A.Y. Deogade, Dr. D.V. Bhope / International Journal of Engineering Research and Applications (IJERA) ISSN: 2248-9622 www.ijera.com
Vol. 2, Issue 5, September- October 2012, pp.561-565

Investigation Of Effect Of Non Central Circular Holes On Stresses In Rotating Disc

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ABSTRACT

In the present work the problem of circular disc with a central hole and a symmetrical array of non-central holes subjected to rotation are analyzed using Finite Element Method. The Stress Concentration Factors (SCF) are derived for various geometric parameters like, R_2/R_1, $d/2R_1$, $R_b/(R_2-R_1)$ and number of holes (N). It is seen that, as the number of holes increases, the SCF decreases. The results are compared with the analytical solution given by H.E.Ang and C.L.Tan[1].

Keywords: Disc, Non central hole, Stress concentration, FEM

NOTATION

A, B, C - positions at edges of holes shown in fig.1.
R_1 - Radius of central hole (bore) of disc
R_2 - Outer radius of disc
R_b - Pitch circle radius of holes
d - Hole diameter
N - Number of holes
E - Modulus of Elasticity
υ - Poisson's ratio
ρ - Density of material
ω - Angular velocity of disc
σ_θ - Hoop stress or tangential stress
σ_r - Radial stress
$(\sigma_\theta)_A$ - Hoop stress at point A
$(\sigma_\theta)_B$ - Hoop stress at point B

I. INTRODUCTION

There are machines elements which rotate while performing the required functions. These include flywheels, thin rings, circular discs, pulley rims, cylinders and spherical shells. Due to rotation, centrifugal stresses are developed in these elements.

Rotating disc shown in fig.1 found in numerous industrial applications. High centrifugal stresses, radial and hoop stresses occur at the non-central holes needed to bolt discs together rotating at high speed, such as compressor and turbine rotors of aircraft engines, flywheel, gears, etc.

In the present work the problem of circular disc with a central hole and a symmetrical array of non-central holes subjected to rotation is analyzed by using Finite Element Method. The finite element approach is used to evaluate the stresses in the rotating disc with central hole by varying R_2/R_1, $d/2R_1$, $R_b/(R_2-R_1)$ and number of holes (N) and to study the effect of variation of these parameters in the stress concentration factor in the disc.

II. INTRODUCTION TO PROBLEM

In this project, effect of non central holes in disc with central hole is investigated considering various geometrical parameters of the disc. The disc is shown in fig.1.

The various geometric parameter ratios & their variation are as follows,

R_2 / R_1 = 3, 4, 5, 6, 7, 8, 9, 10
$d / 2R_1$ = 0.025, 0.05, 0.075, 0.1, 0.125, 0.15, 0.175, 0.2, 0.225, 0.25
$R_b/(R_2 - R_1)$ = 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0
N = 4, 6, 8, 10, 12,14,16,18, 20, 24, 28, 32, 36, 40

The disc is made up of steel and its mechanical properties are tabulated in Table.1.

Table 1: Properties of Material

Modulus of Elasticity (MPa)	E	200×10^3
Poisson's ratio	ν	0.3
Density of material (Kg/mm^3)	ρ	7800

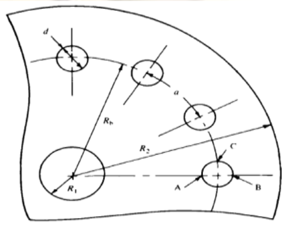

Fig.1. Circular disc with a central hole and a symmetrical array of non-central holes

III. LITERATURE REVIEW

i) Investigation by H.E. ANG and C.L. TAN, Carleton University, Ottawa, Canada[1]

This paper studied the problem of a circular disc with a central hole and a symmetrical array of non-central holes subjected to rotation using the boundary integral equation method.

The aim of this paper is to present and highlight some features of the results for the local stresses at the edge of the non-central holes in thin circular discs subjected to rotation. The computed stresses at these two points $(\sigma_\theta)_A$ and $(\sigma_\theta)_B$, are presented. For generality, they are expressed in terms of stress factors K_A, and K_B, respectively. In the case of disc rotation, they are defined as,

$$K_A = (\sigma_\theta)_A / \sigma_o$$

$$\text{and} \quad K_B = (\sigma_\theta)_B / \sigma_o$$

Where,

$$\sigma_o = \frac{3+\upsilon}{8}\rho\omega^2\left(R_1^2 + 2R_2^2 - \frac{1+3\upsilon}{3+\upsilon}R_1^2\right) \quad \ldots (1)$$

is the hoop stress at the bore of a plane disc with the same bore and outer radii as the problem treated; the material density, ρ, Poisson's ratio, and speed of rotation are also having the same values. For the results corresponding to the radial tension load case,

$$K_A=(\sigma_\theta)_A/\bar{\sigma} \quad \ldots\ldots\ldots\ldots(2a)$$

$$\text{And } K_B=(\sigma_\theta)_B/\bar{\sigma} \quad \ldots\ldots(2b)$$

$$K_A= \alpha - \beta N \quad \ldots\ldots\ldots (3a)$$

$$K_B = \xi - \eta N \ ; 20 \le N \le 40 \quad \ldots\ldots\ldots(3b)$$

The coefficients α, β, ξ, η so obtained are listed in for all the cases treated. It should be remarked that the values of K_A and K_B calculated using equation and the tabulated coefficients, deviated, in general, by less than 0.5 percent from the corresponding BIE computed values.

In terms of the ligament efficiency of the perforated part of the disc,$\sum$, which may be defined as,

$$\sum=\frac{a-d}{a} \quad \ldots\ldots (4)$$

These values correspond to the following ranges of $\sum$ for the respective radius ratios of the disc: 0.3634-0.823 for R_2/R_1 = 2; 0.523-0.878 for R_2/R_1 = 3; and 0.618-0.906 for R_2/R_1 = 4. For each and every combination of the above set of geometric parameter values, the stress distributions in the disc were obtained for the load case of the disc rotating at speed ω and subjected to uniform radial tension $\bar{\sigma}$ at the outer periphery. Poisson's ratio, υ was taken to be 0.3 throughout.

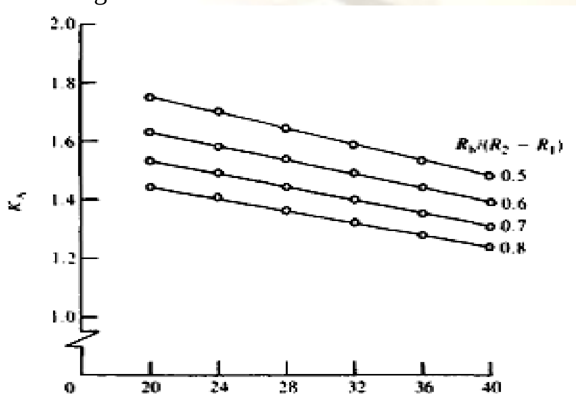

Fig.2: Variation of K_A with N; R_2/R_1=2, $d/2R_1$=0.05

Figure 2 shows the typical variations of K_A with the number of holes, N, for the different values of R_b, considered: the variations for K_B show identical features. These trends were found to be true for all the geometric cases and the loading conditions treated.

The tabulated coefficients can be conveniently interpolated to obtain intermediate values for the stress factors for discs which are within the range of R_2/R_1, and $R_b/(R_2-R_1)$ ratios and N considered for the values of N less than 20 as shown in Table 2 and Table 3.

Table 2: Coefficients of stress factor Equations, equation (3). For the case of disc rotation: $d/2R_1$ = 0.05

R_2/R_1	$R_b/(R_2 - R_1)$	α	β $(\times 10^{-3})$	ξ	η $(\times 10^{-3})$
2.0	0.5	2.0157	13.35	1.8581	11.86
	0.6	1.8723	12.10	1.7318	10.67
	0.7	1.7457	10.90	1.6366	10.14
	0.8	1.6475	10.17	1.5713	9.80
3.0	0.5	1.5051	6.96	1.4292	6.23
	0.6	1.3583	5.75	1.3045	5.68
	0.7	1.2622	4.93	1.2035	4.37
	0.8	1.1619	4.08	1.1176	3.94
4.0	0.5	1.2737	4.06	1.2318	3.71
	0.6	1.1615	3.12	1.1270	2.90
	0.7	1.0664	2.46	1.0375	2.31
	0.8	0.9837	2.06	0.9566	1.99

Table 3: Coefficients of stress factor Equations, equation (3). For the case of disc rotation: $d/2R_1$ = 0.075

R_2/R_1	$R_b/(R_2 - R_1)$	α	β $(\times 10^{-3})$	ξ	η $(\times 10^{-3})$
2.0	0.5	2.0672	18.64	1.8142	14.93
	0.6	1.9275	17.22	1.7193	14.45
	0.7	1.8071	16.01	1.6585	14.62
	0.8	1.7199	15.53	1.6392	14.84
3.0	0.5	1.5711	11.46	1.4456	9.49
	0.6	1.4457	9.90	1.3403	8.30
	0.7	1.3310	8.90	1.2373	7.55
	0.8	1.2265	7.57	1.1580	6.96
4.0	0.5	1.3136	7.39	1.2459	6.43
	0.6	1.1926	5.74	1.1420	5.29
	0.7	1.0995	4.93	1.0505	4.43
	0.8	1.0141	4.21	0.9715	3.93

ii) Investigation by H.FESSLER and T.E.THORPE, University of Nottingham, Nottingham

This paper studied the frozen stress photo elastic technique to determine the hoop and radial stresses at the hole boundaries of sixty different hole configuration in flat discs. Rotating discs are frequently required in engineering, but special problems arise in gas turbine discs because these rotate at high speeds and must be as light as possible.

iii) **Investigation by CERLINCA Delia Aurora, University of Suceava**

The paper studied for an axially symmetrical disk, with or without a central hole; there are analytical relations between stresses, strains, depending on disk geometry and angular velocity. It is known that the static condition a keyway induces increased stresses acting as a stress concentrator.

iv) **Investigation by H. FESSLER and T. E. THORPE University of Nottingham, Nottingham**

This paper investigates the frozen-stress photo elastic technique used to determine the stresses around reinforced non-central holes in discs. Both circular bosses and a thickened annulus were used as reinforcements. Calculations by Green, Hooper, and Hetherington were found to predict the mean stresses (through the thickness) well but these calculations cannot predict the peak stresses. The latter were not significantly less than the values of the unreinforced perforated discs. The stresses in a taper disc were lower than in a disc of constant thickness.

IV. VARIATION OF SCF WITH RESPECT TO GEOMETRICAL PARAMETERS EVALUATED BY FE APPROACH

Principal stress contours for some ratios R_2/R_1, $R_b/(R_2-R_1)$, $d/2R_1$, N are shown in fig.3 to fig.6 as an illustration.

Fig.3: Principal Stress Contour (σ_θ) for R_2/R_1 =3, $R_b/(R_2-R_1)$=0.8, $d/2R_1$=0.2,N=24

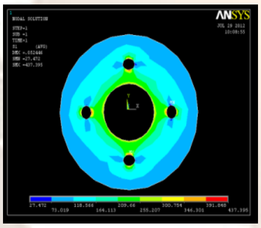

Fig.4: Principal Stress Contour (σ_θ) for R_2/R_1=3, $R_b/(R_2-R_1)$=0.9, $d/2R_1$= 0.225 , N=4

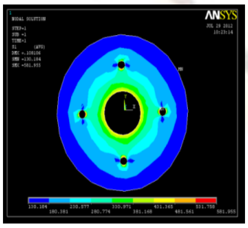

Fig.5: Principal Stress Contour (σ_θ) for R_2/R_1= 4, $R_b/(R_2-R_1)$= 0.8, $d/2R_1$=0.2, N=4

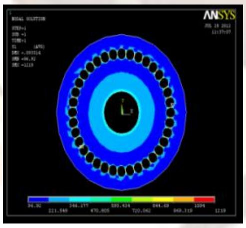

Fig.6: Principal Stress Contour (σ_θ) for R_2/R_1=4, $R_b/(R_2-R_1)$=1,$d/2R_1$=0.25, N=36

An effort is made to show the variation of SCF with respect to geometrical parameter of disc i, e, $R_b/(R_2-R_1)$, R_2/R_1, $d/2R_1$ and number of holes (N). These variations are shown in Fig.7 to Fig. 14.

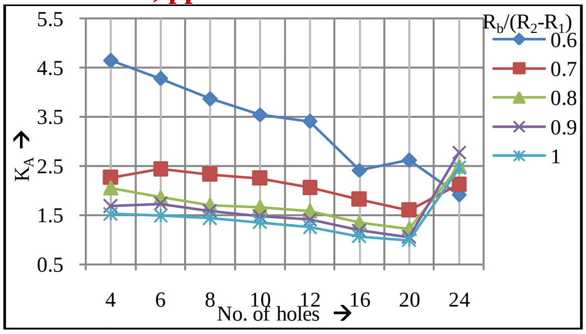

Fig.7: Variation of SCF with Respect to Number of Holes for R_2/R_1=3

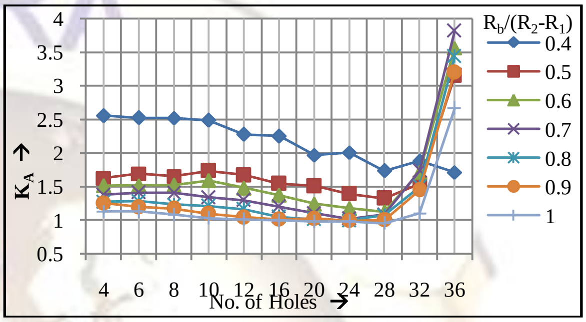

Fig.8: Variation of SCF with Respect to Number of Holes for R_2/R_1=4

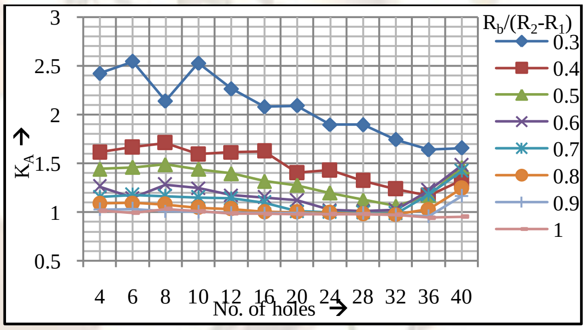

Fig.9: Variation of SCF with Respect to Number of Holes for R_2/R_1=5

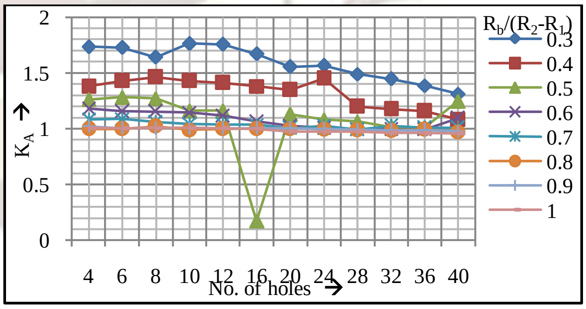

Fig.10: Variation of SCF with Respect to Number of Holes for R_2/R_1=6

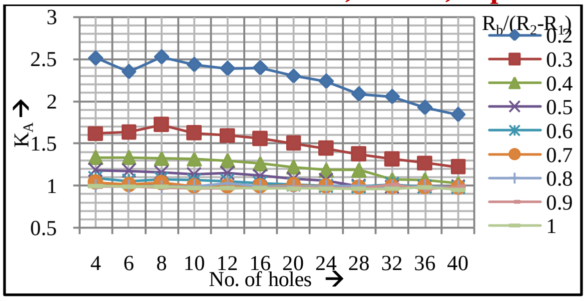

Fig.11: Variation of SCF with Respect to Number of Holes for R_2/R_1=7

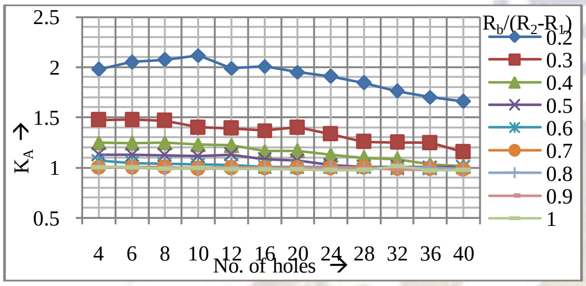

Fig12: Variation of SCF with Respect to Number of Holes for R_2/R_1=8

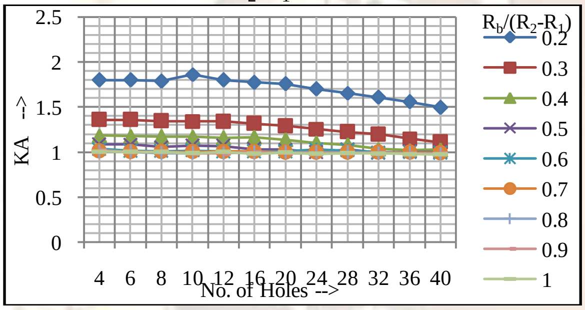

Fig.13: Variation of SCF with Respect to Number of Holes for R_2/R_1=9

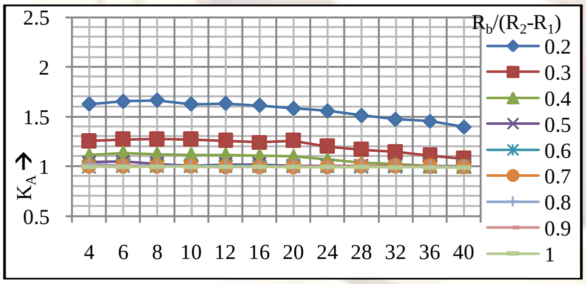

Fig.14: Variation of SCF with Respect to Number of Holes for R_2/R_1=10

V. COMPARISON OF FE RESULTS WITH ANALYTICAL RESULTS REPORTED BY H.E. ANG & C.L.TAN[1]

The physical geometries and SCF given by H.E.ANG and C.L.TAN[1] and evaluated by FE approach are given in Table 3 and Table 4 for the ratio of $d/2R_1$=0.05 and $d/2R_1$=0.075

Table 4: Comparison of FE and Analytical Results for $d/2R_1$=0.05

R_2/R_1	$R_b/(R_2-R_1)$	R_b	$d/2R_1$	d	N	σ_θ|FEM pt. A (MPa)	σ_θ|THE (MPa)	K_A	K_A| The.$
3	0.6	24	0.05	2	20	485.858	260.032	1.868	1.243
					24	498.441	260.032	1.916	1.220
					28	471.526	260.032	1.813	1.197
					32	476.938	260.032	1.834	1.174
					36	460.247	260.032	1.769	1.151
					40	460.83	260.032	1.772	1.128
	0.7	28	0.05	2	20	464.486	260.032	1.786	1.163
					24	436.965	260.032	1.680	1.143
					28	471.523	260.032	1.813	1.124
					32	414.451	260.032	1.593	1.104
					36	397.778	260.032	1.529	1.084
					40	390.733	260.032	1.502	1.065
	0.8	32	0.05	2	20	402.29	260.032	1.547	1.080
					24	397.121	260.032	1.527	1.063
					28	373.352	260.032	1.435	1.047
					32	365.164	260.032	1.404	1.031
					36	354.958	260.032	1.365	1.015
					40	343.583	260.032	1.321	0.998
4	0.5	30	0.05	2	20	721.946	457.622	1.577	1.192
					24	699.263	457.622	1.528	1.176
					28	706.564	457.622	1.543	1.160
					32	714.737	457.622	1.561	1.143
	0.6	36	0.05	2	20	685.121	457.622	1.497	1.099
					24	625.976	457.622	1.367	1.086
					28	656.061	457.622	1.433	1.074
					32	622.804	457.622	1.360	1.061
					36	602.582	457.622	1.316	1.049
					40	572.771	457.622	1.251	1.036
	0.7	42	0.05	2	20	608.248	457.622	1.329	1.017
					24	599.54	457.622	1.310	1.007
					28	560.41	457.622	1.224	0.997
					32	541.045	457.622	1.182	0.9876
					36	530.183	457.622	1.158	0.9778
					40	528.798	457.622	1.155	0.968
	0.8	48	0.05	2	20	552.879	457.622	1.208	0.9425
					24	547.907	457.622	1.197	0.9342
					28	546.596	457.622	1.194	0.9260
					32	537.205	457.622	1.173	0.9177
					36	509.025	457.622	1.112	0.9095
					40	490.968	457.622	1.072	0.9013

Table 5: Comparison of FE and Analytical Results for $d/2R_1=0.075$

R_2/R_1	$R_b/(R_2-R_1)$	R_b	$d/2R_1$	d	N	σ_θ|FEM pt. A (MPa)	σ_θ| THEO. (MPa)	K_A	K_A|The.$
3	0.6	24	0.075	3	20	485.858	260.032	1.8684	1.2477
					24	498.441	260.032	1.9168	1.2081
	0.7	28	0.075	3	20	464.486	260.032	1.7862	1.153
					24	436.965	260.032	1.6804	1.1174
	0.8	32	0.075	3	20	402.29	260.032	1.5470	1.0751
					24	397.121	260.032	1.5271	1.0448
4	0.5	30	0.075	3	20	721.946	457.622	1.5776	1.1658
					24	699.263	457.622	1.5280	1.1362
					28	706.564	457.622	1.5439	1.1066
					32	714.737	457.622	1.5618	1.0771
	0.6	36	0.075	3	20	685.121	457.622	1.4971	1.0778
					24	625.976	457.622	1.3678	1.0548
					28	656.061	457.622	1.4336	1.0318
					32	622.804	457.622	1.3609	1.0089
	0.7	42	0.075	3	20	608.248	457.622	1.3291	1.0009
					24	599.54	457.622	1.3101	0.9811
					28	560.41	457.622	1.2246	0.9614
					32	541.045	457.622	1.1822	0.9417
	0.8	48	0.075	3	20	552.879	457.622	1.2081	0.9299
					24	547.907	457.622	1.1972	0.9130
					28	546.596	457.622	1.1944	0.8962
					32	537.205	457.622	1.1739	0.8793

$ - Stress Concentration Factor reported by H.E. Ang and C.L. Tan.

It is seen from the Table 4 & 5 that the SCF determined by FE approach and with analytical approach differ by 62 to 84%.This confirms that the analytical solution represented C.L.Tan and H.E.Ang may need some correction factors depend upon geometry of perforated disc. Hence, it can be suggested that the analytical approach may needs some refinement to predict the actual SCF in perforated disc.

VI. DISCUSSION AND CONCLUSION

1. It is observed that as the number of holes increases the SCF decreases for most cases. But for some cases it is observed that the SCF increases abruptly after certain number of holes. This may be due to very less pitch width between the two consecutive holes. Thus there is a limit for the maximum number of holes to keep the working stresses within safe limits.
2. As it is also seen that as the pitch circle radius of non central holes increases, the SCF decreases. Thus the non central holes very close to the central hole should be avoided in practice.
3. As the diameter of non central holes increases the SCF decreases. This may be due to the smoothening of stress lines over larger curvature of hole. Thus larger diameter of non central hole is preferred in actual practice.

The Stress Concentration Factors derived for various geometric parameters like, R_2/R_1, $d/2R_1$, $R_b/(R_2-R_1)$ and number of holes (N) can serve as a guideline for designing the perforated rotating disc.

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