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A Geometric Programming Model for Production Rate Optimization of Turning Process with Experimental Validation

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ABSTRACT
Every manufacturing unit wants to maximize its production rate because production rate directly affects the profit and the growth of the manufacturing unit. There are many approaches for achieving the same, some of the methods are experimental and some are based on very lengthy and time consuming statistical techniques. The manufacturing firms want a quick approximate solution to the optimization problem, so as to gain a competitive advantage in the market. In this research paper, a geometric programming based approach to maximize the production rate of the turning process within in some operating constraints is proposed. It is achieved by taking production time as the objective function of the optimization problem and then minimizing the same. It involves mathematical modeling for production time of turning process, which is expressed as a function of the cutting parameters which include the cutting speed and feed rate. Then, the developed mathematical model was optimized with in some operating constraints. The results of the experimental validation of the model reveal that the proposed method provides a systematic and efficient technique to obtain the optimal cutting parameters that will maximize the production rate of turning process.

Keywords - geometric programming, optimization, production rate, model.

Nomenclature:

C_{01} = constant = $\left(\frac{\pi dl}{1000}\right)$

C_{02} = constant = $\left(\frac{\pi dl\, t_c}{1000\, Z^{\frac{1}{n}}}\right)$

C_{11} = constant

C_0 = machine cost per unit time ($/min.)

C_m = machining cost per piece ($/piece)

C_t = tool cost ($/cutting edge)

d = diameter of the work piece (mm.)

f = feed rate (mm/revolution)

F = cutting Force (N)

l = length of the work piece (mm.)

n, a, b and p are constants.

P_t = production time per piece (min./piece)

R= nose radius of the tool (mm)

R_a =average surface roughness (µm)

t_c = tool changing time (min.)

t_h = tool handling time (min.)

t_m = time required to machine a work piece = $\frac{\pi dl}{1000\, vf}$ (min.)

T = tool life (min.)

v = the cutting speed (m/min.)

Z = constant

η = efficiency of cutting

$\lambda_{01}, \lambda_{02}$ *and* λ_{11} are lagrange multipliers.

1. Introduction

Traditionally, the selection of cutting conditions for machining was left to the machine operator. In such cases, the experience of the operator plays a major role, but even for a skilled operator it is very difficult to attain the optimum values each time. Machining parameters in metal cutting are cutting speed, feed rate and depth of cut. The setting of these parameters determines the quality characteristics of machined parts. The first necessary step for process parameter optimization in any metal cutting process is to understand the principles governing the cutting processes by developing an explicit mathematical model, which may be of two types: mechanistic and empirical [1]. To determine the optimal cutting parameters, reliable mathematical models have to be formulated to associate the cutting parameters with the cutting performance. However, it is also well known that reliable mathematical models are not easy to obtain [2-3]. The technology of metal cutting has grown substantially over time owing to the contribution from many branches of engineering with a common goal of achieving higher machining process efficiency. Selection of optimal machining condition is a key factor in achieving this condition [4]. In any multi-stage metal cutting operation, the manufacturer seeks to set the process-related controllable variables at their optimal operating conditions with minimum effect of uncontrollable or noise variables on the levels and variability in the output. To design and implement an effective process control for metal cutting operation by parameter optimization, a manufacturer seeks to balance between quality and cost at each stage of operation resulting in improved delivery and reduced warranty or field failure of a product under consideration. Gilbert [5] presented a theoretical analysis of optimization of machining process and proposed an analytical procedure to determine the cutting speed for a single pass turning operation with fixed feed rate and depth of cut by using two different objectives (i) maximum production rate and (ii) minimum machining cost. Hinduja et. al [6] described a procedure to calculate the optimum cutting conditions for machining operations with minimum cost or maximum production rate as the objective function. For a given combination of tool and work material, the search for the optimum was confined to a feed rate versus depth-of-cut plane defined by the chip-breaking constraint. Some of the other constraints considered include power available, surface finish and dimensional accuracy. In any optimization problem, it is very crucial to identify the prime objective called as the objective function or optimization criterion. In manufacturing processes, the most commonly used objective function is the specific cost [7].

Walvekar and Lambert used geometric programming for the selection of machining variables. The optimum values of both cutting speed and feed rate were found out as a function of depth of cut in multi pass turning operations [8]. Wu et. al. analyzed the problem of optimum cutting parameters selection by finding out the optimal cutting speed which satisfies the basic manufacturing criterion [9]. Basically, this optimization procedure, whenever carried out, involves partial differentiation for the minimization of unit cost, maximization of production rate or maximization of profit rate. These manufacturing conditions are expressed as a function of cutting speed. Then, the optimum cutting speed is determined by equating the partial differentiation of the expressed function to zero. This is not an ideal approach to the problem of obtaining an economical metal cutting. The other cutting variables, particularly the feed rate also have an important effect on cutting economics. Therefore, it is necessary to optimize the cutting speed and feed rate simultaneously in order to obtain an economical metal cutting operation [10]. Once the reliable model for turning operations has been constructed, an optimization algorithm is then applied to the model for determining optimal cutting parameters. The geometric model for machined parts and various time and cost components of the multistage turning operation are also given in the optimization process.

In the optimization of cutting parameters, several methods are used. Some are based on extensive experimentation which is quite laborious and lengthy process and its result may also vary in different conditions. Testing of materials like tool life test may require large amount of metals and considerable tool wear, so, it cannot be used for precious metals. The aim of this research paper is the construction of a mathematical model describing the objective function in terms of the cutting parameters with some operating constraints, then; the mathematical model is optimized by using geometric programming approach. The developed model and program can be used to determine the optimal cutting parameters to satisfy the objective of obtaining maximum production rate of turning process under different operating constraints. The results of the mathematical model were obtained by using MS-Excel. This research paper proposes a very simple, effective and efficient way of optimizing the production rate of the turning process with some operating constraints such as the maximum cutting speed, maximum feed rate, power requirement, surface roughness. This paper also highlights the advantages of using geometric programming optimization technique over other optimization techniques

2. Mathematical Modeling for Optimization:

The maximum production rate for turning process is obtained when the total production time is minimal. So, the objective function is to minimize the total production time of turning operation. The production time to produce a part by turning operation can be expressed as follows:

P_t = Machining Time + Tool Changing Time + Set-up Time (1)

$$P_t = t_m + (t_c)\frac{t_m}{T} + t_h \qquad (2)$$

The Taylor's tool life (T) used in Eq. (2) is given by:

$$T = \left(\frac{Z}{f^p\, v}\right)^{\frac{1}{n}} \qquad (3)$$

Where, n, p and Z depend on the many factors like tool geometry, tool material, work piece material, etc.

On substituting Eq. (3) in Eq. (2), we get

$$P_t(v,f) = C_{01}v^{-1}f^{-1} + C_{02}v^{\left(\frac{1}{n}\right)-1}f^{\left(\frac{p}{n}\right)-1} + t_h \qquad (4)$$

t_h does not depend on cutting speed or feed rate. So, the modified objective function from Eq. (4) can be written as:

$$P_t(v,f) = C_{01}v^{-1}f^{-1} + C_{02}v^{\left(\frac{1}{n}\right)-1}f^{\left(\frac{p}{n}\right)-1} \qquad (5)$$

2.1 Machining constraints:

There are many constraints which affect the selection of the cutting parameters. These constraints arise due to various considerations like the maximum cutting speed, maximum feed rate, power limitations, surface finish, surface roughness and the temperature at the tool-chip interface.

2.1.1 Maximum cutting speed:

The increasing of cutting speed also increases the tool wear, therefore, the cutting speed has to be kept below a certain limit called the maximum cutting speed.

$$v \leq v_{max\,.} \qquad (6)$$

$$C_{11}v \leq 1 \qquad (7)$$

Where

$$C_{11} = \frac{1}{v_{max\,.}} \qquad (8)$$

By the method of primal and dual programming of geometric programming, the maximum value of dual function or the minimum value of primal function is given by:

$$v(\lambda) = \left[\frac{C_{01}}{\lambda_{01}}(\lambda_{01}+\lambda_{02})\right]^{\lambda_{01}} \left[\frac{C_{02}}{\lambda_{02}}(\lambda_{01}+\lambda_{02})\right]^{\lambda_{02}} \left[\frac{C_{11}}{\lambda_{11}}\right]^{\lambda_{11}} \quad (9)$$

Subject to the following constraints:

$$\lambda_{01} + \lambda_{02} = 1 \quad (10)$$

$$-\lambda_{01} + \left\{\left(\frac{1}{n}\right) - 1\right\}\lambda_{02} + \lambda_{11} = 0 \quad (11)$$

$$-\lambda_{01} + \left\{\left(\frac{p}{n}\right) - 1\right\}\lambda_{02} = 0 \quad (12)$$

And the non-negativity constraints are:

$$\lambda_{01} \geq 0, \lambda_{02} \geq 0 \text{ and } \lambda_{11} \geq 0 \quad (13)$$

On adding Eq. (10) and Eq. (12), we get

$$\lambda_{02} = n \quad (14)$$

From Eq. (10) and Eq. (14), we get

$$\lambda_{01} = 1 - n \quad (15)$$

From Eq. (11), (14) and (15), we get

$$\lambda_{11} = 1 - p \quad (16)$$

Therefore, the maximum value of dual function and the minimum value of primal function is given by:

$$v(\lambda) = \left(\frac{C_{01}}{1-n}\right)^{1-n} \left(\frac{C_{02}}{n}\right)^{n} \{C_{11}(1-p)\}^{1-p} \quad (17)$$

Now,

$$\lambda_{11} = \frac{C_{11} v}{v(\lambda)} \quad (18)$$

From Eq. (17) and Eq. (18), we get the optimum values of cutting speed as:

$$v = \frac{\lambda_{11}\left(\frac{C_{01}}{1-n}\right)^{1-n}\left(\frac{C_{02}}{n}\right)^{n}(C_{11})^{1-p}}{C_{11}} \quad (19)$$

And,

$$\lambda_{01} = \frac{C_{01} v^{-1} f^{-1}}{v(\lambda)} \quad (20)$$

Therefore, from (19) and (20), we get optimum feed rate as:

$$f = \frac{(C_{01} \times C_{11})}{(1-n)\times\lambda_1 \times \left(\frac{C_{01}}{1-n}\right)^{1-n}\left(\frac{C_{02}}{n}\right)^{n}(C_{11})^{1-p}} \quad (21)$$

2.1.2 Maximum feed rate:

In rough machining operations, feed rate is taken as a constraint to achieve the maximum production rate.

$$f \leq f_{max}. \quad (22)$$

$$C_{11} f \leq 1 \quad (23)$$

Where

$$C_{11} = \frac{1}{f_{max}.} \quad (24)$$

Following the same procedure as described for the first constraint, we get the following values:

$$\lambda_{01} = 1 - n \quad (25)$$

$$\lambda_{02} = n \quad (26)$$

$$\lambda_{11} = 1 - p \quad (27)$$

$$f = \frac{\lambda_{11}\left(\frac{C_{01}}{1-n}\right)^{1-n}\left(\frac{C_{02}}{n}\right)^{n}(C_{11})^{1-p}}{C_{11}} \quad (28)$$

$$v = \frac{(C_{01} \times C_{11})}{(1-n)\times\lambda_1 \times \left(\frac{C_{01}}{1-n}\right)^{1-n}\left(\frac{C_{02}}{n}\right)^{n}(C_{11})^{1-p}} \quad (29)$$

2.1.3 Power constraint:

The maximum power available for the turning operation will be a constraint in the turning operation, which has to be taken in to consideration. The power available for the turning operation is given by:

$$P = \frac{F \times v}{6120\eta} \leq P_{max}. \quad (30)$$

$$C_{11} v \leq 1 \quad (31)$$

Where

$$C_{11} = \frac{F}{6120\eta P_{max}.} \quad (32)$$

Following the same procedure as described for the first constraint, we get the following values:

$$\lambda_{01} = 1 - n \quad (33)$$

$$\lambda_{02} = n \quad (34)$$

$$\lambda_{11} = 1 - p \quad (35)$$

$$v = \frac{\lambda_{11}\left(\frac{C_{01}}{1-n}\right)^{1-n}\left(\frac{C_{02}}{n}\right)^{n}(C_{11})^{1-p}}{C_{11}} \quad (36)$$

$$f = \frac{(C_{01} \times C_{11})}{(1-n)\times\lambda_1 \times \left(\frac{C_{01}}{1-n}\right)^{1-n}\left(\frac{C_{02}}{n}\right)^{n}(C_{11})^{1-p}} \quad (37)$$

2.1.4 Surface roughness:

Surface roughness can be used as a constraint in finishing operations. Therefore, it becomes a very important factor in determining finish cutting conditions. Surface roughness can be expressed in terms of feed as follows:

$$R_a = \frac{f^2}{32R} \quad (38)$$

$$R_a C_{11} \leq \frac{f^2}{32R} \quad (39)$$

Following the same procedure as described for the first constraint, we get the following values:

$$\lambda_{01} = 1 - n \quad (40)$$

$$\lambda_{02} = n \quad (41)$$

$$\lambda_{11} = \frac{(1-p)}{2} \quad (42)$$

$$v = \frac{\left[\frac{(1-p)}{2}\right]\left[\frac{C_{01}}{1-n}\right]^{1-n}\left[\frac{C_{02}}{n}\right]^{n}[C_{11}]^{1-p}}{C_{11}} \quad (43)$$

$$f = \frac{C_{01} \times C_{11}}{(1-n)\left\{\frac{(1-p)}{2}\right\}\left\{\left[\frac{C_{01}}{1-n}\right]^{1-n}\left[\frac{C_{02}}{n}\right]^{n}[C_{11}]^{1-p}\right\}^{2}} \quad (44)$$

2.1.5 Chip-tool interface temperature constraint:

The temperature at the chip tool interface should be with in a permissible limit otherwise overheating may lead to excessive tool wear damage to metal. The temperature at the chip tool interface is represented by

$$Q_i = k\, v^a f^b \leq Q_u \quad (45)$$

Or

$$C_{11}\, k\, v^a f^b \leq 1 \quad (46)$$

Where $C_{11} = \frac{1}{Q_u}$

By using the method of primal and dual programming of geometric programming, the maximum value of dual function or the minimum value of primal function is given by:

$$v(\lambda) = \left[\frac{C_{05}}{\lambda_{01}}(\lambda_{01}+\lambda_{02})\right]^{\lambda_{01}} \left[\frac{C_{06}}{\lambda_{02}}(\lambda_{01}+\lambda_{02})\right]^{\lambda_{02}} \left[\frac{C_{11}}{\lambda_{11}}\right]^{\lambda_{11}} \quad (47)$$

Subject to the following constraints:

$$\lambda_{01} + \lambda_{02} = 1 \quad (48)$$

$$-\lambda_{01} + \left\{\left(\frac{1}{n}\right) - 1\right\}\lambda_{02} + a\,\lambda_{11} = 0 \quad (49)$$

$$-\lambda_{01} + \left\{\left(\frac{p}{n}\right) - 1\right\}\lambda_{02} + b\,\lambda_{11} = 0 \qquad (50)$$

On solving we get the following values of lagrange multipliers:

$$\lambda_{01} = 1 - \frac{an}{ap+bn-b} \qquad (51)$$

$$\lambda_{02} = \frac{an}{ap+bn-b} \qquad (52)$$

$$\lambda_{11} = \frac{n-1}{ap+bn-b} \qquad (53)$$

So,

$$v(\lambda) = \left(\frac{C_{05}}{1-\frac{an}{ap+bn-b}}\right)^{1-\frac{an}{ap+bn-b}} \left(\frac{C_{06}}{\frac{an}{ap+bn-b}}\right)^{\frac{an}{ap+bn-b}} \left(\frac{C_{11}}{\frac{n-1}{ap+bn-b}}\right)^{\frac{n-1}{ap+bn-b}} \qquad (54)$$

On solving we get the optimum values of cutting speed and feed rate as:

$$v = \left(\frac{\lambda_{11} v(\lambda)^{1-b} {\lambda_{01}}^{b}}{C_{11}\, {C_{03}}^{b}}\right)^{a-b} \qquad (55)$$

$$f = C_{05}\, v(\lambda) \left(\frac{C_{11} {C_{03}}^{b}}{\lambda_{11}\, v(\lambda)^{1-b}}\right)^{a-b} \qquad (56)$$

2.2 Experimental validation of the mathematical model:

For validation of the mathematical model, experimental values were used from [11] and [12]. The values taken from the Chen-Tsai cutting model and experimental data were incorporated in the mathematical model developed to analyze the variations in the production time against the cutting speed and feed rate. The missing values required for analysis were assumed suitably. The values of the various parameters used in the experimental validation are as follows:

a = b = 1
d = 50 mm.
n = 0.9
l = 300 mm
p = 1
R= 1.2 mm.
R_a = 10 µm.
t_c = 0.5 min.
t_h = 0.5 min.
t_1 = 45 min.
t_2 = 0.75 min. /piece.
t_3 = 0.5min.
η = 0.85.
Z = 10

3. Results and Discussion:

3.1 Figures:

The figures obtained from the implementation of the mathematical model are as follows:

Table 1: Experimental values of cutting speed (v) and feed rate (f):

S. No.	Cutting speed (v) m/min.	Feed rate (f) mm./rev.
1	105	0.02
2	115	0.04
3	125	0.06
4	135	0.08
5	145	0.10
6	155	0.12
7	165	0.14
8	175	0.16
9	185	0.18
10	195	0.20

Table 2: Values of constants:

S. No.	Parameter	Formulae	Value
1	C_{01}	C_{01}= constant = $\left(\frac{\pi dl}{1000}\right)$	47
2	C_{02}	C_{02} = constant = $\left(\frac{\pi dl}{1000\, Z^{\frac{1}{n}}}\right)$	18

Table 3: Optimum cutting parameters for minimum production time:

S.No.	Optimum cutting speed (v) (m/min.)	Optimum feed rate (f) (mm. /rev.)	Optimum Production time (P_t) (min./piece)
1	155	0.12	28

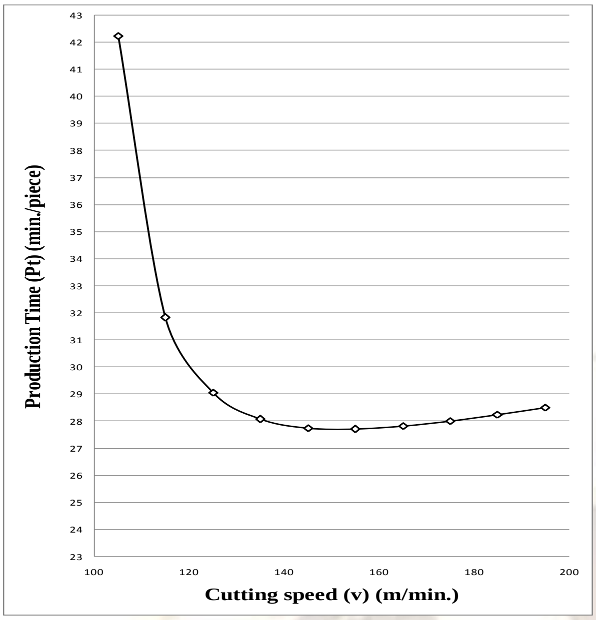

Figure 1: Variation of production time versus cutting speed

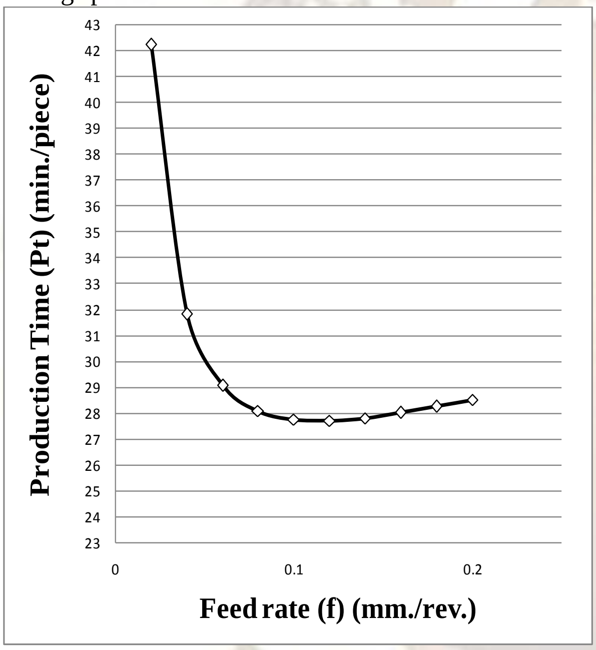

Figure 2: Variation of production time versus feed rate

3.2 Analysis of results:

The curves obtained between production time and cutting speed reveal that a smaller value of cutting speed results in a high production time. It is due to the fact that a smaller cutting speed increases the production time of parts. Also, it will decrease the profit rate due to the production of a lesser number of parts. However, if the cutting speed is too high, it will also lead to a high production time due to excessive tool wear and increased machine downtime. The optimum cutting speed is somewhere between "too slow" and "too fast" which will yield the minimum production time and the production rate will be maximum at the same cutting speed.

The curves between the production time and the feed rate indicate that a small feed rate will result in high production time. A smaller feed rate means the number of revolutions should be increased. The more the number of revolutions, the more will be the production time. Even a very high feed rate is not advisable as it will increase the tool wear and surface roughness resulting in increased machining time and machine downtime resulting in high production time. So, the optimum feed rate is somewhere between "too small" and "too high" which will result in the minimum production time and the production rate will be maximum at the same feed rate.

4. Conclusion:

In this research paper, the cutting speed and feed rate were modeled for the maximum production rate of a turning operation. The maximum cutting speed, the maximum feed rate, maximum power available and the surface roughness was taken as constraints. The results of the model reveal that the proposed method provides a systematic and efficient methodology to obtain the maximum production rate for turning. It can be concluded from this study that the obtained model can be used effectively to determine the optimum values of cutting speed and feed rate that will result in maximum production rate. The developed model saves a considerable time in finding the optimum values of the cutting parameters. It has been shown that the method of geometric programming can be applied successfully to optimize the production rate of turning process. The coefficients n, p and Z of the extended Taylor's tool life equation are not described in depth for all cutting tool and work piece combinations. Obtaining these coefficients experimentally requires lot of time, resources and then, the analysis of the obtained values increases the complexity of the process.

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