

Putting the values of c_1, and c_2 from equations (1.82)-(1.83) into equations(1.63)-(1.66) which corresponds to the stresses and displacements for orthotropic elastic half space medium-II($x \le 0$) and then taking limit $\omega \to \infty$ and then integrate, we get

$$\tau_{12}^{(N)} = \frac{F_1P_5d_{66}m_1}{\pi}\left\{1+\left(\frac{d_{66}+d_{12}}{d_{11}m_1^2-d_{66}}\right)\right\}\left(\frac{-y}{(y^2+m_1^2x^2)}\right) + \frac{F_1P_4d_{66}m_2}{\pi}\left\{1+\left(\frac{d_{66}+d_{12}}{d_{11}m_2^2-d_{66}}\right)\right\}\left(\frac{-y}{(y^2+m_2^2x^2)}\right) \quad (1.104)$$

$$\tau_{11}^{(N)} = \frac{F_1P_5}{\pi}\left\{d_{11}m_1^2\left(\frac{d_{66}+d_{12}}{d_{11}m_1^2-d_{66}}\right)-d_{12}\right\}\left(\frac{-m_1x}{(y^2+m_1^2x^2)}\right) + \frac{F_1P_4}{\pi}\left\{d_{11}m_2^2\left(\frac{d_{66}+d_{12}}{d_{11}m_2^2-d_{66}}\right)-d_{12}\right\}\left(\frac{-m_2x}{(y^2+m_2^2x^2)}\right) \quad (1.105)$$

$$v^{(N)}(x,y) = \frac{F_1P_5}{\pi}\tan^{-1}\left(\frac{y}{m_1x}\right)+\frac{F_1P_4}{\pi}\tan^{-1}\left(\frac{y}{m_2x}\right) \quad (1.106)$$

$$u^{(N)}(x,y) = \frac{-F_1P_4m_1}{2\pi}\left(\frac{d_{66}+d_{12}}{d_{11}m_1^2-d_{66}}\right)\log(y^2+m_1^2x^2) - \frac{F_1P_5m_2}{2\pi}\left(\frac{d_{66}+d_{12}}{d_{11}m_2^2-d_{66}}\right)\log(y^2+m_2^2x^2) \quad (1.107)$$

5. Solution for transversely isotropic elastic solid half space Medium-II ($x \le 0$), Case (b)

The equilibrium equations in Cartesian coordinate system (x_1, x_2, x_3) in absence of body forces are

$$\frac{\partial\tau_{11}}{\partial x_1}+\frac{\partial\tau_{12}}{\partial x_2}+\frac{\partial\tau_{13}}{\partial x_3}=0 \quad (2.1)$$

$$\frac{\partial\tau_{21}}{\partial x_1}+\frac{\partial\tau_{22}}{\partial x_2}+\frac{\partial\tau_{23}}{\partial x_3}=0 \quad (2.2)$$

$$\frac{\partial\tau_{31}}{\partial x_1}+\frac{\partial\tau_{32}}{\partial x_2}+\frac{\partial\tau_{33}}{\partial x_3}=0 \quad (2.3)$$

where τ_{ij} are components of stress tensor.

The stress-strain relations for a transversely isotropic elastic medium are

$$\begin{bmatrix}\tau_{11}\\ \tau_{22}\\ \tau_{33}\\ \tau_{23}\\ \tau_{13}\\ \tau_{12}\end{bmatrix}=\begin{bmatrix}d_{11} & d_{12} & d_{13} & 0 & 0 & 0\\ d_{12} & d_{11} & d_{13} & 0 & 0 & 0\\ d_{13} & d_{13} & d_{33} & 0 & 0 & 0\\ 0 & 0 & 0 & d_{44} & 0 & 0\\ 0 & 0 & 0 & 0 & d_{44} & 0\\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2}(d_{11}-d_{12})\end{bmatrix}\begin{bmatrix}e_{11}\\ e_{22}\\ e_{33}\\ 2e_{23}\\ 2e_{13}\\ 2e_{12}\end{bmatrix} \quad (2.4)$$

where e_{ij} are the components of the strain tensor and are related with displacement components (u_1, u_2, u_3) by the relations

$$e_{ij}=\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right), 1 \le i,j \le 3 \quad (2.5)$$

The two suffix quantity d_{ij} are the elastic moduli for the transversely isotropic elastic medium. We shall write (x_1, x_2, x_3)=(x,y,z), (u_1, u_2, u_3)=(u,v,w)

The equilibrium equations in terms of the displacement components can be obtained from equations (2.1)-(2.4) by using (1.34) and for the present two dimensional problem are

$$d_{11}\frac{\partial^2 u}{\partial x^2}+\frac{1}{2}(d_{11}-d_{12})\frac{\partial^2 u}{\partial y^2}+\frac{1}{2}(d_{11}+d_{12})\frac{\partial^2 v}{\partial x\,\partial y}=0 \quad (2.6)$$

$$\frac{1}{2}(d_{11}+d_{12})\frac{\partial^2 u}{\partial x\partial y}+\frac{1}{2}(d_{11}-d_{12})\frac{\partial^2 v}{\partial x^2}+d_{11}\frac{\partial^2 v}{\partial y^2}=0 \quad (2.7)$$

Applying Fourier Transform as defined by (1.37)-(1.39) on equations (2.6) and (2.7), we get

$$\frac{d^2\bar{u}}{dx^2}=k^2\left(\frac{d_{11}-d_{12}}{2d_{11}}\right)\bar{u}+ik\left(\frac{d_{11}+d_{12}}{2d_{11}}\right)\frac{d\bar{v}}{dx} \quad (2.8)$$

$$\frac{d^2\bar{v}}{dx^2}=k^2\left(\frac{2d_{11}}{d_{11}-d_{12}}\right)\bar{v}+ik\left(\frac{d_{11}+d_{12}}{d_{11}-d_{12}}\right)\frac{d\bar{u}}{dx} \quad (2.9)$$

The above equations (2.8)-(2.9) can be written in the following vector matrix equation form as

$$\frac{dN_1}{dx}=A_1N_1 \quad (2.10)$$

where

$$N_1=\begin{bmatrix}\bar{u}\\ \bar{v}\\ \frac{d\bar{u}}{dx}\\ \frac{d\bar{v}}{dx}\end{bmatrix}, A_1=\begin{bmatrix}0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ k^2R_1 & 0 & 0 & ikR_2\\ 0 & k^2R_3 & ikR_4 & 0\end{bmatrix} \quad (2.11)$$

Where R_1, R_2, R_3 and R_4 are as follow

$$R_1=\frac{d_{11}-d_{12}}{2d_{11}},\quad R_2=\frac{d_{11}+d_{12}}{2d_{11}},\quad R_3=\frac{1}{R_1},\quad R_4=R_2R_3 \quad (2.12)$$

Applying eigen value method to solve equation (2.10), we try a solution of the matrix equation (2.10) of the form

$$N(x,k)=E(k)e^{sx} \quad (2.13)$$

where s is a parameter and E(k) is a matrix of the type 4×1.Substituting the value of N from equation (2.13) into equation (2.10),we get the following characteristic equation.

$$s^4-2k^2s^2+k^4=0 \quad (2.14)$$

After solving the characteristic equation (2.14) , we get the repeated eigenvalues as

$s=s_1=s_2=-s_3=-s_4=|k|$

An eigen vector X_1, corresponding to the eigenvalue $s=s_1=|k|$ of multiplicity 2 is found to be

$$X_{1,}=\begin{bmatrix}i|k|\\ k\\ ik^2\\ k|k|\end{bmatrix} \quad (2.15)$$

Second eigen vector X_2 corresponding to the eigenvalue $s=s_2=|k|$ can be obtained as by Ross [23]

$$X_2=\begin{bmatrix}i\left\{|k|x-4\left(\frac{d_{11}}{d_{11}+d_{12}}\right)\right\}\\ k\left(x-\frac{1}{|k|}\right)\\ i|k|\left\{|k|x-\left(\frac{3d_{11}-d_{12}}{d_{11}+d_{12}}\right)\right\}\\ k|k|x\end{bmatrix} \quad (2.16)$$

An eigen vector X_3, corresponding to the eigenvalue $s=s_3=-|k|$ of multiplicity 2 is found to be

$$X_{3,} = \begin{bmatrix} -i|k| \\ k \\ ik^2 \\ -k|k| \end{bmatrix} \tag{2.17}$$

Similarly second eigen vector X_4 corresponding to the eigenvalue $s = s_4 = -|k|$ can be obtained as by Ross [23]

$$X_4 = \begin{bmatrix} -i\left\{|k|x + 4\left(\frac{d_{11}}{d_{11}+d_{12}}\right)\right\} \\ k\left(x + \frac{1}{|k|}\right) \\ i|k|\left\{|k|x + \left(\frac{3d_{11}-d_{12}}{d_{11}+d_{12}}\right)\right\} \\ -k|k|x \end{bmatrix} \tag{2.18}$$

Thus, a general solution of (2.10) for a transversely isotropic elastic medium is

$$N_1 = (D_1X_1 + D_2X_2)e^{|k|x} + (D_3X_3 + D_4X_4)e^{-|k|x} \tag{2.19}$$

Where D_1, D_2, D_3 and D_4 are coefficients to be determined from boundary conditions and they may depend upon k. From (2.11) and (2.15)-(2.19), we obtain

$$\bar{u} = i\left[\left\{D_1|k| + D_2\left(|k|x - \frac{4d_{11}}{d_{11}+d_{12}}\right)\right\}e^{|k|x} - \left\{D_3|k| + D_4\left(|k|x + \frac{4d_{11}}{d_{11}+d_{12}}\right)\right\}e^{-|k|x}\right] \tag{2.20}$$

$$\bar{v} = k\left[\left\{D_1 + D_2\left(x - \frac{1}{|k|}\right)\right\}e^{|k|x} + \left\{D_3 + D_4\left(x + \frac{1}{|k|}\right)\right\}e^{-|k|x}\right] \tag{2.21}$$

$$\frac{d\bar{u}}{dx} = i\begin{bmatrix} \left\{D_1k^2 + D_2|k|\left(|k|x - \left(\dfrac{3d_{11}-d_{12}}{d_{11}+d_{12}}\right)\right)\right\}e^{|k|x} \\ + \left\{D_3k^2 + D_4|k|\left(|k|x + \left(\dfrac{3d_{11}-d_{12}}{d_{11}+d_{12}}\right)\right)\right\}e^{-|k|x} \end{bmatrix} \tag{2.22}$$

$$\frac{d\bar{v}}{dx} = k|k|\left[(D_1 + D_2x)e^{|k|x} - (D_3 + D_4x)e^{-|k|x}\right] \tag{2.23}$$

Inversion of equation (2.20) and (2.21) gives the displacements in the following integral forms

$$u(x,y) = \frac{i}{2\pi}\int_{-\infty}^{\infty}\begin{bmatrix} \left\{D_1|k| + D_2\left(|k|x - \frac{4d_{11}}{d_{11}+d_{12}}\right)\right\}e^{|k|x} \\ -\left\{D_3|k| + D_4\left(|k|x + \frac{4d_{11}}{d_{11}+d_{12}}\right)\right\}e^{-|k|x} \end{bmatrix} e^{-iky}\,dk \tag{2.24}$$

$$v(x,y) = \frac{1}{2\pi}\int_{-\infty}^{\infty} k\left[\left\{D_1 + D_2\left(x - \frac{1}{|k|}\right)\right\}e^{|k|x} + \left\{D_3 + D_4\left(x + \frac{1}{|k|}\right)\right\}e^{-|k|x}\right]e^{-iky}\,dk \tag{2.25}$$

For the transformed stresses for plane strain deformation parallel to xy plane of a transversely isotropic elastic medium, using equations (1.37)-(1.39) and (2.4)-(2.5), we get

$$\bar{\tau}_{11} = (-ik)d_{12}\bar{v} + d_{11}\frac{d\bar{u}}{dx} \tag{2.26}$$

$$\bar{\tau}_{12} = \frac{1}{2}(d_{11} - d_{12})\left(\frac{d\bar{v}}{dx} - ik\bar{u}\right) \tag{2.27}$$

where $\bar{u}$, $\bar{v}$ used in equations (2.26)-(2.27) are Fourier Transform of u(x, y) and v(x, y).

Putting the values of $\bar{u}$ and $\bar{v}$ from equations (2.20) and (2.21) and their derivatives into equations (2.26)-(2.27) we obtain $\bar{\tau}_{11}$, $\bar{\tau}_{12}$, as

$$\bar{\tau}_{11} = i\begin{bmatrix} k^2(d_{11} - d_{12})D_1e^{|k|x} + \left\{(d_{11} - d_{12})k^2x + |k|\left(\frac{-3d_{11}^2 + 2d_{11}d_{12} + d_{12}^2}{d_{11}+d_{12}}\right)\right\}D_2e^{|k|x} \\ + k^2(d_{11} - d_{12})D_3e^{-|k|x} + \left\{(d_{11} - d_{12})k^2x + |k|\left(\frac{3d_{11}^2 - 2d_{11}d_{12} - d_{12}^2}{d_{11}+d_{12}}\right)\right\}D_4e^{-|k|x} \end{bmatrix} \tag{2.28}$$

$$\bar{\tau}_{12} = (d_{11} - d_{12})\begin{bmatrix} k|k|D_1e^{|k|x} + \left(k|k|x - \frac{2d_{11}k}{d_{11}+d_{12}}\right)D_2e^{|k|x} \\ -k|k|D_3e^{-|k|x} - \left(k|k|x + \frac{2d_{11}k}{d_{11}+d_{12}}\right)D_4e^{-|k|x} \end{bmatrix} \tag{2.29}$$

Inversion of equations (2.28)-(2.29) gives the stresses in the following integral forms for a transversely isotropic elastic medium

$$\tau_{11} = \frac{i}{2\pi}\int_{-\infty}^{\infty}\begin{bmatrix} k^2(d_{11} - d_{12})D_1e^{|k|x} + \left\{(d_{11} - d_{12})k^2x + |k|\left(\dfrac{-3d_{11}^2 + 2d_{11}d_{12} + d_{12}^2}{d_{11} + d_{12}}\right)\right\}D_2e^{|k|x} \\ +k^2(d_{11} - d_{12})D_3e^{-|k|x} + \left\{(d_{11} - d_{12})k^2x + |k|\left(\dfrac{3d_{11}^2 - 2d_{11}d_{12} - d_{12}^2}{d_{11} + d_{12}}\right)\right\}D_4e^{-|k|x} \end{bmatrix} e^{-iky}\,dk, \tag{2.30}$$

$$\tau_{12} = \left(\frac{d_{11} - d_{12}}{2\pi}\right)\int_{-\infty}^{\infty}\begin{bmatrix} k|k|D_1 e^{|k|x} + \left(k|k|x - \frac{2d_{11}k}{d_{11} + d_{12}}\right)D_2 e^{|k|x} \\ -k|k|D_3 e^{-|k|x} - \left(k|k|x + \frac{2d_{11}k}{d_{11} + d_{12}}\right)D_4 e^{-|k|x}\end{bmatrix} e^{-iky}\, dk \qquad (2.31)$$

The displacements and stress components for transversely isotropic elastic half space medium II (x≤0) are now obtained as from equations (2.24)-(2.25) and (2.30)-(2.31)

$$u(x, y) = \frac{i}{2\pi}\int_{-\infty}^{\infty}\left[\left\{D_1|k| + D_2\left(|k|x - \frac{4d_{11}}{d_{11}+d_{12}}\right)\right\}e^{|k|x}\right]e^{-iky}\, dk \qquad (2.32)$$

$$v(x, y) = \frac{1}{2\pi}\int_{-\infty}^{\infty} k\left[\left\{D_1 + D_2\left(x - \frac{1}{|k|}\right)\right\}e^{|k|x}\right]e^{-iky}\, dk \qquad (2.33)$$

$$\tau_{11} = \frac{i}{2\pi}\int_{-\infty}^{\infty}\left[k^2(d_{11} - d_{12})D_1 + D_2\left\{(d_{11} - d_{12})k^2x + |k|\left(\frac{-3d_{11}{}^2 + 2d_{11}d_{12} + d_{12}{}^2}{d_{11}+d_{12}}\right)\right\}\right]e^{|k|x}e^{-iky}\, dk \qquad (2.34)$$

$$\tau_{12} = \left(\frac{d_{11}-d_{12}}{2\pi}\right)\int_{-\infty}^{\infty}\left[k|k|D_1 + \left(k|k|x - \frac{2d_{11}k}{d_{11}+d_{12}}\right)D_2\right]e^{|k|x}e^{-iky}\, dk \qquad (2.35)$$

5.1 Normal Line-load

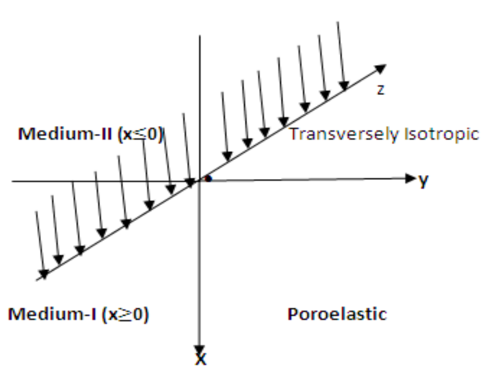


***Fig.2 A Normal Line-load F_1, per unit length, acting in the positive x-direction on the interface x=0 along z-axis*.**

Consider a normal line-load F_1 , per unit length, acting in the positive x-direction on the interface x=0 along z-axis. Since the half spaces (poroelastic half space and transversely isotropic elastic half space) are assumed to be in welded contact along the plane x=0, the continuity of the stresses and the displacements give the following boundary conditions at x=0:

$$\sigma_{12} = \tau_{12} \qquad (2.36)$$

$$\sigma_{11} - \tau_{11} = -F_1\delta(y) \qquad (2.37)$$

$$U_1(x, y) = u(x, y) \qquad (2.38)$$

$$U_2(x, y) = v(x, z) \qquad (2.39)$$

where $\delta(y)$ in equation (2.37) is the Dirac delta function.

Also, if we assume that the interface is impermeable, the hydraulic boundary condition x=0 is

$$\frac{\partial p}{\partial x} = 0 \qquad (2.40)$$

Now using equations (1.25)-(1.29), and (2.32)-(2.40), we get the following system of equations

$$mB_1 + |k|B_2 - |k|B_3 - m_2 i|k|D_1 + 2m_1m_2 iD_2 = 0 \qquad (2.41)$$

$$B_1 + B_2 + im_2D_1 + \frac{i}{|k|}m_3D_2 = \frac{F_1}{K^2} \qquad (2.42)$$

$$B_1 + B_2 + (2v_\mu - 2)B_3 + 2GiD_1 - \frac{2Gi}{|k|}D_2 = 0 \qquad (2.43)$$

$$B_1 m + B_2|k| + |k|(1 - 2v_\mu)B_3 - 2Gi|k|D_1 + 8Gim_1D_2 = 0 \qquad (2.44)$$

$$ma^1B_1 + |k|b^1B_3 = 0 \qquad (2.45)$$

where

$$a^1 = \frac{i\omega}{2\eta c}, b^1 = \frac{2}{3}(1 + v_\mu)Bk^2,\ m_1 = \frac{d_{11}}{d_{11}+d_{12}},\ m_2 = d_{11} - d_{12},$$

$$m_3 = \left(\frac{-3d_{11}{}^2 + 2d_{11}d_{12} + d_{12}{}^2}{d_{11}+d_{12}}\right) \qquad (2.46)$$

Solving the system of equations (2.41)-(2.45) using (2.46), we obtain

$$B_1 = \frac{-|k|b^1}{ma^1}B_3 \qquad (2.47)$$

$$B_2 = -pB_3 + 2Gi\ D_1 - \frac{8Gi}{|k|}m_1D_2 \qquad (2.48)$$

$$B_3 = \frac{4\mathcal{G}}{(p-q)} D_1 - \frac{2\mathcal{G}}{|k|}\left(\frac{1+4m_1}{(p-q)}\right) D_2 \qquad (2.49)$$

$$D_1 = \frac{F_1}{ik^2}\left\{\frac{F(\alpha-r)-t(q-r)}{(tE-Fs)(q-\alpha)(\alpha-r)}\right\} \qquad (2.50)$$

$$D_2 = \frac{|k|}{it}\left[\frac{F_1}{k^2(q-\alpha)} + isD_1\right] \qquad (2.51)$$

where

$$\alpha = \frac{-|k|b^1}{ma^1}, p = \frac{m\alpha}{|k|} + (1-2\nu_\mu), r = \frac{m\alpha}{|k|} - 1, q = \alpha + 2\nu_\mu - 2, E = \left(\frac{2G-m_2}{(q-\alpha)} - \frac{2m_2}{(\alpha-r)}\right)$$

$$F = \left(\frac{2G+m_3}{(q-\alpha)} + \frac{m_3 - 2m_1m_2}{(\alpha-r)}\right), s = \left[\frac{4G}{p-q} + \frac{2G-m_2}{(q-\alpha)}\right], \quad t = \left[\frac{2G(1+4m_1)}{(p-q)} + \frac{(2G+m_3)}{(q-\alpha)}\right] \qquad (2.52)$$

5.1.1. Undrained state, $\omega \to \infty$

Putting the values of B_1, B_2 and B_3 from equations (2.47)-(2.49) into equations(1.24)-(1.29) which corresponds to the stresses ,displacements and pore pressure for poroelastic half space medium-I($x \geq 0$) and then taking limit $\omega \to \infty$ and then integrate, we get

$$\sigma_{11}{}^{(N)} = \frac{-F_1}{\pi} q_{10} \frac{x}{x^2+y^2} - \frac{F_1 q_9}{\pi} \frac{x(x^2-y^2)}{(x^2+y^2)^2} \qquad (2.53)$$

$$\sigma_{22}{}^{(N)} = \frac{F_1(q_{10} - 2q_9)}{\pi} \frac{x}{x^2+y^2} + \frac{F_1 q_9}{\pi} \frac{x(x^2-y^2)}{(x^2+y^2)^2} \qquad (2.54)$$

$$\sigma_{12}{}^{(N)} = \frac{-F_1}{\pi}(q_{10} - q_9)\frac{y}{x^2+y^2} - \frac{2F_1 q_9}{\pi} \frac{yx^2}{(x^2+y^2)^2} \qquad (2.55)$$

$$2GU_2{}^{1(N)} = \frac{F_1}{\pi}\left((q_{10} + q_9(2\nu_\mu - 2)\right)\tan^{-1}\frac{y}{x} + \frac{F_1 q_9}{\pi}\frac{xy}{x^2+y^2} \qquad (2.56)$$

$$2GU_1{}^{1(N)} = -\frac{F_1}{2\pi}(q_{10} + q_9(1-2\nu_\mu))\log(x^2+y^2) + \frac{F_1 q_9}{\pi}\frac{x^2}{x^2+y^2} \qquad (2.57)$$

$$p = \frac{2F_1(1+\nu_\mu)Bq_9}{3\pi}\frac{x}{x^2+y^2} \qquad (2.58)$$

Putting the values of D_1, and D_2 from equations (2.50)-(2.51) into equations(2.32)-(2.35) which corresponds to the stresses and displacements for transversely isotropic elastic half space medium-II($x \leq 0$) and then taking limit $\omega \to \infty$ and then integrate, we get

$$u^{(N)}(x,y) = \frac{-F_1}{2\pi}\{q_7 - 4m_1q_8\}\log(x^2+y^2) - \frac{F_1 q_8}{\pi}\frac{x^2}{x^2+y^2} \qquad (2.59)$$

$$v^{(N)}(x,y) = \frac{F_1}{\pi}(q_7 - q_8)\tan^{-1}\frac{y}{x} - \frac{F_1 q_8}{\pi}\frac{xy}{x^2+y^2} \qquad (2.60)$$

$$\tau_{11}{}^{(N)} = \frac{-F_1}{\pi}(m_2q_7 + m_3q_8)\frac{x}{x^2+y^2} + \frac{F_1 m_2 q_8}{\pi}\frac{x(x^2-y^2)}{(x^2+y^2)^2} \qquad (2.61)$$

$$\tau_{12}{}^{(N)} = \frac{-F_1}{\pi}(m_2q_7 - 2m_1m_2q_8)\frac{y}{x^2+y^2} + \frac{2F_1 m_2 q_8}{\pi}\frac{yx^2}{(x^2+y^2)^2} \qquad (2.62)$$

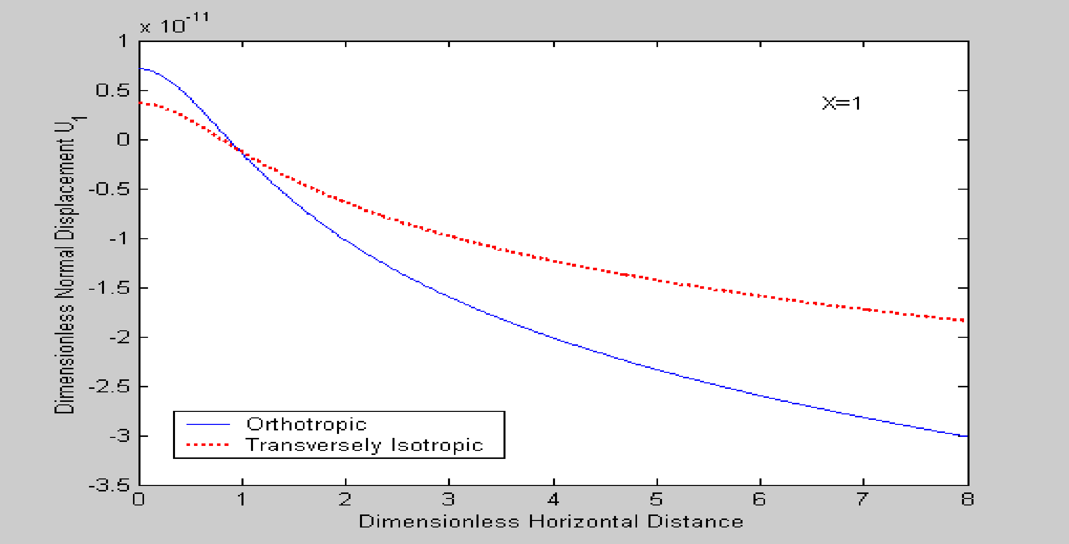
Where q_9, q_{10}, q_7, q_8 are as follow

$$q_1 = \left[\frac{4G}{3-4\nu_\mu} + \frac{2G-m_2}{2\nu_\mu - 2}\right], \quad q_2 = \left[\frac{2G(1+4m_1)}{3-4\nu_\mu} + \frac{2G+m_3}{2\nu_\mu - 2}\right], \quad q_3 = \left[\frac{2G-m_2}{2\nu_\mu - 2} - 2m_2\right],$$

$$q_4 = \left[\frac{2G+m_3}{2\nu_\mu - 2} + (m_3 - 2m_1m_2)\right], \quad q_5 = \frac{\{q_4 - q_2(2\nu_\mu - 1)\}}{2\nu_\mu - 2}, \quad q_6 = q_2q_3 - q_1q_4, \quad q_7 = \frac{q_5}{q_6},$$

$$q_8 = \frac{\{1 + q_1q_7(2\nu_\mu - 2)\}}{q_2(2\nu_\mu - 2)}, q_9 = \frac{\{4Gq_7 - 2G(1+4m_1)q_8\}}{(3-4\nu_\mu)},$$

$$q_{10} = \{-(1-2\nu_\mu)q_9 + 2Gq_7 - 8Gm_1q_8\} \qquad (2.63)$$



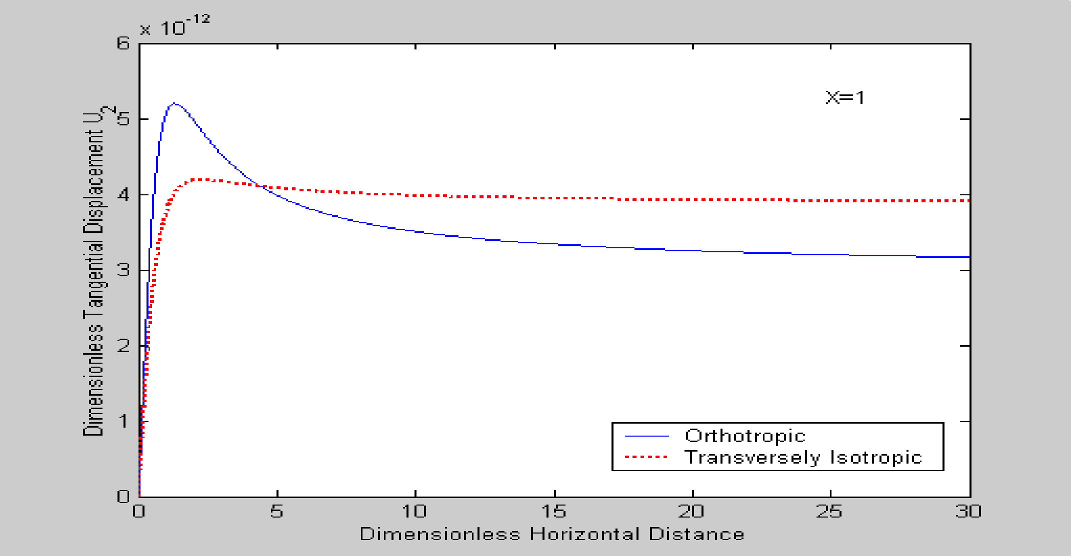


Fig. 3 (previous page) and Fig.4 (above)

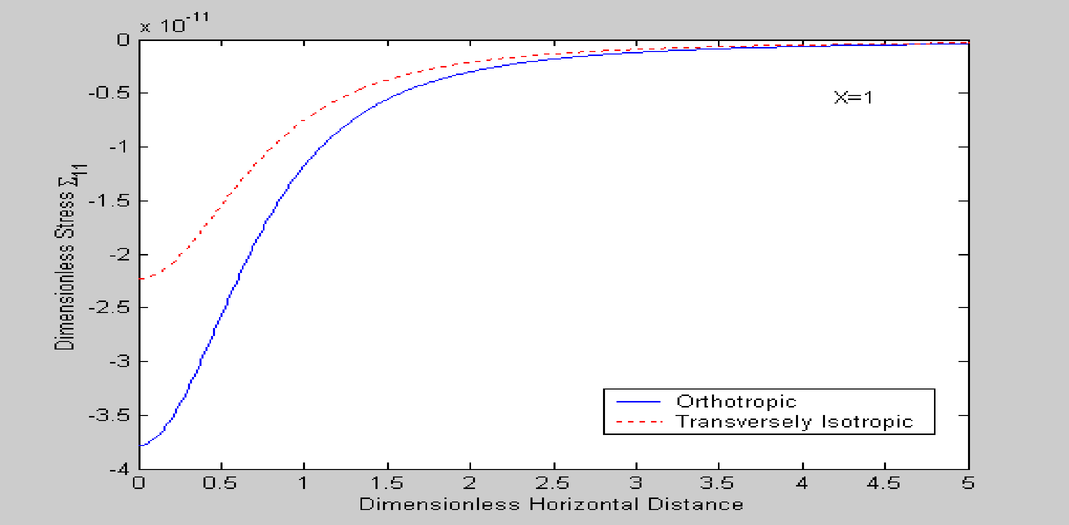


Fig. 5

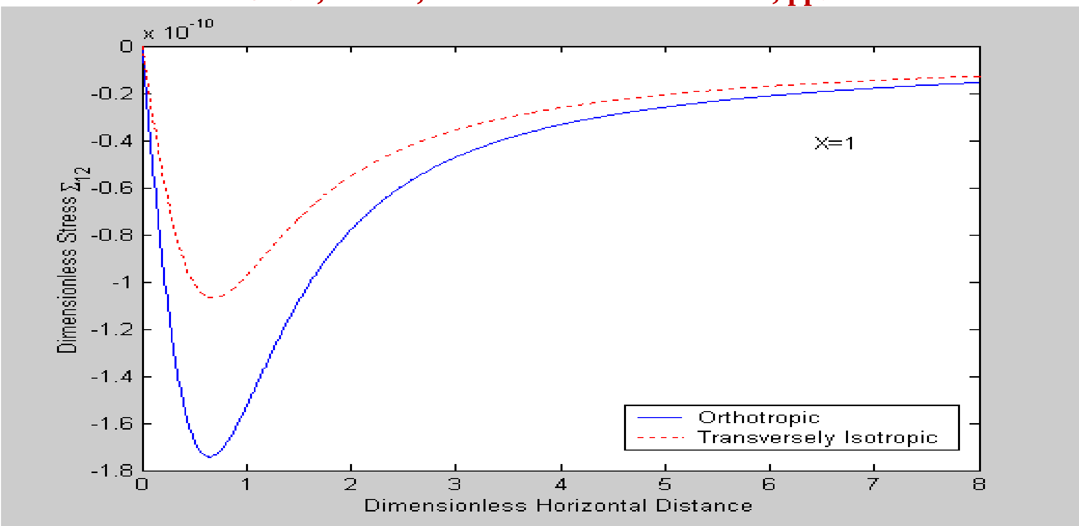


Fig. 6

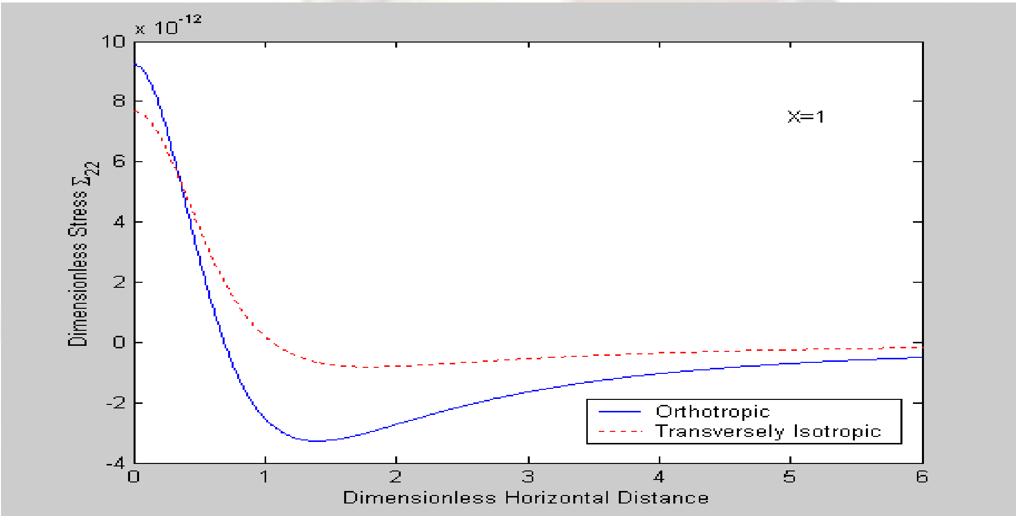
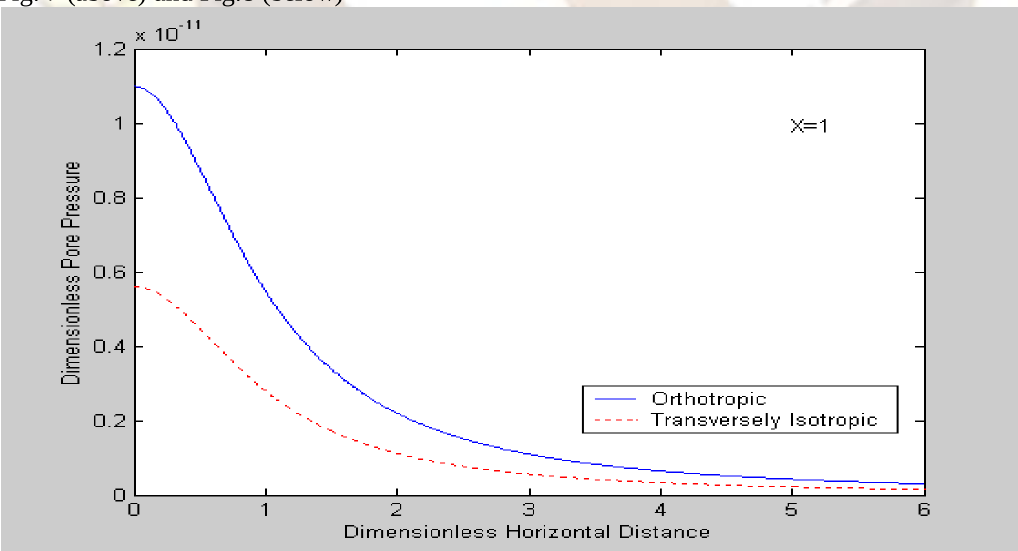


Fig. 7 (above) and Fig.8 (below)



6. Numerical results and discussion

We define the following dimensionless quantities

$X = \frac{x}{h}$, $Y = \frac{y}{h}$, And $\Sigma_{ij} = \frac{\sigma_{ij}}{G}$, ,$1 \leq i, j \leq 2$, $U_i = \frac{U_i^{1}}{h}$, i=1, 2.

For numerical computation we use elastic constants given by Love ([4]) for Topaz material and for poroelastic half space, values of poroelastic constants correspond to Ruhr Sandstone (Wang[3]).We have plotted graphs in figures (3-8) for the variation of the displacements, stresses and pore pressure against the horizontal distance X for a fixed value of X=1. Figure (3) shows the variation of normal displacement (U_1) and figure (4) shows the variation of tangential displacement (U_2) respectively. Figure (5-7) correspond to the variation of stresses against the dimensionless horizontal distance X. Figure (8) correspond to the variation of dimensionless pore pressure against the dimensionless horizontal distance X . From these figures, it is concluded that the anisotropy is affecting the deformation substantially

7. Appendix ($x > 0$)

$$\int_{-\infty}^{\infty} e^{-|k|x} e^{-\iota ky} \, dk = \frac{2x}{y^2 + x^2},$$

$$\int_{-\infty}^{\infty} |k| e^{-|k|x} e^{-\iota ky} \, dk = \frac{2(x^2 - y^2)}{(y^2 + x^2)^2},$$

$$\int_{-\infty}^{\infty} \frac{k}{|k|} e^{-|k|x} e^{-\iota ky} \, dk = \frac{-2\iota y}{y^2 + x^2},$$

$$\int_{-\infty}^{\infty} k e^{-|k|x} e^{-\iota ky} \, dk = \frac{-4\iota xy}{(y^2 + x^2)^2},$$

$$\int_{-\infty}^{\infty} \frac{1}{k} e^{-|k|x} e^{-\iota ky} \, dk = -2\iota \tan^{-1}\left(\frac{y}{x}\right),$$

$$\int_{-\infty}^{\infty} \frac{1}{|k|} e^{-|k|x} e^{-\iota ky} \, dk = -\log(y^2 + x^2)$$

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