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The Effects of Concentration and Hall Current on Unsteady Flow of a Viscoelastic Fluid in a Fixed Plate

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ABSTRACT

The present Study deals with the effects of concentration and Hall current on unsteadyflow of a viscoelastic fluid in a fixed plate. The resultant equations have been solved analytically. The velocity, temperature and concentration distributions are derived, and their profiles for various physical parameters are shown through graphs. The coefficient of Skin friction, Nusselt number and Sherwood number at the plate are derived and their numerical values for various physical parameters are presented through tables. The influence of various parameters such as the thermal Grashof number, mass Grashof number, Schmidt number, Prandtl number, Hall Parameter, Hartmann number, Chemical Reaction parameter and the frequency of oscillation on the flow field are discussed. It is seen that, the velocity increases with the increase in Gc and Gr,and it decreases with increase in Sc, M, m, Kc, and Pr, temperature decreases with increase in Pr, Also, the concentration decreases with the increase in Sc and Kc.

KEY WORDS: Hall Current, Viscoelastic, Porous medium, Magnetohydrodynamics.

1.0 INTRODUCTION

There exist flows which are caused not only by temperature differences but also by concentration differences. There are several engineering situations wherein combined heat and mass transport arise such as dehumidifiers, humidifiers, desert coolers, and chemical reactors etc. Magnetohydrodynamics is currently undergoing a period of great enlargement and differentiation of subject matter. It is important in the design of MHD generators and accelerators in geophysics, underground water storage system, soil sciences, astrophysics, nuclear power reactor, solar structures, and so on.

Moreso, the interest in these new problems generates from their importance in liquid metals, electrolytes and ionized gases. On account of their varied importance, these flows have been studied by several authors – notable amongst them are Singh and Singh (2012) they investigated MHD flow of Viscous Dissipation and Chemical Reaction over a Stretching porous plate in a porous medium numerically. Kai- Long (2010) studied Heat and mass transfer for a viscous flow with radiation effect past a non-linear stretching sheet. Hall Effect on MHD mixed convective flow of a viscous incompressible fluid past a vertical porous plate immersed in a porous medium with Heat Source/Sink was studied by Sharma *et al.* (2007).
Saleh*et al.* (2010) examined Heat and mass transfer in MHD viscoclastic fluid flow through a porous medium over a stretching sheet with chemical reaction. Das and Jana (2010) examined Heat and Mass transfer effects on unsteady MHD free convection flow near a moving vertical plate in a porous medium. The effects of chemical reaction, Hall current and Ion – Slip currents on MHD micropolar fluid flow with thermal Diffusivity using Novel Numerical Technique was studied by Motsa and Shateyi (2012). Aboeldahab and Elbarby (2001) examined Hall current effect on Magnetohydrodynamics free convection flow past a Semi – infinite vertical plate with mass transfer. Hall current effect with simultaneous thermal and mass diffusion on unsteady hydromagnetic flow near an accelerated vertical plate was investigated Acharya*et al.* (2001). Takhar (2006) studied Unsteady flow free convective flow over an infinite vertical porous platedue to the combined effects of thermal and mass diffusion, magnetic field and Hall current.
Exact Solution of MHD free convection flow and Mass Transfer near a moving vertical porous plate in the presence of thermal radiation was investigated by Das (2010). Anjali Devi and Ganga examined effects of viscous and Joules dissipation and MHD flow, heat and mass transfer past a Stretching porous surface embedded in a porous medium. Sonth*et al.* (2012) studied Heat and Mass transfer in a viscoelastic fluid over an accelerated surface with heat source/sink and viscous dissipation. Shateyi*et al.* (2010) investigated The effects of thermal Radiation, Hall currents, Soret and Dufour on MHD flow by mixed convection over vertical surface in porous medium. Effect of Hall currents and Chemical reaction and Hydromagnetic flow of a stretching vertical surface with internal heat generation / absorption was examined by Salem and El-Aziz (2008).

2.0 MATHEMATICAL FORMULATION

We consider the unsteady flow of a viscous incompressible and electrically conducting viscoelastic fluid over afixed porous plate with oscillating temperature. The x- axis is assumed to be oriented vertically upwards along the plate and the y-axis is taken normal to the plane of the plate. It is assumed that the plate is electrically non – conducting and a uniform magnetic field of straight B_0 is applied normal to the plate. The induced magnetic field is assumed constant. So that $\vec{B}=(0,B_0,0)$,. The plate is subjected to a constant suction velocity.

1. The equation of conservation of charge $\nabla\cdot J=0$, gives constant.

2. $$\vec{J}=\omega_e\tau_e(\vec{J}\times\vec{E})=\sigma\left(\vec{V}\times\vec{B}+\frac{\nabla P_e}{en_e}\right) \quad (1)$$

Equation (1) reduces to

$$\left.\begin{aligned} J_{x^*}&=\frac{\sigma B_0}{(1+m^2)}\left(mu^*-\omega^*\right)\\ J_{y^*}&=\frac{\sigma B_0}{(1+m^2)}\left(u^*-m\omega^*\right)\end{aligned}\right\} \quad (2)$$

Where

$m=\omega_e\tau_e$

is the Hall parameter.

The governing equations for the momentum, energy and concentration are as follows;

$$\frac{\partial u}{\partial t}+v_0\frac{\partial u}{\partial y}=v\frac{\partial u}{\partial y}-k_1\frac{\partial^3 u}{\partial y^2\partial t}-\sigma B_0^{\,2}\frac{(u+m\omega)}{\rho(1+m^2)}+g\beta(T-T_d)+g\beta^*(C-C_d)-\frac{vu}{k^*} \quad (3)$$

$$\frac{\partial\omega}{\partial t}+v_0\frac{\partial\omega}{\partial y}=v\frac{\partial^2\omega}{\partial y^2}-k_1\frac{\partial^3\omega}{\partial y^2\partial t}-\sigma B_0^{\,2}\frac{(\omega-mu)}{\rho(1+m^2)}-\frac{u\omega}{k^*} \quad (4)$$

$$\frac{\partial T}{\partial t}+v_0\frac{\partial T}{\partial y}=\frac{K_T}{\rho Cp}\frac{\partial^2 T}{\partial y^2} \quad (5)$$

$$\frac{\partial C}{\partial t}+v_0\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}-KrC \quad (6)$$

The boundary conditions of the problem are:

$$u=0,\omega=0,T=T_d+(T_0-T_d)e^{i\omega t},C=C_d+(C_0-C_d)e^{i\omega t}aty=0$$
$$u\to 0,\omega=0,T=0,C=0aty=1 \quad (7)$$

Where u and v are the components of velocity in the x and y direction respectively, g is the acceleration due to gravity, β and β^* are the coefficient of volume expansion, k_0 is the kinematic viscoelasticity, ρ is the density, μ is the viscosity, ν is the kinematic viscosity, K_T is the thermal conductivity, Cp is the specific heat in the fluid at constant pressure, σ is the electrical conductivity of the fluid, μ_e is the magnetic permeability, D is the molecular diffusivity, T_0 is the temperature of the plane and T_d is the temperature of the fluid far away from plane. C_0 is the concentration of the plane and C_d is the concentration of the fluid far away from the plane.

And $v=-v_0$, the negative sign indicate that the suction is towards the plane.

Introducing the following non-dimensionless parameters

$$\left.\begin{aligned}&\eta=\frac{v_0y}{v},t=\frac{v_0^{\,2}t}{4v},u=\frac{u}{v_0},\omega=\frac{\omega^*}{v_0},\theta=\frac{T-T_\infty}{T_\omega-T_\infty},C=\frac{C-C_\infty}{C_\omega-C_\infty}\\ &Gr=\frac{g\beta v(T_\omega-T_\infty)}{v_0^{\,2}},Gc=\frac{g\beta cv(C_\omega-C_\infty)}{v_0^{\,2}},M=\frac{\sigma B_0^{\,2}v}{\rho v_0^{\,2}},\Pr=\frac{\mu Cp}{K_T}\\ &Sc=\frac{v}{D},K=\frac{k_1v_0^{\,2}}{4v^2},k=\frac{k^*v_0^{\,2}}{v^2}\end{aligned}\right\} \quad (8)$$

Substituting the dimensionless variables in (8) into (3) to (6), we get (dropping the bars)

$$\frac{1}{4}\frac{\partial u}{\partial t}-\frac{\partial u}{\partial\eta}=\frac{\partial^2 u}{\partial\eta^2}-\frac{K\partial^3 u}{4\partial\eta^2\partial t}-\frac{M(u+m\omega)}{(1+m^2)}-\frac{u}{k}+Gr\theta+GcC \quad (9)$$

$$\frac{1}{4}\frac{\partial\omega}{\partial t}-\frac{\partial\omega}{\partial\eta}=\frac{\partial^2\omega}{\partial\eta^2}-\frac{K\partial^3\omega}{4\partial\eta^2\partial t}-\frac{M(\omega-mu)}{(1+m^2)}-\frac{u}{k} \quad (10)$$

$$\frac{1}{4}\frac{\partial\theta}{\partial t}-\frac{\partial\theta}{\partial\eta}=\frac{1}{\Pr}\frac{\partial^2\theta}{\partial\eta^2} \quad (11)$$

$$\frac{1}{4}\frac{\partial C}{\partial t}-\frac{\partial C}{\partial\eta}=\frac{1}{Sc}\frac{\partial^2 C}{\partial\eta^2} \quad (12)$$

The corresponding boundary conditions are

$$u(0,t)=0,\omega(0,t)=0,\theta(0,t)=e^{i\omega t},C(0,t)e^{i\omega t}aty=0$$
$$u(1,t)=\omega(1,t)=\theta(1,t)=C(1,t)=0aty=1 \quad (13)$$

Equations (9) and (10) can be combined into a single equation by introducing the complex velocity.

$$U = u(\eta,t) + i\omega(\eta,t) \quad (14)$$

Where

$$i = \sqrt{-1}$$

Thus,

$$\frac{1}{4}\frac{\partial U}{\partial t} - \frac{\partial U}{\partial \eta} = \frac{\partial^2 U}{\partial \eta^2} - \frac{K\partial^3 U}{4\partial \eta^2 \partial t} - \frac{M(1-im)U}{(1+m^2)} - \frac{U}{k} + Gr\theta + GcC \quad (15)$$

With boundary conditions:

$$t^* > 0 : U(0,t) = 0, \theta(0,t) = e^{i\omega t}, C(0,t)e^{i\omega t} at\eta = 0$$
$$U(1,t) = \theta(1,t) = C(1,t) \to 0, at\eta = 1 \quad (16)$$

where Gr is the thermal Grashof number, Gc is the mass Grashof number, Sc is the Schmidt number, Pr is the Prandtl number, Rm is the Reynold number, K is the viscoelastic Parameter and S is the pearmeability.

3.0 METHOD OF SOLUTIONS

To solve (11), (12) and (15) subject to the boundary conditions (16), we assume solutions of the form

$$U(\eta,t) = U_1(\eta)\varepsilon e^{i\omega t} \quad (17)$$

$$\theta(\eta,t) = \theta_1(\eta)\varepsilon e^{i\omega t} \quad (18)$$

$$C(\eta,t) = C_1(\eta)\varepsilon e^{i\omega t} \quad (19)$$

where $U_1(\eta)$ $\theta_1(\eta)$ and $C_1(\eta)$ are to be determined.

Substituting (17) to (19) into (11), (12) and (15), Comparing harmonic and non harmonic terms, we obtain

$$U'' + \frac{U'}{L_1} - L_3 U = -\frac{Gr\theta}{4L_1 e^{i\omega t}} - \frac{GcC}{4L_1 e^{i\omega t}} \quad (20)$$

$$\theta_1'' + \Pr\theta_1' - \frac{1}{4}i\omega\theta_1 = 0 \quad (21)$$

$$C_1'' + ScC_1' - \frac{1}{4}i\omega C_1 - KcScC_1 = 0 \quad (22)$$

and boundary conditions give

$$\left.\begin{array}{l}\eta = 0 : U_1(0) = \theta(0) = C(0) = 1 \\ \eta = 1 : U_1(1) = 0, \theta(1) = 0, C(1) \to 0\end{array}\right\} \quad (23)$$

where the primes connotes differentiation with respect to η.

Solving (20) to (22) under the boundary conditions (23) and (25).And substituting the obtained solutions into (17) to (19) respectively. Then
the velocity distribution can be expressed as

$$U(\eta,t) = \left[A_5 e^{m_5\eta} + A_6 e^{-m_6\eta} + A_8 e^{m_3\eta} + A_9 e^{-m_4\eta} + A_{10} e^{m_1\eta} + A_{11} e^{-m_2\eta}\right]\varepsilon e^{\text{int}} \quad (24)$$

And, the temperature field is given by

$$\theta(\eta,t) = \left(A_3 e^{m_3\eta} + A_4 e^{-m_4\eta}\right)\varepsilon e^{i\omega t} \quad (25)$$

Similarly, the concentration distribution gives

$$C(\eta,t) = \left(A_1 e^{m_1\eta} + A_2 e^{-m_2\eta}\right)\varepsilon e^{i\omega t} \quad (26)$$

The Skin friction, Nusselt number and Sherwood number is obtained by differentiating (24) to (26) and at evaluated at $\eta = 0$ respectively.

$$-\left.\frac{\partial U(\eta,t)}{\partial \eta}\right|_{\eta=0} = \left[m_5 A_5 + m_6 A_6 + m_3 A_8 + m_4 A_9 + m_1 A_{10} + m_2 A_{11}\right]\varepsilon e^{i\omega t} \quad (27)$$

$$-\left.\frac{\partial \theta(\eta,t)}{\partial \eta}\right|_{\eta=0} = \left(m_3 A_3 + m_4 A_4\right)\varepsilon e^{i\omega t} \quad (28)$$

$$-\left.\frac{\partial C(\eta,t)}{\partial \eta}\right|_{\eta=0} = \left(m_1 A_1 + m_2 A_2\right) \quad (29)$$

4.0 RESULTS AND DISCUSSION

The effects of concentration and Hall current on unsteady flow of a viscoelastic fluid in a fixed plate has been formulated and solved analytically. In order to understand the flow of the fluid, computations are performed for different parameters such as Gr, Gc, Sc, Pr, ω, Kc, M, and m.

4.1 Velocity profiles

Figures 1 to 7connote the velocity profiles; figure 8 depicts the temperature profiles and figures9& 10display the concentration profiles with varying parameters respectively.

The effect of velocity for different values of (Pr = 0.71, 1, 3) is displayed in figure 1, the graph show that velocity decreases with increase in Pr.
The effect of velocity for different values of (Sc = 0.18, 0.78, 2.01, 2.65) is given in figure 2, the graph show that velocity decreases with the increase in Sc.
Figure 3 denotes the effect of velocity for different values of (m = 2, 4, 6, 10), it is seen that velocity decreases with the increase in m.
The effect of velocity for different values of (M = 1, 3, 6, 10) is shown in figure 4, it depict that velocity decreases with increase in M.

Figure 5 depicts the effect of velocity for (Gr = 1, 2, 3, 5), the graph connote that velocity increases with the increase in Gr.

The effect of velocity for different values of (Gc = 2, 3, 4, 8) is presented in figure 6, it is seen that velocity increases with the increase in Gc.

The effect of velocity for different values of (Kc = 1, 2, 3, 4) is displayed in figure 7, it is clear that velocity decreases with the increase in Kc.

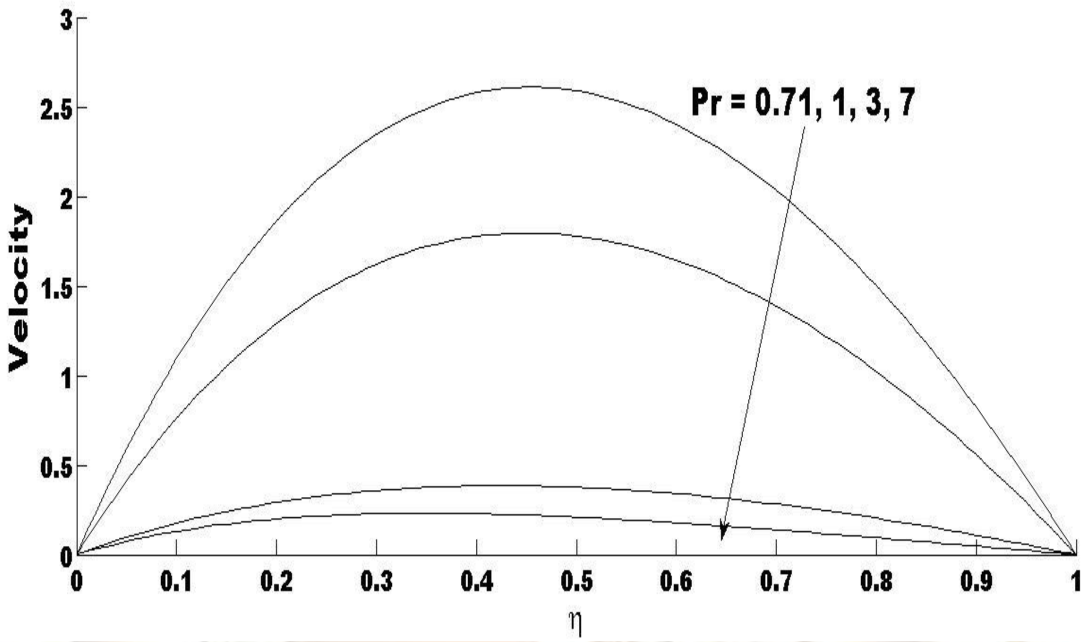

Figure 1. Velocity profiles for different values of Pr.

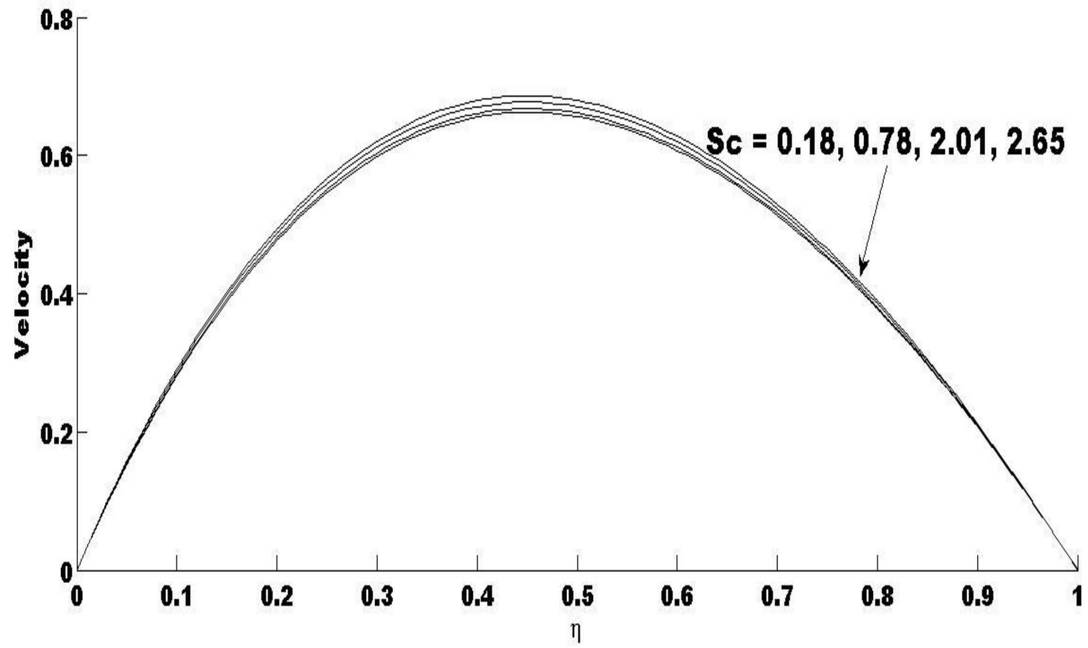

Figure 2. Velocity profiles for different values of Sc.

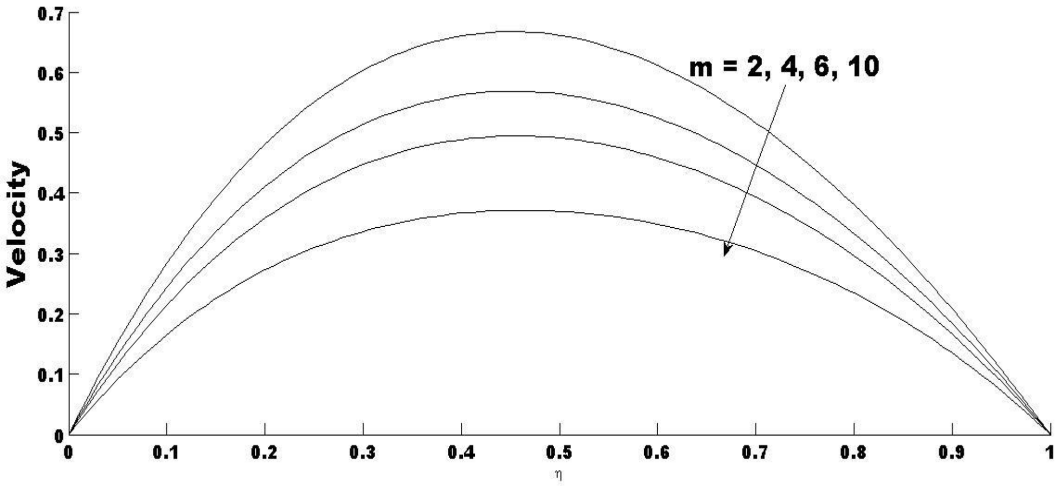

Figure 3. Velocity profiles for different values of m.

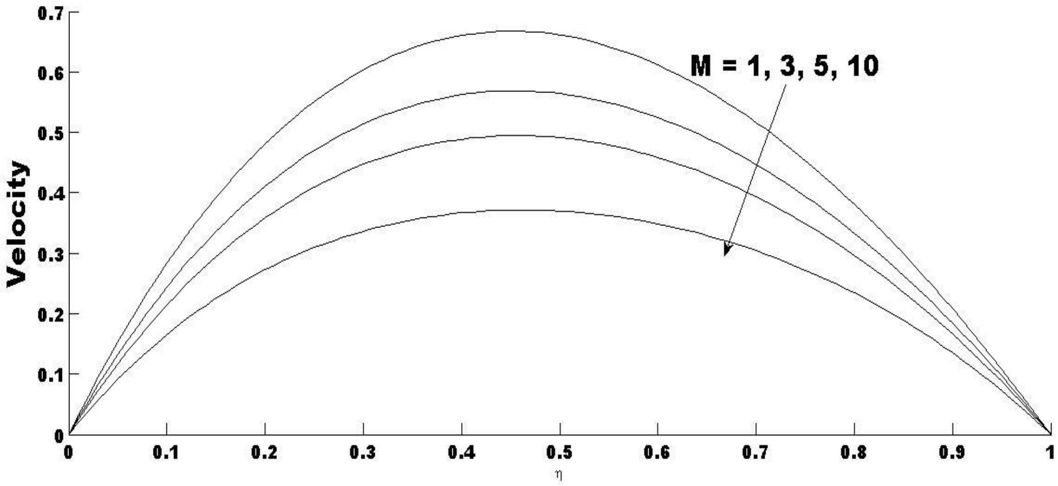

Figure 4. Velocity profiles for different values of M .

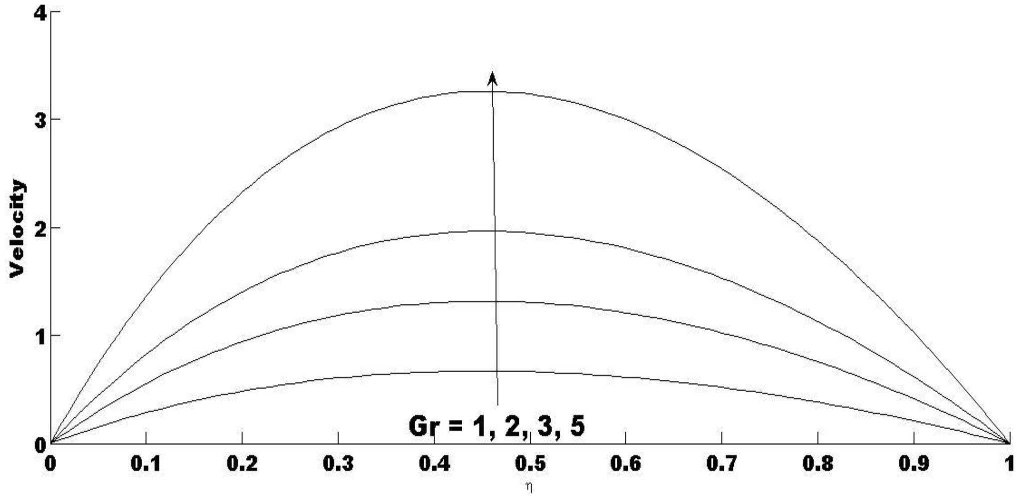

Figure 5. Velocity profiles for different values of Gr.

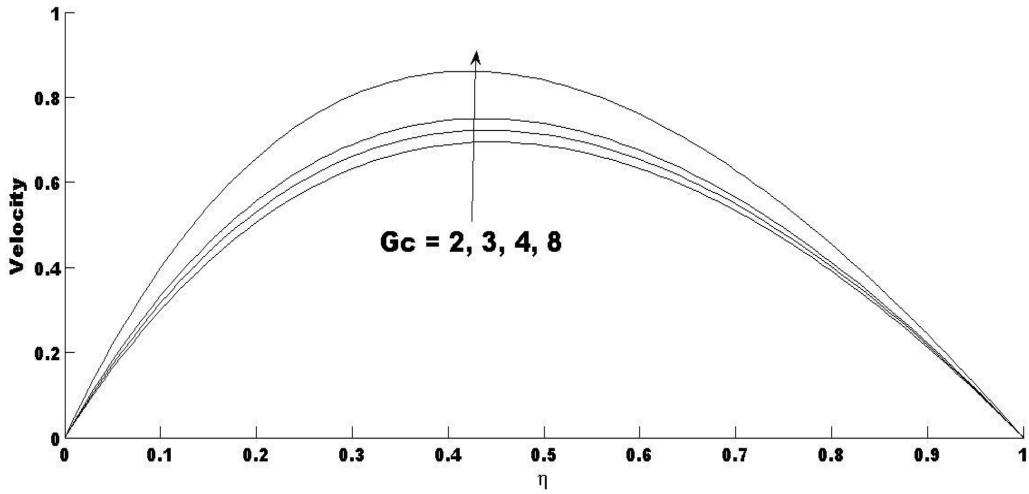

Figure 6. Velocity profiles for different values of Gc.

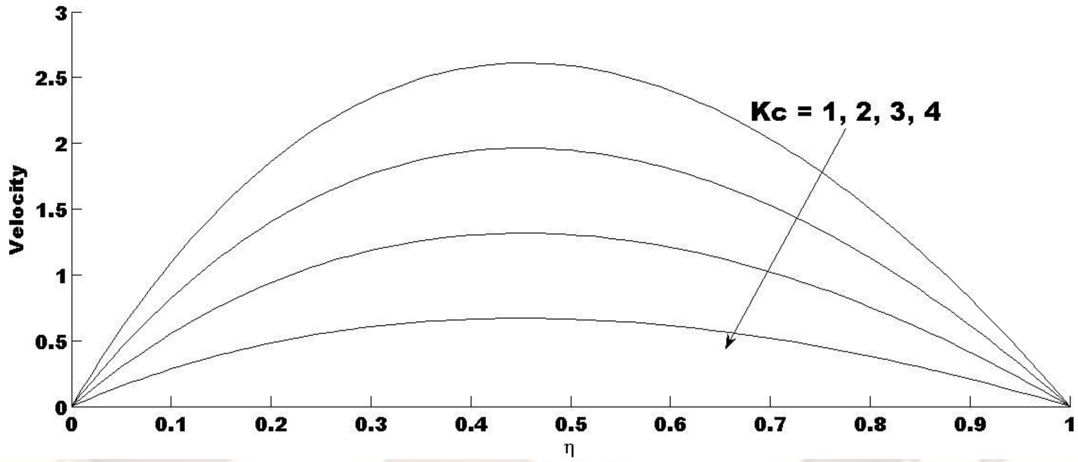

Figure 7. Velocity profiles for different values of Kc.

4.2 **Temperature profiles**

In figure 8, the effect of temperature for different values of (Pr = 0.71, 1, 3, 7) is given. The graph show that temperature increases with increasing Pr.

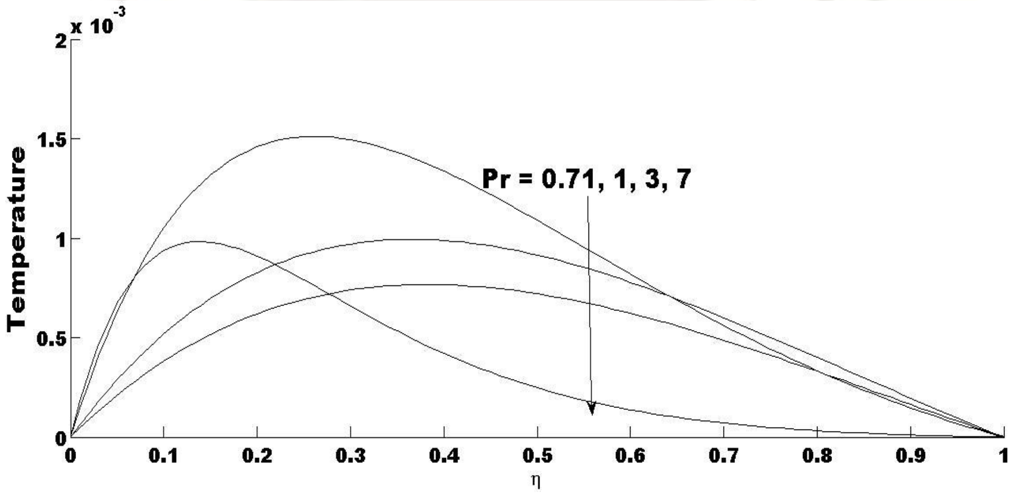

Figure 8. Temperature profiles for different values of Pr.

4.3 Concentration profiles

Figure 9 depicts the effect of concentration for (Sc = 0.3, 0.78, 1, 2.01), it is seen that concentration increases with the increase in Sc. The effect of concentration for (Kc= 0, 0.5, 1, 3) is given in figure 10, it is seen that concentration decreases with the decrease in Kc.

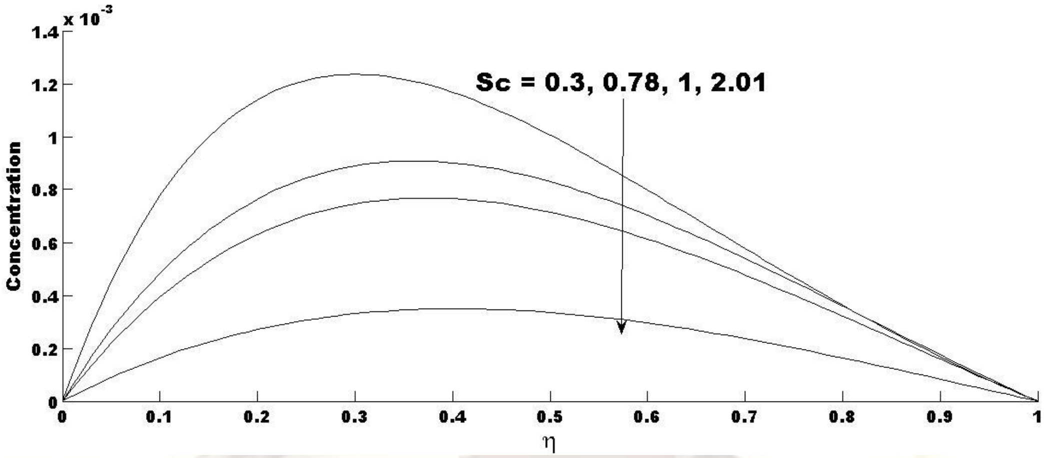

Figure 9. Concentration profiles for different values of Sc.

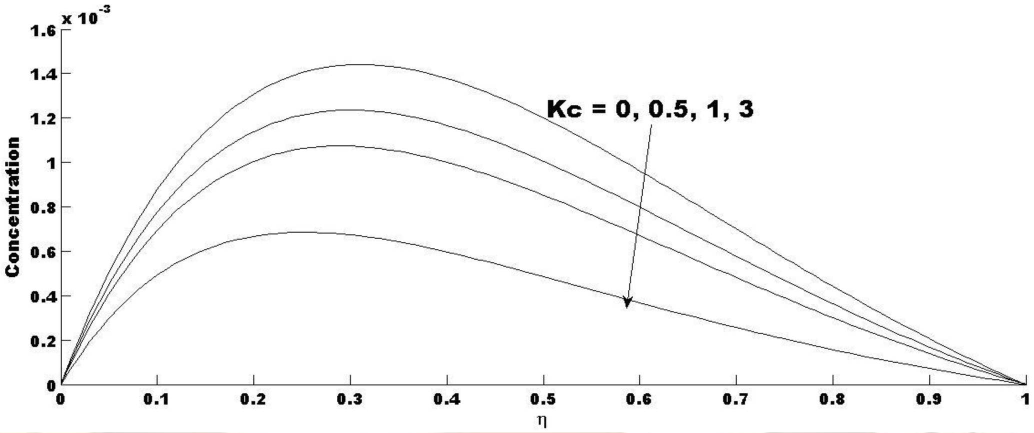

Figure 10. Concentration profiles for different values of Kc.

Table 1 to 3 represent the Skin friction, Nusselt number and Sherwood number respectively.
Table 1 depicts that the Skin friction increases with increase in Gc and Grand decreases with increase in Sc, m, M, Kc, and Pr.
Table 2 represents that the Nusselt number decreases with increase in Pr, and decreases with increase in ω
Table 3 shows that the Sherwood number decreases with increase in Sc and ω .

Table 1: Skin friction τ

ω	Gc	Sc	Kc	Gr	M	Pr	M	τ
1	1	0.3	0.1	1	1	0.71	0.5	6.1082
3	1	0.3	0.1	1	1	0.71	0.5	5.5219
1	1	0.6	0.1	1	1	0.71	0.5	5.7548
1	4	0.3	0.1	1	1	0.71	0.5	6.5671
1	1	0.3	1	1	1	0.71	0.5	6.3654
1	1	0.3	0.1	4	1	0.71	0.5	6.8760
1	1	0.3	0.1	1	3	3	0.5	5.9006
1	1	0.3	0.1	1	1	0.71	2	5.0126
1	1	0.3	0.1	1	1	0.71	0.5	6.8741

Table 2: Nusselt number

ω	Pr	Nu
1	0.71	0.8181
4	0.71	0.7400
1	3	0.7830

Table 3: Sherwood number

ω	Sc	Sh
1	0.3	0.4015
5	0.3	0.3890
4	0.6	0.3977

5.0 SUMMARY AND CONCLUSION

We have examined and solved the governing equations for the effects of concentration and Hall current on unsteady flow of a viscoelastic fluid in a fixed plate analytically. In order to point out the effect of physical parameters namely; thermal Grashof number, mass Grashof number, Schmidt number, Prandtl number, Chemical reaction parameter, Hartmann number, Hall parameter and

the frequency of oscillation on the flow field and the following conclusions are drawn:
*The velocity increases with the increase in Gc and Gr.
*The velocity falls due to increasing in Sc, ω , M, m, Kc, and Pr.
*The temperature decreases with increase in Pr,
*The concentration reduces with the increase in Sc and Kc.
*The Skin friction increases with increase in Gc and Gr and decreases with increase in Sc, M, m,Kc, and Pr.
*The rate of heat transfer decreases with increase in Pr and ω ,
*The Sherwood number decreases with the increase in Sc and ω .

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