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SNR & BER Optimization For Pre-DFT Combining In Coded SIMO-OFDM Systems

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Abstract

For coded SIMO-OFDM systems, pre-DFT combining was previously shown to provide a good trade-off between error-rate performance and processing complexity. *Max-sum SNR* and *max-min SNR* are two reasonable ways for obtaining these combining weights. In this letter, we employ multi objective optimization to further reveal the suitability and limitation of these two criteria. Our results show that: 1) Neither maxsumS*NR* nor *max-min SNR* is universally good; 2) For better error-rate performance, the means for weight calculation should be adapted according to the capability of the error-correcting code used, and multi objective optimization can help in the determination.

Index Terms— SIMO, OFDM, pre-DFT combining, convex optimization, multiobjective optimization.

I. INTRODUCTION

ORTHOGONAL frequency division multiplexing (OFDM) combined with multiple receive antennas, namely, single-input multiple-output (SIMO) OFDM, has recently been investigated for use in wireless communication systems. It can provide high spectrum efficiency and high data rate for information transmission. On one hand, OFDM divides the entire channel into many parallel sub channels which increases the symbol duration and therefore reduces the inter-symbol interference (ISI) caused by multipath propagation. Besides, since the subcarriers are orthogonal to each other, OFDM can utilize the spectrum very efficiently. On the other hand, SIMO along with combining techniques takes advantage of receive spatial diversity and therefore further enhances the performance.

It is known that subcarrier-based maximum ratio combining (MRC) performs the best for coded SIMO-OFDM systems; however, it requires high processing complexity. Pre-discrete Fourier transform (DFT) combining was then developed, in which only one DFT block is necessary at the receiver [1]. It was previously shown to provide a good trade-off between error-rate performance and processing complexity. In this letter, we employ multiobjective optimization to reveal the suitability and limitation of two previously-proposed criteria for obtaining the pre-DFT combining weights, i.e., maximization of the sum of subcarrier signal-to-noise ratio (SNR) values (called *max-sum SNR* hereafter) [1] and maximization of the minimum subcarrier SNR value (called *max-min SNR* hereafter) [2]. Our results show that neither *max-sum SNR* nor *maxmin SNR* is universally good. Furthermore, for better error rate performance, the means for weight calculation should be adapted according to the capability of the error-correcting code used, and multiobjective optimization can help in the determination. Monte Carlo simulations are finally provided to verify the correctness of these sayings. Throughout the letter, we use boldface letters, boldface letters with over bar, lower-case letters, and upper-case letters to denote vectors, matrices, time-domain signals, and frequency domain signals, respectively. Besides, $(\cdot)^T$, $(\cdot)^H$, trace$(\cdot)$, rank$(\cdot)$, and diag$\{\cdot\}$ are used to represent the matrix transpose, matrix Hermitian, matrix trace, matrix rank calculation, and diagonal matrix with its main diagonal being the included vector, respectively.

II. PRE-DFT COMBINING IN SIMO-OFDM SYSTEMS

We consider an SIMO-OFDM system with M receive antennas. Define an $N \times 1$ signal vector $\mathbf{S}(k) = [S(kN)\ S(kN{+}1)\ \cdots\ S(kN + N - 1)]^T$ as the kth OFDM data block to be transmitted, where N is the number of subcarriers. This data block is first modulated by the inverse DFT (IDFT). With matrix representation, we can write the output of the IDFT as $\mathbf{s}(k) = [s(kN)\ s(kN+1)\ \cdots\ s(kN+N-1)]^T = \bar{\mathrm{F}}^H\mathbf{S}(k)$, where $\bar{\mathrm{F}}$ is an $N \times N$ DFT matrix with elements $[\bar{\mathrm{F}}]_{p,q} = (1/\sqrt{\mathrm{N}})\exp(-j2\pi pq/N)$ for $p, q = 0, 1, \cdots, N-1$ and $j = \sqrt{-1}$. A cyclic prefix (CP) is inserted afterwards and its length (L_{cp}) is chosen to be longer than the maximum length of the multipath fading channel (L). Also define an $N \times 1$ vector $\mathbf{h}_m = [h_m(0)\ h_m(1)\ \cdots\ h_m(L-1)\ 0\ \cdots 0]^T$, where $h_m(l)$ represents the lth channel coefficient for the mth receive antenna, with $l = 0, 1, \cdots, L-1$ and $m = 0, 1, \cdots, M-1$. Collecting all channel vectors from the M different receive antennas, we construct an $N \times M$ channel matrix $\bar{\mathrm{h}} = [\mathbf{h_0}\ \ \mathbf{h}_1\ \cdots\ \mathbf{h}_{M-1}]$, and its frequency response as

$\bar{\mathrm{H}} = [\mathbf{H}_0\, \mathbf{H}_1 \cdots \mathbf{H}_{M-1}] = \bar{\mathrm{F}}\, \bar{\mathrm{h}}$ (1)

with $\mathbf{H}m = \bar{\mathrm{F}}\mathbf{h}_m$ .In an ordinary OFDM signal reception process, after CP removal and DFT demodulation, the resultant $N \times 1$ signal vector from the mth receive antenna, denoted by $\mathbf{R}_m(k)$, can be shown to be

$$\mathbf{R}(k) = \mathrm{diag}\{\mathbf{S}(k)\}\mathbf{H}_m + \mathbf{N}_m(k) \quad (2)$$

where $\mathbf{N}_m(k)$ is an $N \times 1$ complex Gaussian noise vector with zero mean and equal variance for each element. For the considered SIMO scenario, we can collect the M received signal vectors and form an $N \times M$ received signal matrix as

$$\bar{\mathrm{R}}(k) = [\,\mathbf{R}_0(k)\, \mathbf{R}_1(k) \cdots \mathbf{R}_{M-1}(k)]. \quad (3)$$

Let $\mathbf{w} = [w_0\, w_1 \cdots w_{M-1}]$ be an $M \times 1$ weight vector.

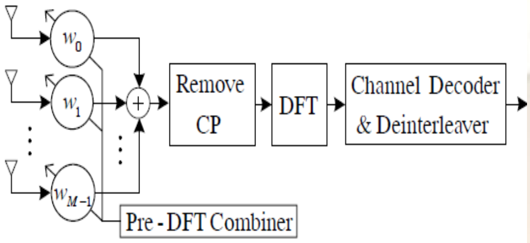

Fig. 1 Block diagram of OFDM diversity receiver with pre-DFT combining

With (1)-(3), the pre-DFT combining operation and the resultant $N \times 1$ signal vector can be expressed as

$$\mathbf{Y}(k) = \bar{\mathrm{R}}(k)\,\mathbf{w} = \mathrm{diag}\{\mathbf{S}(k)\}\,\bar{\mathrm{H}}\mathbf{w} + \bar{\mathrm{N}}(k)\mathbf{w} \quad (4)$$

with $\bar{N}(k) = [\mathbf{N}_0(k)\, \mathbf{N}_1(k) \cdots \mathbf{N}_{M-1}(k)]$. Fig. 1 is the block diagram of a simplified OFDM receiver performing pre- DFT combining. In [1], $\mathbf{w}$ was calculated based on *max-sum SNR*. For that case, the optimum $\mathbf{w}$ can be shown to be the solution of the following optimization problem:

$$\max_{\mathbf{w}}\ \mathbf{w}^H \bar{\mathrm{H}}^H \bar{\mathrm{H}}\mathbf{w} \text{ subject to } \mathbf{w}^H\mathbf{w} = 1 \quad (5)$$

in which $\mathbf{w}^H \bar{\mathrm{H}}^H \bar{\mathrm{H}}\mathbf{w}$ indicates the sum of the signal power in all N subcarriers. As an alternative, pre-DFT combining based on *max-min SNR* was proposed in [2]. Define a $1 \times M$ vector γ_n as the nth row of the channel matrix $\bar{\mathrm{H}}$ given in (1), with $n = 0, 1, \cdots, -1$. For that approach, the optimization of $\mathbf{w}$ can be described as

$$\max_{\mathbf{w}} \max_{n} |\gamma_n \mathrm{w}|^2 \text{subject to } \mathbf{w}^H\mathbf{w} = 1 \quad (6)$$

in which $|\gamma_n \mathrm{w}|^2$ indicates the signal power of the nth subcarrier after pre-DFT combining.
It is understood that while *max-sum SNR* tends to help the good, *max-min SNR* tends to help the bad. Both criteria are reasonable for obtaining the pre-DFT combining weights. Nevertheless, two questions are naturally raised: 1) Is one of the two criteria strictly superior to the other? 2) Can we further improve the error-rate performance with pre-DFT combining? We try to answer these questions through the use of multiobjective optimization in the following.

II. MULTIOBJECTIVE OPTIMIZATION FOR PRE-DFT COMBINING

Although *max-sum SNR* and *max-min SNR* are both practical, they are normally in conflict with each other, i.e., an improvement in one leads to deterioration in the other, which will be shown later in this section. This motivates the use of multiobjective optimization for gaining further insight into the two problems for the case at hand can be stated as follows:

$$\max_{\mathrm{w}} \mathbf{g}(\mathbf{w}) = \begin{bmatrix} g_1(\mathrm{w}) \\ g_2(\mathrm{w}) \end{bmatrix}, \text{ subject to } \mathbf{w}^H\mathbf{w} = 1 \quad (7)$$

with $g_1(\mathbf{w}) = \max_{n} |\gamma_n \mathrm{w}|^2$ and $g_2(\mathbf{w}) = (\mathbf{w}^H \bar{\mathrm{H}}^H \bar{\mathrm{H}}$ $\mathbf{w})$/in which $g_2(\mathbf{w})$ is normalized for convenience during numerical calculation. With (7), we can generally look for some good trade-offs, rather than a single solution of either *max-sum SNR* or *max-min SNR*. For this problem, a solution is optimal if there exists no other solution that gives enhanced performance with regard to both $g_1(\mathbf{w})$ and $g_2(\mathbf{w})$ - Pareto optimizers. The set of Pareto optimizers is called the Pareto front [3].However; there is no systematic manner to find the Pareto front in (7). Instead, we use a simple and popular way, i.e., the weighted-sum method, to approach to the solution set. This essentially converts the multiobjective optimization problem into a single objective problem. Mathematically speaking, the objective function in this circumstance is changed to be a linear combination of the two objectives as

$$\max_{\mathrm{w}} \lambda g_1(\mathbf{w}) + (1 - \lambda) g_2(\mathbf{w}), \text{ subject to } \mathbf{w}^H\mathbf{w} = 1 \quad (8)$$

where $\lambda \in [0, 1]$ is a parameter determining the relative importance between *max-sum SNR* and *max-min SNR*. Solving (8) yields the solution that gives the best compromise for a typical λ Next, we show that (8) can be efficiently evaluated via convex optimization techniques. Without loss of generality, we can recast the optimization problem in (8) to be

$$\max_{\mathrm{W}} \lambda [\min \mathrm{trace}(\Gamma_n \mathbf{W})] + (1 - \lambda)[\mathrm{trace}(\mathbf{QW})]$$

subject to $\mathrm{trace}(\mathbf{W}) = 1$, $\mathrm{rank}(\mathbf{W}) = 1$, $\mathbf{W} \geq \mathbf{0}$ (9)

with $\Gamma_n = \gamma_n^H \gamma_n$ and $\mathbf{Q} = \bar{\mathrm{H}}^H \bar{\mathrm{H}}$

In (9), $\mathbf{W}$ is an $M \times M$matrix to be determined and the inequality $\mathbf{W} \geq \mathbf{0}$ means that $\mathbf{W}$ is symmetric positive semi definite. Instead of solving the above nondeterministic polynomial-time hard (NP-hard) problem directly, we seek an approximation of the solution.

By dropping the nonconvex rank-one constraint, this weighted sum objective function can be relaxed to

$$\max_{\mathbf{W}} \lambda [\min \operatorname{trace}(\Gamma_n \mathbf{W})] + (1-\lambda)[\operatorname{trace}(\mathbf{QW})]$$

subject to trace($\mathbf{W}$) = 1, $\mathbf{W} \geq \mathbf{0}$. (10)

Let z_1 and z_2 be two scalars. The relaxation is equivalent to

$$\max_{\mathbf{W}} \lambda z_1 + (1-\lambda) z_2$$

subject to $\operatorname{trace}(\Gamma_n \mathbf{W}) \geq z_1$, $\operatorname{trace}(\mathbf{QW}) \geq z_2$

subject to $\operatorname{trace}(\mathbf{W}) = 1$, $\mathbf{W} \geq \mathbf{0}$.

which becomes convex. It is not difficult to see that (11) can be categorized to be a semi definite programming problem. The optimal choice of $\mathbf{W}$, i.e., $\mathbf{W}_{opt}$, can be obtained systematically using the efficient interior point method [4], and then a randomization step is used to produce an approximated solution to (7). In general, the complexity from weight calculation can be ignored as compared with the complexity saving from the reduction of DFT components [1], [2].

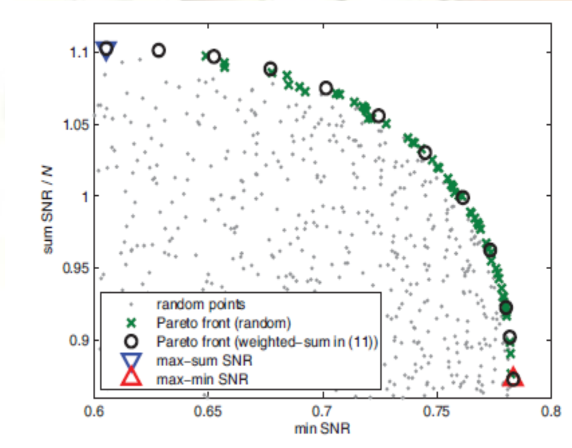

Fig. 2. Pareto front for *max-sum SNR* and *max-min SNR* with SNR=15 dB, M = 2, N = 64, Lcp = 16, L = 2, and λ = [0: 0.1: 0.8 0.9: 0.05: 1].

An example of a typical Pareto front solved via (11) is illustrated in Fig. 2. To obtain the entire approximation set, the search is repeated with various values of λ. We clearly see the trade-off between *max-sum SNR* and *max-min SNR*. Besides, the weighted-sum method along with the convex formulation can efficiently approach the Pareto front, as expected.

IV. SIMULATIONS AND DISCUSSION

A comparison of the bit-error-rate (BER) performance with different pre-DFT combining is made by Monte Carlo simulations carried out regarding a 1 × 2 coded OFDM system. Quadrature phase-shift keying (QPSK) is used for modulation. Besides, N = 64, L_{cp} = 16, and L = 2(independently generated with the Rayleigh distribution) are set. Convolution codes with different error-correcting capabilities (different minimum free distance $d$free) are used for error protection. At the receiver, the Viterbi algorithm with hard decision is employed for decoding. Figs. 3 and 4 present the corresponding BER performance. From these figures, we have the following observations: For the case of higher error-correcting capability (Fig. 3), *max-sum SNR* performs slightly better than *max-min SNR*. Note that *maxsumSNR* generally focuses on the good and ignores the bad. With the relatively large amount of error protection, the low sub carrier SNR values may be compensated. Together with the "boosted" high-SNR subcarriers, *max-sum SNR* provides better BER performance in this case. On the contrary, for the case of lower error-correcting capability (Fig. 4), *maxminSNR* outperforms *max-sum SNR*, especially in the high SNR region. The small amount of error protection makes each subcarrier equally essential. *Max-min SNR* usually does a good job in balancing the subcarrier SNR values, and thus gives better BER performance. Moreover, it is interesting to note that in either Fig. 3 or Fig. 4, the weighted-sum method which successfully captures the advantages of both *max-sum SNR* and *max-min SNR* is superior to these two previously-proposed criteria. By varying λ, there exist some cases in which a lower BER can be achieved. That is to say, multiobjective optimization can be employed to form some better pre-DFT combining weights over the pure *max-sum SNR* and *max-minSNR*. By means of exhaustive simulations, we find that the effect of *max-min SNR* is more substantial than that of *maxsumSNR* in most circumstances.

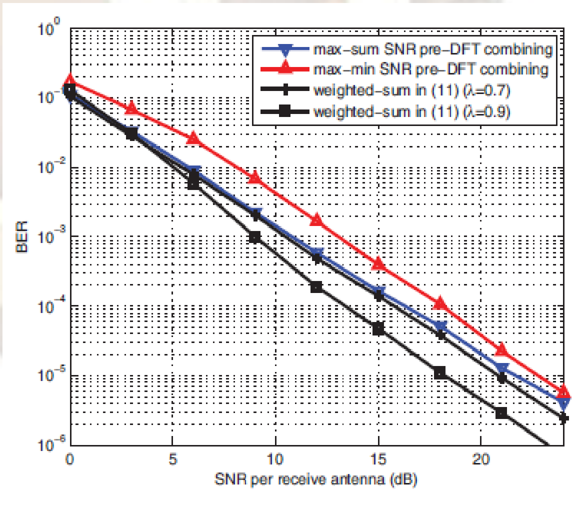

Fig. 3. BER for 1/2-rate convolutional-coded SIMO-OFDM with generator sequence $([247\ 371])_8$ and $d$free = 10 [5].

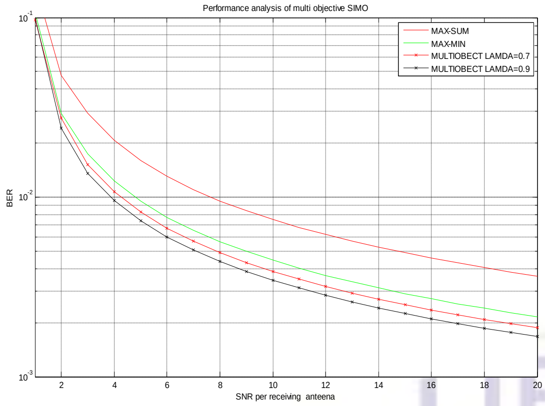

Fig. 4 BER for 3/4-rate convolutional-coded SIMO-OFDM with generator sequence ([1 1 1 0],[3 0 0 1],[3 2 0 2])$_8$ and $d_{\text{free}} = 3$ [5].

Simulation result for code rate 1/3 with minimum distance 15 is also calculated in this calculation λ is taken very close to 1 is used which improves the BER value which is shown in Fig 5

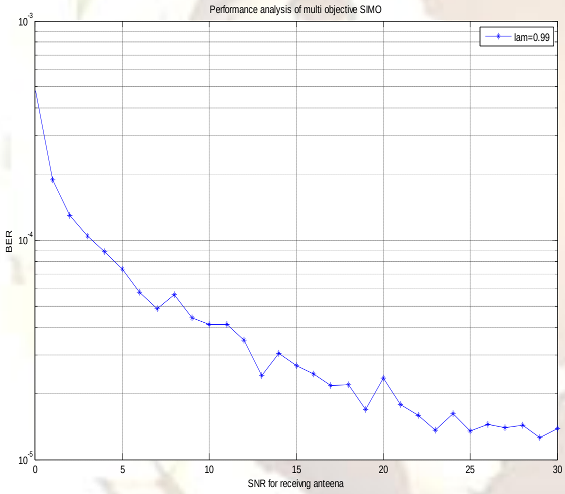

Fig. 5 BER for 1/3-rate convolutional-coded SIMO-OFDM with generator sequence [117 127 155]$_8$ And $d_{\text{free}} = 15$[5]

V. CONCLUSIONS

This letter has discussed and compared the error-rate performance for coded SIMO-OFDM systems with different pre-DFT combining. Our results show that multiobjective optimization can be used to determine some better pre-DFT combining weights, which are generally superior to both *maxsum SNR* and *max-min SNR* for achieving a lower BER.

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