Effects Of Zero Locations On The Tracking Performance Of Feedforward Trajectory ZPETC

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ABSTRACT

The coupling of electronic and hydraulic technology is becoming increasingly common, especially in electro hydraulic actuators. Hydraulic actuator is widely used in industrial applications because it exhibits linear movements, fast response, smooth reversal and accurate positioning of heavy load. Reducing the position tracking error of the hydraulic actuator system is a challenging task but advances in transducer logic and control capabilities have resulted in cylinders that transmit high forces with a high degree of positioning accuracy. One of the method in controlling the position variation of the electro-hydraulic actuator is through the implementation of Zero Phase Error Tracking Control (ZPETC). The electro-hydraulic actuator system mathematical model was approximated using system identification technique with non-minimum phase system being considered and the controller parameters of the obtained model are determined using comparing coefficients method. The controller was applied to two types of third-order non-minimum phase plant; the first plant was having a zero outside and far from the unity circle and the second plant was having a zero outside the unity circle but much nearer to the circle compared to the first plant. All the studies were done using Matlab Simulink environment and were validated with the real-time system.

Keywords - Feed-forward Control, ZPETC, Real-Time Control, Pole-Placement, Digital Tracking Control

I. INTRODUCTION

In tracking control system, achieving a perfect tracking is what it is all about. Zero tracking error is the objective of a perfect tracking system and one of the methods in achieving this is by using a feed-forward controller. Feed-forward is a term describing an element or pathway within a control system, which passes a controlling signal from a source in the control system's external environment, often a command signal from an external operator, to a load elsewhere in its external environment. A control system, which has only feed-forward behavior, responds to its control signal in a pre-defined way without responding to how the load reacts. With feed-forward control, the disturbances are measured and accounted for before they have time to affect the system.

There are two fundamental problems in servo control, one is the tracking control problem and the other is the point to point problem. The objective of tracking control is to follow a desired path as closely as possible, so unity gain and zero phase shift are needed for overall system in the relevant frequency band; e.g., automated arc welding (certain trajectory) and servo turning table (uncertain trajectory). While the point to point problem is concerned with moving the object from one point to another [1].

The natural non-linear property of hydraulic cylinder had challenge researchers in designing suitable controller for motion control or tracking control. In the past few years, researchers have investigated the use of digital feed-forward controller to improve the performance of servo system. The feed-forward controller is capable of cancelling all the poles and zeros hence creating a unity overall transfer function but in most real-world process, it is difficult to get this type of perfect system. A non-minimum phase zero located on or outside the unity circle will cause the system to be unstable and to overcome this problem, many methods have been introduced. One of the method is called Stable Phase-Zero Cancelling (SPZC) which was proposed by Masayoshi Tomizuka but this method was unable to eliminate the phase error and the gain error left by the zero outside the unity circle. Realizing this, Tomizuka proposed another method that has the ability to cancel not only all the poles and zeros but it is also capable of eliminating the phase error left by the SPZC hence increasing the controller's performance and this new method is called the Zero Phase Error Tracking Control (ZPETC) [2]. On such designs, the zero phase error tracking controller (ZPETC) cancels the closed-loop poles and cancellable zeros, at the same time, eliminates phase error induced by non-cancellable zeros. The main objective in feed-forward ZPETC is to find the optimum gain filter so that the overall gain is close to unity. The ZPETC can provide the overall system with frequency characteristics such that phase is zero for all frequencies and the gain is unity at only zero frequency. To avoid the unwanted phase error, Yeh and Hsu [3], Mustafa [4] and Adnan [5] used ZPETC without factorization of zeros. In this method, gain filter is proposed as Eq. (1)

$$F_g(z, z^{-1}) = \sum_{k=0}^{n_\alpha} \alpha_k (z^k + z^{-k}) \tag{1}$$

where n_α is the order of the filter. The value α in Eq. (1) is solved using comparing coefficient method.

As for the plants, shows in Fig. (1), both models uses different sampling time, the first model was obtained using 40ms sampling time while the second model was obtained using 50ms sampling time. A minimum phase model can be obtained using bigger sampling time whereas the non-minimum phase model can be obtained using smaller sampling time [6]. Not only that, the transfer function for both third order discrete-time model are different; one represents the zero located outside and far from the unity circle while the other one represents the zero located outside but much nearer to the unity circle. The reason why these two plant models were used in this study is to check whether the position of zero outside the unity circle affects the performance of obtaining the desired output.

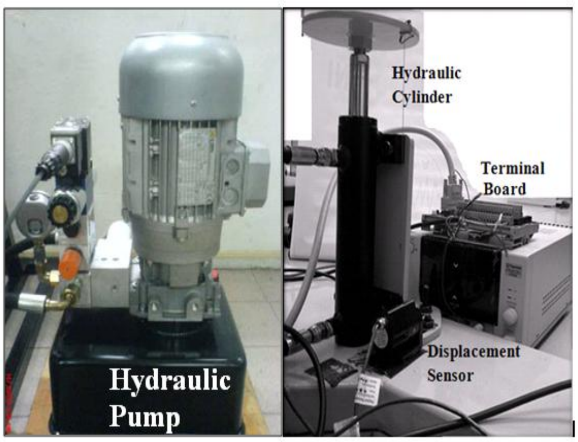

Figure 1.Electro-Hydraulic Actuator

II. METHODOLOGY

A. ZPETC Without Factorization of Zeros

Tomizuka proposed a tracking control system with two-degrees-of-freedom (2-DOF) controller as in Fig. 2.

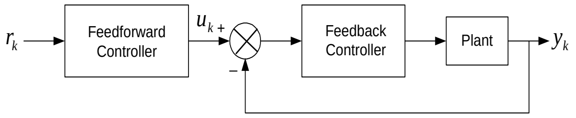

Figure 2. Tomizuka 2-DOF Controller

Without the feed-forward controller, the reference signal continuously varying and mixed with the closed-loop system dynamics, which keeps the tracking error inside the system without trying to eliminte it. The feed-forward controller is needed so that the reference signal can be pre-shaped by the feed-forward controller, so that more emphasis to the frequency components that were not properly taken care off by the feedback system can be provided [7].

The closed-loop system transfer function, *Gcl* without feedforward controller can be given in terms of discrete-time model

$$G_{cl}(z^{-1}) = \frac{z^{-d} B_c(z^{-1})}{A_c(z^{-1})} \tag{2}$$

$$A_c(z^{-1}) = 1 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_{n_a} z^{-n_a}$$
$$B_c(z^{-1}) = b_0 + b_1 z^{-1} + b_2 z^{-2} + \cdots + b_{n_b} z^{-n_b}$$

where $n_a \geq n_b$ and *d* is a time delay. The factor $B(z^{-1})$ can be factorised into minimum and non-minimum phase factors.

$$B_c(z^{-1}) = B_c^+(z^{-1}) B_c^-(z^{-1}) \tag{3}$$

where $B_c^+(z^{-1})$ represents the minimum phase factor and $B_c^-(z^{-1})$ represents the non-minimum phase factor.

The ZPETC proposed by Tomizuka can be divided into three blocks [8].

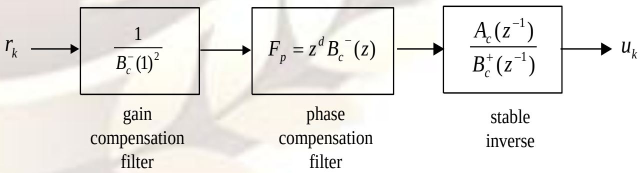

Figure 3. Conventional ZPETC Block Diagram

The modified version of the ZPETC implemented as feed-forward controller used in this paper is without the factorization of zeros as in Fig. 4.

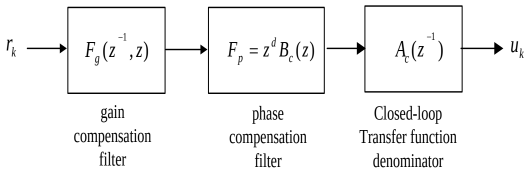

Figure 4. ZPETC without factorization of zeros

Same with others ZPETC, this design mainly focused on the selection of appropriate gains compensation filter to ensure that the overall gain is unity.

The same approach taken by Yeh and Hsu [3], Mustafa [4] and was used to ensure that the gain compensation filter, F_g as in Eq. (1) does not introduce any phase error that might jeopardize the whole objective of this study. The cost function to represent the error between the desired and actual frequency response is given by

$$J(\alpha_k) = \left\| 1 - B_c(z^{-1}) B_c(z) \sum_{k=0}^{n_\alpha} \alpha_k (z^k + z^{-k}) \right\|_{l_2} \tag{4}$$

The design objective is to find a set of α_k so that the cost function of Eq. (4) is minimized. For finite α_k, the cost function cannot be made zero for all frequencies. Minimizing Eq. (4) will result in the rising of Eq. (5).

$$B_c(z^{-1}) B_c(z) \sum_{k=0}^{n_\alpha} \alpha_k (z^k + z^{-k}) = 1 \tag{5}$$

The optimal set of α_k can be obtained by expanding Eq. (5) to
polynomial of positive and negative power of *z*, and then compare the coefficients of the same power.

B. Plant Model

Two plant models were used in the study, both was a 3rd order discrete-time models.
The first model (Plant 1) is given in Eq. (6)

$$\frac{B_o(z^{-1})}{A_o(z^{-1})} = \frac{0.0039z^{-1} + 0.0029z^{-2} - 0.0055z^{-3}}{1 - 1.907z^{-1} + 0.9414z^{-2} - 0.0341z^{-3}} \tag{6}$$

with its transfer function is given the pole-zero plot in Fig. 5 by using the sampling time of 40ms.

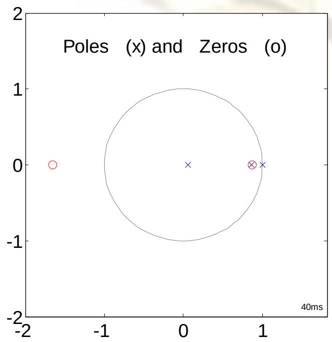

Figure 5. Pole-zero plot of Plant 1

The second model (Plant 2) is given in Eq. (7)

$$\frac{B_o(z^{-1})}{A_o(z^{-1})} = \frac{0.008704z^{-1} + 0.003684z^{-2} - 0.008833z^{-3}}{1 - 1.58z^{-1} + 0.3938z^{-2} - 0.1861z^{-3}} \tag{7}$$

with its transfer function is given the pole-zero plot in Fig. 6 by using the sampling time of 50ms.

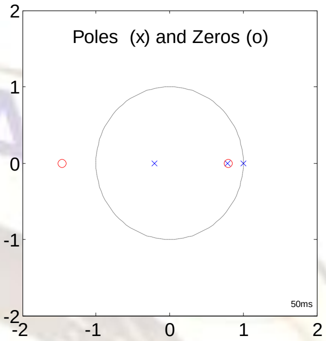

Figure 6. Pole-zero plot of Plant 2

Eq. (6) can be simplified as

$$\frac{B_o(z^{-1})}{A_o(z^{-1})} = \frac{0.0039z^{-1}(1 + 0.7576z^{-1} - 1.4253z^{-2})}{1 - 1.907z^{-1} + 0.9414z^{-2} - 0.0341z^{-3}} \tag{8}$$

From Eq. (8), the optimal set of α_k for the 10th order gain compensation filter is obtained as follows:

$$B_c(z^{-1}) = 1 + 0.7576z^{-1} - 1.4253z^{-2}$$
$$B_c(z) = 1 + 0.7576z - 1.4253z^2 \tag{9}$$

Implementing these two function into Eq. (5) to minimize the cost function,

$$(1+0.7576z^{-1} - 1.4253z^{-2})(1+0.7576z - 1.4253z^2)$$

$$\sum_{k=0}^{n_\alpha} \alpha_k (z^k + z^{-k}) = 1$$

And

$$\left[3.6055 - 0.3222(z+z^{-1}) - 1.4253(z+z^{-1})\right] \bullet$$
$$\left[2\alpha_0 + \alpha_1(z+z^{-1}) + \alpha_2(z^2+z^{-2}) + \alpha_3(z^3+z^{-3}) + \cdots + \alpha_{10}(z^{10}+z^{-10})\right] = 1$$
(10)

By expanding Eq. (10) to polynomial of positive and negative power of z and compare the coefficients of the same power, the following equation (in matrix form) is obtained:

$$\begin{bmatrix} 7.2109 & -0.6444 & -2.8506 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -0.6444 & 2.1801 & -0.3222 & -1.4253 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -2.8506 & -0.3222 & 3.6054 & -0.3222 & -1.4253 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1.4253 & -0.3222 & 3.6054 & -0.3222 & -1.4253 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1.4253 & -0.3222 & 3.6054 & -0.3222 & -1.4253 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1.4253 & -0.3222 & 3.6054 & -0.3222 & -1.4253 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1.4253 & -0.3222 & 3.6054 & -0.3222 & -1.4253 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1.4253 & -0.3222 & 3.6054 & -0.3222 & -1.4253 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1.4253 & -0.3222 & 3.6054 & -0.3222 & -1.4253 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1.4253 & -0.3222 & 3.6054 & -0.3222 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1.4253 & -0.3222 & 3.6054 \end{bmatrix} \begin{bmatrix} \alpha_0 \\ \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \\ \alpha_5 \\ \alpha_6 \\ \alpha_7 \\ \alpha_8 \\ \alpha_9 \\ \alpha_{10} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (11)$$

By solving Eq. (11), values of α obtained as listed in Table 1.

Table 1. Optimal set of α_k for 10^{th} order gain compensation filter of Eq. (6) for plant 1 and Eq. (7) for plant 2.

k	***Plant 1***	***Plant 2***
	α_k	α_k
0	0.3579	0.5666
1	0.3776	0.0362
2	0.4692	0.7405
3	0.3097	0.0310
4	0.3157	0.4782
5	0.2284	0.0238
6	0.2078	0.3001
7	0.1497	0.0160
8	0.1243	0.1749
9	0.0753	0.0081
10	0.0559	0.0803

It is clear from the Table 1 above that the values of α are converging to zero. If the same technique is used for Eq. (7) which is the second plant, the data obtained is given in Table 1.

The optimal set of α_k that were obtained by solving Eq. (11) are entirely different between the two plants due to the different location of zero that were discussed earlier and is used in the gain compensation filter, F_g as in Fig. 4.

C. Simulation Studies

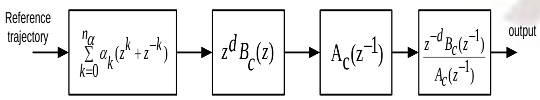

Figure 7. Trajectory ZPETC Structure

Fig. 7 is the trajectory zero phase error tracking control without factorisation of zeros with its controller parameters are determined by using comparing coefficient method and due to the effect of poles cancellation, the control structure was simplified as given in Fig. 8. It can be seen that only the zero polynomial equation of the plant model is needed.

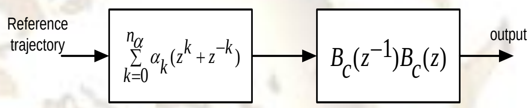

Figure 8. Tracking control structure for simulation studies

D. Real-Time Studies

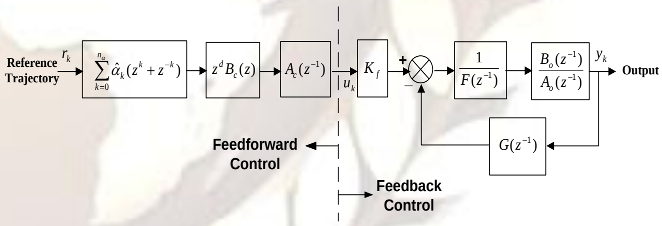

Figure 9. Tracking structure for real-time studies

The tracking control structure that used for real-time studies is shown in Fig. 9. The structure is divided into two parts: feed-forward control; and feedback control. The feed-forward control block is using the trajectory ZPETC where the controller parameters are determine using the proposed comparing coefficients method. The controller parameters as given in Table I are used in the trajectory ZPETC structure. The feedback control block is using pole-placement method to determine its controller parameters. The controllers parameters used are as follows:

Model 1:
$t_1 = 1 - 0.87z^{-1}$
$K_f = 96.59$
$F(z^{-1}) = 1 + 0.9366z^{-1} + 0.4823z^{-2}$
$G(z^{-1}) = 24.5043 + 74.5365z^{-1} - 2.9675z^{-2}$

Model 2:
$t_1 = 1 - 0.82z^{-1}$
$K_f = 50.63$
$F(z^{-1}) = 1 + 0.21896z^{-1} - 0.2165z^{-2}$
$G(z^{-1}) = 62.1614 - 6.9397z^{-1} - 4.5606z^{-}$

The literature on calculating the given parameters is available in [9,10,11].

III. Results And Discussion

In order to determine the correct filter order to be used in the trajectory ZPETC, the frequency response of the ZPETC given in Figure 8 are plotted using the transfer functions of Model 1 and Model 2. The resulting frequency responses are given in Figure 10 and Fig. 11. From Figure 10, it can be observed that an approximate overall unity gain can be achieved when using filter order, N≥30

From Fig. 11, it can be observed that an approximate overall unity gain can be achieved when using filter order, N≥20. Thus, the degree of difficulty is harder for Model 1.

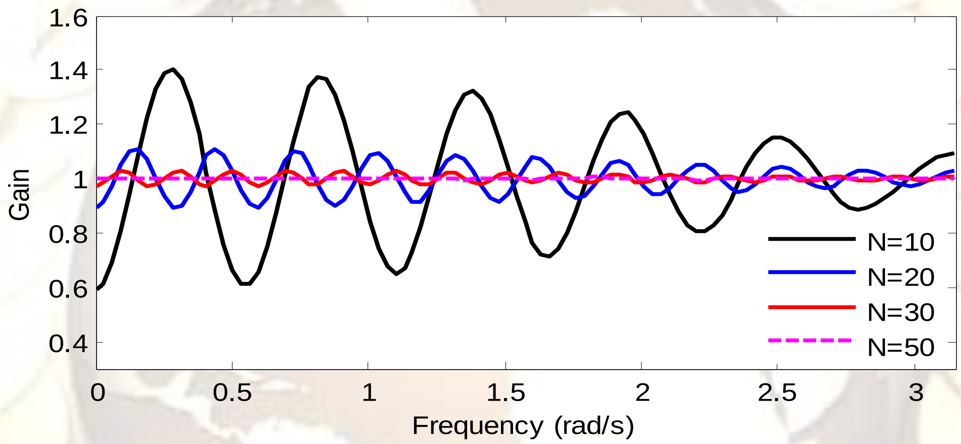

Figure 10. Frequency response of 10th,20th,30th and 50th filter order ZPETC using Model 1

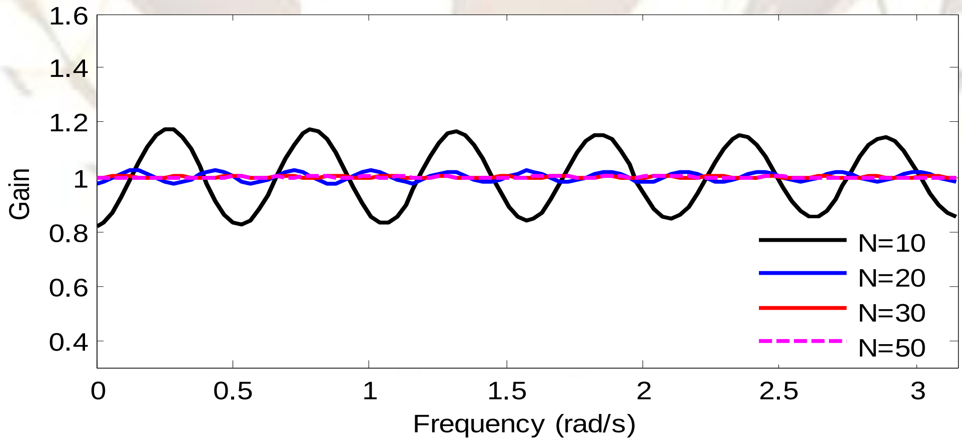

Figure 11. Frequency response of 10th,20th,30th and 50th filter order ZPETC using Model 2

A . Simulation Results

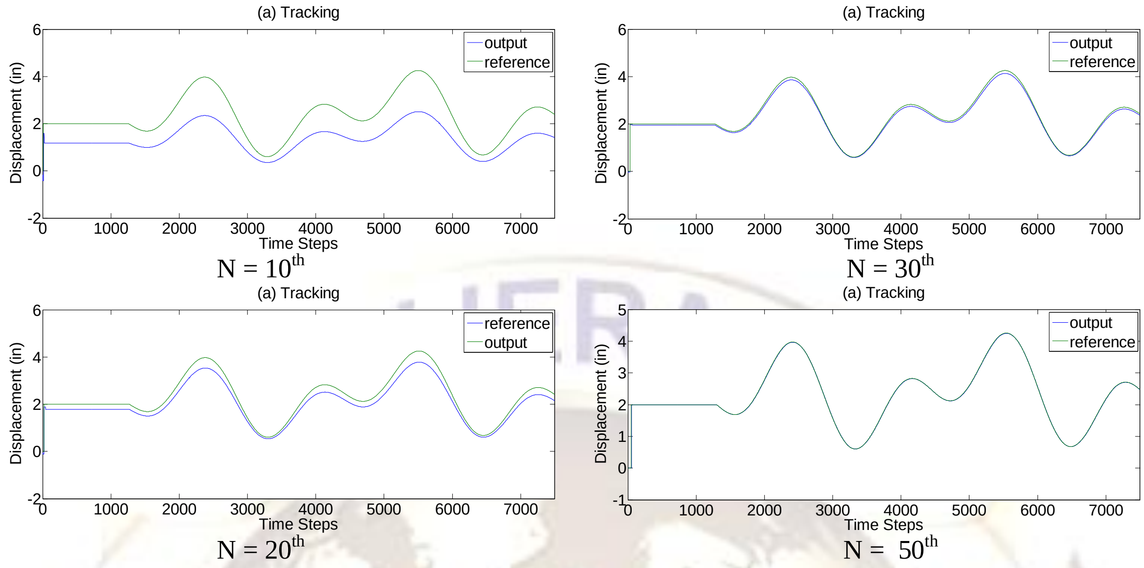

Figure 12. Simulation result using 10th , 20th, 30th and 50th order ZPETC applied to Plant 1 model

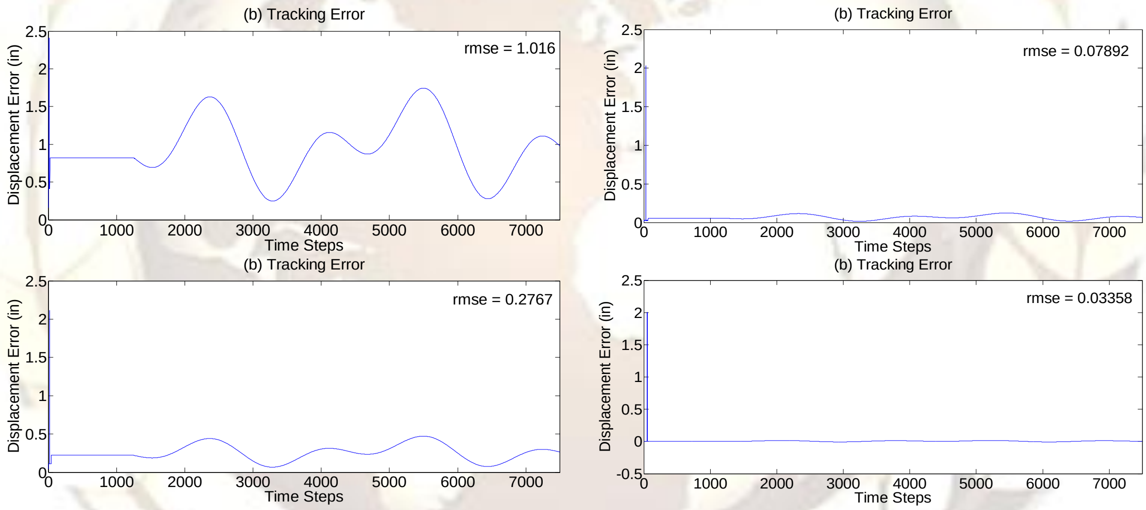

Figure 13. Simulation Tracking Error using 10th , 20th, 30th and 50th order ZPETC applied to Plant 1 model

In Fig.12, optimal set of α_k for all order gain compensation filter as in Table 1 was used and the result of tracking performance is poor compared to the other higher order gain. In Fig. 12, when optimal set of α_k was increased to 20th and the increment of performance can be clearly seen in the figure. In Fig. 13, the root mean square error (RMSE) for the tracking error decreased significantly hence indicating a much better tracking performance. As the α_k optimal set increased, the RMSE continues to drop from 0.2767 inch to 0.07892 inch and finally to 0.03358 inch for the optimal set of α_k for 50th order.

Using the same technique, plant 1 is replaced with plant 2 where the function differ as shown in Eq. (7) and the frequency response as in Fig. 11 is obtained. From the transfer function of plant 2 above, it can be seen that the location of zero outside the unity circle; may it be nearer to or farther from the unity circle really affect the performance of the overall system. Plant 2 (zero located outside but near the unity circle) really shows improvement on achieving the unity gain.

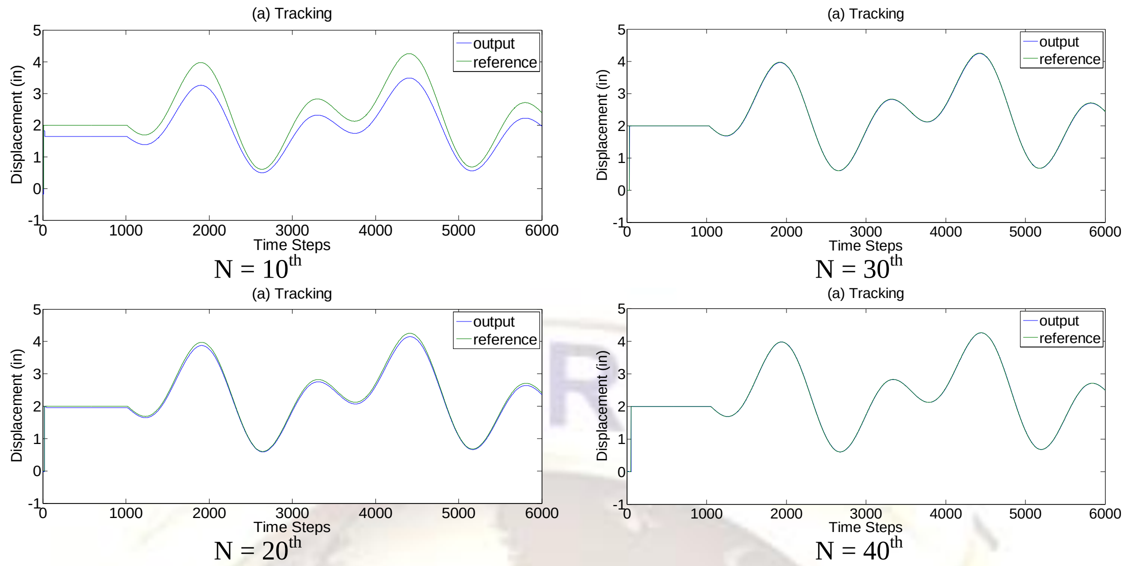

Figure 14. Simulation result using 10th , 20th, 30th and 50th order ZPETC applied to Plant 2 model

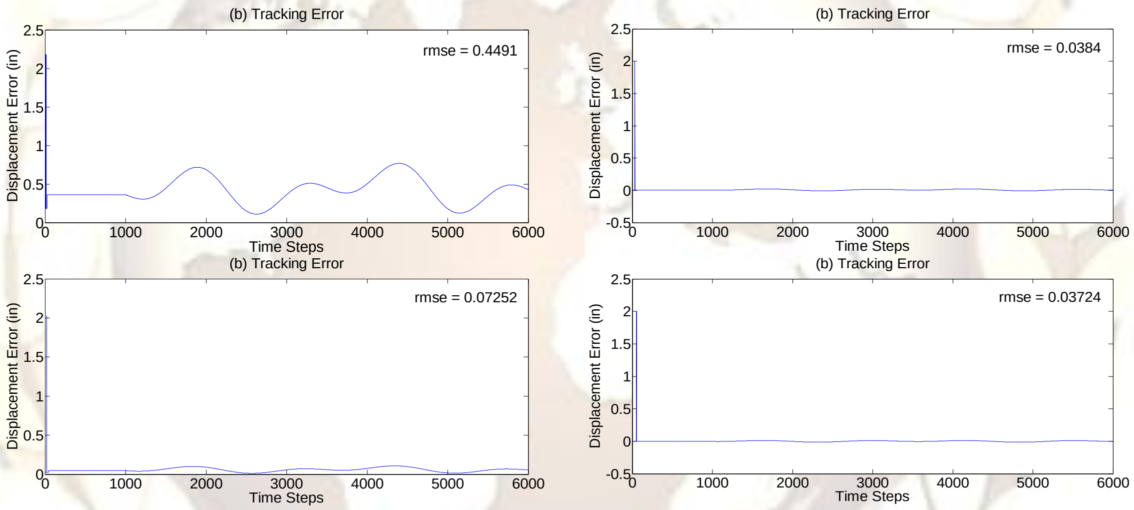

Figure 15. Tracking Error using 10th , 20th, 30th and 50th order ZPETC applied to Plant 2 model

For plant 2 model, the overall performance of the system increased for all optimal set of α_k. Based on Fig. 14 and 15, as expected where the RMSE dropped steadily throughout the orders indicating its increment in tracking performance. In Fig. 15, the RMSE reduced from 0.4491 inch for 10th order to almost zero. When using the 20th order, the RMSE dropped significantly to 0.07252 inch and from there on, the RMSE dropped slightly as it approach zero. The overall performance for plant 2 is much better as compared to plant 1 thus indicating the zero position outside the unity circle do affect the performance of the controller.

B. Experimental Results

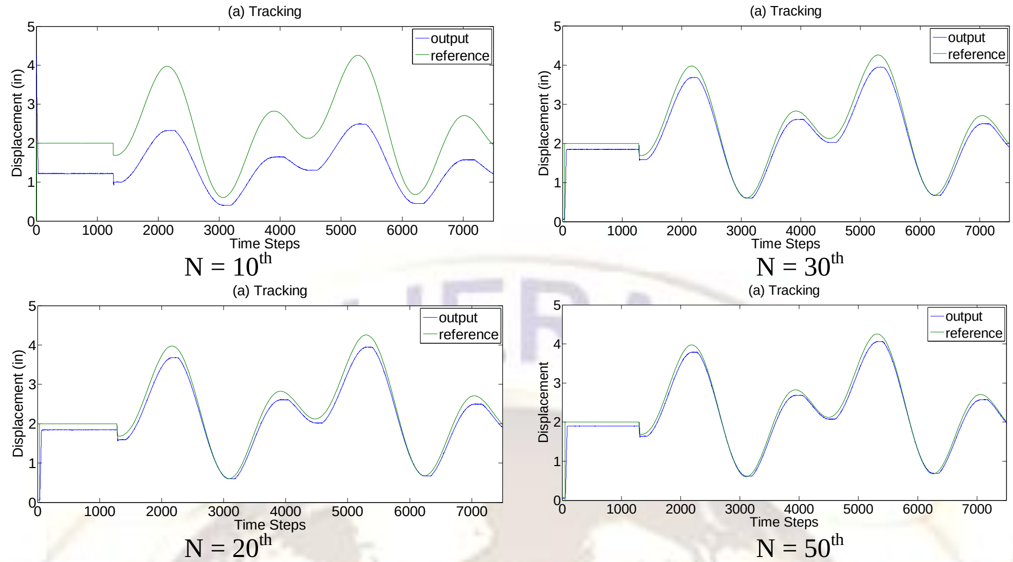

Figure 16. Experimental result using 10th , 20th, 30th and 50th order ZPETC applied to Plant 1 model

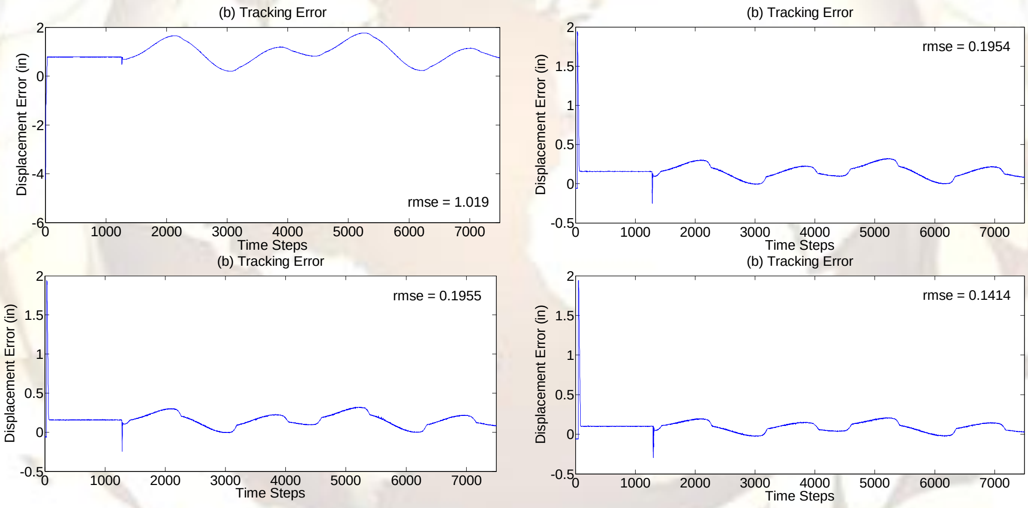

Figure 17. Tracking Error using 10th , 20th, 30th and 50th order ZPETC applied to Plant 1 model

Experimental results for plant 1 does not contradict with the simulation results found earlier as can be seen from Fig. 16. In Fig. 17, the RMSE dropped from 1.019 inch for the 10th order to 0.1414 inch for the 50th order but as expected from an experiment, results for the 20th and 30th order contradict the findings made earlier through the simulation tests due to the exterior factor like plant-model mismatch. The overall performance still support the findings made earlier even with this minor abnormality.

Plant 1 model is substituted with plant 2 model and the following set of results is obtained.

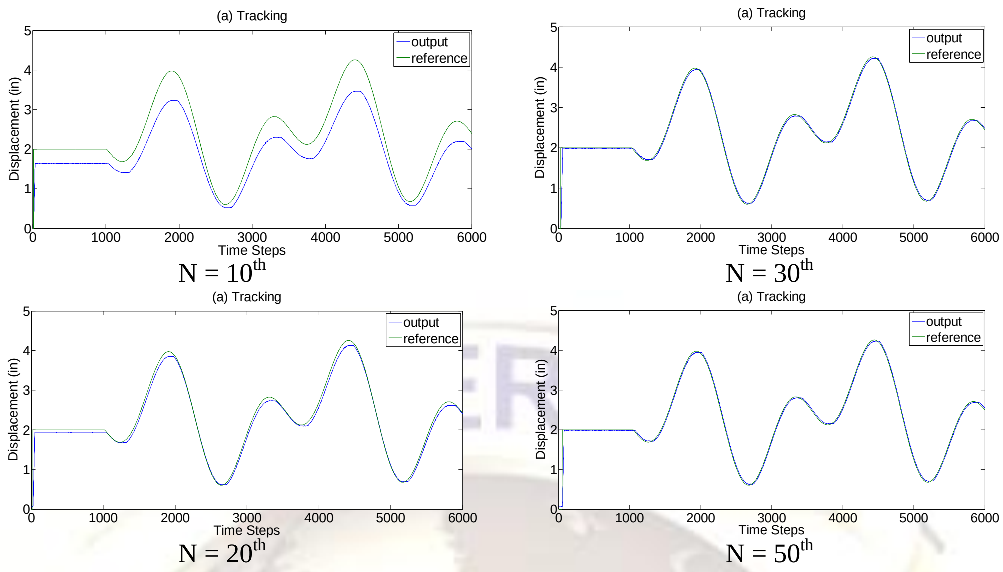

Figure 18. Experimental result using 10^{th} , 20^{th}, 30^{th} and 50^{th} order ZPETC applied to Plant 2 model

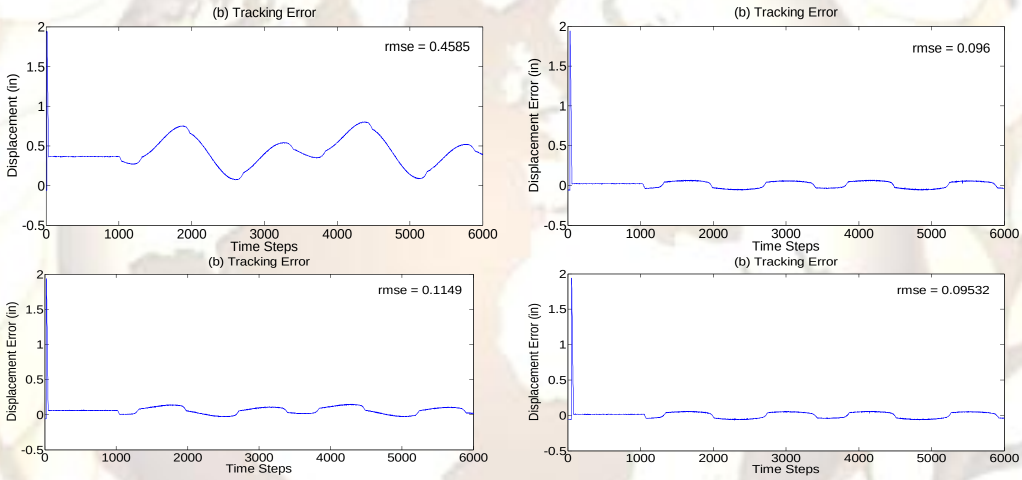

Figure 19. Tracking Error using 10^{th} , 20^{th}, 30^{th} and 50^{th} order ZPETC applied to Plant 2 model

Fig. 18 and 19 shows the tracking results and traking error for plant 2. The results shows that an increment in tracking performance as compared to plant 1 thus support the findings made earlier that the zero location outside the unity circle that is positioned much nearer to the unity circle gives a much better tracking performance for the controller as the gain compensation filter is increased. The RMSE for all results are summarized in Table 2 for simulation and for experimental.

Table 2. Tracking performance summary for simulation and experimental

Gain Filter Order (N)	**Simulation**		**Experimental**	
	RMSE (inch)		RMSE (inch)	
	Plant 1 (40ms model)	Plant 2 (50ms model)	Plant 1 (40ms model)	Plant 2 (50ms model)
10	1.01600	0.44910	1.01900	0.45850
20	0.27670	0.07252	0.19540	0.11490
30	0.07892	0.03840	0.19550	0.09600
50	0.03358	0.03324	0.14140	0.09532

IV. CONCLUSION

The studies on the development of new algorithm to determine the controller parameters by using the comparing coefficient method were presented. It is proven from this paper that by varying the compensation filter gains, the tracking performance of ZPETC increased by taking into consideration the zero position outside the unity circle for the plant model. The higher the order for the gain compensation filter, the better the tracking performance gets and the position of zero outside the unity circle do affect the performance of the controller where the tracking performance for a plant with zero much nearer to the unity circle is much better when compared to the plant with a zero much farther from the unity circle.

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REFERENCES

[1] X. Liu, Y. Wu, and Y. Liu, “Digital Servo Control Based on Adaptive Zero Phase Error Tracking Controller & Zero Phase Filter”, Chinese Control and Decision Conference, China, 2010.

[2] M. Tomizuka, Zero Phase Error Tracking Algorithm for Digital Control ASME Journal of Dynamic Systems, Measurement, and Control, vol. 109, pp. 65-68, 1987

[3] S.S. Yeh, & P.L. Hsu, “An Optimal and Adaptive Design of the Feedforward Motion Controller”. IEEE/ASME Transactions on Mechatronic, 4(4), pp.428–439,1999.

[4] Mustafa, M.M., “Trajectory-Adaptive Digital Tracking Controllers for Non-minimum Phase System Without Factorization of Zeroes”, IEEE Proc.Control Theory Appl.,vol.149, no. 2,pp. 157-162,2002.

[5] R. Adnan, A.M. Samad, and M. Mustafa, “Real-Time Control of Non-Minimum Phase System Using Trajectory Adaptive ZPETC”, 7th International Colloquium on Signal Processing & Its Application, pp. 78-82,Malaysia, 2011.

[6] Astrom, K.J. and Wittenmark, B.,"Computer-Controlled Systems:Theory and Design", 3rd ed., Prentice Hall, Englewood Cliffs, N.J.,1997.

[7] Ko. R.C., Good, M.C. and Halgamuge, S.K., “Adaptive Calibration of Feedforward Controller for Laser Profiling Machines”, Proc. Information, Decision and Control Conference (IDC ‘99), Adelaide, Australia, pp. 157–162, 1999.

[8] Tomizuka, M., “On the Design of Digital Tracking Controllers”, ASME J. of Dyn. Syst., Meas., and Control, vol. 115, pp.412–418, 1993

[9] H. Ismail, N. Ishak, M. Tajjudin, M. H. F. Rahiman & R. Adnan, “Robust Trajectory Tracking Controller for XY Table”, 2012 IEEE 8th International Colloquium on Signal Processing & Its Application, pp. 153-158, Malaysia, 2012.

[10] H. Ismail, N. Ishak, M. Tajjudin, M. H. F. Rahiman & R. Adnan, ”Positioning and Tracking Control of an XY Table”, 2011 IEEE International Conference on Control System, Computing and Engineering, pp. 343-347, Malaysia, 2011.

[11] N. Ishak, M. Tajjudin, H. Ismail, M. H. F. Rahiman, Y. M. Sam & R. Adnan, “Tracking Control for Electro-Hydraulic Actuator Using ZPETC”, 2011 IEEE Control and System Graduate Research Colloquium,pp.94-97,Malaysia,2011.